TRANSACTIONS

OF THE

AMERICAN INSTITUTE

OF

ELECTRICAL ENGINEERS

JANUARY TO JUNE 24, 1919

VOL. XXXVIII, PART I

PUBLISHED BY THE
AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
33 WEST THIRTY-NINTH STREET
NEW YORK, N. Y., U. S. A.
1919

Copyright, 1919
by the
American Institute of Electrical Engineers

Press of McIlroy & Emmet, 22 Thames Street, New York

OFFICERS AND BOARD OF DIRECTORS

1918-1919

PRESIDENT
COMFORT A. ADAMS

JUNIOR PAST-PRESIDENTS
H. W. BUCK
E. W. RICE, JR.

VICE-PRESIDENTS
W. B. JACKSON
RAYMOND S. KELSCH
F. B. JEWETT
JOHN B. TAYLOR
A. H. BABCOCK
HAROLD PENDER

MANAGERS
C. E. SKINNER
JOHN B. FISKEN
N. A. CARLE
CHARLES S. RUFFNER
CHARLES ROBBINS
E. H. MARTINDALE
WALTER A. HALL
WILLIAM A. DEL MAR
WILFRED SYKES
WALTER I. SLICHTER
G. FACCIOLI
FRANK D. NEWBURY

TREASURER
GEORGE A. HAMILTON

SECRETARY
F. L. HUTCHINSON

HONORARY SECRETARY
RALPH W. POPE

DIRECTOR OF THE LIBRARY
HARRISON W. CRAVER

GENERAL COUNSEL
PARKER and AARON
52 Broadway, New York

PAST PRESIDENTS—1884-1918.

*NORVIN GREEN, 1884-5-6.
*FRANKLIN L. POPE, 1886-7.
T. COMMERFORD MARTIN, 1887-8.
EDWARD WESTON, 1888-9.
ELIHU THOMSON, 1889-90.
*WILLIAM A. ANTHONY, 1890-91.
ALEXANDER GRAHAM BELL, 1891-2.
FRANK JULIAN SPRAGUE, 1892-3.
*EDWIN J. HOUSTON, 1893-4-5.
*LOUIS DUNCAN, 1895-6-7.
FRANCIS BACON CROCKER, 1897-8.
A. E. KENNELLY, 1898-1900.
CARL HERING, 1900-1.
CHARLES P. STEINMETZ, 1901-2.
CHARLES F. SCOTT, 1902-3.
BION J ARNOLD, 1903-4.
JOHN W. LIEB, 1904-5.
SCHUYLER SKAATS WHEELER, 1905-6.
SAMUEL SHELDON, 1906-7.
*HENRY G. STOTT, 1907-8.
LOUIS A. FERGUSON, 1908-09.
LEWIS B. STILLWELL, 1909-10.
DUGALD C JACKSON, 1910-11.
GANO DUNN, 1911-12.
RALPH D. MERSHON, 1912-13.
C. O. MAILLOUX, 1913-14.
PAUL M. LINCOLN, 1914-15.
JOHN J. CARTY, 1915-16.
H. W. BUCK, 1916-17.
E. W. RICE, JR., 1917-18.

*Deceased.

LOCAL HONORARY SECRETARIES

GUIDO SEMENZA,
N. 10 Via S. Radegonda, Milan, Italy.

T. P. STRICKLAND,
N. S. W. Government Railways, Sydney, N. S. W.

JAMES S. FITZMAURICE,
Perth, West Australia.

HENRY GRAFTIO,
Ministry of Ways of Communication, Petrograd, Russia.

HARRY PARKER GIBBS,
Tata Hydroelectric Power Supply Co., Ltd., Bombay, India.

ROBERT JULIAN SCOTT,
Christchurch, New Zealand.

W. G. T. GOODMAN,
Adelaide, South Australia.

L. A. HERDT,
McGill Univ., Montreal, Que.

A. S. GARFIELD,
45 Boulevard Beausejour, Paris, 16 E. France

JOHN W. KIRKLAND,
Johannesburg, South Africa.

STANDING COMMITTEES

EXECUTIVE COMMITTEE

COMFORT A. ADAMS Chairman
N. A. CARLE
GEORGE A. HAMILTON
F. B. JEWETT
HAROLD PENDER
C. E. SKINNER
JOHN B. TAYLOR

FINANCE COMMITTEE

N. A. CARLE, Chairman
WALTER A. HALL
CHARLES ROBBINS

MEETINGS AND PAPERS COMMITTEE

W. I. SLICHTER, Chairman
L. W. CHUBB, Secretary
B. A. BEHREND
L. T. ROBINSON
HARRIS J. RYAN
and the chairmen of the technical committees

EDITING COMMITTEE

HENRY H. NORRIS
Chairman
L. W. CHUBB
M. G. LLOYD
L. T. ROBINSON
W. I. SLICHTER

COMMITTEE ON SAFETY CODES

FARLEY OSGOOD, Chairman
J. C. FORSYTH
H. B. GEAR
H. O. LACOUNT
JOHNSTON LIVINGSTON
M. G. LLOYD
R. W. E. MOORE
HENRY N. MULLER
H. R. SARGENT
G. A. SAWIN
A. M. SCHOEN
GEORGE F. SEVER
H. S. WARREN

BOARD OF EXAMINERS

P. L. RHODES, Chairman
H. N. LATEY
H. H. NORRIS
CLAYTON H. SHARP
H. S. WYNKOOP

SECTIONS COMMITTEE

WALTER A. HALL, Chairman
C. FRANCIS HARDING
Vice-Chairman
L. T. ROBINSON
A. M. SCHOEN
J. LLOYD WAYNE, *and the Chairmen of all Institute Sections, ex-officio.*

COMMITTEE ON STUDENT BRANCHES

(Sub-Committee of Sections Committee.)

C. FRANCIS HARDING
Chairman
P. H. DAGGETT
ALEXANDER M. GRAY
A. C. LANIER
CHARLES F. SCOTT

MEMBERSHIP COMMITTEE

H. A. PRATT, Chairman
H. C. CARPENTER
R. W. KRASS
WILLS MACLACHLAN
J. FRANKLIN STEVENS
G. E. STOLTZ, and the chairmen of local membership committees.

PUBLIC POLICY COMMITTEE

CALVERT TOWNLEY,
Chairman
H. W. BUCK
N. A. CARLE
JOHN W. LIEB
C. E. SKINNER
L. B. STILLWELL

HEADQUARTERS COMMITTEE

N. A. CARLE, Chairman
WILLIAM A. DEL MAR
F. L. HUTCHINSON

ELECTRICAL ENGINEERING SERVICE COMMITTEE

WILLIAM A. DEL MAR,
Chairman
WALTER A. HALL
DUGALD C. JACKSON
G. L. KNIGHT
A. S. McALLISTER

U. S. NATIONAL COMMITTEE OF THE INTERNATIONAL ELECTROTECHNICAL COMMISSION

C. O. MAILLOUX, President
F. B. CROCKER,
Vice-President
A. E. KENNELLY, Secretary
COMFORT A. ADAMS
B. A. BEHREND
JAMES BURKE
JOHN J. CARTY
GANO DUNN
H. M. HOBART
DUGALD C. JACKSON
JOHN W. LIEB
A. S. McALLISTER
M. I. PUPIN
E. B. ROSA
DAVID B. RUSHMORE
CHARLES F. SCOTT
CLAYTON H. SHARP
C. E. SKINNER
ELIHU THOMSON

COMMITTEE ON CODE OF PRINCIPLES OF PROFESSIONAL CONDUCT

GEORGE F. SEVER,
Chairman
A. H. BABCOCK
A. S. McALLISTER
GANO DUNN
SCHUYLER S. WHEELER

EDISON MEDAL COMMITTEE

Appointed by the President for terms of five years.

CHARLES F. BRUSH
C. C. CHESNEY
N. W. STORER
CARL HERING, Chairman
HARRIS J. RYAN
ROBERT LINDSAY
W. C. L. EGLIN
BANCROFT GHERARDI
E. W. RICE, Jr.
COMFORT A. ADAMS
L. A. FERGUSON
S. W. STRATTON
FREDERICK BEDELL
L. T. ROBINSON
CALVERT TOWNLEY

Elected by the Board of Directors from its own membership for terms of two years.

F. B. JEWETT
HAROLD PENDER
JOHN B. TAYLOR
CHARLES ROBBINS
C. E. SKINNER
W. I. SLICHTER

Ex-Officio

COMFORT A. ADAMS,
President
GEORGE A. HAMILTON,
Treasurer
F. L. HUTCHINSON,
Secretary

STANDARDS COMMITTEE

L. T. ROBINSON, Chairman
E. J. CHENEY, Secretary
P. G. AGNEW
G. A. ANDEREGG
FREDERICK BEDELL
JOSEPH BIJUR
L. F. BLUME
JAMES BURKE
G. A. BURNHAM
N. A. CARLE
P. H. CHASE
H. H. CLARK
F. P. COX
WILLIAM A. DEL MAR
A. M. DUDLEY
E. J. EDWARDS
H. W. FISHER
W. S. GORSUCH
F. C. HANKER
H. M. HOBART
HENRY D. JAMES
A. E. KENNELLY
G. L. KNIGHT
F. A. LAWS
J. L. McQUARRIE
C. O. MAILLOUX
W. L. MERRILL
L. F. MOREHOUSE
FRANK D. NEWBURY
H. S. OSBORNE
HAROLD PENDER
CHARLES ROBBINS
CLAYTON H. SHARP
C. E. SKINNER
ARTHUR B. SMITH
JOHN B. TAYLOR
R. B. WILLIAMSON

TECHNICAL COMMITTEES

POWER STATIONS

PHILIP TORCHIO, Chairman
W. L. BIRD
N. A. CARLE
W. L. R. EMMET
W. S. GORSUCH
F. C. HANKER
JAMES T. LAWSON
H. P. LIVERSIDGE
S. A. MOSS
I. E. MOULTROP
FRANK D. NEWBURY
H. S. PUTNAM
J. D. ROSS
F. A. SCHEFFLER
R. F. SCHUCHARDT
H. L. WALLAU
W. F. WELLS

TRANSMISSION AND DISTRIBUTION

EDWARD B. MEYER, Chairman
J. E. WOODBRIDGE, Vice-Chairman
A. O. AUSTIN
H. W. BUCK
WALLACE S. CLARK
W. H. COLE
A. W. COPLEY
E. E. F. CREIGHTON
H. B. DWIGHT
G. FACCIOLI
H. W. FISHER
L. C. NICHOLSON
D. W. ROPER
HARRIS J. RYAN
A. E. SILVER
PAUL SPENCER
PERCY H. THOMAS

TRACTION AND TRANSPORTATION

C. F. UEBELACKER, Chairman
A. H. ARMSTRONG
A. H. BABCOCK
REINIER BEEUWKES
W. H. BLOOD, Jr.
R. H. DALGLEISH
WILLIAM A. DEL MAR
L. V. B. DEER
W. G. GORDON
JOHN MURPHY
RALPH H. RICE
GUY A. RICHARDSON
CHARLES F. SCOTT
E. R. SHEPARD
N. W. STORER
H. M. WARREN

INDUSTRIAL AND DOMESTIC POWER

A. G. PIERCE, Chairman
H. W. COPE
JAMES DIXON
A. M. DUDLEY
R. H. GOODWILLIE
A. L. HADLEY
H. D. JAMES
W. C. KALB
E. H. MARTINDALE
W. L. MERRILL
S. D. SPRONG
L. E. UNDERWOOD
H. WEICHSEL
R. B. WILLIAMSON

LIGHTING AND ILLUMINATION

C. E. CLEWELL, Chairman
G. N. CHAMBERLIN
J. R. CRAVATH
BASSETT JONES
P. G. NUTTING
R. C. POWELL
E. B. ROSA
W. D'A. RYAN
CLAYTON H. SHARP
W. M. SKIFF
JOHN B. TAYLOR

ECONOMICS OF ELECTRIC SERVICE

WILLIAM B. JACKSON, Chairman
PHILANDER BETTS
H. V. BOZELL
C. L. CORY
LEAVITT L. EDGAR
W. S. GORSUCH
RALPH G. HUDSON
A. H. KRUESI
J. F. MEYER
S. G. RHODES
ALLARD SMITH

PROTECTIVE DEVICES

D. W. ROPER, Chairman
P. H. CHASE
W. H. COLE
E. E. F. CREIGHTON
JAMES C. DOW
E. M. HEWLETT
D. MacGAHAN
A. A. MEYER
N. L. POLLARD
F. E. RICKETTS
R. F. SCHUCHARDT
PHILIP TORCHIO
H. R. WOODROW

ELECTROCHEMISTRY AND ELECTROMETALLURGY

CARL HERING, Chairman
E. R. BERRY
JOSEPH L. DIXON
J. R. EDMANDS
F. A. J. FITZGERALD
F. A. LIDBURY
J. A. SEEDE
JOHN B. TAYLOR
W. R. WHITNEY
J. L. YARDLEY

ELECTROPHYSICS

F. W. PEEK, Jr., Chairman
P. G. AGNEW
H. D. ARNOLD
FREDERICK BEDELL
L. W. CHUBB
A. C. CREHORE
E. P. HYDE
CHESTER W. RICE
C. E. SKINNER
JOHN B. TAYLOR

TELEGRAPHY AND TELEPHONY

DONALD McNICOL, Chairman
E. F. W. ALEXANDERSON
O. B. BLACKWELL
R. E. CHETWOOD
A. W. COPLEY
E. H. CRAFT
CHARLES E. DAVIES
LOUIS M. EVANS
W. E. McGOVERN
KEMPSTER B. MILLER
ARTHUR B. SMITH
JOHN STONE STONE
JOHN B. TAYLOR
J. LLOYD WAYNE
F. A. WOLFF

IRON AND STEEL INDUSTRY

EUGENE FRIEDLAENDER, Chairman
CHARLES W. BURROWS
F. B. CROSBY
F. D. EGAN
E. S. JEFFERIES
W. C. KENNEDY
D. M. PETTY
A. G. PIERCE
J. C. REED
WILLIAM T. SNYDER
WILFRED SYKES
R. B. WILLIAMSON
J. H. WILSON
J. D. WRIGHT

MARINE

H. A. HORNOR, Chairman
W. P. JAMES, Secretary
I. H. ANDERSON
R. H. BARCLAY
H. D. BARTER
ERIK H. BERG
L. C. BROOKS
A. M. CANDY
A. R. CHEYNEY
M. W. DAY
E. A. [illegible]
C. L. DOUGHERTY
G. R. EDGAR
W. L. R. EMMET
R. H. FENKHAUSEN
H. C. FORD
R. E. FRICKEY
W. R. FURLONG
E. G. GALLAGHER
H. L. HIBBARD
GUY HILL
M. G. LLOYD
D. M. MAHOOD
W. F. MEYERHOFFER
I. H. OSBORNE
R. W. OSGOOD
ARTHUR PARKER
H. A. PIERCE, Jr.
L. H. PORTER
J. S. SCHLESINGER
W. S. SCHMIDER
S. H. SHIRLAW
H. M. SOUTHGATE
ELMER A. SPERRY
WILFRED SYKES
W. H. THAU
HENRY THOMAS
F. F. TOWNSEND
A. E. WALLER
F. W. WOOD

USE OF ELECTRICITY IN MINES

K. A. PAULY, Chairman
FRANK H. ARMSTRONG
GRAHAM BRIGHT
M. A. CHANDLER
W. H. CLARK
[illegible]
L. C. ILSLEY
[illegible]
A. B. KISER
CHARLES LEGRAND
H. M. WARREN
E. H. WOODWARD

ELECTRICAL MACHINERY

ALEXANDER M. GRAY, Chairman
B. A. BEHREND
JAMES BURKE
A. CHURCHWARD
A. M. DUDLEY
CHARLES ROBBINS
[illegible]
R. B. WILLIAMSON

INSTRUMENTS AND MEASUREMENTS

S. G. RHODES, Chairman
P. G. AGNEW
CHARLES W. BURROWS
F. P. COX
F. V. MAGALHAES
HAROLD PENDER
T. S. PERKINS
CLAYTON H. SHARP

EDUCATIONAL

V. KARAPETOFF, Chairman
J. M. BRYANT
R. E. DOHERTY
C. R. DOOLEY
CHARLES S. HOWE
E. A. LOEW
C. E. MAGNUSSON
E. B. MERRIAM
CHESTER W. RICE
W. I. SLICHTER
W. E. WICKENDEN

Sections and their Officers

A. E. F. (FRANCE)

ATLANTA
A. M. SCHOEN, chairman
Thomas C. Taliaferro, secretary

BALTIMORE
J. B. Whitehead, chairman
L. M. POTTS, secretary

BOSTON
Ira M. Cushing, chairman
Ira M. Cushing, secretary

CHICAGO
C. A. Keller, chairman
A. F. Riggs, secretary

CLEVELAND
C. S. Ripley, chairman
R. W. Knowles, secretary

DENVER
Fred J. Rankin, chairman
Robert B. Bonney, secretary

DETROIT-ANN ARBOR
George E. Lewis, chairman
A. S. Albright, secretary

ERIE
Clayton P. Yoder, chairman
Scott S. Hill, secretary

FORT WAYNE
P. C. Morganthaler chairman
O. B. Rinehart, secretary

INDIANAPOLIS-LAFAYETTE
G. B. Schley, chairman
William H. Beck, secretary

ITHACA
Alexander Gray, chairman
John G. Pertsch, Jr., secretary

KANSAS CITY, MO.
W. F. Barnes, chairman
W. F. Barnes, secretary

LOS ANGELES
J. H. Cunningham, chairman
Clem A. Copeland, secretary

LYNN
L. E. Underwood, chairman
Fred E. Ewart, secretary

MADISON
James W. Watson, chairman
L. E. A. Kelso, secretary

MILWAUKEE
F. A. COFFIN, chairman
F. J. Mayer, secretary

MINNESOTA
Meyer Barnert, chairman
J. D. Marshall, secretary

PANAMA
W. L. Hersh, chairman
M. P. Benninger, secretary

PHILADELPHIA
W. F. James, chairman
E. B. Tuttle, secretary

PITTSBURGH
W. R. Work, chairman
J. G. Carroll, secretary

PITTSFIELD
Neil Currie, Jr., chairman
N. F. Hanley, secretary

PORTLAND, ORE.
R. M. Boykin, chairman
W. D. Scott, secretary

ROCHESTER
R. H. Manson, chairman
W. C. Freeman, secretary

ST. LOUIS
J. L. Hamilton, chairman
H. G. Hake, secretary

SAN FRANCISCO
J. C. Clark, chairman
A. G. Jones, secretary

SCHENECTADY
K. A. Pauly, chairman
L. F. Millham, secretary

SEATTLE
J. Harisberger, chairman
T. C. Smith, secretary

SPOKANE
George Nixon, chairman
G. H. Hoppin, secretary

TOLEDO
W. A. Hill, chairman
Max Neuber, secretary

TORONTO
A. H. Hull, chairman
Ernest V. Pannell, secretary

UTAH
A. S. Peters, chairman
H. T. Plumb, secretary

URBANA
A. R. Knight, chairman
E. H. Waldo, secretary

VANCOUVER
R. F. Hayward, chairman
T. H. Crosby, secretary

WASHINGTON, D. C.
J. E. Smith, chairman
Milton Flanders, secretary

Branches and their Officers

*CLEMSON AGRICULTURAL COL.

*COLORADO STATE AGRICULTURAL COLLEGE

*COLORADO, UNIV. OF

GEORGIA SCHOOL OF TECHNOLOGY
R. S. Griffith, chairman
H. L. Richards, secretary

*HIGHLAND PARK COL.

*IDAHO, UNIV. OF

*IOWA, UNIV. OF

*KANSAS STATE AGR. COL.

*KANSAS UNIV. OF

KENTUCKY, STATE UNIV OF
A. D. Hall, chairman
C. A. Timmer, secretary

LAFAYETTE COLLEGE
Frank Wilson Ross, chairman
Chas. Specner Allen, secretary

*LEHIGH UNIVERSITY

*LEWIS INSTITUTE

MAINE UNIV. OF
Ray C. Hopkins, chairman
Lloyd R. Douglass, secretary

MASSACHUSETTS INST. OF TECH.
Harmon B. Deal, chairman
Florence Fogler, secretary

*MICHIGAN AGRIC. COL.

MICHIGAN, UNIV. OF
P. Verschoor, chairman
A. L. Christie, secretary

MINNESOTA, UNIV. OF
Oscar C. Lee, chairman
J. Robert Heinemann, secretary

MISSOURI UNIV. OF
A. C. Lanier, chairman
D. P. Savant, Secretary

*MONTANA STATE COL.
J. A. Thaler, secretary

*NEBRASKA, UNIV. OF

*NORTH CAROLINA COL. OF AGR. AND ENGG.

NORTH CAROLINA, UNIV. OF
E. C. Ballentine, chairman
John E. Lear, secretary

*NORTH DAKOTA, UNIV. OF

*NORWICH UNIVERSITY

OHIO NORTHERN UNIV.
J. E. Summers, chairman
C. K. South, secretary

OHIO STATE UNIVERSITY
R. L. Shumaker, chairman
G. E. Park, secretary

*OKLAHOMA AGRICULTURAL AND MECH. COL.

*OKLAHOMA, UNIV. OF

OREGON AGR. COL.
Lawrence Fudge, chairman
Otto L. Cantrall, secretary

*PENN. STATE COLLEGE

Branches and their Officers—*Continued*

*PITTSBURGH, UNIV. OF

PURDUE UNIVERSITY
C. F. Harding, chairman
A. N. Topping, secretary

*QUEEN'S UNIVERSITY (ONT.)

RENSSALAER POLY. INST.
W. J. Williams, chairman

*ROSE POLYTECHNIC INST.

*STANFORD UNIV.

SYRACUSE UNIV.
W. P. Graham, chairman
R. A. Porter, secretary

*TEXAS, UNIV. OF

THROOP COLLEGE OF TECHNOLOGY
Mark Sawyer, chairman
Russell Otis, secretary

VIRGINIA POLYTECHNIC INSTITUTE
J. A. Carr, chairman
L. D. Fry, secretary

VIRGINIA, UNIV. OF
J. E. B. Stuart, chairman
J. Arthur Evans, secretary

WASH. STATE COL. OF
C .E. Guse, chairman
Ralph C. Guse, secretary

*WASHINGTON UNIV.
Walter J. Skrainka, chairman
Harold T. Lange, secretary

*WASHINGTON, UNIV. OF
M. A. Whitman, chairman
Walter H. Weber, secretary

*WEST VIRGINIA UNIV.

*WORCESTER POLY. INST.

YALE UNIVERSITY
P. C. O'Brien, chairman
Donald F. Hine, secretary

*Inactive due to the war

Institute Representatives

On Board of Award, John Fritz Medal.

W. W. BUCK
E. W. RICE, Jr.
JOHN J. CARTY
COMFORT A. ADAMS

On Board of Trustees, United Engineering Society

CALVERT TOWNLEY
SAMUEL SHELDON
L. T. ROBINSON

On Library Board of United Engineering Society

EDWARD D. ADAMS
SAMUEL SHELDON
F. L. HUTCHINSON
W. I. SLICHTER
ALFRED W. KIDDLE

On Electrical Committee of National Fire Protection Association

The chairman of the Institute's Committee on Safety Codes

On Joint Committee on Engineering Education

CHARLES F. SCOTT
SAMUEL SHELDON

On Council of American Association for the Advancement of Science

W. S. FRANKLIN
JOHN B. TAYLOR

On Advisory Board of American Year-Book

EDWARD CALDWELL

On U. S. National Committee of the International Illumination Commission

A. E. KENNELLY
CLAYTON H. SHARP
C. O. MAILLOUX

On Pan-American Engineering Committee

GANO DUNN
JOHN H. FINNEY
CALVERT TOWNLEY
WILLIAM McCLELLAN
CHARLES W. STONE

On National Joint Committee on Overhead and Underground Line Construction

FARLEY OSGOOD
PERCY H. THOMAS

On American Committee on Electrolysis

BION J. ARNOLD
PAUL-WINSOR
F. N. WATERMAN

On Engineering Council

H. W. BUCK
E. W. RICE, Jr.
C. E. SKINNER
N. A. CARLE
COMFORT A. ADAMS

On Electrical Engineering Section of Engineering Division, National Research Council

COMFORT A. ADAMS

On Engineering Foundation Board

E. W. RICE, Jr.
F. B. JEWETT

On Naval Consulting Board

BENJAMIN G. LAMME
FRANK J. SPRAGUE

On American Engineering Standards

COMFORT A. ADAMS
H. M. HOBART
N. A. CARLE

On Commission of Washington Award

CHARLES F. SCOTT
JOHN PRICE JACKSON

CONTENTS.

MEETING AT NEW YORK, JANUARY 10, 1919

Aeronautics in the United States, 1918—By George O. Squier (*Illustrated*)........ 1

MEETING AT NEW YORK, FEBRUARY 19, 20 AND 21, 1919

Welding Mild Steel—By H. M. Hobart (*Illustrated*)........ 65
Welding as a Process in Ship Construction—By S. V. Goodall........ 113
Fusion in Arc Welding—By O. H. Eschholz (*Illustrated*)........ 121
The General Equations of the Electric Circuit—By Charles P. Steinmetz (*Illustrated*)........ 191
Review of Work of Sub-Committee on Wave Shape Standard of the Standards Committee—By Harold S. Osborne (*Illustrated*)........ 261
Radio Telephony—By E. B. Craft and E. H. Colpitts (*Illustrated*)........ 305
Theory of the Transient Oscillations of Electrical Networks and Transmission Systems—By John R. Carson (*Illustrated*)........ 345
Telephone Circuits with Zero Mutual Induction—By William W. Crawford (*Illustrated*)........ 429
Ionization of Occluded Gases in High Tension Insulation—By G. B. Shanklin and J. J. Matson (*Illustrated*)........ 489
The Dielectric Strength of Air Films Entrapped in Solid Insulation and a Practical Application of the Problem for Alternator Coils and Cables—By F. Dubsky. (*Illustrated*)........ 537
Abnormal Voltages Within Transformers—By L. F. Blume and A. Boyajian. (*Illustrated*)........ 577

MEETING AT BOSTON, MASS., MARCH 14, 1919

Some Possibilities of Steam Railroad Electrification As Affecting Future Policies—By Calvert Townley........ 621
Electromagnetic Theory of the Telephone Receiver—A. E. Kennelly and H. Nukiyama. (*Illustrated*)........ 651
The Absolute Measurement of the Intensity of Sound—By Arthur Gordon Webster. (*Illustrated*)........ 701

MEETING AT NEW YORK, APRIL 11, 1919

Present Status of Industrial Lighting Codes—By G. H. Stickney........ 725

MEETING AT BOSTON, MASS., MAY 7, 1919

Utilizing the Time Characteristics of Alternating Current—By Henry E. Warren. (*Illustrated*)........ 767

MEETING AT LAKE PLACID CLUB, N. Y., JUNE 24, 1919

Cooperation—President's Address—By C. A. Adams........ 783
Transmission Line Relay Protection—By H. R. Woodrow, D. W. Roper, O. C. Traver and P. MacGahan. (*Illustrated*)........ 795
Grounding the Neutral of Generating and Transmission Systems—By H. R. Woodrow. (*Illustrated*)........ 837
Grounded Neutral Transmission Line—By W. E. Richards (*Illustrated*)........ 853

ANNUAL REPORTS OF TECHNICAL COMMITTEES:

Power Stations Committee 871
Transmission and Distribution Committee 876
Industrial and Domestic Power Committee 881
Lighting and Illumination Committee 885
Committee on Economics of Electric Service 891
Protective Devices Committee 893
Electrochemistry and Electrometallurgy Committee 895
Electrophysics Committee 896
Telegraphy and Telephony Committee 897
Marine Committee 905
Committee on Use of Electricity in Mines 908
Iron and Steel Industry Committee 909
Instruments and Measurements Committee 912
Educational Committee 915
High-Tension Single-Conductor Cable for Polyphase Systems—By W. S. Clark and G. B. Shanklin. (*Illustrated*) 917

Presented at the 345th Meeting of the American Institute of Electrical Engineers, New York, January 10, 1919.

Copyright 1919. By A. I. E. E.

AERONAUTICS IN THE UNITED STATES, 1918

BY GEORGE O. SQUIER

ABSTRACT OF PAPER

Major General George O. Squier, Chief Signal Officer of the United States Army, reviews in this address the development of Military Aeronautics in the United States up to the date of the armistice, November 11, 1918. The War Department's first heavier-than-air flying machine was produced approximately ten years ago, but for the eight years that followed, the development lagged—in fact less than a million dollars was appropriated for aeronautics in the entire eight years. Then under the pressure of war, the United States by necessity plunged into a gigantic aircraft program—and the accomplishments today are numbered by the score. The Liberty Engine, acknowledged now to be the standard for the world, was produced; an industry new to the United States was developed, and other tremendous strides taken in the science of aerial navigation.

I—Introduction

ALMOST exactly ten years ago (December, 1908) the undersigned had the honor of addressing, from this same platform, the American Society of Mechanical Engineers on the subject of Military Aeronautics, at its annual meeting. The preceding summer months had witnessed the first public flights of the Wright airplane at Fort Myer, Virginia.

The Signal Corps of the United States Army, under date of December 23, 1907, had issued an advertisement and specification for a heavier-than-air flying machine, and the sealed proposals under this specification were opened formally at Washington at twelve o'clock noon on February 1, 1908. The main requirement before acceptance, by the Government, was an endurance test in which the flying machine must remain continuously in the air for one hour without landing.

We all remember the world-wide interest which was aroused by the performance of this new and promising realization of the dream of the ages. During the continuance of the experiments, the eyes of the world were centered upon the little flying field just outside of Washington. The President and Cabinet Ministers were in frequent attendance, and the Congress adjourned from day to day whenever a flight was in prospect.

And then an almost inconceivable thing occurred. The Messrs. Wright having thoroughly demonstrated and fulfilled all of the conditions required by that specification, and the first machine having been duly purchased and paid for, during the eight years following the entire appropriations by this government for military aeronautics amounted to less than a million dollars.

European Powers, however, in their quest for military advantage, quickly interpreted what had happened at Fort Myer, and France, in particular, during these years was responsible for the principal advances in construction and design.

I shall not enter here into the causes, some military, some civil, of this utter unpreparedness, but the outbreak of war found the United States with but a handful of fliers and very few training planes. There was no aviation industry in this country, and the number of professional men trained as aeronautical engineers and designers was so small as to be practically negligible. In this respect the problem of developing the air program was unique. The United States had built ships before, had manufactured clothing, guns, munitions, built cantonments, etc.; and had a splendid body of men trained in these professions and employments, but, outside of a few men there was no one in the United States with experience in the design or building of even training planes.

Once the United States actually had entered the war, the pressure from our Allies and a sudden realization of our real situation in aeronautics led Congress to grant for this purpose, in the Act of May 12, 1917, $10,800,000; the Act of June 15, 1917, $31,846,000, and finally the appropriation of $640,000,000, the largest ever made by Congress for one specific purpose, which was put through the House of Representatives' Military Affairs Committee in two sittings, the House itself in one, the Senate Military Affairs Committee in forty-five minutes; and the Senate itself a week later, becoming a law on July 24, 1917, three months and a half after the outbreak of war.

The task you have given me of presenting within the present hour even an outline of the great undertaking which this nation has accomplished during the last nineteen months in developing the Air Service, is indeed impossible to meet.

I hasten to say at the outset that I can only hope to present to you some of the salient features and accomplishments of this effort. Of the hundreds and thousands of men and women who wrought valiantly and loyally to achieve these results, it will be impossible to mention specific names, or to give anything like just credit or mention to individuals or particular corporations.

Where names are mentioned in connection with certain features of the work, it must not be interpreted that there are not many others who are equally meritorious.

The Honor Roll of this Institute comprises no less than 1,317 men who participated in the Allied forces in the Great War. Of these, 976 were officers, 728 were officers in the American Army, 110 in the Signal Corps, and 52 in the Air Service.

Indeed, it is only in the long reaches of future history that we can form a just estimate of the many lines of endeavor which were set in motion in carrying out this enterprise, and which will, it is hoped, go forward into the future to bear their fruits in times of peace.

II—Statistical Résumé of Aircraft Production and Training

THE work involved in training aviators and obtaining aeronautical equipment in the United States has been of such magnitude as to preclude the possibility of presenting here anything more than a brief résumé of the major accomplishments.

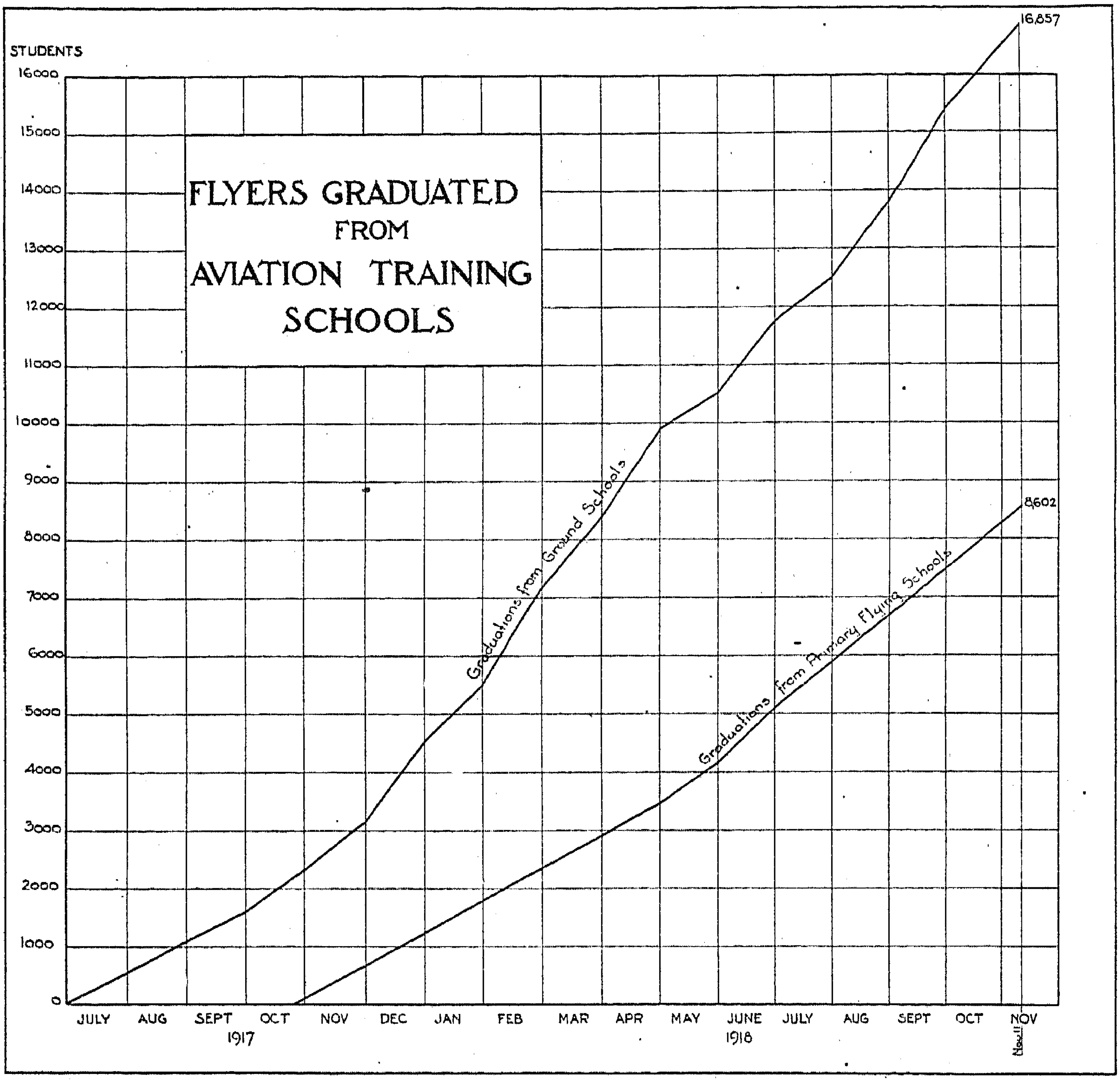


FIG. 1

The data is presented as of six o'clock a. m., November 11, 1918 (the date of the signing of the armistice).

PERSONNEL

More than 8600 fliers have been trained in the United States since its entry into the war. Monthly graduations from ground and flying schools are shown on Fig. 1. The gradually increasing deviation between the two curves on this chart is caused largely by failure of graduates of ground schools to develop into fliers.

There have been training fatalities in the United States, as in all other countries where training has been conducted on a large scale. However, when the fact is realized that our students have flown more than 880,000 hours, which is the equivalent to more than 66,000,000 miles, it will be seen that our training casualties have been astonishingly few. Statistics show that the United States has a lower percentage of fatalities than any other of the allied countries. The monthly average in the United States has been only one fatality for each 3200 hours flown. Training fatalities by months are shown in Fig. 2, and in Fig. 3 are shown the analyzed causes of these fatalities. Two independent investigations made in connection with this paper have developed the fact that more than 90 per cent of training casualties are attributed to the aviator himself. Fig. 4 shows the actual distribution of our fliers overseas.

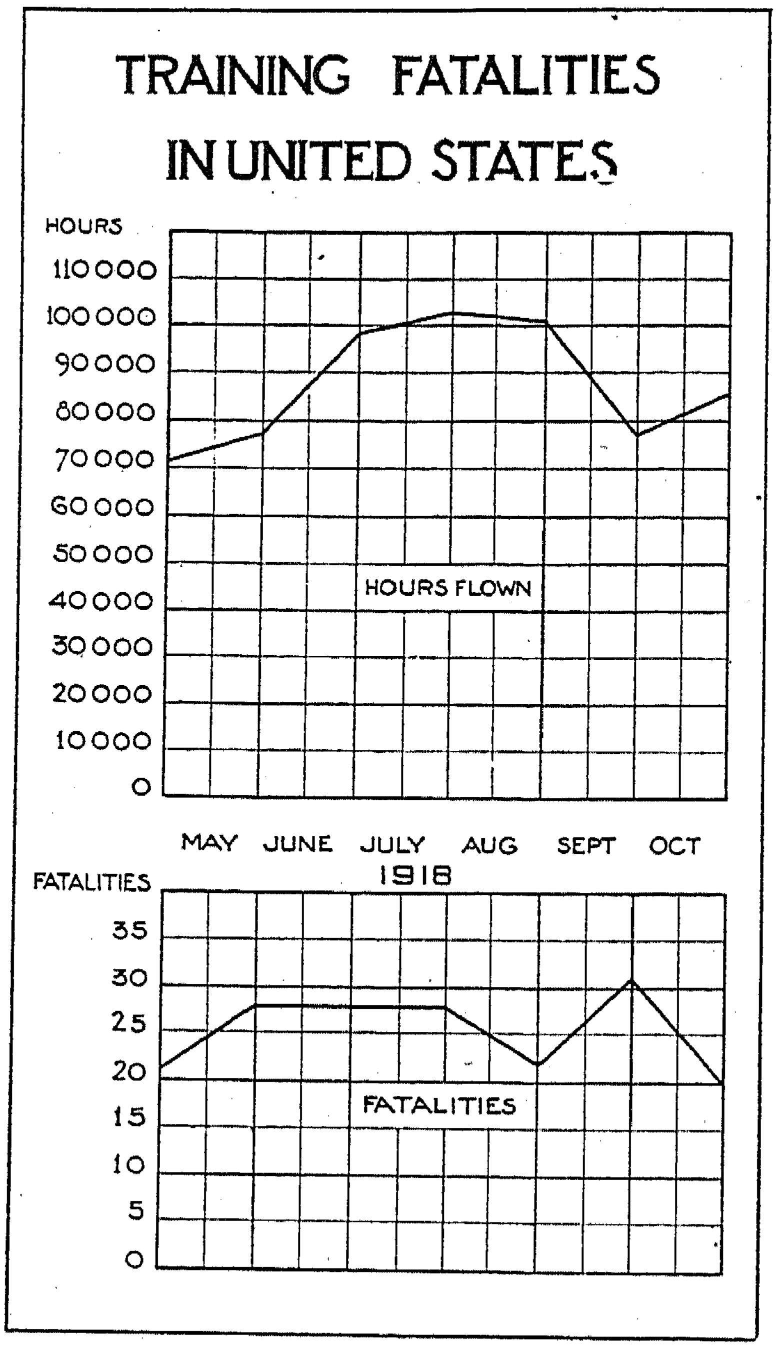


FIG. 2

TRAINING EQUIPMENT

Herewith is presented a tabulation showing the actual production of training material up to November 11, 1918:

Planes		Engines			Propellers
Type	Number furnished	Name	H.P.	Number furnished	Number furnished
Elementary					
JN-4D	3,762	OX-5		83,18	17,830
SJ-1	1,600	A-7-A		2,250	5,173
Advanced					
JN-4H	929				
JN-6H	803	Hispano	150	3,497	5,295
SE-5	6				
S-4B	100	Gnome	100	278	500
S-4C	384				
E-1	18	La Rhone	80	1,107	358
Total	7,602			15,450	29,156

RAW MATERIALS

Some idea of the quantity of wood necessary for airplane and propeller construction may be obtained from the following table of actual deliveries:

	Spruce	
	Board Feet	Totals
U. S. Army	25,472,000	
U. S. Navy	8,667,000	
England	36,877,000	
France	22,929,000	
Italy	9,147,000	
		103,092,000
	Douglas Fir	
U. S. Army	19,193,000	
U. S. Navy	21,746,000	
England	21,226,000	
France	9,460,000	
		71,625,000
	Port Orford Cedar	
U. S. Army	30,000	
U. S. Navy	1,926,000	
England	2,557,000	
		4,513,000
	Mahogany, Central American	
U. S. Army & Navy	4,524,000	
Allies	4,197,815	
		8,721,815
	Mahogany, African	
U. S. Army & Navy	269,000	
Allies	200,935	
		469,935
	American Black Walnut	
U. S. Army & Navy	2,408,000	
Allies	2,096,876	
		4,504,876
	Quartered White Oak	
U. S. Army & Navy	308,000	308,000
	Cherry	
U. S. Army & Navy	618,000	618,000
	Ash	
U. S. Army & Navy	87,000	
Allies	33,565	
		120,565
	Birch	
U. S. Army & Navy	663,000	663,000

In addition to airplane wood, it was found necessary to maintain strict supervision over the production of airplane and balloon fabrics. Of this class of material there had been produced to November 11, 1918, 3,187,000 yards of linen; 7,000,000 yards of cotton airplane fabric; 2,647,000 yards of balloon fabric.

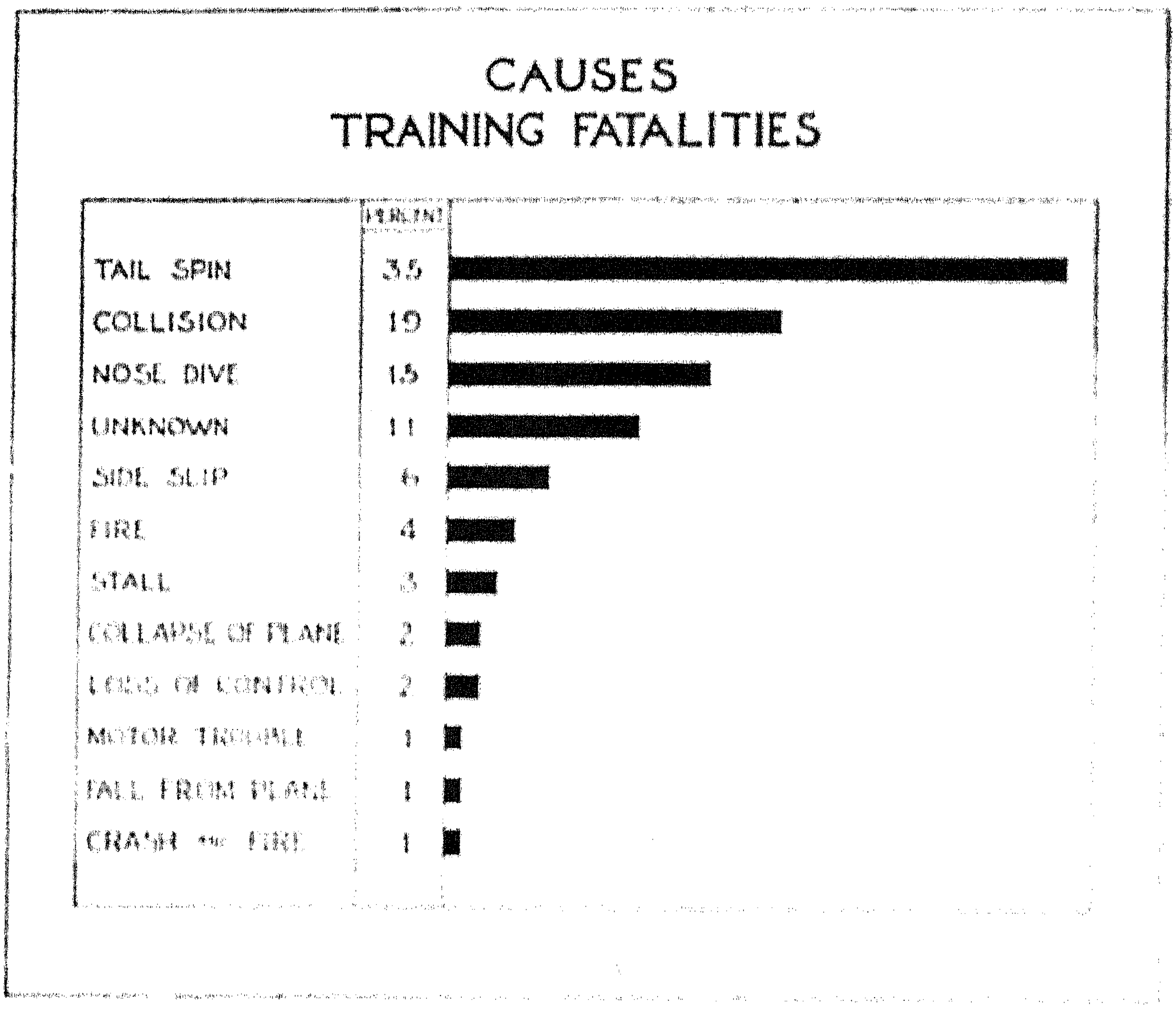


Fig. 3

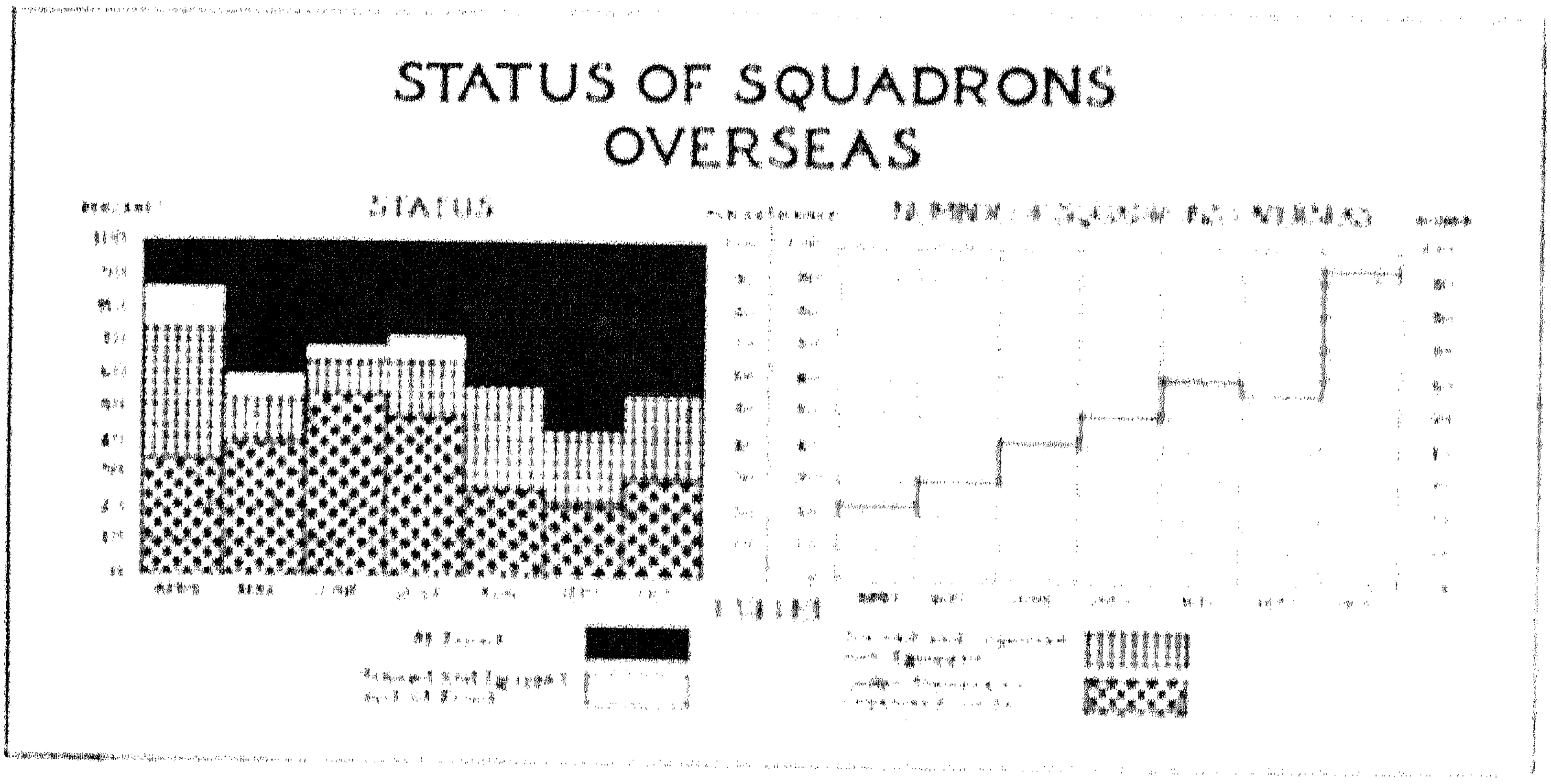


Fig. 4

Accessories

The production of airplane accessories to November 11, 1918 was as follows:

Air Pressure Gages	9,994
Air Speed Indicators	9,051
Altimeters	19,857

Clocks	17,593
Compasses Type "B"	10,179
Fire Extinguishers	12,209
Gasoline Gages	550
Inclinometers	40
Map Cases Rotating	2,417
Oil Pressure Gages	12,187
Oxygen Apparatus	4,318
Oxygen Masks	3,341

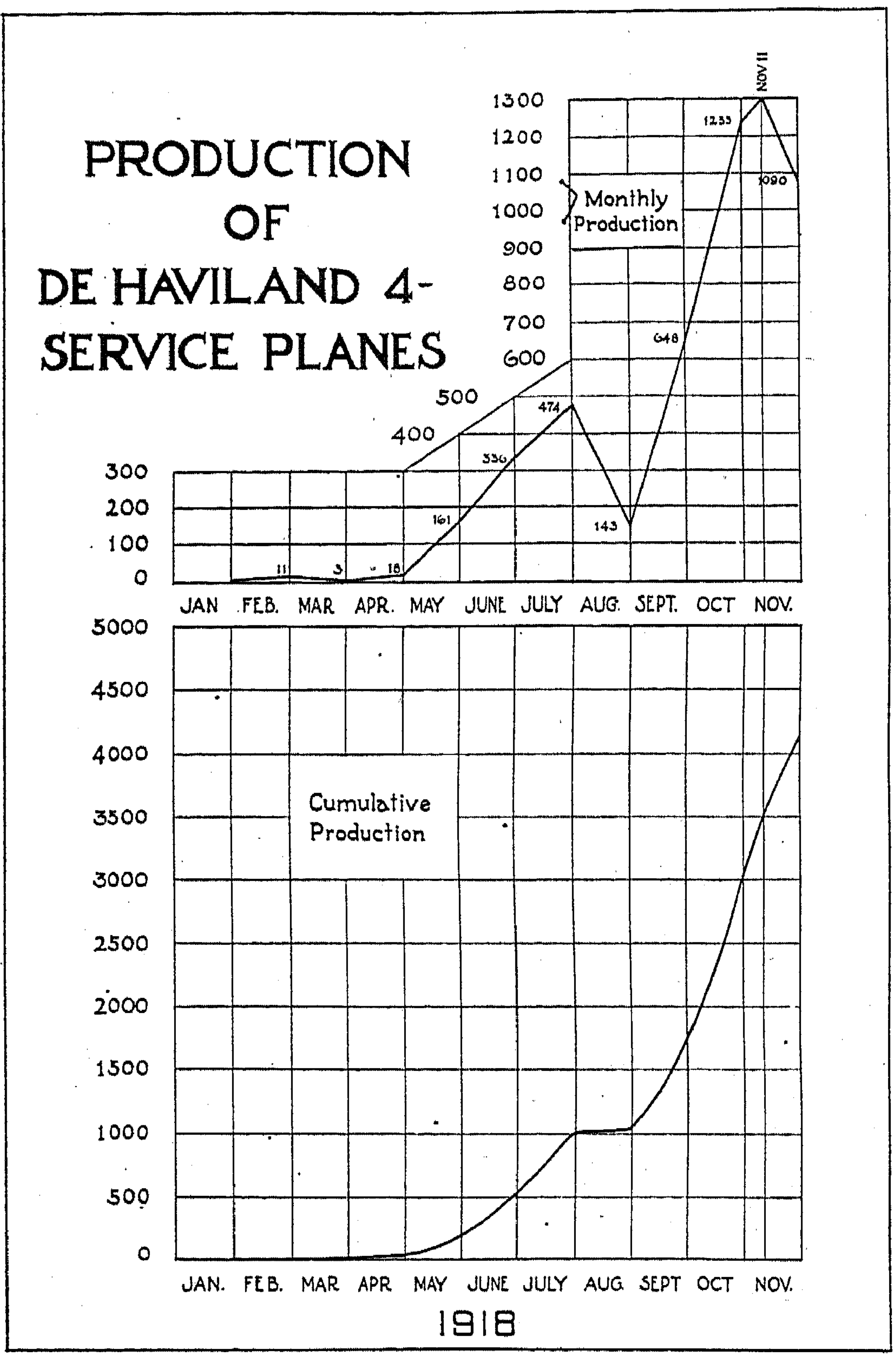


Fig. 5

Oxygen Tanks	12,000
Panel Switches	15,466
Radiator Thermometers	11,984
Tachometers	20,549

Ordnance for use in connection with the air service, such as aircraft machine guns, bombs, etc. were produced under the direction of the Air Service of the Army. Of this type of

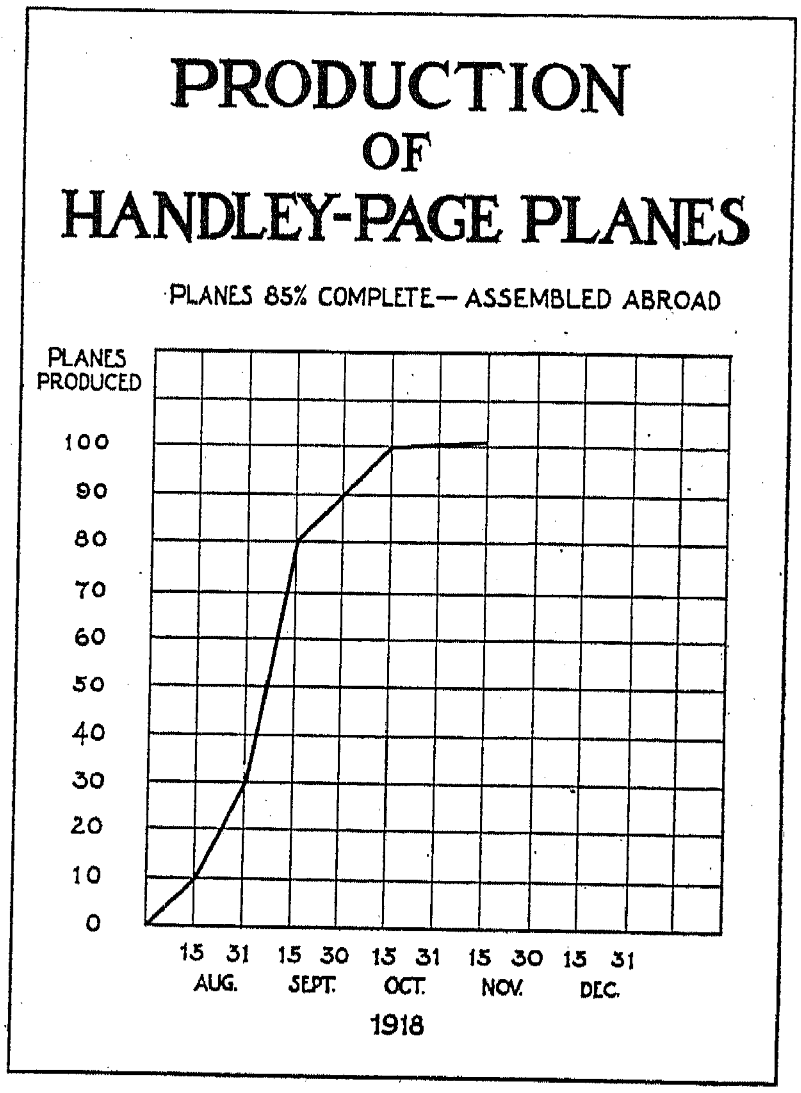
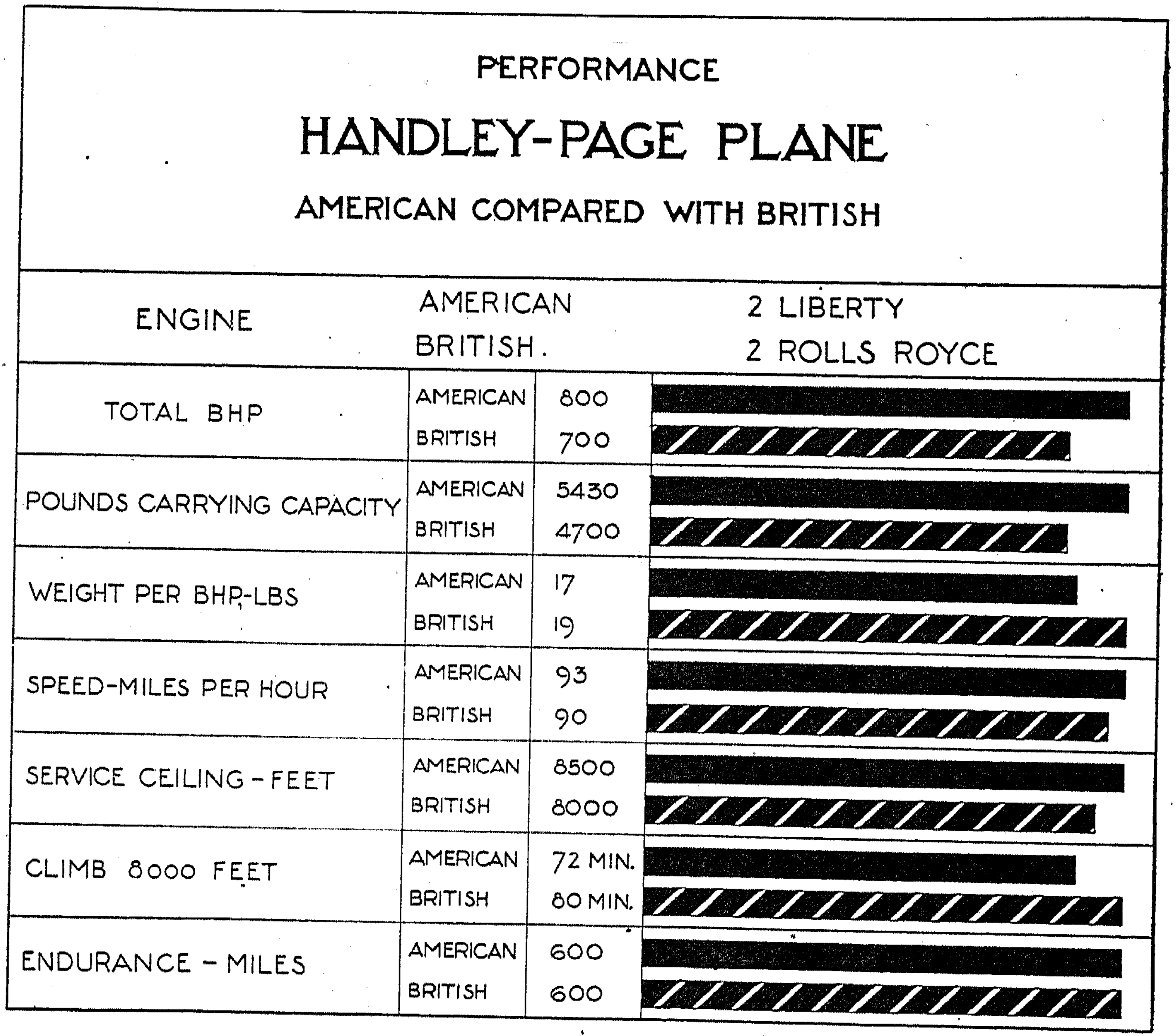


FIG. 6



PERFORMANCE

HANDLEY-PAGE PLANE

AMERICAN COMPARED WITH BRITISH

ENGINE	AMERICAN	2 LIBERTY
	BRITISH.	2 ROLLS ROYCE
TOTAL BHP	AMERICAN	800
	BRITISH	700
POUNDS CARRYING CAPACITY	AMERICAN	5430
	BRITISH	4700
WEIGHT PER BHP-LBS	AMERICAN	17
	BRITISH	19
SPEED-MILES PER HOUR	AMERICAN	93
	BRITISH	90
SERVICE CEILING - FEET	AMERICAN	8500
	BRITISH	8000
CLIMB 8000 FEET	AMERICAN	72 MIN.
	BRITISH	80 MIN.
ENDURANCE - MILES	AMERICAN	600
	BRITISH	600

FIG. 7

material there were manufactured to November 11, 1918, a total of 81,754 machine guns for aircraft, divided as follows:

31,671	Lewis Guns
11,904	Vickers Ground Guns
411	Vickers Aircraft Guns
37,768	Marlin Guns

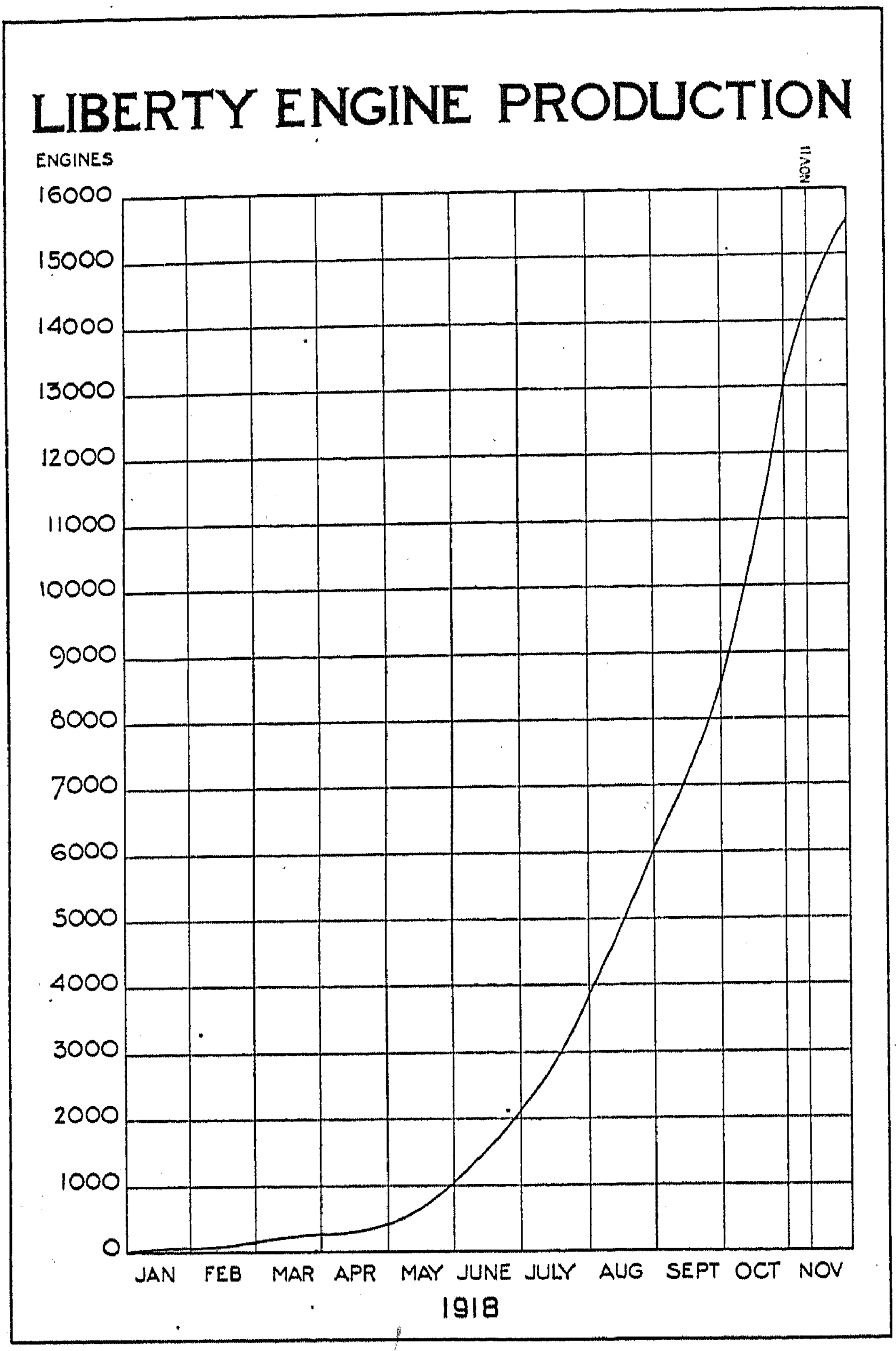
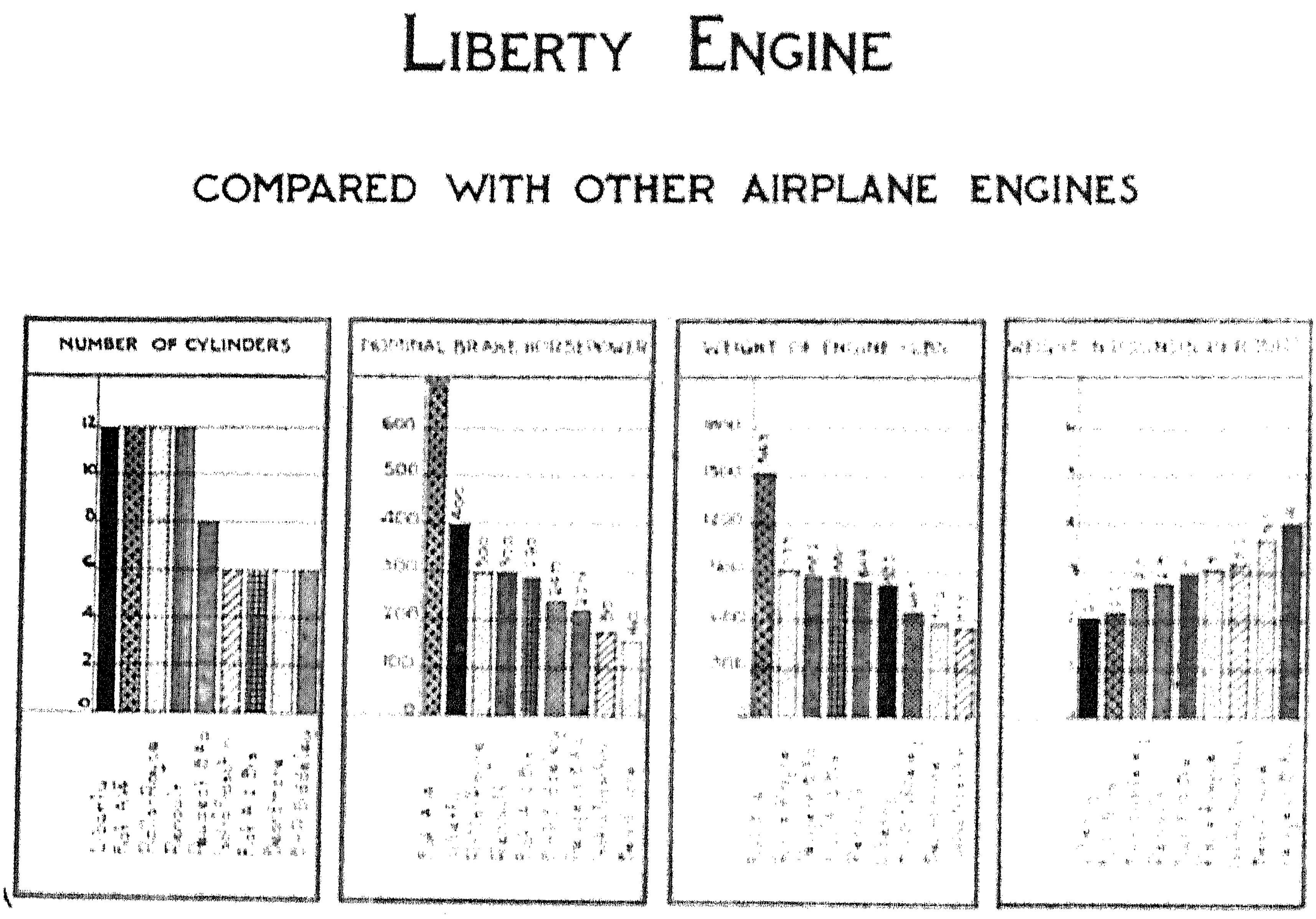


FIG. 8

with the following accessories:

Article	Number Produced
Flexible ring mounts	12,336
Ring sights	13,200
Wing vane sights	12,999
Unit sights	10,104
Auxiliary post and ring	4,585

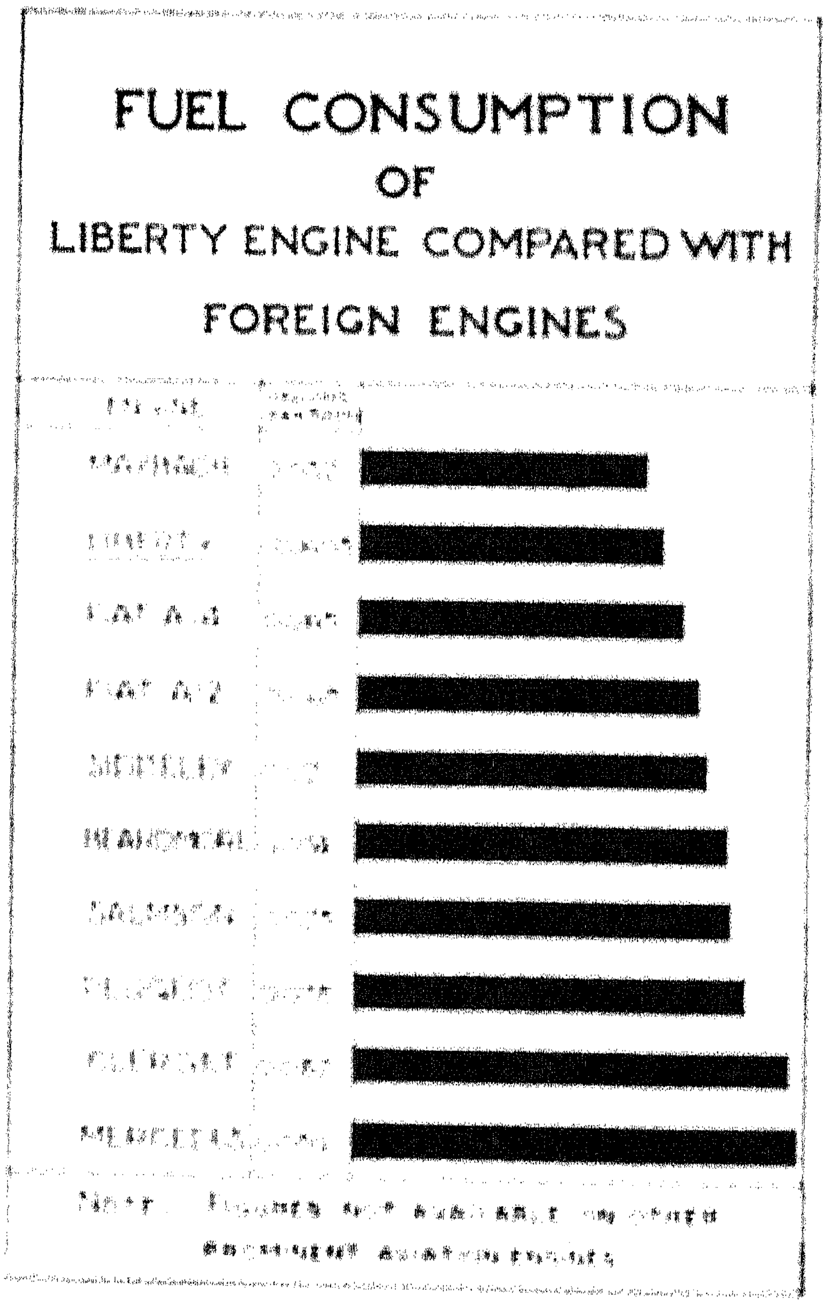
A total of 510,271 bombs to be launched from air craft was produced to November 11, 1918, of which 114,809 were



LIBERTY ENGINE

COMPARED WITH OTHER AIRPLANE ENGINES

Fig. 9



FUEL CONSUMPTION OF LIBERTY ENGINE COMPARED WITH FOREIGN ENGINES

Fig. 10

incendiary bombs; 386,543 were high-capacity demolition bombs, and 8,919 were high-capacity fragmentation bombs.

SERVICE PLANES

DE HAVILAND 4

The production of the De Haviland 4 service plane is shown by months in Fig. 5. It was found necessary to delay production during the month of August 1918 that certain important changes, based on overseas experience with this plane, might be incorporated.

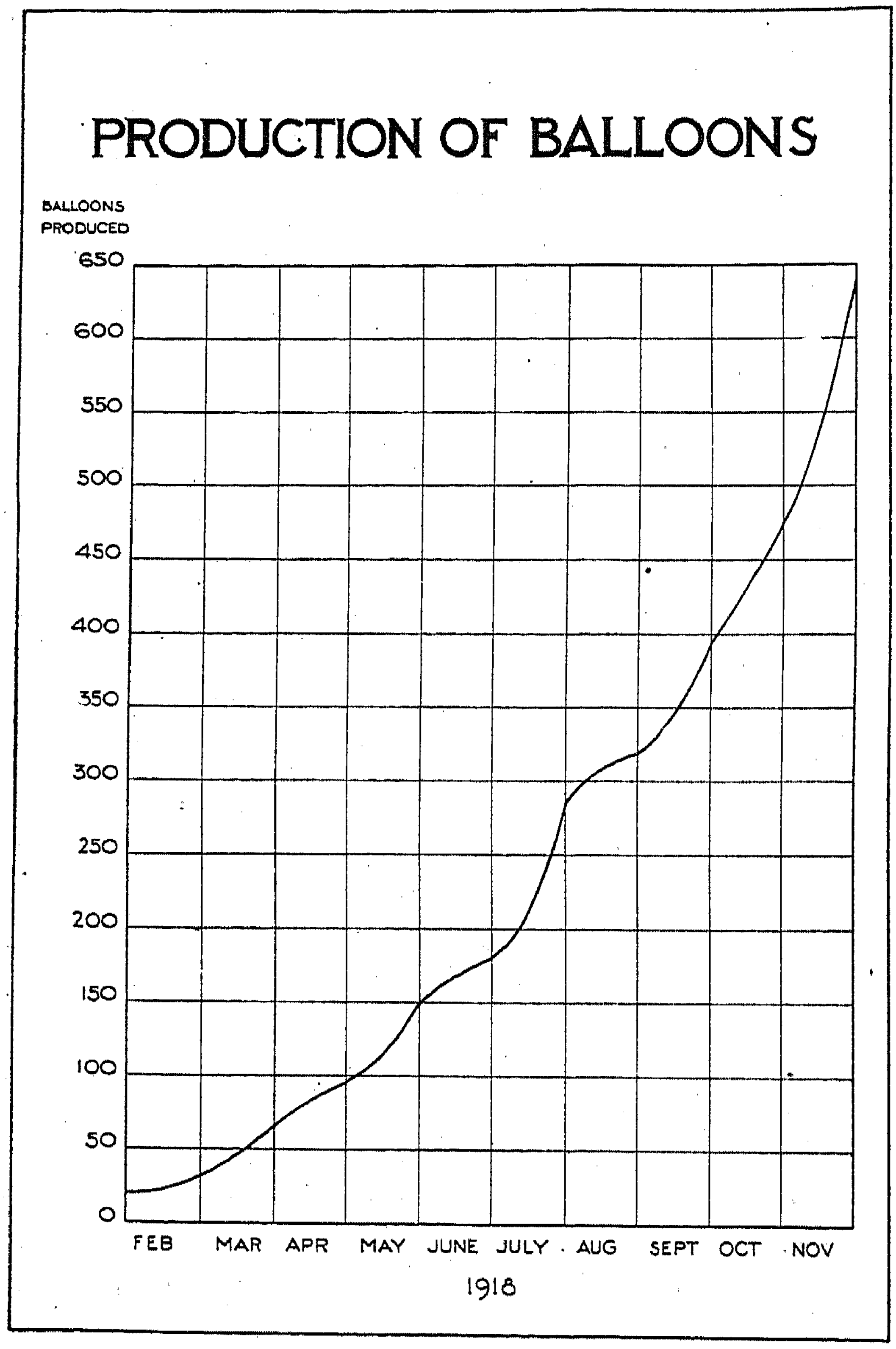


FIG. 11

HANDLEY-PAGE

The production of Handley-Page planes is found in Fig. 6. It should be noted that this production is for planes 85 per cent completed. In other words, planes shipped from this country were not assembled. Fifteen per cent was allowed for assembling abroad. The reason for shipping these planes abroad unassembled is obvious when the size of the assembled plane is taken into consideration. An interesting comparison of the American Handley-Page plane with its British prototype will be found in Fig. 7.

SERVICE ENGINES

More than 16,000 Liberty engines were produced during the calendar year 1918. To November 11, 1918, more than 14,000 Liberty engines were produced, equivalent to 5,700,000 brake horse-power. Fig. 8 shows the production of Liberty engines by months.

A comparison of the weight of the Liberty engine with some of the better known foreign airplane engines is shown in Fig. 9, and fuel consumption in Fig. 10. The basic data for Figs. 9 and 10 came from official communiques from abroad. In the case of the Liberty engine the results of more than thirty tests were averaged.

BALLOONS

The production of service balloons met all service requirements. Fig. 11 presents the production of balloons by months.

TYPES OF AMERICAN MACHINES
DEVELOPED, TESTED AND ADOPTED FOR PRODUCTION

On November 11, 1918, there had been developed, tested and adopted by the Army four airplanes, on which production would have started early in the present calendar year. They were the Lepere or L. U. S. A. C.-11 equipped with the Liberty engine, the U. S. De Haviland 9-A equipped with the Liberty engine, the Martin Bomber equipped with two Liberty engines and the Loening two-seater fighter equipped with the 300-h.p. Hispano-Suiza engine. The characteristics of these new airplanes are as follows:

Lepere or L. U. S. A. C. 11

Type: Two seater fighter Engine: One Liberty

Weight empty with water	2468	pounds
Fuel and oil	475	“
Crew	360	“
Guns, ammunition, etc.	352	“
Gross weight	3655	“
Pounds per b. h. p.	10.15	“
Pounds per square foot	9.33	“
Speed at ground	136	mi. per hr.
Climb to 6500 feet	6	minutes

U. S. De Haviland 9-A

Type: Day bombing and reconnaissance Engine: One Liberty

As a Day Bomber

Weight empty with water	2815	pounds
Fuel and oil	933	"
Crew	360	"
Guns, ammunition, etc.	764	"
Gross weight	4872	"
Pounds per b. h. p.	13.5	"
Pounds per square foot	9.5	"
Speed at ground	121.5	mi. per hr.
Climb to 6500 feet	11	minutes 40 seconds

As Reconnaissance Plane

Weight empty with water	2815	pounds
Fuel and oil	933	"
Crew	360	"
Guns, ammunition, etc.	214	"
Gross weight	4322	"
Pounds per b. h. p.	12	"
Pounds per square foot	8.4	"
Speed at ground	126.2	mi. per hr.
Climb to 6500 feet	7	minutes 30 seconds

Loening

Type: Two seater fighter Engine: One 300 HP Hispano

Weight empty with water	1130	pounds
Fuel and oil	290	"
Crew	360	"
Guns, ammunition, etc.	828	"
Gross weight	2608	"
Pounds per b. h. p.	8.7	pounds
Pounds per square foot	12.1	"
Speed at ground	143.5	mi. per hr.
Climb to 6500 feet	5	minutes 12 seconds

Martin Bomber

Type: Night Bomber Engine: Two Libertys

Weight empty with water	5862	pounds
Fuel and oil	1492	"
Crew	540	"
Guns, ammunition, etc.	1769	"
Gross weight	9663	"
Pounds per b. h. p.	13.4	"
Pounds per square foot	9	"
Speed at ground	113.3	mi. per hr.
Climb to 6500 feet	10	minutes 45 seconds

In order that intelligent comparisons may be made, there are submitted below characteristics of the De Haviland 4 plane:

Engine	One Liberty	
Weight empty with water	2391	pounds
Fuel and oil	457	"
Crew	360	"
Guns, ammunition, etc.	374	"
Gross weight	3582	"
Pounds per b. h. p.	8.95	"
Pounds per square foot	7.12	"
Speed at ground	124.7	mi. per hr.
Climb to 6500 feet	8	minutes

THE NAVAL SEA-PLANE NC-1

One of the most striking accomplishments of the United States in airplane design has been the development by the Navy of the Naval Seaplane or flying boat NC-1. This plane has a central float 46 feet long, a wing span of 126 feet and a total wing area of 2400 square feet. This plane is equipped with three Liberty engines with tractor screws. Fully loaded the plane weighs 22,000 pounds. The weight empty with water is 14,000 pounds.

This plane is the largest seaplane in the world, at the moment, and is an American product throughout. The hull was designed after tank experiments on models. On a recent test this plane made a flight at Rockaway Beach, New York, with fifty-one passengers aboard, getting off in fifty-three seconds. Upon landing it was discovered that the fifty-first passenger was a "stowaway" on board the plane.

III—Remarks on Instruments, Raw Materials and the Training of Personnel

INSTRUMENTS

THE facilities for the manufacture of instruments necessary to give airplanes, pilots and observers their greatest usefulness were extremely limited at the outbreak of the war. The making of these instruments involved practically the formation of an industry new to this country. By June, 1918, twenty types of instruments were in production and being supplied for installation in training, bombing and fighting planes. The list included airspeed indicators, altimeters, compasses, safety belts, oxygen apparatus, radio sets and all other necessary adjuncts for the navigation of the air, and for the safety and comfort of the pilot.

Looking into the future, a reliable and simple "turning indicator" for airplanes is much desired. Furthermore, the development of some form of gyroscopic compass to meet the severe conditions of aircraft service is a problem of great importance. It has been said that the magnetic needle has sense but no power, and the gyroscope has power but no sense.

SPRUCE PRODUCTION

Some idea of the magnitude of the task involved in the procurement of the principal raw material used in the construction of the plane proper, namely spruce, may be obtained from the following figures on production and personnel. On June 30, 1918, the spruce production personnel consisted of 18,305 officers, enlisted men and civilians. Railroads were constructed into the forests of the West Coast and a cut-up plant having a maximum capacity of 9,000,000 board feet of lumber a month was erected at Vancouver Barracks near Portland, Oregon. To November 11, 1918, approximately 174,000,000 board feet of spruce and fir, as shown above, had been shipped of which more than two-thirds went to our Allies.

LINEN, DOPE AND OIL

As it was impossible to produce the millions of yards of linen fabric required for airplane wing and body covering, a suitable cotton substitute was developed and manufactured in this country in quantities sufficient to meet the needs of both

America and the Allies, and at a cost slightly more than half that of the linen fabric formerly used. This accomplishment, theretofore thought impossible, ranks among achievements of the first importance. The dope used on the fabric of airplanes presented many difficulties not only to us but to our Allies. This and other chemicals required soon were being produced in this country in quantities sufficient for their requirements as well as our own.

To meet the extensive demands for a high-grade lubricating oil, castor bean seeds were imported from India and about 108,000 acres planted in this country. Meanwhile, research work with mineral oils was carried on intensively, with the result that lubricant was developed which proved satisfactory in practically every type of airplane engine, except the rotary engine, in which castor oil still is preferred.

TRAINING OF PERSONNEL

The personnel side of the Air Service, including the selection, training, organization, and operation of the flying forces, developed within the fiscal year 1917-18 into an educational system on a scale infinitely larger and more diverse than anyone had anticipated. Teaching men to fly, to send messages by wireless, to operate machine guns in the air, to know artillery fire by its bursts, and to travel hundreds of miles by compass, teaching other men to read the enemy's strategy from aerial photographs, and still others to repair instruments, ignition systems, propellers, airplane wings, and motors, has required a network of flying fields and schools, a large instructional force, and a maze of equipment and curricula.

IV—Some Remarks on the Physical Conception of the Air.

IN a very condensed form I have endeavored to present in the above outline, the salient facts and figures of actual production and deliveries in connection with the air program in the United States up to the moment of signing the armistice. These figures are the cold facts of actual accomplishment of finished product.

I have made no exterpolations to these production curves which would show what flow of Liberty engines and combat planes for instance, we reasonably could expect would result in the succeeding months following the signing of the armistice. I leave this exterpolation to those of you who may be curious.

Behind the production figures of November 11 was mobilized in the United States an industrial army of about 350 firms and corporations employing more than 200,000 men and women. A huge imaginary conduit leading from the factories and assembly shops of America to the fighting front in France was in operation. At any given moment, the number of engines, planes and accessories distributed along the 3000 miles of this imaginary conduit were known accurately and organized in such a way that the flow throughout should be continuous, or that minimum congestion should occur at any point, be it factory door, railway train, embarkation depot, steamship, or debarkation point in Europe.

In order, however, to give an adequate idea of our whole effort, it will be necessary to refer now to certain lines of development, some physical, some chemical and some physiological, which have been set in motion during the progress of this work.

I think we will admit that no world-movement such as we now are witnessing in the development of aerial navigation demands such a close union between science and art. To make a machine to go into the air to transport man is a task set for the hand and brain as never before.

Fifteen years seems a short time when we contemplate what has been accomplished; yet, it was only fifteen years ago, on December 17, 1903, that Wilbur Wright made the first successful flight in a heavier-than-air machine at Kitty Hawk, N. C. To-day Government airplanes have carried the mail between

Washington and New York for seven months, and few of us take the trouble even to watch for these daily flights.

The advent of the first heavier-than-air flying machine begat, as a consequence, a period of intensive interest and study of the physical and dynamical properties of the medium, which are only now in the early beginning, and whose ultimate extensions we can only dimly perceive.

In November, 1916, I had the honor of addressing the National Academy of Sciences at Boston and outlining at that time some outstanding problems for research in connection with aeronautics. Although this was only two years ago, such has been the activity in research in studying air problems that fully two-thirds of the subjects suggested at that time now have been satisfactorily solved and are in actual use in the art.

Technical courses have been established at universities and colleges, and the new profession of Air Engineering is at the moment vying with the older engineering branches.

Everyone knows, of course, that if there were no air there could be no life, but probably very few fully realize its immense importance in almost everything we do. In one condition it is invigorating and gives us a zest for hard work whether mental or physical; in another it leaves us depressed and incapacitated for efficient labor. Numerous manufacturing processes are radically affected by the amount of moisture in the air, and many others by its temperature. Power is transmitted by it; we communicate our thoughts to one another by vibrations of the air; and by its aid we recently have acquired our swiftest mode of travel. Obviously then, a knowledge of the composition, structure, and physical properties of this universal medium is of such vital importance as to justify most painstaking study and research.

In the last few years, for instance, several elements: helium, argon, neon, krypton, xenon, have been found in the atmosphere that previously were unknown and even unsuspected, for they were not required by the Mendeleeff table of the elements as then understood. One of these, argon, amounts to nearly one part in a hundred of the whole atmosphere, and yet through decade after decade of chemical investigations involving countless thousands of air analyses, it, and all its family of gases, remained undiscovered.

Recently, too, means have been found for drawing directly on the atmosphere for an inexhaustible supply of nitrogen compounds used in the production of powerful explosives, fertilizers, and many other things of industrial value.

The first eighteen years of the present century have witnessed a wholly new and startling flood of physical knowledge which has resulted in profoundly modifying our conceptions of electricity and matter, as developed in the nineteenth century. These two great subjects have been brought together and now are recognized as one and the same subject.

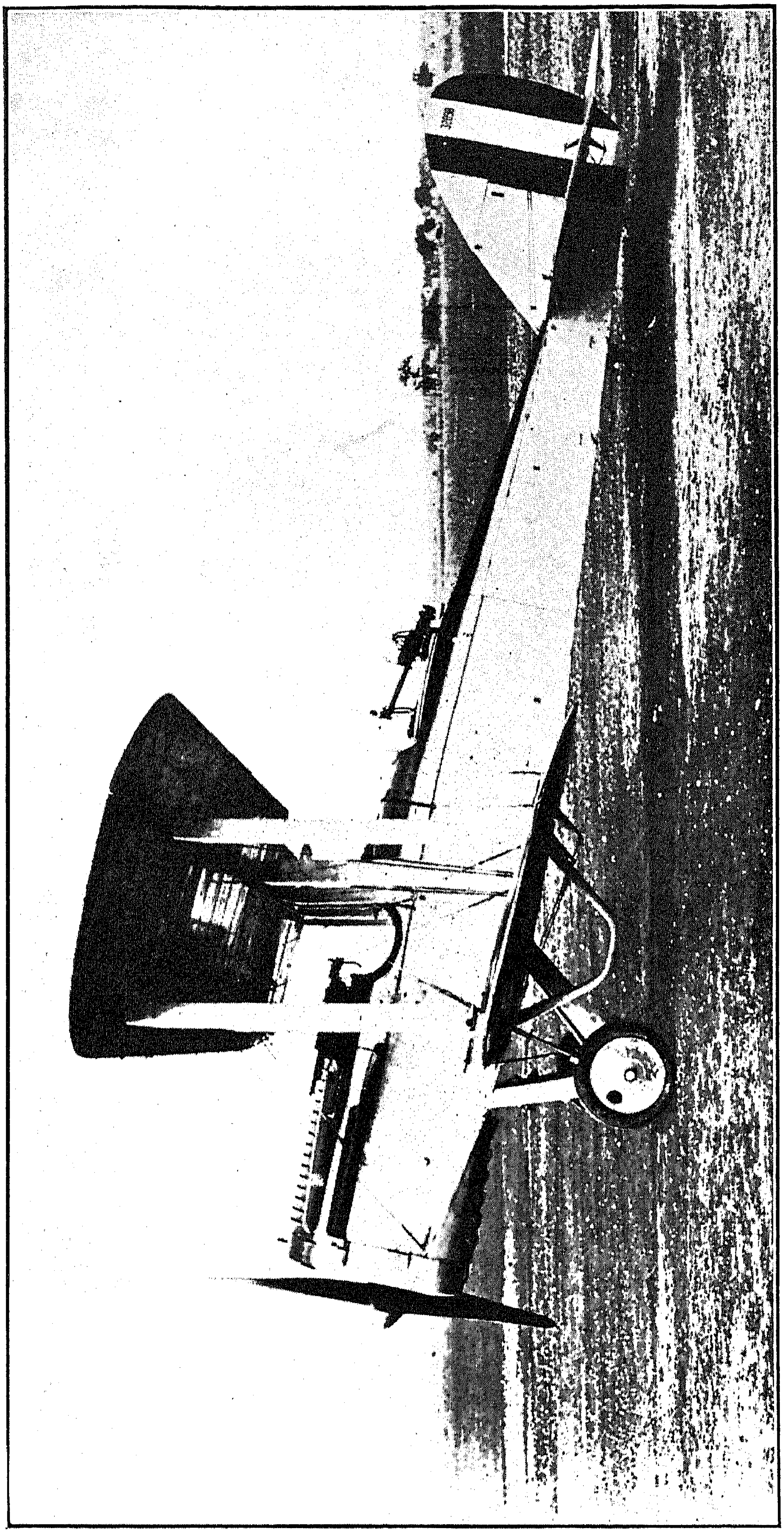
What we called self-induction now is considered to be only inertia. Mass is supposed to be wholly electrical and a vector instead of a scalor quantity. The most fundamental of all physical constants, the electron, has been isolated and its charge measured. Electricity from whatever source is made up of these electrons and even metallic conduction consists of the actual projection of these granular units along the conductor.

A gas like the air, on the present view, is not a continuous medium, but is composed of many individual structures which preserve their indentity and individuality during the normal life of the gas. These largest units are known as molecules, and they are each moving with considerable velocity on the average, the path of each being for the greater part a straight line until two approach near to each other. The forces that these two exert upon each other reach out into the medium which surround them both, creating a region of intense field at sufficient distance away from each so that their integrity is not disturbed.

When the two molecules approach each other, they are deflected off without collision, preventing a catastrophe much as a comet coming into the solar system is merely deflected by the action of the sun, and does not fall into it unless it makes a direct hit, the chances of which are extremely small. The gas pressure of the air upon the walls of the containing vessel merely is the reaction of these deflected molecules that come into close proximity with the more fixed molecules of the container.

If we consider for a moment a cubic centimeter of air under standard conditions, we know that the number of the largest structures therein, the molecules, is a constant, and is about 2.705×10^{19}. It does not matter what kind of a gas we have, whether hydrogen, oxygen, nitrogen or compounds like methane carbonic acid, etc; the number of molecules remains fixed. In fact, we probably can determine the number of molecules in a

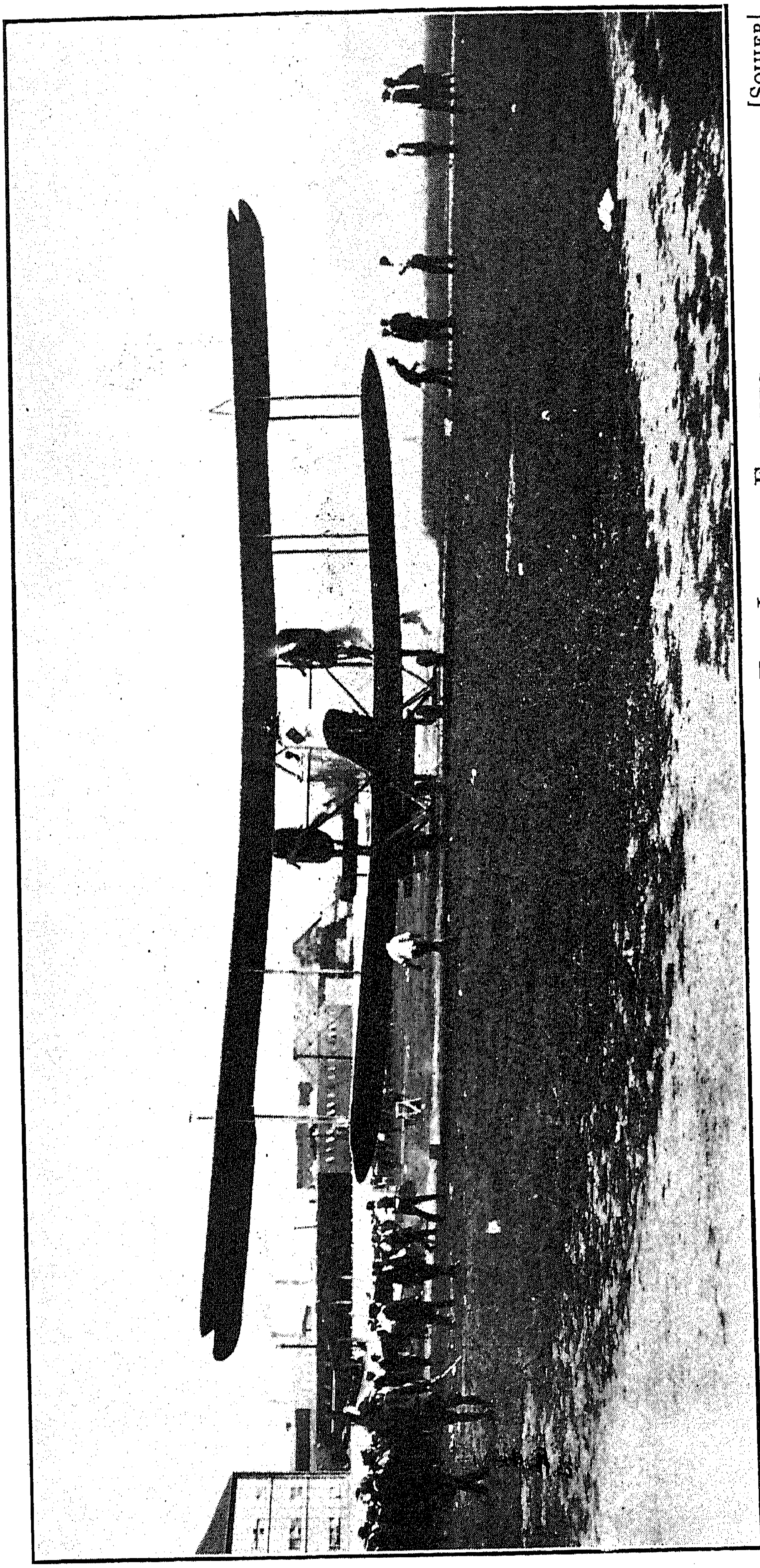
PLATE I.
A. I. E. E.
VOL. XXXVIII, 1919



De Haviland 4—Liberty Engine

[Squier]

PLATE II.
A. I. E. E.
VOL. XXXVIII, 1919



[SQUIER]

HANDLEY-PAGE BOMBING PLANE—TWO LIBERTY ENGINES cubic centimeter of a gas more accurately than we can determine the number of inhabitants in New York City, in spite of the fact that the number is slightly more than twenty-seven billion billion.

By measuring the weight of the cubic centimeter of air and dividing this by the number of molecules contained therein, we obtain the weight of the largest structure composing the air, namely, the molecule. The structure of the molecule, while sufficiently enduring to remain together as a unit in the normal condition of the gas, yet is not strong enough to withstand forces that may be easily brought to bear upon it by chemical means to cause it to separate into still smaller units known as atoms. In the nitrogen and the oxygen of the air there are but two atoms making up each molecule, so that each structure in air can only be divided into two by any means at the disposal of the chemist.

The atoms, however, have been broken up into still smaller units, but it can be done only with great difficulty, comparatively speaking, and until recently, the difficulty was so great that they earned the name atom, signifying that they could not be cut in two. They are far more enduring structures than the molecule.

The only two kinds of things that ever have been obtained thus far by the breaking up of the atoms, of all kinds, are the so-called positive ions and negative electrons, and these remain today as things which never have been broken up into smaller components. Whereas the negative electron has been isolated from gross matter the positive charge never has been dissociated from matter. The present idea of physicists is that we have found an ultimate constituent of all gross matter. The mass of this negative electron has been measured, and the mass of the smallest atom, hydrogen, is 1845 times greater than that of the electron.

We regard the atom as consisting of a positive nucleus about which negative electrons are ceaselessly revolving. The dimensions of the negative and positive constituents of atoms, in comparison with the dimensions of the atoms themselves are like the dimensions of the planets and asteroids in comparison with the size of the solar system. Here then the master physicists with consummate skill have built up for us an alluring conception of miniature solar systems of which all gross matter is composed "A mighty maze! but not without a plan."

In the light of the foregoing conception of a cubic centimeter of air, imagine if you can, what a compilcated thing is the result of an orchestra playing perhaps one hundred instruments and two hundred voices singing in chorus, accompanied by the orchestra. The cubic centimeter of atomic worlds, in this case, becomes subjected from without to a great number of superimposed frequencies representing each and all of the individual instruments, as well as the innumerable harmonics required to indicate the qualities of the human voices; yet our medium is sensitive enough to respond accurately to each and every one of these frequencies simultaneously, and synthetically adds them together and produces a resultant envelope curve of pressure which the marvels of the human ear are able to analyze and interpret into its elements. A medium which has such qualities as to be capable of acting and reacting in this delicate manner, and at the same time when moved bodily with the velocity of a tornado may cut down trees and buildings as if by a giant steel knife, is a medium the possibilities of which for future research we, at present, only dimly perceive.

V—The Production of Helium on a Commercial Scale

ONE of the greatest scientific achievements of the present war from a technical standpoint is the production of helium in balloon quantities. This gas is non-inflammable and has about 92 per cent of the buoyant effect of hydrogen. Its name is due to its having been discovered in the sun's atmosphere through a characteristic line in the solar spectrum, before its presence on the earth, or any of its properties were known. It first was obtained in minute quantities by Ramsay in England some twenty years ago by heating certain radioactive minerals, in which it occurs because it is a disintegration product of radium. Its pre-war scarcity may be appreciated from the fact that, up to two years ago, not more than 100 cubic feet ever had been obtained, and the usual selling price was about $1700 a cubic foot.

Notwithstanding so discouraging an outlook someone in the British Admiralty had imagination enough to propose the large scale separation of helium from certain natural gases in Canada, that contain about one-third of one per cent of it, and experiments were undertaken at the University of Toronto. Soon after the entry of the United States into the war, the Bureau of Mines, learning of the problem from a British confidential memorandum, persuaded the Signal Corps and the Bureau of Steam Engineering of the Navy to approve and finance jointly an experimental program on a large scale. Thanks partly to the unusually rich sources of supply in this country, and partly to the skill of the two commercial companies whose services were enlisted, and to the enthusiasm of the Bureau of Mines Staff and of Mr. Carter, of the Navy, who for a time represented the Army as well in the project, such success was achieved that, at the cessation of hostilities, there was compressed and on the dock ready for floating 147,000 cu. ft. of nearly pure helium, and plants were under construction to give at least 50,000 cu. ft. a day at an estimated cost of not more than ten cents a cubic foot.

The production of a balloon gas that assures safety from fire opens up a new era for the dirigible balloon. In November, 1917, a Zeppelin made the trip from Bulgaria to German East Africa with twenty five tons of medicines and munitions, only to find that the German forces already had been dispersed, and returned safely to its base without landing. With a non-inflammable gas, not only comfortable and expeditious, but also safe transcontinental and transatlantic travel in dirigibles will, it is believed, soon be commonplace.

VI—Meteorological Service of the Army

IN August, 1917, the Chief Signal Officer directed Lieut. Colonel R. A. Millikan to organize an Army Meteorological and Aerological Service, the purpose of which was three-fold:

a.. To provide the American Expeditionary Forces with all the meteorological and aerological information needed.

b. To supply the aviation fields, the coast artillery stations, the ordnance proving grounds, and the gas warfare service within the United States with such meteorological and aerological data as might be useful to them.

c. To undertake for the first time in the history of the world the problem of mapping the upper air currents over the United States, the Atlantic, and Western Europe in aid of aviation, and particularly with reference to transatlantic flight.

In carrying out the first of these projects, there were selected approximately 550 men of high qualifications, most of them physicists or engineers who were given a two months' course in meteorological and aerological theory and observations at College Station, Texas. Three hundred and fourteen of these men were sent overseas where they have been operating as an effective and well organized branch of the work of the A. E. F. In addition to furnishing a general weather forecast for the A. E. F., this service has supplied a meteorological unit to each aviation post, each gas service post, each artillery post, and each sound-ranging post of the American Army in France. Under the effective direction of Major W. R. Blair, one of the most experienced aerologists of the United States, commissioned for the service from the Weather Bureau, about twenty upper-air stations were established in France and England and a forecast based on data furnished by these stations made regularly to the A. E. F.

For the accomplishment of the second element of the program, twenty eight stations manned by 150 men, all carefully picked at the start and well trained at College Station, have been established for furnishing local data as to either surface or upper-air conditions, or both, to the flying fields, artillery posts, and proving grounds in this country. The largest of these stations is that at the Aberdeen Proving Ground, which is manned by twenty two men and furnished to the Bureau of Ordnance all necessary data for the determination of ballistic wind which, in view of the development of high angle fire, has become altogether indispensable for the construction of range tables needed for obtaining accuracy in the work of the artillery.

In carrying out the third element of the program, twenty-six meteorological stations were established, placed at carefully selected points over the whole of the United States, which stations have been manned by trained observers who telegraph to Washington each day observations on wind velocities at all altitudes up to 35,000 feet. In one instance these observations have been carried to 65,000 feet. On the basis of these observations, a daily forecast of upper-air winds is now being issued. The use which such forecasts may serve, both in connection with the aviation mail service and ultimately with the trans-atlantic service, may be seen from the fact that above the level of 10,000 feet, 95 per cent of the winds in both the United States and Europe are from West to East and often attain velocities in excess of 100 miles an hour. On November 6, 1918, at Chattanooga, Tennessee, a velocity of 154 miles an hour at an altitude of 28,000 feet was observed. It is because of this easterly direction of these upper air currents that all of the long flights thus far made have been from West to East. The importance of a forecast of such currents for the purpose of long flights will be appreciated as soon as the foregoing facts are understood. An airplane capable of a velocity of 154 miles an hour in still air either would remain stationary or travel at 308 miles an hour depending on whether it was headed into or with a wind of the velocity of that observed at Chattanooga.

All of the aerological work so far mentioned has been done with the aid of theodolites especially designed by Major W. R. Blair for this service. Sixty of these have been built for the work in this country and twenty shipped abroad.

The problem of exploring the upper-air currents over the Atlantic was at first thought insoluble on account of the absence of fixed bases, but the Meterological Service has developed propaganda balloons which already have flown at an average altitude of 18,000 feet from Omaha to New Jersey, a distance of more than a thousand miles. The success of the project now has made possible the mapping of the upper-air highways across the Atlantic; for arrangements are being made to send up from both coastal stations and from transatlantic steamers, these long-range balloons designed for from two to three-thousand-mile flights, and adjusted to maintain a constant altitude and to drop in Western Europe their records of average winds in these heretofore unchartable regions. The importance of this work for the future of aviation needs no emphasis.

The success which the Meteorological Service has attained would have been wholly impossible had it not been for the intimate and effective cooperation which has been extended to it in all of its projects by Director C. F. Marvin and the entire staff of the United States Weather Bureau.

VII—Physiological Study of the Flier

UNTIL a year and a half ago, interest centered in the development of aircraft and not in the flier. There was little regard for the special fitness of the man on the day and hour he drove his machine.

This was true, not only in the United States, but also in the Allied countries. The pilot was not selected because of any peculiar fitness for flying. It simply was a question of whether or not he had the nerve.

But, about the time the United States entered the war, it became clear that the science of physiology could be applied with advantage to the selection, classification and maintenance of the aviator. Each of the countries in the war now has a fully established air medical service, as an integral part of the air forces. In the British Army there is a separate air medical service with a Surgeon General of Aeronautics. In the American Army this work has been handled effectively by a division of the Surgeon General's office, assigned as part of the Air Service.

The early view that any one who "had the nerve" could fly caused enormous avoidable wastage of life and material. The lesson learned from bitter experience was that it is essential to obtain fliers who are especially fitted for particular work and to keep them in condition to perform their duties at all times.

Nature never intended man to fly in the same sense that she did not intend him for life in a submarine. Conditions are unnatural from the time he leaves the ground until he returns. There are many obstacles to overcome. He flies in an atmosphere deficient in that oxygen which is the "breath of life;" he is subjected in war to the shells of anti-aircraft guns and enemy aircraft; he travels through space high above the ground at rates well in excess of 100 miles an hour. In attaining altitudes and breathing rarefied air, the flier is defying nature.

The pilot is the heart and brain of the whole flying apparatus. Parts of the airplane may break without serious result, but when the pilot breaks, even momentarily, nothing is left to direct the flight. The man and the machine come crashing to the earth.

All this the Air Medical Service had to consider. The service had the advantage of a series of reports of the air medical services of our Allies, which enabled it to attain self-footing in a remarkably brief time. A medical research laboratory, fully equipped, was established by the Army Air Service at the flying fields on Long Island, N. Y.

The keynote of the American Air Medical Service is the handling of the flier as an individual, which naturally brings the work into three main divisions:

First, the selection of the flier:

Second, the classification of the flier, and

Third, the maintenance of the physical efficiency of the flier.

THE SELECTION OF THE FLIER

The outset of the war made it necessary to obtain a large number of military aviators in the shortest possible space of time. The medical problem consisted of selecting men who were physically fit.

It is possible for men to fly in spite of one or more physical handicaps, such as having only one leg or one eye or even being cross-eyed, but such men are not desirable fliers, since men with complete endowments naturally have superior advantages. The man in the flying service would be called upon to negotiate critical emergencies in the air. Instant decision and action, therefore, would be essential for success. So it was seen that only those men with exceptional mental and physical capabilities should be chosen.

In order to accomplish the best results, a comprehensive program was undertaken, providing for the standardization of both tests and examiners. Sixty-seven military units were established, each examining from ten to sixty applicants a day. The figures show that 70.7 per cent of the applicants were qualified. The large number of applicants made it possible to maintain the highest standard in selecting men.

In the same spirit with which the United States determined to supply the American air fighters with as good, if not better, planes than those used by the enemy, it was decided that the American aviators must be as good if not better than those of the enemy. The outcome of any encounter might easily depend upon which combatant possessed the better vision and other special senses, the better nervous system, and the better mental and physical equipment in general.

Each applicant received a complete physical examination embracing all the features ordinarily required of men entering the military service, and, in addition, comprehensive and extensive tests of the special senses of vision, hearing and motion sensing.

CLASSIFICATION OF THE FLIER

It is important to classify the flier for the kind of work he is physically capable of performing. Some men are not able to fly at higher levels than a few thousand feet without serious deleterious effects, while others may operate at much higher altitudes. It is necessary to know a flier's limitations before his training is specialized, for the saving of time and money, and, in fact, the flier himself. It would be an evident waste to train a pilot for combat work and then find he was physically capable of doing only bombing at a relatively low level. To accomplish this classification, branch medical research laboratories were placed in the flying fields, which by appropriate tests on the rebreather machine, developed at the parent institution at Mineola, classified the fliers.

The work of these laboratories demonstrated that sixty-one per cent are capable of flying to 20,000 feet or more, twenty-five per cent should not fly above 15,000 feet, and fourteen per cent were unsafe above 8,000 feet.

The re-breather is used also to determine temporary unfitness for altitudes, due to some intercurrent illness, dietary indiscretion and staleness from prolonged flying.

The necessity for altitude classification is readily understood from the fact that whereas in 1915 flying rarely exceeded 8000 or 10,000 feet, through improved design, scouts of today climb to altitudes of 25,000 feet within a very short time.

Night bombing is carried out at altitudes as low as 300 feet. Day bombing, in order not to reveal the objective of the flight and to guard against concentrated anti-aircraft fire, may call for flights at extremely high altitudes. Reconnaissance machines rarely get to high altitudes, owing to the necessity for more or less close observation of the ground, and machines doing this work accomplish low flying even in the face of highly concentrated anti-aircraft fire, and other enemy activity. Machines cooperating with the artillery making range corrections for batteries, do not often work above 6000 or 8000 feet.

Owing to the fact that all this work is widely diversified, it is essential that each man be placed in that division for which he is best suited. A knowledge of the limitations of a valuable man spells increased efficiency. Just as the pilot is provided with a certain type of plane adapted to the work in hand, so the plane must be provided with a pilot adapted to the work.

Fliers who are forced to undergo abrupt changes in atmospheric pressure and oxygen supply must be singled out. Atmospheric pressure plays an unimportant role; the whole problem resolves itself into a deprivation of the normal oxygen supply.

The Flack bag was the prototype of the rebreathing apparatus which has been developed in the medical research laboratory. By means of this apparatus the aviator rebreathes air confined in a tank, from which he gradually consumes the oxygen. As the percentage of oxygen decreases, the flier, in effect, is slowly ascending to higher altitudes. In the course of twenty-five to thirty minutes he lowers the oxygen content of the air in this tank to 8 or 7 per cent, which is equivalent to attaining altitudes of 25,000 to 28,000 feet.

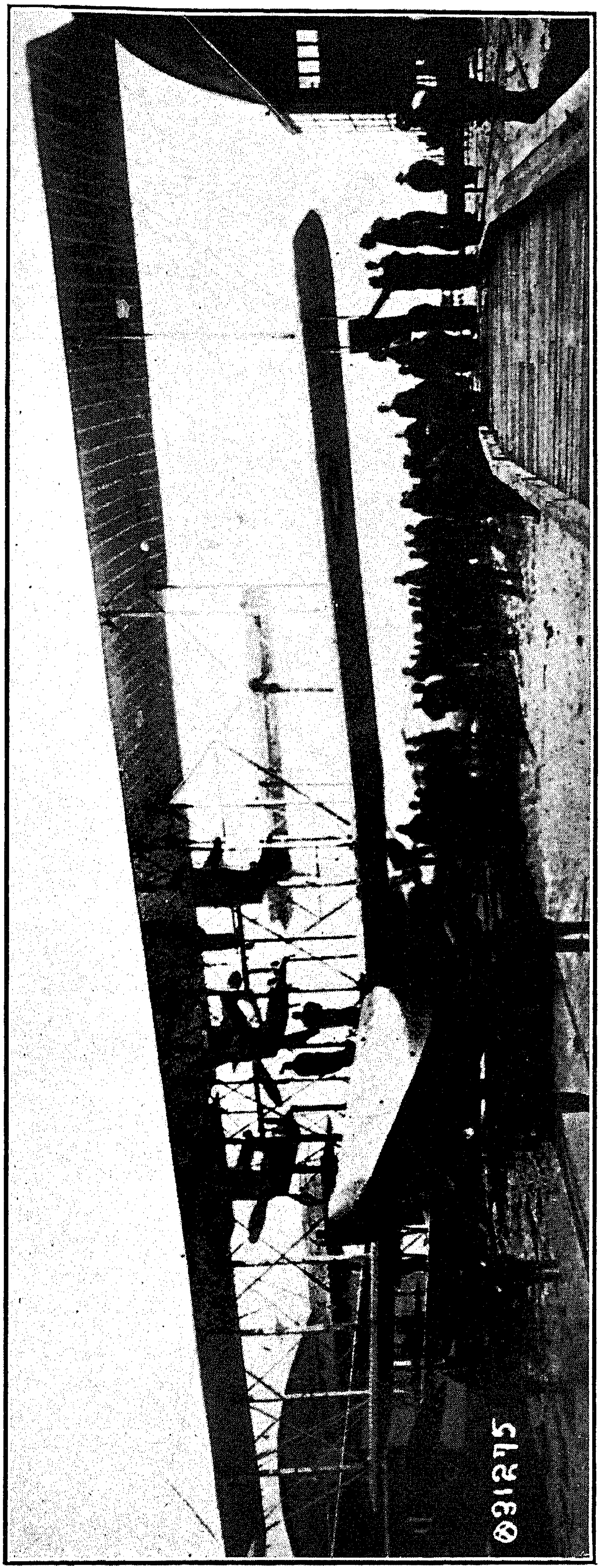
Another means of attaining the same result is by the diluting apparatus, which supplies directly to a mask over the face a mixture of air and nitrogen in whatever proportion is desired. All of these tests have been standardized and confirmed by the low-pressure tank, in which the air is rarefied to correspond to any given altitude.

By a comparison of the percentage of oxygen to which the flier succumbs when on the low-oxygen test, it is possible to determine the altitude at which he would fail were he in the air. This determination is made on the ground, without danger either to the flier or to his machine.

The effect of low oxygen upon the mental processes varies greatly in the individual. He usually becomes mentally inefficient at an altitude at which there is as yet no serious failure of his vital bodily functions. By simple tests of mental alertness during rebreathing it is easy to determine that one flier becomes mentally inefficient at 15,000 feet, in sharp contrast to another who has his full mental powers up to and beyond an altitude of 25,000 feet.

Oxygen-want exaggerates any latent defect of the eyes. Crash reports have demonstrated that a large proportion are due to eye defects.

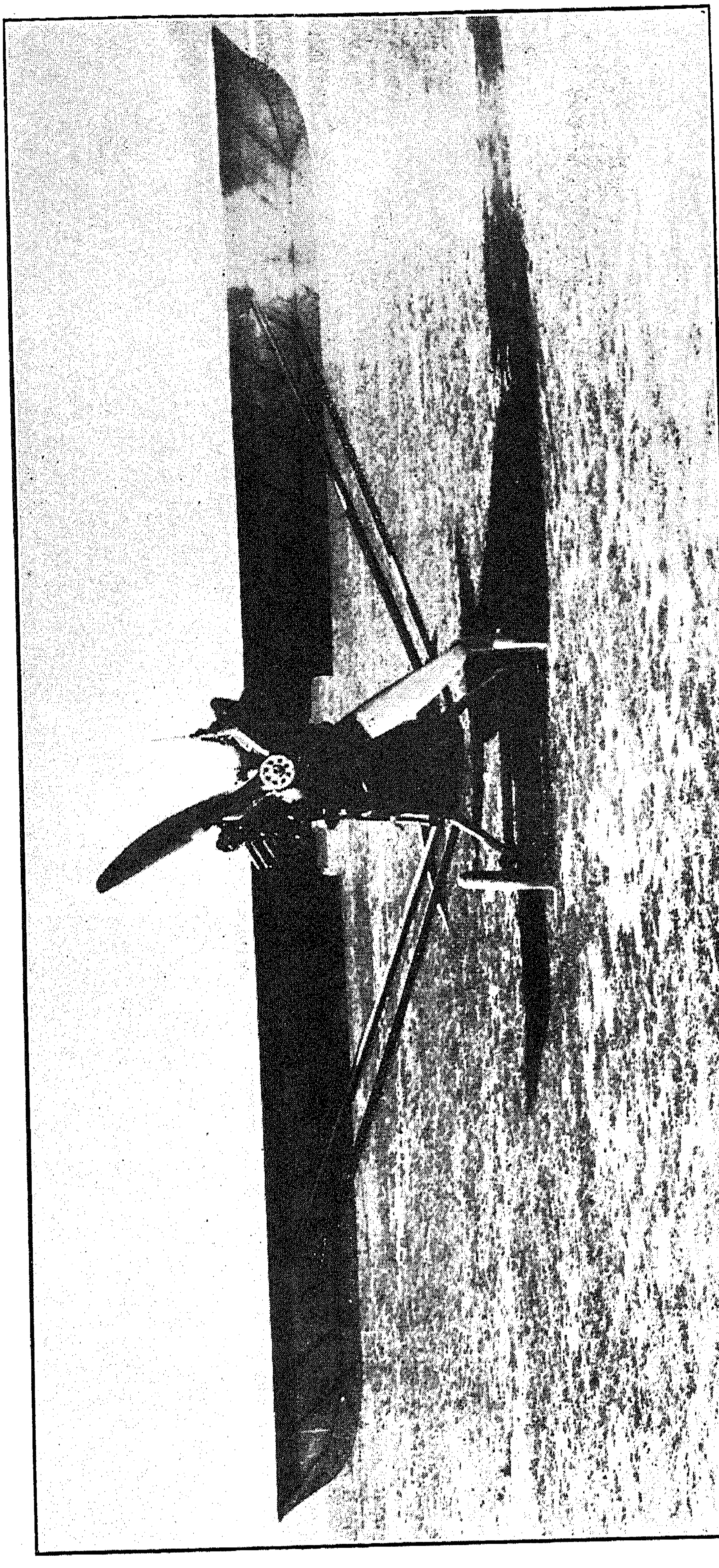
"Stunting" essentially is an internal-ear problem. During and after rapid turning the flier's brain is receiving impulses from his semicircular canals. In a normal individual nothing



N. C. 1 Navy Seaplane (or Flying Boat)—Three Liberty Engines

[Squier]

PLATE IV
A. I. E. E.
VOL. XXXVIII, 1919



LEONING MONOPLANE—300 H. P. HISPANA-SUIZA ENGINE

[SQUIER]

can control or alter the sending or receiving of these impulses, which produce sensations of motion. Fliers vary greatly in their ability to interpret correctly the significance of these impulses. Experience alone enables them to familiarize themselves with their meaning. Those who develop the greatest ability in their interpretation naturally fall into the scout-pursuit class. Those who, in spite of training, are still disturbed or bewildered by stunting are reserved exclusively for straight flying.

The Maintenance of Efficiency

When the work of selection and classification is completed, the great problem of the Air Medical Service is to maintain the fliers on a basis of high physical efficiency.

After a certain amount of continuous service the flier begins to show unmistakable signs of deterioration. Until the Air Medical Service went to work, fliers were permitted to continue until they broke. Their breaking was signalized sometimes by simple failure to return from behind the enemy lines; sometimes by becoming mentally and nervously exhausted to the extent of permanently unfitting them for flying.

The rebreathing test also is valuable in determining this staleness in fliers, caused by frequent exposure to high altitudes. Incipient cases of the deterioration of efficiency caused by oxygen-want are detected easily by means of the rebreather. Then it is possible to ground a man for a certain period to enable him to recover entirely, whereas if this condition were not detected, it might progress to a point where it would be impossible for the man to regain his former efficiency. When staleness becomes marked the flier is liable to faint in the air, thus losing his life and wrecking his machine. Periodical examinations are made to detect staleness.

There are three and only three means by which the flier's usefulness may be terminated; first, by the enemy; second, by the failure of the machine; and third, by the failure of the flier himself.

While it was not possible to arrive at exact percentages, estimates made at the time of our entrance into the war, based upon information from Italy, France and Great Britain, indicated that not more than two per cent. of the aviation losses in active service were caused by the enemy. Failures of the airplane were responsible for only slight losses, due, of course, to the rigid inspection of the machines. Statements from all sources agreed that of the total number of fliers permanently out of the flying service, not more than 8 per cent could be attributed to mechanical shortcomings of the airplane or engine.

The remaining 90 per cent loomed large when it was realized that this proportion represented trouble in the flier himself.

Therefore the "Flight Surgeon" was authorized to keep aviators physically fit and to study the causes of accidents attributed to the failure of the man.

Keeping the flier fit embraces the amount of physical exercise necessary; the provision of proper recreation; the state of fatigue of the individual; the amount of sleep; the provisions for leave or furlough; the food problems; the field conditions bearing on his welfare, and his reexamination at frequent intervals.

It was observed that more than half the injuries in crashes were caused by the flier striking his head against the cowl. The cowl was cut out to give the flier more room in front, and this change has practically eliminated the head injuries. A safety belt was lashed to the machine by a simple rubber shock absorber, and since this has been done the number and extent of injuries to the upper abdomen and ribs have been reduced decidedly. The problem of protecting the flier against the extreme cold of high altitudes in winter was solved by designing electrically warmed clothing. The problem of enabling the flier to withstand the glare of reflected sunlight above cloud banks was solved by furnishing him with the "Noviol" type of goggles. The Dreyer oxygen apparatus is used to compensate for his lack of sufficient oxygen in high altitudes. This apparatus has been so modified and perfected that it is now possible to automatically supply a flier with the correct amount of oxygen for the altitude at which he is flying.

All this has been done toward reducing this "ninety per cent," and much more is being developed. Within the last few months an apparatus has been perfected whereby students may acquire flying experience and training without leaving the ground. This machine, known as the Ruggles Orientator, is a modification of the universal joint, composed of three concentric rings so pivoted as to permit the fuselage, which is pivoted within the innermost ring, to be put through every possible evolution experienced in actual flying, except forward progression.

An analysis of "crash reports" shows that a large number are due solely to failure to come out of the spinning nose dive or tail spin. The student failed in these maneuvers because he had not learned to compensate by sufficient previous experience for his dizziness. The new apparatus gives the flier experience until he becomes familiar with the various sensations.

Another method of educating the student is by means of flying calisthenics. By daily turning and tumbling exercises he becomes accustomed to positions and movements in which at first he is awkward and bewildered.

It has become evident during the last nine months through activities of nutritional survey parties of the Food Division, Surgeon General's Office, that there is a great need in each aviation camp for a nutrition officer. The nervous system is more highly differentiated than the muscular system, and more easily upset by improper food. It has been shown that a nutrition officer with special knowledge of food values should supervise the messes of all cadets and officer fliers in order to keep up efficiency.

Under date of November 20, 1918, General Pershing says in a report to the Secretary of War: "Our aviators have no equals in daring or in fighting ability, and have left a record of courageous deeds that will ever remain a brilliant page in the annals of our Army."

The work of the Air Service Division of the Surgeon General's Office was instituted by Major, now Brigadier General T. C. Lyster, with the assistance of such specialists as Colonel George H. Crabtree, Colonel William H. Wilmer, Lt. Col. E. G. Seibert, Lt. Col. Isaac H. Jones, Lt. Col. Eugene R. Lewis, and Lt. Col. Ralph H. Goldthwaite.

VIII—Activities of the Bureau of Standards for the Air Service

RADIO INVESTIGATIONS

A THOROUGH study of vacuum tubes for use in transmitting, receiving, and amplifying is in progress; a study of insulating materials used in radio apparatus; the development of a permanent contact crystal detector; the development of a high-frequency oscillographic equipment; the furnishing of standards for use in radio development are among the subjects handled in connection with the production of radio apparatus for military purposes.

The facilities of the laboratory and working quarters have been provided for the Research Laboratory of the Science and Research Division of the Signal Corps, and the Intelligence Division of the Signal Corps.

AERODYNAMIC INVESTIGATIONS

The wind tunnel of the Bureau of Standards has been employed in a wide variety of military problems, among which may be mentioned the following:

Bomb-dropping devices have been tested and compared. The head resistance and other characteristics of airplane bombs have been measured in order to provide data for computing the trajectories. Machine gun sights for airplane use, with automatic adjustments for speed have been tested and adjusted in the wind tunnel. Hundreds of Pitot tubes and Pitot-Venturi combinations for measuring the speed of the airplane have been calibrated. The head resistance of many types of engine-radiator sections has been measured. Small variable-pitch propellers for driving at constant speed the generator used in radio signaling, independently of the speed of the airplane or engine were perfected through wind tunnel tests and later tested in large numbers. Finally, the characteristics of new wing-sections and new airplane designs have been determined from wind tunnel tests on models.

AERONAUTIC POWER PLANTS

An altitude laboratory for the study of airplane engine characteristics and performance at all flying altitudes was begun in August, 1917, completed in January, 1918, and has been in service since that time.

Studies have been made of the Liberty "8" and Liberty "12", Hispano Suiza 150 h. p., Hispano-Suiza 180 h. p., and Hispano-Suiza 300 h. p. models.

The results include complete data as to the power developed, fuel consumptions and heat balances at all practicable engine speeds and altitudes up to 30,000 feet; the demonstration of causes of bad carburetion at high altitudes and of the requirements for the best results, together with the development of means for producing these results automatically; the analysis of the effect of fuel composition on engine performance, the results of which served as the basis of specifications for aviation gasolines; the securing of necessary data for the establishment of standard flight test specifications for engines, including variations of horse power with the pressure, density and temperature of the air supplied the carburetor; the measurements of the effect of different compression ratios on engine power at different altitudes; the increase in power possible when using supercharging devices designed to supply the engine with air at pressures greater than that at which the airplane is flying.

AIRPLANE RADIATORS

The necessary apparatus has been designed and constructed to study the heat transfer characteristics of radiator cores used in airplanes. The performance of more than 85 different types of cores has been studied in this apparatus at reduced pressure and high rates of air flow. Head resistance measurements have been made on these cores in the 54-inch wind tunnel. The results of these experiments have shown that certain types of cores in use today may absorb twice or three times as much of the engine power as other types and as much as 25 per cent of the total power developed by the engine.

Experiments on the head resistance of a model fuselage fitted with a nose radiator have shown that the power absorbed by the resistance of a nose radiator may be as much as 40 per cent greater than that absorbed by a radiator more suitably placed. To afford maximum efficiency, a radiator must be designed for its particular air speed and position on the airplane. The results of this investigation give the necessary data for the design or selection of radiators for all air speeds and for various locations.

IGNITION PROBLEMS

Investigations have been made on various problems connected with spark plugs and other ignition apparatus used on airplane engines. This work has resulted in the development of a new porcelain for spark plug insulators which is definitely superior electrically and mechanically to any previous commercial product. Methods of test for spark plugs have been developed as a basis for Bureau of Aircraft Production specifications and several thousand spark plugs have been tested by these methods. The results have led to the use for aviation work of several high-grade commercial porcelains not previously considered for this purpose.

CARBURETION

In the carburetor testing plant the work has included a detail study of the changes in mixture proportions that occur with throttle manipulation under the conditions of atmospheric pressure and temperature existing at and between ground level and 33,000 feet altitude. This work has resulted in the development of suitable control of the mixture under throttle manipulation at any level above the earth's surface; and in the development of methods and devices for automatic correction of the variations in mixture proportions ordinarily occurring in passing from ground level to an altitude of 33,000 feet.

LIGHT ALLOYS

An important group of problems which engaged the attention of the Bureau was that relating to the production of light aluminum alloys and the improvement of their mechanical properties. Thus, new manufacturing methods were developed for a well-known rolling and forging alloy by which both the ease of its production was increased and its physical properties improved. Studies were made of the corrodibility of various alloys, while mechanical and metallurgical tests were made of a variety of different compositions of alloys giving most valuable information for the guidance of the designer of engines and planes.

A light alloys committee was organized to act as a clearing house for information along these lines and to oversee and direct the research work proceeding throughout the country.

AIRPLANE DOPES AND VARNISHES

Specifications for acetate and nitrate dopes were prepared at the Bureau with the cooperation of dope manufacturers, airplane constructors, and representatives of the government. Simultaneously, dopes manufactured in the country were examined in accordance with the specifications and list of products prepared which were approved for use on government aircraft. After much controversy, the use of acetate dopes containing tetrachlorethane was forbidden because of the toxicity of the above compound. Many proposed methods of fireproofing dopes and fabrics have also been examined. A commerical method of preparing and stabilizing lactic acid esters was devised for use in acetate dopes, in order to conserve the products of acetate of lime. A transparent silk fabric for possible use in obtaining low visibility has recently been perfected. Lately work has been in progress on the pigmentation of dopes with a view to protecting the dope film from the deteriorating action of sunlight, and thus prolonging its life, at the same time dispensing with the use of enamels, thus saving one step in production. The use of pigmented dopes will also conserve cellulose acetate.

Investigations of methods of making wooden parts of airplanes water resistant showed that a good grade of spar varnish was satisfactory, and a specification for such a varnish that could be produced at low cost was prepared. The same varnish has been found to be a satisfactory coating for doped linen or cotton, used either as a transparent varnish or as the basis of a vehicle for pigmented coatings. These may be used on both wood and metal.

AIRPLANE FABRICS

Previous to 1916 linen fabric was used exclusively in the covering of airplane wings and found to be satisfactory in every respect. During the present crisis, it became evident that the available supply of linen would not suffice for the demands of the military programs of the Allies, and it became necessary to find a material which would form a satisfactory substitute for linen. The urgency was emphasized by the conditions in Russia, from which the larger portion of the flax formerly had been derived.

As early as January, 1916, the Bureau started investigating the possibilities of substituting cotton for linen airplane fabric and found that the general consensus of opinion among manufacturers and investigators here and abroad was that cotton fabric could not be used for wing coverings. Their difficulties were due to the fact that they attempted to substitute structure of fabric rather than physical properties. The linen fiber has radically different properties from the cotton fiber, and the only hope for successful cotton airplane fabrics was so to change their structure that the ultimate fabric had the same properties as the linen fabric.

The investigation was confined to the study of stresses and stress distribution in fabrics, together with the factors covering the properties of cotton fabric as related to wing covering.

This phase of cotton manufacturing was an entirely new one, and great difficulty was experienced in studying the manufacture from a new angle, on account of the limited time available. Upon the entrance of this country into the war all the facilities of the manufacturers were made available to the Bureau's experts.

In March, 1917, the Bureau was in a position to issue instructions covering the construction of cotton fabrics, some of which proved quite successful. The next important step was to determine the actual performance of these fabrics and to this end samples were placed on Army planes at Langley Field and Navy planes at Pensacola during August, 1917. Later these fabrics were placed on planes manufactured by the Canadian Aeroplane Company of Toronto about the middle of October, 1917, in all cases with the most satisfactory results.

The Grade A cotton fabric now being supplied to the Air Service compares favorably with the linen regarding weight, has a much higher factor of safety, a greater tear resistance, and dopes up to satisfactory tautness. The life of any fabric is dependent entirely upon the life of the dope, and therefore the cotton has as long a life as the linen.

The English Government became concerned about its linen supply, and also adopted the fabrics designed by the Bureau.

As a result of more recent investigations by the Bureau of Standards, another distinct fabric has been evolved which is 25 per cent lighter than any linen fabric now in use, and is materially stronger.

Director S. W. Stratton has placed the services of the entire Bureau of Standard's staff at all times at the disposal of the Air Services of both the Army and Navy in attacking and solving new problems which have constantly been presented, and in fact the Bureau has been during the war practically an adjunct of the War and Navy Departments.

IX—National Advisory Committee for Aeronautics

THE National Advisory Committee for Aeronautics was established by Congress by Act approved March 3, 1915. Under the law the Committee is charged with the supervision and direction of the scientific study of the problems of flight with a view to their practical solution, the determination of the problems which should be experimentally attacked, their investigation and application to practical questions of aeronautics. The Committee is also authorized to direct and conduct research and experiment in aeronautics in such laboratory or laboratories, in whole or in part, as may be placed under its direction.

In addition to its specific duties, the Committee has acted as a clearing house for inventions submitted to the War and Navy Departments with a view to their possible usefulness in the field of aeronautics. These inventions have been studied carefully by the Committee's technical staff and those deemed worthy of development have been called to the attention of the military branches of the Government. During the last year, the Committee has examined approximately seven thousand such inventions and suggestions.

The Committee is recognized by the Aircraft industry as an authoritative source of technical information and as a court of arbitration whose weighed judgment obtains. The National Advisory Committee for Aeronautics is today the only official agency for the development of aeronautics which numbers among its members officers of both the Army and the Navy thus assisting in the coordination of the air activities of these services.

The scope of activity for aeronautical research to be carried out by the Committee in the future has been outlined to include the following investigations:

1. Propellers; 2. Aerofoils, or elementary supporting and controlling surfaces; 3. Combinations of supporting and controlling surfaces; 4. Reaction between the airplane and the propeller system; 5. Structure; 6. Investigations to improve the present instruments for use in airplanes and to develop new instruments to meet future needs; 7. In connection with aviation engines, the problem of supercharging to maintain power at different altitudes; 8. Improvement of engine details, radiation, carburetors, ignition, and fuels and lubricants;

9. Standardization and investigation of materials for aircraft construction; 10. Free flight tests; 11. The study of the relation of the atmosphere to aeronautics.

Some of the more important reports rendered during the last year are:

1. Behavior of Airplanes in Gusts. 2. General Theory of the Blade Screw. 3. The Testing of Balloon Fabrics. 4. Aluminum and its Light Alloys. 5. Carbureting Conditions Characteristic of Aircraft Engines. 6. General Analysis of Airplane Radiator Problems. 7. Effect of Compression Ratio, Pressure, Temperature, and Humidity on Power. 8. Properties and Preparation of Ceramic Insulators for Spark Plugs. 9. Heat Energy of Various Ignition Sparks. 10. A New Process for the Production of Aircraft Engine Fuels.

With the completion of the Committee's aeronautical laboratory at Langley Field, the Army Experimental Air Station, the work of aeronautical research will be expanded, it is hoped, to cover present and future problems vital to the Science and Art of Aeronautics.

The National Advisory Committee for Aeronautics gave valuable service to the Army and Navy both before and especially during the early months of the war. This body of selected men, reporting directly to the Congress, furnished to the Chief Signal Officer a medium for counsel and advice. This Committee inaugurated and recommended the establishment of the Aircraft Board. Prior to the establishment of the Aircraft Board, the Chief Signal Officer enjoyed the counsel of the Advisory Committee which was always ready to assemble without regard to time or place.

Although the statute creating this Committee contemplated a general supervision of aeronautical research, all manner of questions were submitted to its deliberation and judgment during the early period of the war. Whether it was the founding of a permanent aeronautical experimental station which should be a sort of West Point for aeronautics, the assembling of University Presidents to establish curricula for our ground schools, or the search for a practical solution to the stifling situation confronting the Government as a result of the airplane patent controversy, the National Advisory Committee was at all times available to the Army and Navy authorities for counsel and decisions.

X—Work of the Science and Research Division of the Signal Corps

THE Science and Research Division of the Signal Corps was established by order of the Chief Signal Officer on October 22, 1917, and Lieut. Colonel R. A. Millikan placed at its head.

Its personnel, exclusive of the Meteorological Section, consisted on November 11, 1918, of twenty-one commissioned officers, 118 enlisted men, and sixteen civilian employes of scientific and technical qualifications. Four of these officers are abroad and seventeen in the United States.

At the time of the signing of the armistice, the Science and Research Division had in progress sixty-four problems.

A summary of the most important results actually achieved up to that time is given herewith:

1. The Aeronautic Instruments Section, in addition to fulfilling the function specifically assigned to it of writing all the initial specifications for aeronautical instruments in use on planes, and in working continually with the instruments in production in order to detect imperfections and make suggestions for improvement, designed and developed wholly within its organization, through the activity of its chief, Major C. E. Mendenhall, a new and improved Pitot-Venturi tube for use in the determinations of airplane air speed. This instrument was actually put into production and thirty-seven thousand ordered.

2. Major Mendenhall and Lieut. R. C. Williamson have cooperated with the General Electric Company in the development of a new and improved compass, ten thousand of which were produced. This compass is now in use on planes for the American Army.

3. The Science & Research Division cooperated with the Eastman Kodak Company in the development of an entirely new film camera which is in production and which is the only film camera in existence capable of taking the standard size 18 by 24-cm. pictures. This camera is entirely automatic and capable of taking one hundred pictures without refilling.

4. The Science and Research Division with the assistance of the Burke & James Company of Chicago developed a new plate camera which is a modification of the French De Ram. It is

semi-automatic, carrying fifty plates. It has been accepted and is in production now. Both this photographic development and the one mentioned under (3) were made possible solely by the bringing together of a highly trained scientific group in the Photographic Section of the Science & Research Division.

There would have been practically no possibility of the attainment of these results without such a group. This photographic group constituted one of the Sections of the Science & Research Division until about July 1, 1918, when it was transferred to the Production Division. The development of photography during the present war, due to the advent of the airplane has placed it permanently among the indispensable agencies for successful military operations. The older methods of recording personal observations have been superseded entirely by the far more accurate and reliable photographic record. A complete detailed photographic map must now be made daily of each sector immediately in front of an army, and by carefully matching these maps and by intensive study of each individual area, the maze of trenches, entanglements, machine-gun nests and shell-holes can be accurately analyzed. It is worthy of notice that 17,000 photographs were taken by the British army before the operations at St. Quentin in order that a relief map of the sector might be made before the drive against the Germans. By means of this map, every detail of the work to be performed by the British troops was planned. By a series of consecutive photographs taken at regular intervals by a moving-picture camera an accurate and reliable picture of a military road, for instance, showing all details to scale may now be made in a few minutes. With the aid of the camera the detection of camouflage is possible where the human eye would fail. This highly specialized photographic work has now taken its permanent place in the waging of modern warfare.

5. Dr. Gordon S. Fulcher, working in collaboration with the Miller Rubber Company, developed a leak proof tank simultaneously with the development of a similar tank in England—achievements of the utmost importance for the lives of Allied aviators. The Fulcher Tank was ordered placed upon all fighting planes.

6. Major C. E. Mendenhall and Lieut. John T. Tate, with the assistance of the General Electric Company, perfected a trench signalling lamp which, after test at the front, was

ordered sent over in considerable numbers and will probably constitute a standard American projector for the United States Army.

7. Major R. W. Wood developed a telescopic signaling device using a six-volt, two-ampere lamp. This lamp was tested at the front and favorably passed upon by the A.E.F. It has made possible light signaling in broad daylight over a distance of eighteen miles. General Pershing ordered a considerable number of these sent to the A.E.F.

8. Major Wood also developed a secret daylight signaling lamp which has a range of five miles. This also was ready for shipment abroad at the time of the signing of the armistice.

9. Secret signaling at night, with the aid of ultra-violet light, was perfected by this department, working in collaboration with Mr. Norman Marshall. With simple signaling telescopes of the sort mentioned above, using only a six-volt, two-ampere lamp, secret signals have easily been sent six miles. Major Wood also developed new means of adapting this method to the problems of signaling to airplanes, and secret signaling between convoys, and obtained thereby a device which will be of much use in peace as well as in war.

10. W. J. Lester, S. R. Williams, Capt B. J. Sherry and Sergt. W. H. Redman of this Department developed propaganda balloons which have a range of more than a thousand miles. This is an accomplishment which is invaluable to the future development of aviation, particularly with reference to transatlantic flights, whether in peace or in war.

11. Dr. W. F. G. Swann and Dr. Gordon S. Fulcher of the Science and Research Division gave invaluable service to the Balloon Section by making an elaborate and highly important study of the causes of fires in balloons and sending to the balloon officers in the Balloon Section directions for the prevention of such fires.

12. Dr. H. N. Russell of the Science and Research Division and Capt. J. P. Ault developed means for navigating airplanes with the aid of the sextant and an artificial horizon. They also developed means of speedy reduction of observations so that an observer in a plane can locate himself with an average error of not more than ten nautical miles within five minutes after he makes his observation. This achievement is of great value for the problem of long flights.

13. Lieut. John T. Tate and Otto Mohr developed a port-

able landing light for use on aviation fields. This device was officially tested and favorably reported upon by the division of Military Aeronautics. It is now in use at some fields and recommendation has been made by the Division of Military Aeronautics that two complete units be made a part of the permanent equipment of all aviation fields.

14. Dr. Harvey N. Davis of this Division represented the army in the development of the helium program and contributed in a large measure to the great success which has been attained in that work.

15. The Chemical Section under Dr. H. D. Gibbs developed new methods of producing acetone, which will make it possible to obtain this substance for one-fourth the price which the Government is now paying.

16. The Chemical Section, through the activity and energy of Dr. L. E. Wise, developed sensitizing dyes which previously had not been obtainable at all in the United States and which were urgently needed in airplane photography.

17. Capt. Herbert E. Ives and Dr. F. A. Saunders of this Department, in collaboration with I. W. Priest of the Bureau of Standards, made notable contributions in the development of color filters for detecting camouflage and increasing visibility. Forty thousand of these devices now are in use in the Army and Navy.

18. Dr. Wilmer A. Duff's Section worked out the only method which has been developed thus far for the accurate, experimental determination of bomb-trajectories. This is of primary importance in obtaining precision in bombing. It will have peace uses as well as war uses.

19. Dr. Duff's section also developed a bombsight stabilizer which has reduced the main error now made in bombing, namely, the error in the determination of the vertical, by more than three-fold. When it is remembered that a three-fold increase in the accuracy of bombing is precisely equivalent to the multiplication by three of the production of bombing planes, the importance of work of this kind scarcely needs comment.

There are probably a dozen of the nineteen developments mentioned above, each one of which is worth more to this country than the total amount spent upon the establishment and maintenance of the Science and Research Division from its inception up to the present time.

XI—Radio Development Work

IN the question of the engineering achievements of the Signal Corps during the war, the development of radio apparatus forms a large part. Inasmuch as the vacuum tube occupies so prominent a role in almost every kind of radio apparatus, an outline of its development logically precedes discussion of the radio sets.

VACUUM TUBES

The application to radio inter-communication of the vacuum tube—perhaps more properly called the thermionic tube or bulb—is one of the most interesting developments in the whole field of applied science. For not only has it made possible what has been justly heralded as one of the most spectacular achievements of the whole war—the airplane radiophone—but the confidence growing out of the extensive experience with the vacuum tube in warfare, coupled with its extreme adaptability, have resulted in a rapidly increasing amount of radio development involving its use.

PRE-WAR HISTORY

The vacuum tube was known in various forms before the war. Following extensive experiments with the so-called "Edison effect", Fleming, some years ago, produced the well known Fleming valve—a current rectifying device, capable therefore of being used as a detector of radio signals. This device contains two elements: an incandescent filament emitting electrons, and a plate upon which an alternating voltage is impressed, both placed within an evacuated bulb. Later Dr. Lee DeForest introduced an important modification by placing a wire mesh or "grid" between the filament and the plate. A small voltage variation on this grid produces the same current change through the tube as would a much larger voltage variation on the plate, thus adding amplifying properties to the detector characteristics of the Fleming valve. DeForest called his device the "audion". Later, with superior facilities for evacuation available and with a more intimate knowledge of the laws of thermionic emission from hot bodies, improvements and modifications were made in the audion or vacuum tube by both the General Electric Company and the Western Electric

Company, the latter designating their product as "vacuum tube", and the former the "pliotron".

In addition to acting as detectors and amplifiers, as mentioned above, vacuum tubes can function in two other important ways:

1. As Oscillators. In properly designed circuits containing inductance and capacity they will act as radio frequency generators, for use in transmitting or receiving radio signals.
2. As Modulators. By suitable connection to an oscillator circuit or antenna, they can be made to vary the power radiated so that the envelopes of the waves transmitted shall have any desired wave form, as for example, the speech waves from an ordinary telephone transmitter.

The most striking use made of vacuum tubes prior to the time we entered the war was the transmission of speech by radio from Washington to Paris and Honolulu, during the experiments carried out by the American Telephone and Telegraph Company and the Navy Department. Vacuum tubes were used as the radio frequency generator for transmitting, and for detector and amplifier in receiving.

When the United States entered the war, vacuum tubes already were in use by the Allied forces for various signaling purposes. The French particularly had been quick to recognize the military value of vacuum tubes and had, previous to June, 1917, developed very creditable tubes and apparatus. In America, tubes were in limited use as "repeaters" on telephone lines, and as detectors and amplifiers in laboratories and radio stations. The total production, however, in this country did not exceed three or four hundred a week.

DEVELOPMENTS DURING THE WAR

Early in our participation in the war, it became evident that vacuum tubes would be required in very large quantities in order to meet the growing demands for radio communication and signaling. It was equally evident that service conditions, not hitherto anticipated, would require great mechanical strength, freedom from disturbance under extreme vibration, and uniformity of product sufficient to make possible absolute interchangeability of the tubes in sets, without the necessity of readjusting when changing tubes. To these conditions must be added that of minimum size consistent with dependable operation.

To make such a device, with its complicated, yet accurately constructed metallic system, within a practically perfect vaccum, is no small problem even when made in the laboratory, on the individual unit basis, by a skilled operator who appreciates the delicacy of the job. To turn out tubes by the thousands by factory methods involves almost infinitely greater difficulties. How well certain companies, in collaboration with the Signal Corps, have succeeded in solving these difficulties is indicated by the fact that recently the total rate of production in the United States of high quality standardized tubes was considerably in excess of one million a year. This rate of production could be made many times greater on short notice.

As an example of the difficulties which this quantity production has involved may be mentioned that of evacuation. The degree of vacuum required is such that unusual methods of exhaust are necessary. The heating of the tubes in electric ovens is supplemented by heating of the elements of the tube by excessive filament and plate electrical power input. Molecular pumps are employed, necessitating an extremely large number of pumps to handle quantity production. Special treatment of metal parts prior to assembly is employed to reduce the gas given off by them during the exhaust process.

Another problem is that of making the complicated metallic structure of all tubes exactly alike, in order to insure identical electrical properties. As an indication of progress in this direction, it may be stated that one company is prepared to manufacture, in quantity, a certain tube in which the clearance between filament and grid is only three hundredths of an inch, the allowable variation being of course only a small percentage of this.

Manufacturing in quantity involves careful inspection. The problem of specifying definitely the required performance of tubes, the development of adequate testing specifications, the placing of standardized testing and inspection methods, personnel, and equipment in the various factories so that tubes manufactured at different times and places would, after passing inspection, be uniform and interchangeable—these questions were entirely new and have been solved almost entirely by the Signal Corps Engineers.

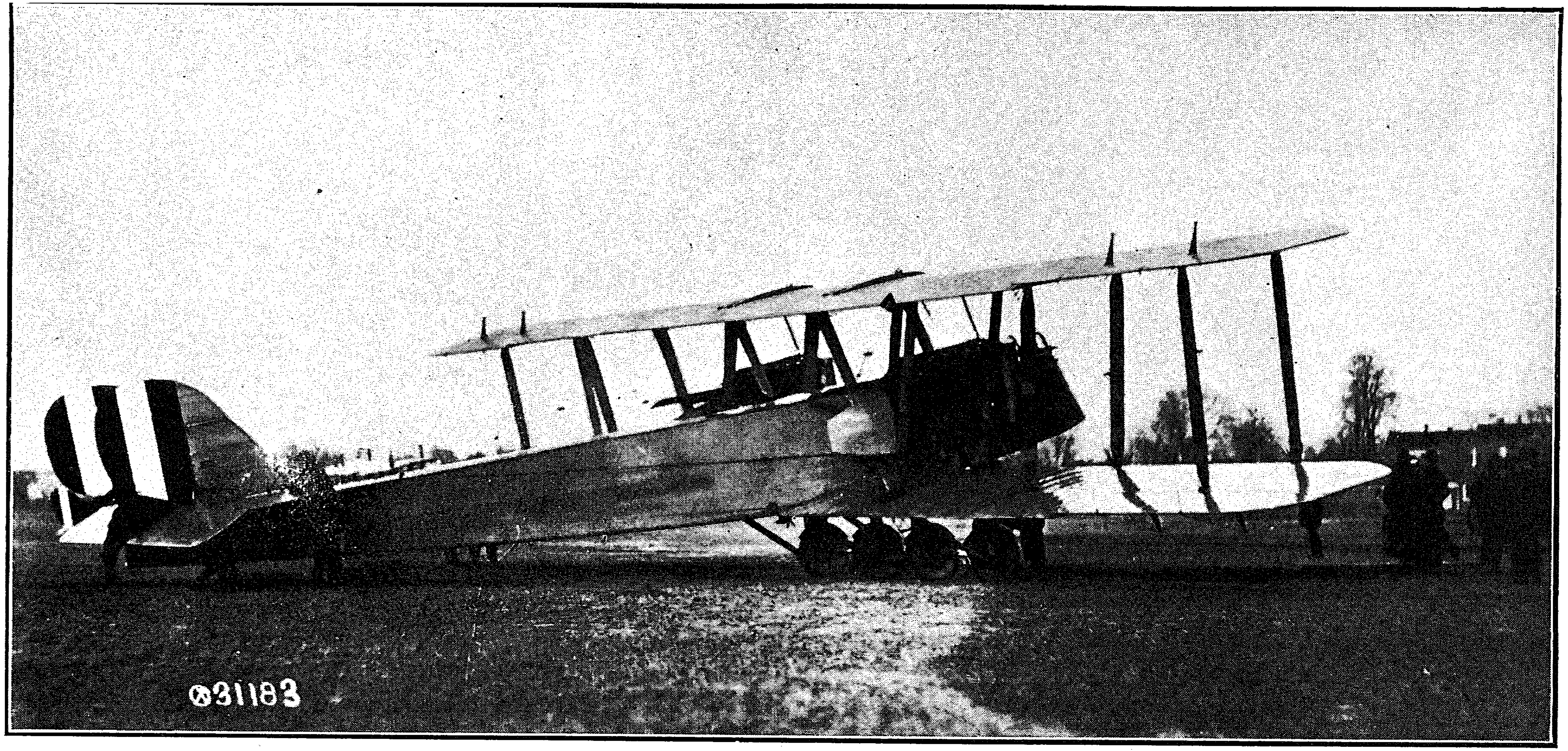
PRESENT STATE OF THE ART

Tubes developed by the Signal Corps may be divided into two general classes: the tungsten filament types as developed and manufactured by the General Electric Company and the De Forest Radio Telephone and Telegraph Company, and the coated filament or Wehnelt Cathode types as developed and manufactured by the Western Electric Company. The coated filament tubes so far have proven superior to the tungsten filament tubes for Signal Corps use. Both classes have been standardized as regards base, exterior dimensions, filament current and voltage, and in addition, plate voltage and output for transmitting tubes; and amplifying power and detecting power for receiving tubes. Except in certain special cases, the Signal Corps uses two types of tubes, one for transmitting, and another for receiving. The French and the British have been using one type for both transmitting and receiving, but present tendencies of the British are toward different tubes for different duties.

Vacuum tubes are now employed for electric wave detection, radio frequency and audio frequency amplification, radio telephony, particularly in the airplane radiophone, continuous wave radio telegraphy, voltage and current regulators on generators, and for other miscellaneous purposes. However, varied as are the applications at present, the uses, actual and potential, growing out of war development work have proved that the art of Vacuum Tube Engineering, and the application of its products to radio engineering, telephone and telegraph engineering, and particularly to electrical engineering in general, are still in their early infancy. That vacuum tubes in various forms and sizes will, within a few years, become widely used in every field of electrical development and application is not to be denied.

The engineering advancement accomplished in less than two years represents at least a decade under the normal conditions of peace, and our profession will, it is hoped, profit by this particular salvage of war which offers perhaps the most striking example extant of a minimum "time-lag" between the advanced "firing line" of so-called pure physics, and applied engineering.

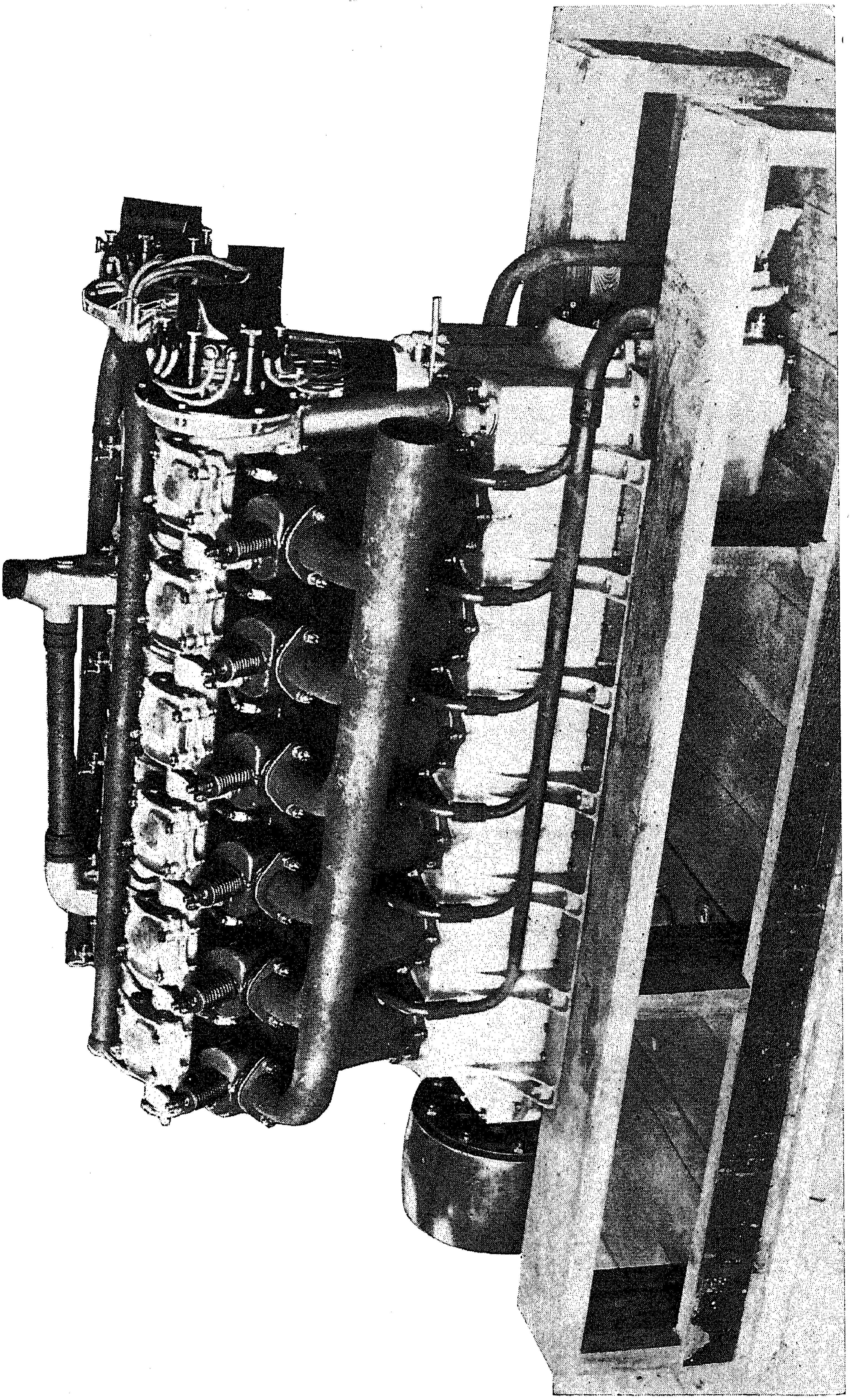
The Chief Signal Officer considers that the work of standardization and quantity production of vacuum tubes accomplished during the last eighteen months under the pressure of military necessity, represents an advance in the art of electrical engineering which will prove of inestimable industrial and scientific value to this country, and to the engineering world at large.



Martin Bombing Plane—Two Liberty Engines

[Squier]

PLATE VI.
A. I. E. E.
VOL. XXXVIII, 1919



[SQUIER]

LIBERTY ENGINE—12 CYLINDERS

AIRPLANE RADIO TELEPHONE SET

Prior to April, 1917, a few experiments had been made, in which speech had been transmitted from airplane to ground by radio methods; but the apparatus involved was hopelessly crude. On May 22, 1917, the Chief Signal Officer called a conference in his office at which the project of evolving a "voice command" equipment for airplanes, which should meet all the severe requirements of the military service, and which should be thoroughly standardized for quantity production, was definitely set in motion against time. The plan for accomplishing this was the same as in the evolution of the Liberty engine, and from the beginning this project was regarded as one of the major creative efforts in the development of the American Air Service. The present airplane radiophone, therefore, is the result of a period of intensive development work begun shortly after we entered the war. The services of the Western Electric engineers were enlisted, and under the direction of the Signal Corps, rapid development resulted in successful tests as early as August, 1917. Speech was exchanged between airplanes twenty-five miles apart in October, and sample sets were sent at once to the Army in France for trial. Several thousands sets were ordered and have been completed and distributed to flying fields here and to the Air Service in France.

The satisfactory performance of this apparatus has resulted in a new type of military unit known as a voice-commanded squadron. The commander of an air fleet directs the movements of the individual units in any manner desired; the effectiveness of the squadron as a military machine is thereby enormously increased.

Other uses are in communicating information from airplanes to ground stations, and in directing one or more airplanes from a ground station. Innumerable applications will be evolved as the possibilities are realized.

The essential elements of the Airplane Radiophone are the power equipment, the radio equipment, and the antenna.

a. The power equipment includes a double-voltage direct-current generator driven by an air fan, with a vacuum-tube voltage regulator.

b. The radio equipment consists of the vacuum tube transmitting and receiving set, and the special telephone transmitters and receivers. Those of you who have heard the terrific roar of a Liberty engine will realize the difficulty of talking in an airplane in flight. The development of a transmitter which is affected by the human voice, and not by the enormously greater engine and wind noises, is one of the principal features of this set. Similarly, to shield the ears of the aviator from the same noises required a special combination of sound-insulating materials surrounding the telephone receivers and suitable for use within an aviator's helmet.

c. The antenna originally consisted of a flexible copper wire several hundred feet long, unreeled by the aviator and trailing almost horizontally behind the airplane. Modified antenna using much shorter wires fixed to the framework are now used.

The operation of the sets is extremely simple, all adjustments being made before leaving the ground. The only manipulation required of the aviator is that of the change-over switch to change from talking to listening.

Airplane Radio Telegraph Set

The principal use of radio communication made during the war was in sending radio telegraph signals from observation airplaines, for controlling artillery fire. The French developed a set which consisted of several units, making the installation and operation complicated. The Signal Corps developed a self-contained set, which has been demonstrated to be far superior to any other airplane set.

It consists of three units—first, the 200-watt, 900-cycle alternator, driven by a regulating air fan, and containing in a stream line case attached to the generator, all the elements of the radio set. This radio apparatus is of the synchronous spark type, with four spark tones and nine wave lengths. The weight of the complete unit is only twenty three pounds, and size only six inches by six inches by twenty-six inches. The regulating air fan maintains the speed of the generator within four per cent of 4500 rev. per min., with air velocities between 60 and 200 miles an hour.

The remaining units in the complete set are a variometer or tuning coil, with antenna ammeter attached, and the antenna system, comprising a reel, insulated bushing, and training antenna. When it is realized that voltages of thirty thousand or more are produced by this set, the difficulties of insulation in such restricted spaces will be appreciated.

Ranges of communication of one hundred miles have been accomplished with this set.

Airplane Direction Finder

One of the principal problems of airplane navigation has been the evolution of a suitable compass, particularly for night bombing work. Magnetic and gyroscopic compasses have limitations at present which make impossible reliable air navigation by dead reckoning.

The use of directional effects of loops or coils for receiving radio signals have resulted in the development of a radio compass for airplanes which gives positive information to the aerial navigator, and enables him either to locate his position by triangulation with respect to two beacon land stations, or to fly at any given angle with respect to a certain beacon station.

The apparatus consists of two principal parts—the antenna coils, and the tuning and amplifying apparatus. The antenna coils are mounted in the fuselage of the Handley-Page airplane, with suitable means for rotating in azimuth. The amplifier is extremely sensitive, consisting of a detector and six-stage amplifier. A novel feature of the amplifier is the use of iron-core transformers for frequencies of 100,000 cycles.

The direction of the beacon land radio station is determined by maximum strength of signals, in a highly ingenious manner developed originally by the British. The precission of the directional effect is remarkable. In fact the radio direction finder may well be called a radio eye, by which the aerial navigator sees one or more radio lighthouses which are sending identifying signals to guide him on this way. These lighthouses, furthermore, have certain advantages over the normal light-house in that their ranges may be much greater, and they are not invisible in the day time nor obscured by fog and mist.

The remarkable advances made during the last eighteen months have resulted in the application of radio communication to practically every phase of military aviation. Commercial and military possibilities have, however, hardly been touched as yet. It is believed that radio apparatus soon will be as essential on aircraft as it now is on ocean going steamships, and that its use will enormously increase the effectiveness of aircraft for all purposes.

XII—The Liberty Aircraft Engine

FOREWORD

AS we look back on the record of accomplishment in the problem of obtaining large numbers of high powered aviation engines for our Army and Navy Air Services both in this country and abroad, it seems to those of us who were in close contact with the work and the difficulties, more like a fairy tale than the statement of hard facts which it is in reality. On the face of things, it certainly would seem to be the height of presumption to assume that this country could, following its almost total neglect of aviation development in previous years, hope to design, develop and produce in unprecedented quantities an acceptable aircraft engine of greater power than had yet been evolved by any of the European nations, even under their spurs of governmental encouragement and tremendous war demands. Yet just that and nothing else was the only thing to do, and the story of its doing is one of the most brilliant chapters in the history of our Country's part in the Great War.

INITIAL SITUATION

At the time this country entered the war, in April 1917, there were being built in the United States only four makes of engines that were developed so far as to be considered of any military value, and even these were useful only for primary training. We had no engines at all suitable for services on the battle front, or even for the advanced training of pilots. The largest engine of domestic manufacture developed about 220 horse power and had not proved satisfactory when judged from the standpoint of combat service requirements. The others ranged in power from 90 h. p. to 135 h. p. It, therefore, was evident that the existing American engines could be used for preliminary instruction purposes only, and that their further manufacture should be limited to the training requirements. This was done, with the result that by far the greater portion of the primary training of pilots has been conducted with the Curtis OX-5 90-h. p. engine, the quantity production of which was early obtained. This engine was particularly valuable, owing to the very satisfactory training plane which had been designed around it. The Hall-Scott A7-A 100-h. p. engine was also extensively used at first until the production of the Curtiss engine could be brought to a point to meet all demands for primary training.

Two European engines, the Gnome 100-h. p. and the Hispano-Suiza 150-h. p., were being put into production in this country early in 1917, by the General Vehicle Company and the Wright-Martin Aircraft Corporation respectively. These concerns had obtained contracts with Great Britain and France, but were experiencing considerable difficulty in getting production under way. The production of the first Gnome engine was not completed until a period of more than nine months had elapsed, during all of which time English experts were present in the contractor's plant and aiding the manufacture. In the case of the Hispano-Suiza, notwithstanding the assistance of a group of French experts sent over to help get it into production in the minimum time, thirteen months were required to get the first production engine on the test block, and another month before the first delivery was made. Preparation periods were filled with the most costly experiments and the development of methods and tools for performing the intricate operations required by the design of these engines. All of which expense and delay had cost our Allies dearly.

The Gnome and Hispano-Suiza engines represented the highest product of European design and were in a perfected and standardized state, according to foreign practise and conditions, when their production was undertaken in this country. Nevertheless, the changes involved in adapting them to manufacture by American methods, and the development of expert workers for those operations which could not readily be so adapted, required so much time that the advances made in aeronautical engineering rendered such engines largely obsolete for service at the front, by the time they could be produced in sufficient numbers to supply any material portion of the requirements.

These two engines were, however, of unquestioned value for advanced training purposes, the Hispano-Suiza in particular being a dominant factor in this work. Later the Le Rhone 80-h. p. was put into production by the Union Switch & Signal Company and, by proper utilization of the lessons learned in the case of the other two foreign engines, reasonably satisfactory progress was made in manufacture. This engine, too, was used for advanced training work.

LIBERTY ENGINE INCEPTION AND EXPERIMENTAL DEVELOPMENT

One of the serious mistakes which the Allies had fallen into at the time the United States entered the war was the development of a multiplicity of types of engines and planes which made it impossible to have a large number of any one of these types. As a further consequence, the trained personnel on the ground to operate and repair the machines had grown to such a proportion that it was estimated that from thirty to fifty men were required on the ground to keep each one of the many types of planes in the air on the fighting line. Manifestly, unless this large number of trained men per fighting plane could be reduced by some means it would be hopeless to expect within a reasonable time to put into the air thousands of flying planes, because a single thousand planes on this basis would require from 30,000 to 50,000 men in attendance.

The experience obtained in getting two foreign engines into production in this country and under our manufacturing methods, so thoroughly demonstrated the futility of attempting any such solution of our service engine problem as to at once eliminate its continued consideration. It was realized that copies of foreign designs could not be available in time and in adequate size or numbers to answer the demand for an overwhelming air force at the front. American air performance would have been very small indeed if limited by such a handicap as this.

Moreover, in spite of the fact that a technical commission was at once organized and despatched for the purpose of getting first hand information on the front and in the aircraft centers of Europe, it was unthinkable that this country should sit idly by and wait perhaps months for the final definite report which should decide the nature and extent of our part in the Allied aircraft program. Since the most successful airplanes are designed around specific engines, and the engine involves the greatest expenditure of time and effort in development, it was apparent to a few of us who were in close contact with the situation that it would in all likelihood be possible to design, develop and produce an entirely new American engine, embodying characteristics which would render it particularly adapted to manufacture under American conditions, in less time than would be required by the commission to determine the particular European engines that offered the best approximation to the various exacting requirements of service and production, plus the time to get it in production in this country. It was decided, therefore, that our efforts should be directed along both these channels simultaneously and in addition to purchase in Europe whatever service equipment might be available, to tide over the interval while we were getting into production.

It is apparent that the fundamental unit of engine design or construction is the cylinder, and that the evolution of engine power rested mainly with the unit power capacity of that cylinder which could be taken as representing the largest practicable size sanctioned by the state of the art at that time. Starting with the foundation of this cylinder then, it was the most direct reasoning to conclude that all requirements of the service for engines of varying power capacities could be most logically met by combining these unit cylinders into groups of whatever number were required to produce the several sizes of engines desired. This was done, and the cylinder size of five inches bore by seven inches stroke was selected, after a careful examination of the performance of both American and European engines of the then most modern design, as being the largest that could be relied upon to give satisfactory service. While originally designed to produce approximately 28 h. p., this cylinder has been so developed as to yield at present more than 40 h. p. as the result of somewhat increasing the speed and altering the functioning characteristics. The standard unit cylinders were to be used in engines having four or more cylinders each and yielding the following power output:

Number of Cylinders.	Original Rated h.p.
4	110
6	165
8	225
12	335

Thus, for the first time in the history of the aeronautic engine for military uses, a truly comprehensive design plan was evolved which in a simple and direct manner provided for the production of a whole line of engines of wide power capacities, but composed of units that were highly standardized and therefore could be really manufactured, instead of being merely built. This was, and is, the only way in which this country could meet the requirements of this most vital part of the program.

It must not be thought, however, that such a revolutionary decision was easy to make, or adhere to when made, in the face of all kinds of adverse criticism, some of which seemed to be based upon adequate grounds. The development of new types of aircraft engines heretofore had been such a time-consuming matter that it was generally regarded as a thing to be avoided if any ready-developed engine could be found anywhere which would at all answer the requirements. Thus the numerical preponderance of opinion was against the possibility of a standardized American engine being designed, developed, and produced in large numbers, in anything like as short a time as would be required to duplicate some European engine. The nation may well render thanks that its destinies in this particular were guided by such a far-seeing and courageous group of men, who had the ability to formulate such plans and then to enforce their realization.

You are all familiar to some extent with the history of the design and construction of this engine:—how Lieutenant Colonels J. G. Vincent, of the Packard Co., and E. J. Hall, of the Hall-Scott Motor Car Co., laid down the general features and got out the first assembly drawings personally between mid-day of May 29 and the afternoon of May 31, 1917, working in Col. E. A. Deed's apartment in a Washington hotel in response to a request for a report on the aircraft engine situation, which came from Howard E. Coffin, Chairman of the Aircraft Production Board; the Chief Signal Officer of the Army; Colonel Edward A. Deeds, Chief of the Equipment Division, Signal Corps, and Colonel Sidney D. Waldon, Assistant Chief:—how the order to build ten sample eights and twelves was given as the result of the approval by the joint conference of the Army and Navy Technical Board and the Aircraft Production Board:—how the first engine, an eight-cylinder, was built in one month as a result of the enthusiastic cooperation of some ten manufacturers, each of whom produced those parts for which they were best fitted:—how the first sample twelve-cylinder finished its official fifty-hour endurance test eighty-two days from the time the order for samples was given, and that the total elapsed time during this test was only about fifty five hours, a record-breaking performance:—and how the success of this endurance test definitely removed the engine from the experimental stage to the realm of proved engines.

A very gratifying endorsement of this standardized engine project came from the late Colonel R. C. Bolling, whose untimely death in France cost the American Air Service one of its most valuable officers, and Col. V. E. Clark, and Lieut. Col. Howard C. Marmon, members of the commission sent abroad to ascertain the requirements, and which returned about this time, to the effect that a 400 h. p. engine was absolutely demanded at the front for the types of airplanes which it had been decided this country should supply, and that no engine of this size then existed in Europe.

Those of us who are familiar with the difficulties and disappointments involved in the design, development and perfection of any form of intricate mechanical device can readily appreciate the really remarkable accomplishment represented in the Liberty. Therefore, it is not at all surprising that the representatives of the Allies were for some time unable to believe the full truth of this accomplishment. They never had been able to obtain such action and were, naturally, only fully convinced after many varied and exhaustive tests. So well recognized did the value of the Liberty engine become, however, that the Allies had on order at the time of signing the armistice 16,741 Liberty engines, and were constantly endeavoring each to increase their rate of monthly delivery. Airplanes were being designed around this engine in all Allied countries and it was fast becoming the predominating aeronautical engine of the Allied cause.

It is of interest in this connection to note that this standardized engine already has been tested in the twenty-four cylinder model, and shown results which prove that the original basic idea will provide for engines of any size which would have been required for any probable increase in airplane size during years of continuation of the war. The sixteen cylinder was also proved by the success of the larger engine.

The experimental development of the Liberty has been in charge of a department entirely separate from that dealing with its production, the Airplane Engineering Department under Lieut. Col. J. C. Vincent and Lieut. Col. Howard C. Marmon. The work of this department has resulted in a continuous improvement of the power output and performance characteristics of the Liberty twelve cylinder, to such an extent that 526 h. p. have been obtained with special fuel and detail changes; certainly a remarkable increase from the

335 h. p. which the original design was intended to yield. While the weight of the service engine per horse power has remained at approximately two pounds, the maximum present development had reduced this figure to one and two thirds pounds.

LIBERTY PRODUCTION—DEVELOPMENT AND PRODUCTION

The record of production and production-development of the Liberty conclusively proves the wisdom of the decision to concentrate all efforts on this one engine for the major part of our program of combat engines. In common with all similar machines, many possible improvements and cost reductions become evident as the manufacturing processes and tools are being evolved and as experience is being acquired in the actual production. Also, experience in the operating and adjustment of these engines led to alterations being made which resulted in increasing the power output; so that when we were advised that more power was desired than the 335 h. p. which the original Liberty was designed to produce, the necessary steps already were known and the delay incidental to putting them into effect was small. The resulting power increase to about 375 h. p. answered requirements for several months when advices were again received that more power was needed, and we then again made such alterations as were required to increase the power to around 420 h. p. The weight of some of the parts was increased at this time in order that the reliability might not be reduced.

The problem of production was placed in the hands of a separate department in charge of Lieut. H. H. Emmons, U.S.N., and the continuous assistance of such men as Henry M. Leland, of the Lincoln Motors Co.; C. Harold Wills, of the Ford Motor Co.; F. F. Beall, O. E. Hunt and Edward Roberts, all of the Packard Motor Co., were obtained by Major J. G. Heaslet, in charge of the Detroit district. The work of Mr. Wills which has the greatest value was the perfection of a new process for the forging of cylinder blanks in sufficient quantities to supply all manufacturers, and which was so very effective that the problem of cylinder forging supply and cost was reduced to a minor consideration. The wisdom of the action taken is evidenced by the production of 1100 Liberty 12s in one year from the day when Messrs. Vincent and Hall first met and started a preliminary drawing, and over 15,000 by the end of another six months.

It is perhaps to be expected that many criticisms would be leveled at an engine evolved under the conditions obtaining at that time, and registering the large success which it represents, but all such have proved to be the result of misinformation regarding the conditions to be met, the fundamental ideas of design, or of lack of appreciation of the difficulties encountered in creating at a single stroke and without previous experience, an aeronautical power plant so much larger than was then in existence.

The Liberty engine stands today as an achievement which for daring, constructive imagination and farsightedness will ever be a cause of pride to the American people.

XIII—A Granular Theory of Armies

IN these days we are face to face with the so-called granular theory of electricity and matter, whose fundamental unit is the electron. From one angle of view, as I shall hope to show, there is also a granular theory of armies.

The history of the military operations of the past disclose the fact that the actual combatant troops are specially selected men most fit physically for the hardships of war. In general terms the actual fighting has been restricted to selected persons from the male population with definite maximum and minimum limits for age and also definite requirements for the vital physical functions.

In the language of the physicist we may say that the male population of a nation is carefully segregated by a sort of "band filter" which picks out for combatant warfare only those whose age cycles fall between say eighteen and thirty years. These selected men are then brought to a further common standard by intensive physical training before they are permitted actual combatant service. This elevation of the military unit to a common physical standard makes possible, as will be readily seen, the formation of larger units such as companies, battalions, brigades, divisions, corps and armies and their direction by a centralized command in the tactics of war.

Invoking the law of averages, we may say furthermore that,—*the one unchanging factor in warfare is the individual physical strength of a man.*

The soldiers of Ceasar's army were physically no stronger nor weaker than our own. On the average, a battalion or company of soldiers from one nation can march, for instance, just as far in a day as a battalion or company from any other nation, and this statement was equally true before the Christian era, and is not likely to change in the future.

In consequence of this fact, military supremacy must be looked for primarily in the weapons and agencies provided by scientists and engineers and placed in the hands of these combatant units to multiply their military strength.

To illustrate, the day in the fourteenth century, when Berthold Schwartz or whoever actually invented gunpowder, put together charcoal, sulphur and saltpeter, he, by a scientific act, and at a single stroke, exerted more influence upon the development of warfare and indeed the history of the world since that time, than many armies could accomplish by any mere physical qualities which they possessed.

Stated in another way, if our enemies in the recent great conflict had been made up wholly of civilizations like those of Turkey and Bulgaria, nearly all of the agencies which I have been describing during the past hour would have been utterly impossible of either development or production by them. There were no Liberty engines nor airplane radio sets evolved or made in this quarter, nor could there be such without outside aid.

It follows from this, that those civilizations which by their scientific and engineering training can create and manufacture these agencies, will and must control the fortunes of war in the future, or far better, apply these same agencies, in a potential way, to banish war from the world for all time.

The present war has made aviation in four years what it would have taken two decades at least to accomplish in times of peace, and has multiplied the reach and capabilities of the physical military unit to an extent which is only now in the beginning. May we not fervently hope that the further development of aerial navigation will place in our hands such potential destructive powers as to go a long way toward making war impossible again?

I can say no more at the present time.

* * * * * *

IN conclusion, the courteous invitation of the Board of Directors of this Institute to address you on this occasion, suggested that I may have some message to deliver to the American Engineer.

The organization of the American Air Service requiring, *inter alia*, the foundation of a new industry under the adverse conditions above outlined, involved a creative project worthy of the broad conceptions for which the United States is noted.

When the United States entered the war, it was evident that the time was fast approaching when the reservoirs of raw material for the allies were to be found only in the United States and that America's effort should be so organized from the beginning as to furnish a continuous flow of this necessary raw material, not only for our own air program but for those of our Allies as well. It was clear, for instance, that the spruce, the fabric, the dope and the oil must be produced as a part of our program on a scale to supply also the air programs of our Allies. It was always the general Allied cause therefore which controlled the decisions in founding this industry, rather than the needs of the United States alone, and this is obviously the only correct point of view.

It is believed that the major decisions which had to be made by those in authority were accurately made, and promptly executed, and that as a consequence the record of the United States Air Service during the war shows a creditable performance. If America will but press on into the future, building upon the sound foundations now erected, she may lead the world in the development and utilization of aerial navigation for the triumphs of peace.

By a wise policy of readjustment, utilizing immediately our machines and our surplus aviators for the rapid expansion of the aerial mail and special passenger services, it will be possible to salvage for the nation a greater percentage of the money and energy invested for strictly war purposes than from any other feature of our war activities.

As a steward of the people, and in all humility therefore, I must not shrink from answering, as best I may, your request for a message which applies equally to peace and war.

The general principles which governed in the making of this enterprise in all of its ramifications of material and personnel, and to which the success attained must be attributed, may perhaps be formulated as follows:

VICTORY CREED

To foster individual talent, imagination and initiative, to couple with this a high degree of cooperation, and to subject these to a not too minute direction; the whole vitalized by a supreme purpose which serves as the magic key to unlock the upper strata of the energies of men.

Presented at the Seventh Midwinter Convention of the American Institute of Electrical Engineers, at a Joint Session with the American Institute of Mining Engineers, New York, February 19, 1919.

Copyright 1919. By A. I. E. E.

WELDING MILD STEEL

BY H. M. HOBART

Abstract of Paper

The probability that welding could, with advantage, be more widely used in ship construction than had heretofore been the case, led to the formation, in 1917, of a Welding Committee, which at first was conducted under the auspices of the Standards Committee of the Institute. The Welding Research Sub-Committee, formed in 1918, was a sub-committee of the Metallurgical and Electrical Sections of the Engineering Division of the National Research Council, and the Welding Committee, (under the chairmanship of Professor C. A. Adams) came under the direction of the Emergency Fleet Corporation. The paper is in large part based on the work done by the Welding Research Sub-Committee up to January of this year.

THIS paper deals principally with investigations undertaken by the Welding Research Sub-committee of the Welding Committee of the Emergency Fleet Corporation. The general object of the investigations has been to extend the use of welding in the construction of merchant ships and, specifically, to provide a definite basis for obtaining the best economy and efficiency in employing welding in place of riveting in the construction of the hulls of such ships.

Composition of Ship-Plate Steel. The chemical composition of the steel employed in such hull construction varies with the thickness of the plates. Through the courtesy of Mr. H. Jasper Cox, of Lloyd's Register of Shipping, the following information may be given concerning the kind of steel plate employed in American Shipyards in 1918 for the hull construction of merchant ships.

Lloyd's requirements do not relate to the chemical composition. They require a tensile strength of 58,000 lb. per square inch (40.75 kg. per sq. mm.) for their lower limit and 72,000 lb. per square inch (50.59 kg. per sq. mm.) for their upper limit. For the information of the Committee, Lloyd's obtained from their surveyors at various works data of the carbon content, which are as follows:

Works		Carbon Content for Plates ¼ in. thick	Carbon Content for Plates 1 in. thick
A		0.14	0.23
B		0.14	0.25
C		0.19	0.25
D		0.20	0.30
E	Upper Limit	0.30	0.35
	Lower Limit	0.24	0.29
F	Upper Limit	0.25	0.30
	Lower Limit	0.21	0.27
G	Upper Limit	0.25	0.35
	Lower Limit	0.22	0.28

For shapes, Works H employ:

Shapes about ½ in. thick, 0.24 per cent to 0.30 per cent carbon.
Shapes about 1 in. thick, 0.28 per cent to 0.35 per cent carbon.

Small shapes such as:

2½×2¼× ¼-in. angles, about 0.15 per cent carbon.
4×4× ⅞-in. angles, about 0.20 per cent carbon.

From several tons of half-inch thick (12.7 mm.) plate from the yard of the Chester Shipbuilding Company, which was employed in making many sample welds in an investigation designated the Wirt-Jones Tests, seven analyses were made at the Bureau of Standards. The maximum and minimum percentages of each of the impurities for these seven samples were as follows:

	Maximum per cent.	Minimum per cent.
Carbon	0.25	0.24
Manganese	0.46	0.45
Phosphorus	0.043	0.039
Sulphur	0.031	0.027
Silicon	0.052	0.024

For this material the Bureau of Standards reports:

Yield point, 37,900 lb. per sq. in.
Ultimate tensile strength, 63,600 lb. per sq. in.
Elongation in 2 in., 38.6 per cent.

The following manufacturer's data apply to about ten tons of half-inch ship plate supplied by the Worth Steel Company, of Claymont, Del., and to be used for testing electrodes:

Chemical Analysis (Ladle Analysis)

Carbon	0.29 per cent.
Manganese	0.37 per cent.
Phosphorus	0.015 per cent.
Sulphur	0.032 per cent.

Physical Properties

Tensile strength—lb.per sq. inch	67,400
Elongation, per cent in 8 inches	25.25 per cent.

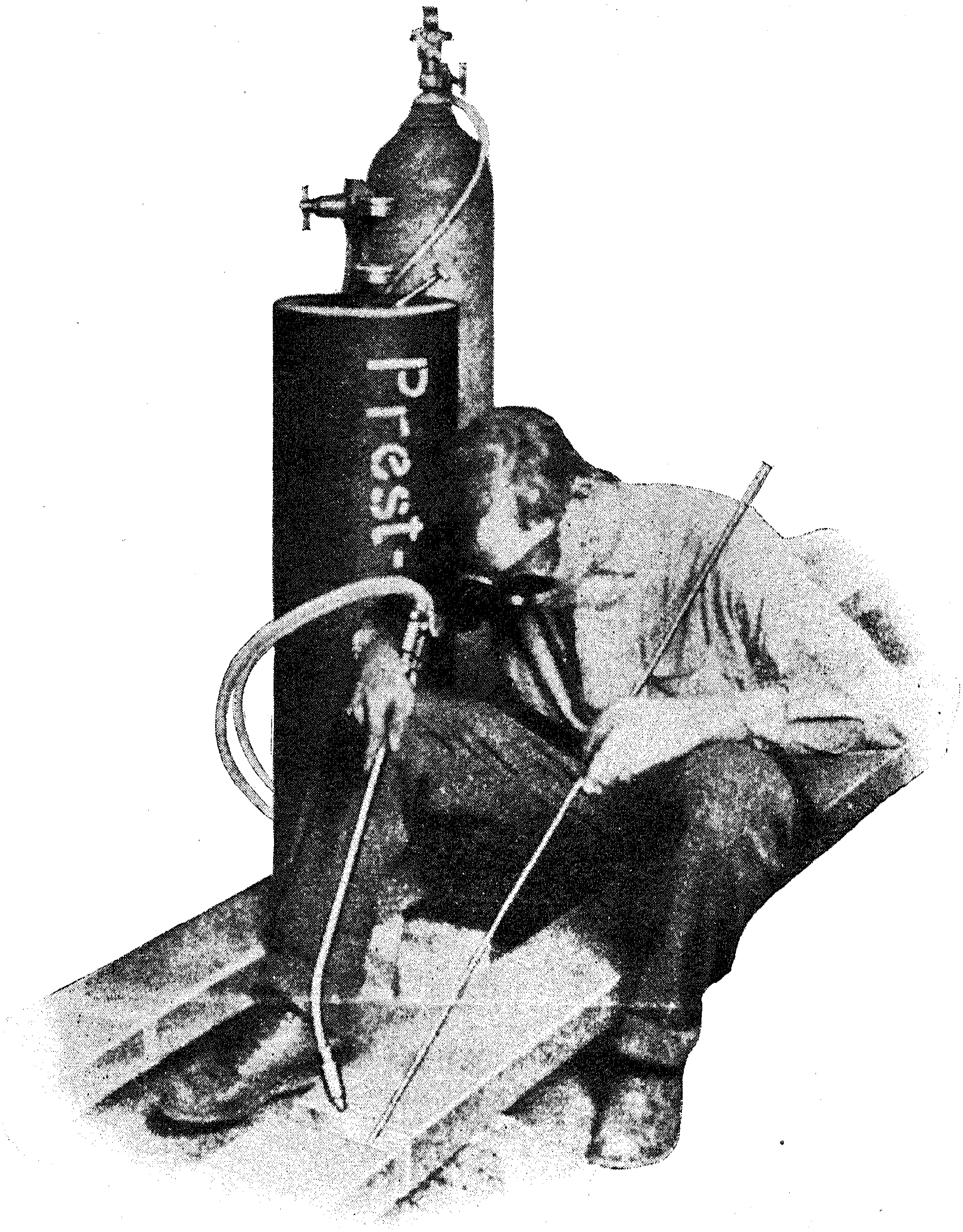


FIG. 1—GAS WELDING [HOBART]

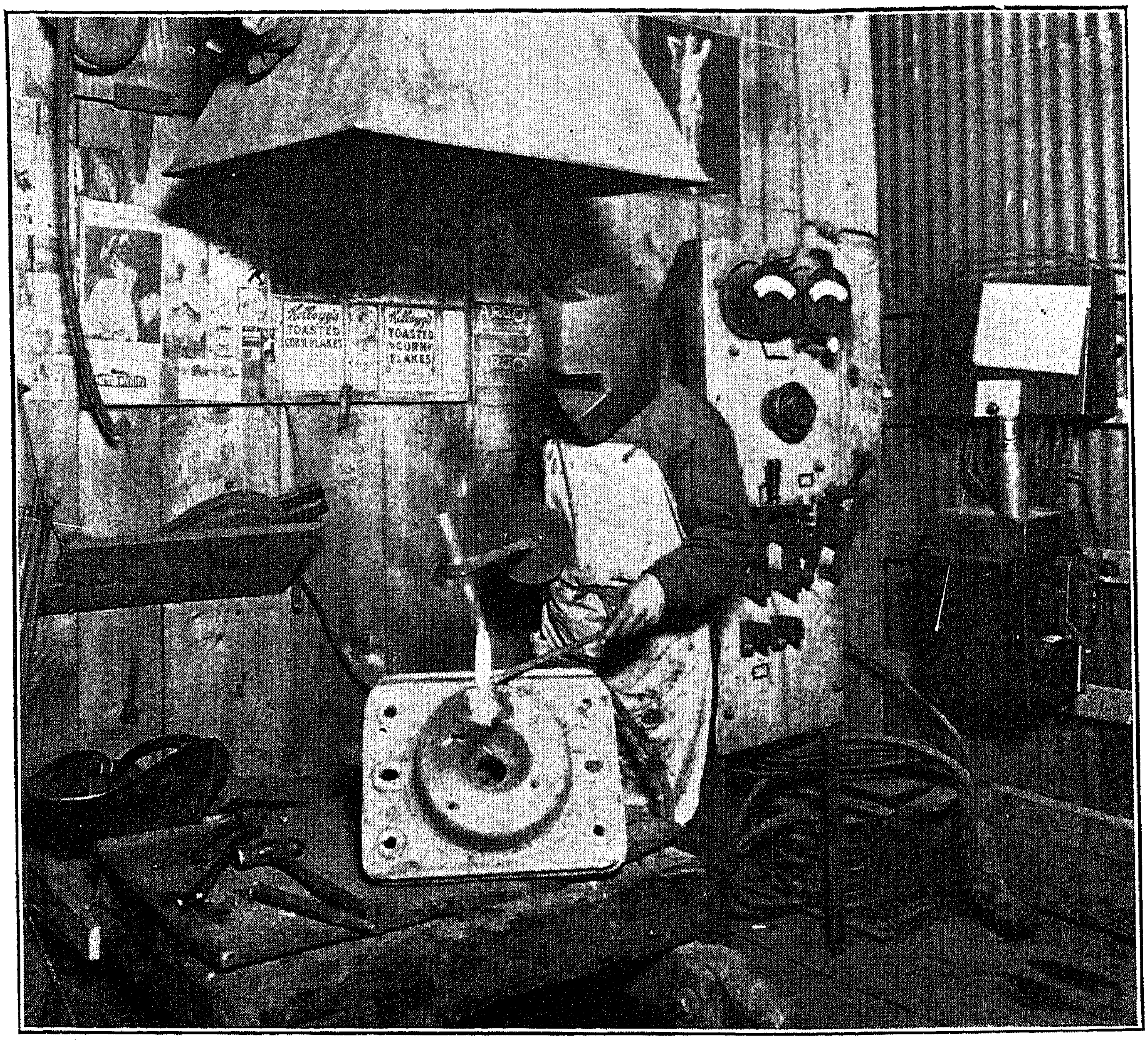


FIG. 2—CARBON-ARC WELDING

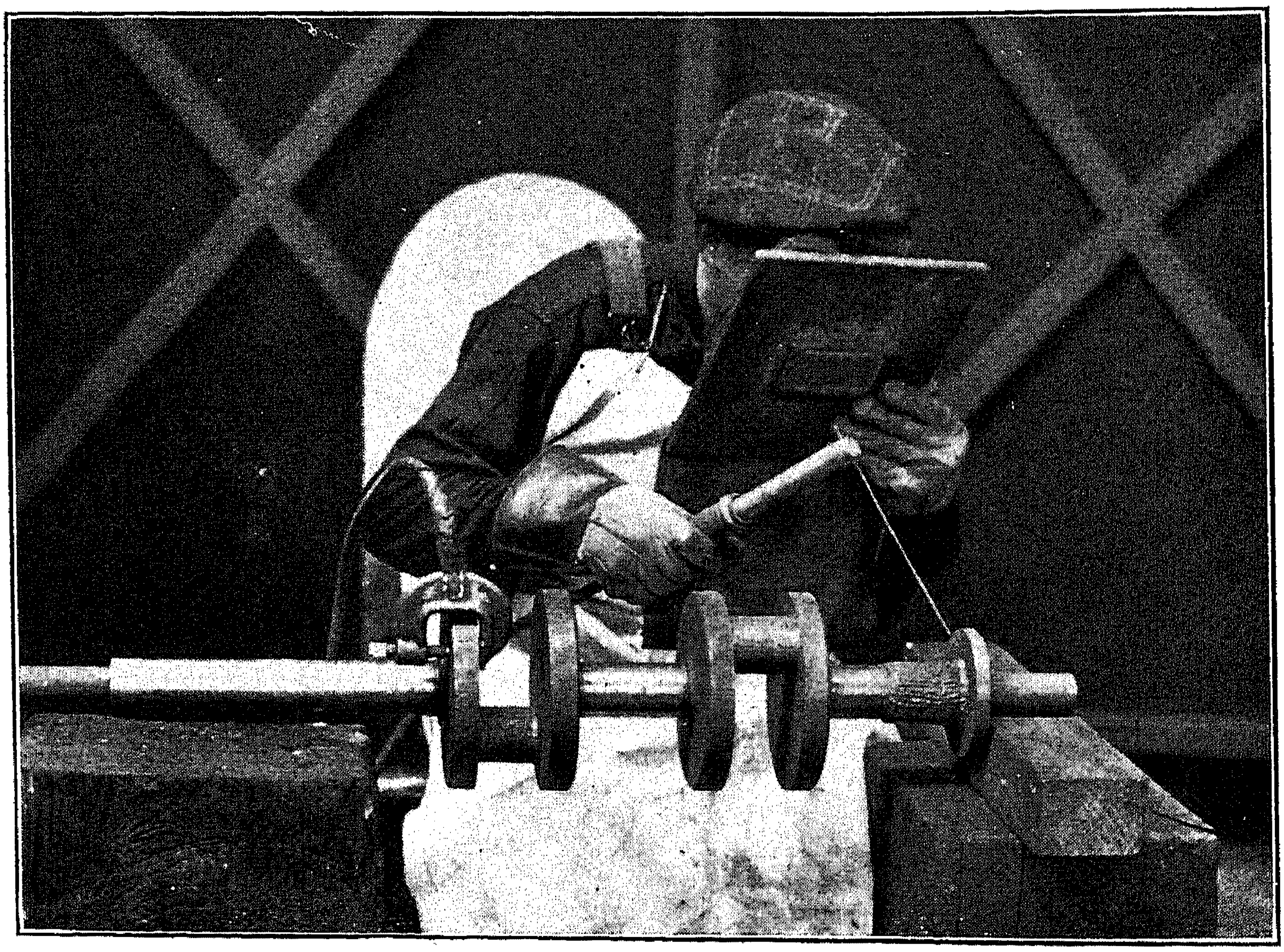


FIG. 3—METAL-ARC WELDING

[HOBART]

Another lot of about 1½ tons of ½-in. and 1-in. ship plate from another source was analyzed by the Electrical Testing Laboratories with the following result, four analyses being made for each thickness:

	Maximum per cent		Minimum per cent	
	½ in.	1 in.	½ in.	1 in.
Carbon	0.24	0.28	0.22	0.26
Manganese	0.44	0.53	0.40	0.47
Phosphorus	0.033	0.033	0.028	0.027

The specifications of the American Society for Testing Materials for structural steel for ships (serial designation A 12–16, p. 98, A. S. T. M. *standard*, 1918) are in abstract as follows:

Phosphorus (acid steel), Not over 0.06 per cent.
Phosphorus (basic steel), Not over 0.04 per cent.
Sulphur, Not over 0.05 per cent.
Tensile strength, between 58,000 and 68,000 lb. per sq. inch.
Elongation, min. per cent in 8 in. 1,500,000/tensile strength.

From the above data we have a good idea of the kind of steel in connection with which it was the Committee's first and specific task to investigate welding.

Two kinds of welding are under investigation at present:

1. Fusion welding.
2. Spot welding.

These are totally different kinds of welding. The fundamental difference is that while in fusion welding no pressure is employed, the success of spot welding is entirely dependent upon the application of both heat and pressure. For the spot welding of thick plates, the required pressure is very great.

The main features of each of these two kinds of welding will now be stated:

FUSION WELDING

The term fusion welding is employed to cover gas welding and electric-arc welding.

Gas welding is usually effected by simultaneously fusing with an oxyacetylene flame (1) the material at and near the surfaces which it is desired to join, and (2) some material (which is usually similar in composition) in the form of a rod, the tip of which is subjected to the heat of the flame. The oxy-acetylene flame is directed with one hand and the welding rod is manipulated with the other hand. The operation is illustrated in Fig. 1.

Electric-arc welding may be subdivided into several classes. The two broadest classes are·

a. Carbon-arc welding
b. Metal-arc-welding

In carbon-arc welding, an arc is established between a carbon or graphite electrode (usually a *graphite* electrode) and the two pieces of steel which it is desired to join. This graphite electrode is manipulated with one hand and a welding rod is fed into the weld by the other hand. The operation of carbon-arc welding is illustrated in Fig. 2. The manual activities in carbon-arc welding are seen to be quite similar to those in gas welding. In neither case is it necessary for the material of the welding rod to traverse the arc.[1]

In metal-arc welding, we find a fundamental difference in this latter respect, since in metal-arc welding of mild steel, the arc, instead of having a graphite electrode for one terminal of the circuit, is established between a steel welding rod (or welding electrode) and the two steel parts requiring to be joined. The operator in Fig. 3 is employing metal-arc welding to build up an incorrectly machined crank-shaft journal. There is always a distance of a matter of a tenth of an inch (2.5 mm.) or more between the end of the welding rod and the work. This distance is bridged by an electric arc. The form in which the steel exists during its passage from one end of the arc to the other is at present the subject of investigation by several independent experimenters.

Their conclusions are awaited with interest. The material cannot pass as a continuous liquid stream, since then there could be no interruptions in the metallic circuit and hence there could be no arc. It can pass as a series of liquid drops, and these can even momentarily short-circuit the arc, the duration of the short-circuit being too brief to be apparent to the operator or ordinary observer unaided by special apparatus. Or the drops can be so minute as to be incapable of effecting a short-circuit. If this should be the case, we can conceive of the metal passing as a stream of finely-divided liquid. Still another possibility is that the steel may pass as a highly-heated gas and condense on the opposite surfaces. It is suggested by physicists that, in its passage through the arc, the steel may undergo instantaneous transformations of which no human knowledge at present exists.

---

[1] Both for carbon-arc welding and gas welding, the edges of the parts to be joined sometimes may be so designed as to obviate the need for any additional material; in other words, no welding rod is necessary in such cases.

There would appear to be more of these complex possibilities in metal-arc welding than in gas welding or in carbon-arc welding. Nevertheless, it is precisely metal-arc welding which is at present proving very attractive to engineers. It is too early to return a verdict as to whether this wide-spread tendency toward metal-arc welding is based on sound premises or whether there ultimately may not be a reaction (for certain kinds of work) back to carbon-arc welding. It may be that there has been undue precipitancy in the general stampede which has taken place from carbon-arc welding (which was the first to be developed) to metal-arc welding, which is a later development.

SPOT WELDING

Spot welding, as developed for use in ship construction, consists in bringing into good contact, by hydraulic or pneumatic pressure, over-lapping portions of the plates or parts requiring to be joined, and in sending through the spot of contact a sufficiently large current to heat the plates or parts at this point to a welding temperature. The weld is effected by the combination of pressure and heat.

Several large spot welders have been built. With one of these (which was an experimental machine), sufficient pressure and current were available to weld together three one-inch thick plates. The usual construction of commercial spot welders for use in shipbuilding is similar in general appearance to so-called bull-riveters. The largest spot welder yet built for actual use in ship fabrication has a six-foot (1.8 m.) gap. This outfit is a large stationary machine to which the steel plates and shapes must be brought. It is planned that bulkheads, frames, floors, and other parts shall be constructed with it and shall then be transported by cranes to their places in the ship. This six-foot-gap machine is designed with capacity to weld two three-quarter-inch-thick plates. It provides a pneumatic pressure of 60,000 lb. and a current of 50,000 amperes and welds simultaneously two spots, each of some 1½ inch diameter, in about 30 seconds. With less current a longer time is required, and vice-versa. This particular spot welder, which is shown in Fig. 4, is known as a Duplex Welder. This name is due to the feature that *two* spots are *simultaneously* welded, the current crossing the plate in one direction between two electrodes, and then back again between two other electrodes.

Two transformers, one located on each side of the plate, are comprised in the outfit. The arrangement is indicated, diagrammatically in Fig. 5, in which:

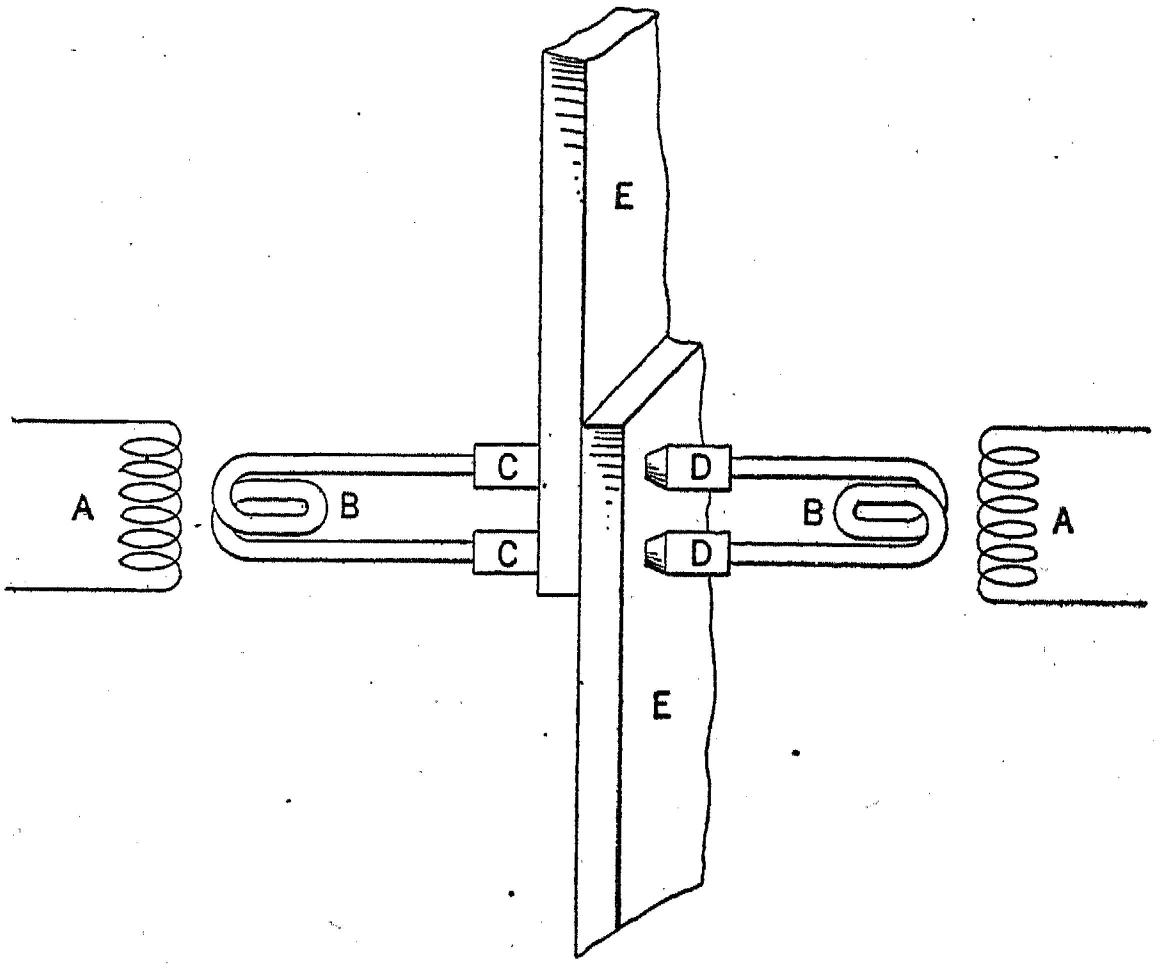


FIG. 5—WIRING DIAGRAM OF DUPLEX SPOT WELDER

A A represents the two primaries.
B B represents the two secondaries (which, in the actual construction, have only one turn each.)
C C and *D D* represent the electrodes between which the current flows and between which the pressure is exerted.
E E represents the two plates to be joined.

The chief object of this duplex feature is to eliminate the large reactance drop of a conducting loop some six feet long and one foot wide when traversed by some 50,000 amperes of 60-cycle current. This amounts to approximately 25 volts. Mr. J. M. Weed, the designer of the machine, reports the interesting fact that the presence in the gap of the plates to be

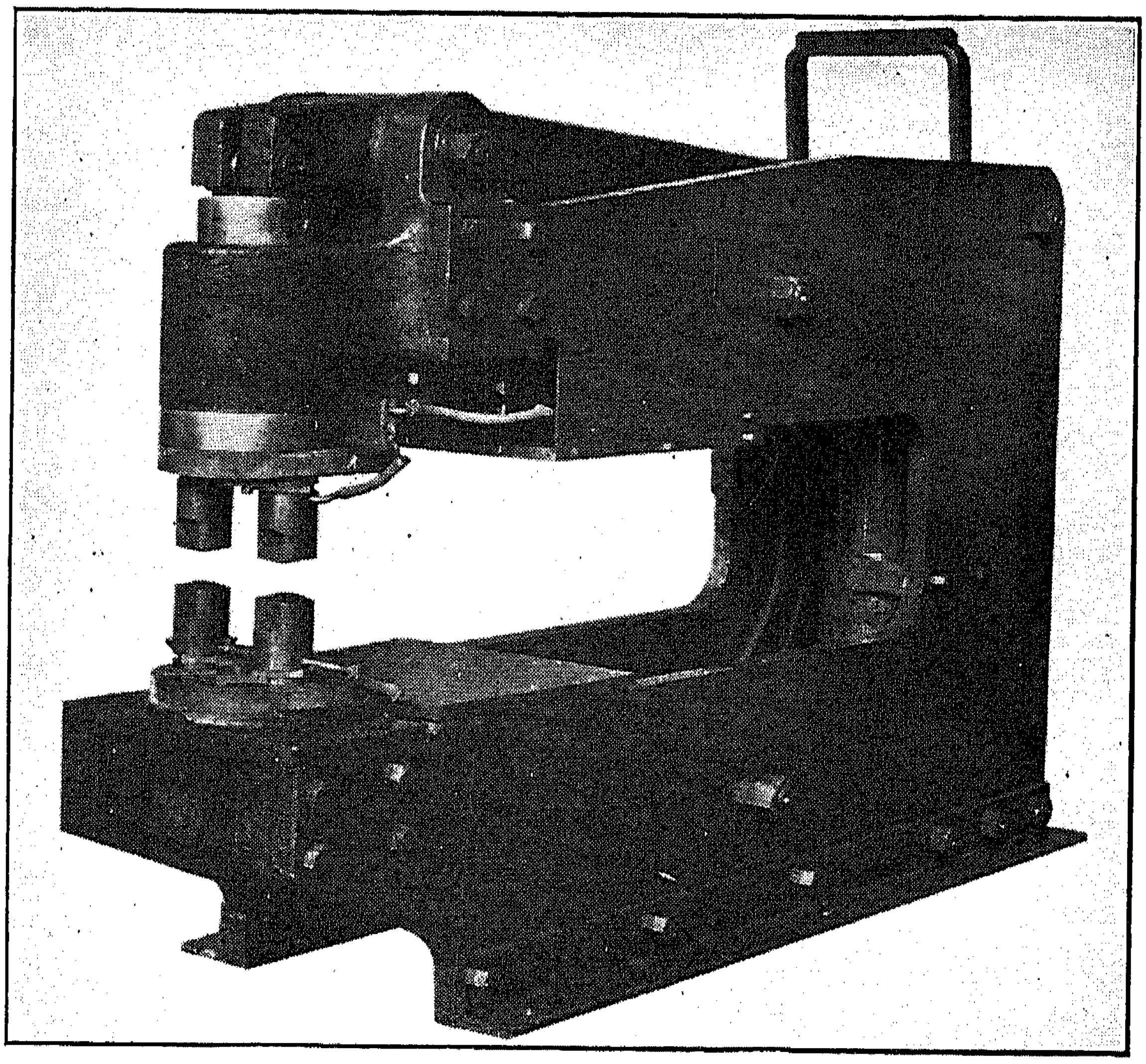
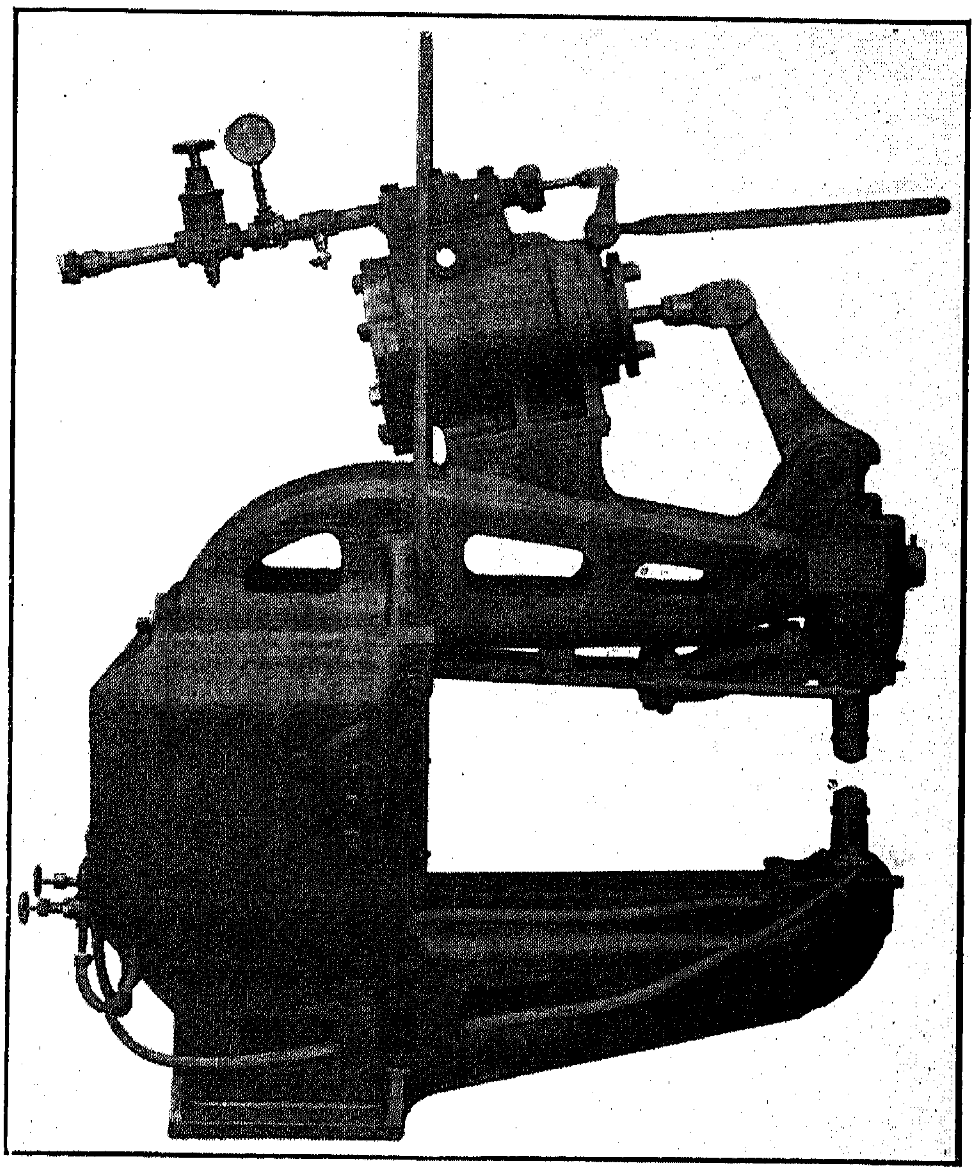


FIG. 4—DUPLEX SPOT WELDER



[HOBART]

FIG. 6—PORTABLE SPOT WELDER WITH 27-INCH GAP

welded, only decreased the current some ten per cent. This six-foot-gap machine weighs six tons.

In some other large spot welders of somewhat reduced size and capacity, the duplex feature is not employed and only one spot is welded at each application of the current. A portable welder of this type and having a 27-inch (68.5 cm.) gap, is shown in Fig. 6. This machine weighs only 2800 lb. (1271 kg.). In this case only one transformer is employed, and the circuit connections are those shown in Fig. 7, in which:

A represents the primary of the transformer.

B represents its secondary.

C and *D* represent the electrodes.

E and *F* represent the two plates to be welded.

In all spot welders for welding thick plates, the electrodes are water-cooled.[2]

When, in the Spring of 1918, Prof. C. A. Adams, of the Welding Committee of the Emergency Fleet Corporation, appointed several of us to be members of a Welding Research Sub-committee, we found ourselves facing a task of great importance and of enormous magnitude. It was desired that our investigations should be directed chiefly to the application of welding in the construction of the hulls of merchant ships. Much interior work on ships was already being performed very successfully with fusion welding and it appeared strongly indicated that the time was ripe for the extension of the appli-

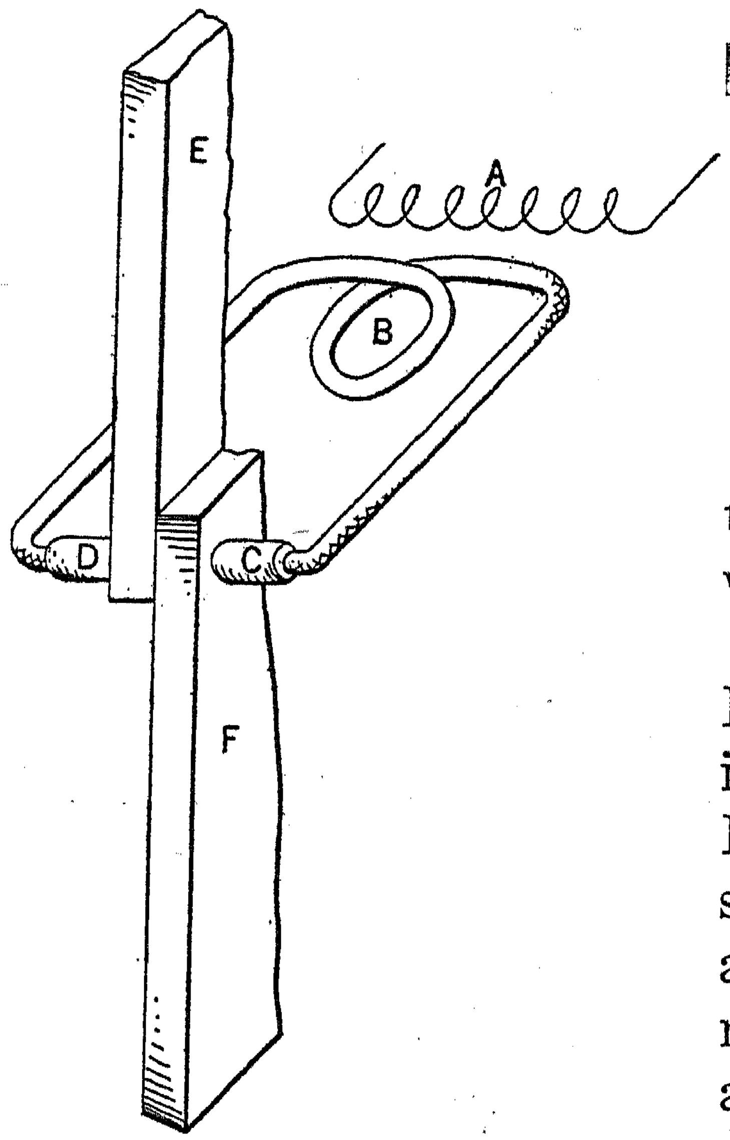


FIG. 7—WIRING DIAGRAM OF PORTABLE SPOT WELDER WITH 27-INCH GAP

cation to the hulls of ships, with the prospect of producing work not only fully equal (and probably superior) to that obtained by riveting, but also distinctly quicker and cheaper. At the time a welded barge was already nearly completed in England. The Welding Research Sub-committee ascertained that several American railways had for some time employed fusion welding extensively in routine repairs of locomotives and that a matter of possibly a couple of thousand arc welders were at that time employed by American Railways. The extensive and successful use of fusion welding for locomotive repairs, in itself constituted strong evidence of the ability of such welds to withstand vibration and shock in addition to their proved excellence with respect to tensile strength.

---

[2] Excellent discussions of the subject of spot welding and descriptions of several spot welders built for use in ship construction are given in the four following papers in the *General Electric Review*, December, 1918: Research in Spot Welding of Heavy Plates by W. L. Merrill, p. 919; Spot Welding and Some of its Applications to Ship Construction by H. A. Winne, p. 923; An Electrically Welded Freight Car by Jos. A. Osborne, p. 912; Some Recent Developments in Machines for Electric Spot Welding as a Substitute for Riveting by J. M. Weed, p. 928.

Any doubts entertained by the Committee related chiefly to the question of which of many ways in which it had been demonstrated that good fusion welding could be done, was the best way. Furthermore, as regards such mild-steel plates as are employed in the construction of merchant ships, it was soon demonstrated that while sound and quite ductile welds could be depended upon for plates of not over one-half-inch thickness, there was less certainty of good results with plates of greater thickness. But at that time there was no general recognition of the most suitable current to be employed for welding. It was rare to find more than 150 amperes used, even for the heaviest work, and as low as 100 to 125 amperes was found to be frequently employed for welding plates of half-inch thickness.

It now has been quite conclusively shown that stronger and more ductile welds of half-inch-thick plates are obtained by using at least 200 amperes. The author believes that fully 300 amperes should be used for butt-welding three-quarter-inch-thick plates and a matter of at least 400 amperes for one-inch-thick plates. These are some twice as great currents as have heretofore usually been employed in arc-welding plates of these thicknesses.

In view of this subsequent experience, it is clear that the disappointing lack of strength and ductility in certain welds of thick plates made nearly a year ago was a practically certain consequence of using such small currents.

It would be easy to yield to the temptation to enter discursively upon comments and opinions regarding the many points on which experienced welding specialists hold widely divergent opinions. All these specialists are producing thoroughly reliable work, but this is not saying that they are all producing nearly as good work as could be produced under the most appropriate conditions for each case. Indeed, the author's observations lead him to the conclusion that while excellent arc welding is being done on a wide scale, there is a margin for improvement over the present average quality, which, so far as it can be expressed by a sort of resultant of such physical characteristics as:

a. Bending and torsion tests
b. Tensile strength
c. Elongation at fracture

may be assessed as amounting to at least 25 per cent.

The author has attempted to make a list of some of the points which are the subject of discussion, and (while usually not going at much length into the questions), to make reference, in some instances, to the views and evidence on each side of a question. It has been the author's thought that such a summarized presentation might constitute the foundation for an instructive discussion.

The list is as follows:

PAGE
1. Bare and Covered Welding Wire.................. 72
2. Salvaging Welding Wire.......................... 72
3. Preferable Kind of Covering for Welding Wire...... 73
4. Preferable Composition for Bare Welding Wire...... 73
5. Composition of Metal Deposited in Weld........... 76
6. Polarity.. 77
7. Direct-current Versus Alternating-current for Arc Welding.................................... 78
8. Periodicity for Alternating-current Arc Welding..... 78
9. Bare or Covered Electrodes for Alternating-current Arc Welding................................ 79
10. Relative Speeds of Alternating- and Direct-current Welding.................................... 79
11. Comparative Quality of Arc Welds Made with Alternating Current and with Direct Current.......... 80
12. Power Factor for Alternating-current Arc Welding.. 80
13. Alternating-current Welding is Essentially a Single-phase Load.................................. 81
14. Spot Welding is a Single-phase Load............... 81
15. Ductility of Arc and Gas Welds.................. 82
16. Respective Fields of Gas and Arc Welding.......... 83
17. Relative Ductility of Welds with Bare and Covered Electrodes.................................. 87
18. Speed of Arc Welding............................ 87
19. Suitable Current for Given Cases................. 89

20. Effect on Arc Welding of Voltage Employed........ 90
21. Preferable Size of Electrodes.................... 90
22. Automatic Machinery for Arc Welding............. 91
23. Carbon Arc Welding.............................. 91
24. Preparation of Welding Edges.................... 92
25. Quality of Overhead Arc Welding................. 93
26. Number of Layers to be Employed................. 93
27. Rigid vs. Non-rigid Methods of Welding.......... 93
28. Consequences of Different Lengths of Arc........ 94
29. Spot and Arc Welding............................ 94
30. Condition of Surfaces to be Welded.............. 96
31. Preheating, Heat Treatment, and Hammering while Cooling.............................. 96
32. Time Required to Train Welders.................. 96
33. Question of Need for Special Machines for Welding . 97
34. Technique of Testing Welds...................... 97

1. BARE AND COVERED WELDING WIRE

The kinds of electrodes advocated and actually used in the art range all the way from the cheapest fence wire, costing but a few cents per pound (say at present prices, some 10 cents per lb.) up to carefully treated and covered electrodes, certain types of which cost over five times as much as bare electrodes. Some makes of covered electrodes are, however, obtainable at reasonable cost (such as, at present prices, some 20 cents to 25 cents per lb.). One type of electrode must yield results uniformly superior to those obtained with another type in order to afford economic justification for a five-times greater price.

2. SALVAGING WELDING WIRE

As to bare electrodes it is generally considered that uniformity is very essential. An operator may be getting along very nicely but will suddenly come to bad places in the welding wire. Heretofore it has been considered necessary to reject such wire. The claim is now made by some people that by merely dipping the electrode wire in suitable material, it may be salvaged. Thus in the Welding Committee's Specification for Electrode Wire (given at page 75 of this paper) occurs a note to the effect that "If electrodes to the above specification sputter or flow unevenly, they may be dipped in milk of lime (whitewash) before welding. This dipping may be done in quantity on stock on hand and allowed to dry, or the welder may keep a pot of solution on hand into which the electrode may be dipped immediately before welding." This method of salvaging electrode wire was developed by the General Electric Research Laboratory.

Also, it has been demonstrated by Mr. E. Wanamaker that the application, by dipping, of a kind of coating which he has developed (and the precise composition of which he will doubtless contribute to the discussion of this paper), permits of doing good work with electrodes which would otherwise be useless.

3. PREFERABLE KIND OF COVERING FOR WELDING WIRE

With regard to covered electrodes, while some claim that a thin covering obtained by dipping, accomplishes the desired purpose, others contend that it is desirable to provide a thick covering of appropriate material, which, in turn, is suitably impregnated. Moreover, even for covered electrodes, the usual belief is that the greatest care should be given to the composition and quality of the welding wire to which the covering is applied. In other words, it is not generally held that the use of inferior wire salvaged as indicated under (2), will permit of obtaining the best quality of welds. It is important that the covering shall be so designed as to be consumed at a definite rate as compared with the rate of consumption of the enclosed welding wire. A consequence is that any particular gage of *covered* welding wire must be used within rather close current limits.

For *overhead welding* one firm exploiting covered electrodes supplies a special (and additionally high-priced) grade in which the covering is impregnated with a more viscous material than is used for the electrodes which the firm supplies for other welding operations.

4. PREFERABLE COMPOSITION FOR BARE WELDING WIRE

There is a great diversity of practise as to the preferred composition of bare electrodes suitable for welding mild steel plates. As instances of extremes it may be said that amongst widely used electrodes, while one type consists of almost pure iron, other types have nearly 0.2 per cent of carbon and 0.5 per cent of manganese, and still other types run very much higher than this in manganese. This is quite aside from the subject of special compositions for welding high-carbon steel and for welding cast iron. It is anticipated that quantitative measurements will indicate superiority in tensile strength for some

TABLE I.
COMPOSITION OF WELDING ELECTRODES FOR METAL-ARC WELDING

Trade designation of electrode	Carbon	Manganese	Phosphorus	Sulphur	Silicon	Remarks
Page Steel & Wire Co.						
Armco	0.01	0.025	0.005	0.025	0.005	
Wilson Welders & Metals Co.						
Grade, No. 6	0.15 to 0.23	0.60 to 0.75	less than 0.04	less than 0.04		Also 0.25 per cent copper.
Grade, No. 9	0.30 to 0.40	about 1.00	less than 0.04	less than 0.04		
Grade, No. 8	0.17 to 0.22	0.30 to 0.45	less than 0.04	less than 0.04		
Grade, No. 17	0.10	0.30 to 0.45	0.06	0.06		
Quasi Arc Co.	0.08 to 0.12	0.45 to 0.55	0.00 to 0.06	0.00 to 0.06	0.05 to 0.08	Flux covering of blue asbestos fibre (Croeodilite) enclosing percentage of aluminum or other metal in form of fine wire capable of giving strong reducing action.
Roebling Co.	0.16	0.56	0.032	0.024	0.016	
Toncan wire	0.10	0.16	0.01	0.046	trace	
Electric Arc Cutting & Welding Co.	0.25	0.30	0.05	0.05	0.05	
Siemund Wenzel Co.	0.10 and under	0.30 to 0.50	0.05 and under	0.05 and under	trace	
Norway-iron Wire	0.05	0.02	0.025	0.007	0.08	
Double Arc Co., of England	0.085	0.35	.....	0.054	0.108	Flux covered.
T. Scott Anderson Co., of England	0.057	0.32	.....	0.026	0.014	Flux covered.
E.A.Jones & Co.,of England	0.22	0.25	0.001	0.026	0.024	Nickle-plated and flux covered.
Engineering and Equipment Co., of England	0.12	0.51	.....	0.08	0.016	Flux covered.
Central Steel & Wire Co.						
Swedox	0.05	0.18	0.04			
The Spencer Wire Co.						
Basic open-hearth steel electrode	00.06	0.12 to .20	0.013 and under	0.03 and under		

compositions and superiority in ductility for other compositions. Mr. R. E. Wagner has exhibited some very ductile welds made with electrodes containing small percentages of magnesium and of boron sub-oxide.

It is only within the last few months that there have been available any specifications for use in establishing the merits of welding wire. These are now available in the Welding Committee's specification setting forth a "Standard Procedure for Testing Welding Electrodes." This specification, which is given in Appendix A to this paper, was prepared by the Welding Research Sub-committee in collaboration with Prof. H. L. Whittemore representing the Bureau of Standards and with representatives of manufacturers of welding electrodes.

In Table I are given the compositions of various electrodes in current use. Prior to publication, the data in this Table was submitted to the manufacturers concerned, who, in six instances, improved the opportunity to correct the data to conform with their latest practise. The American Steel and Wire Co. has requested the omission from the accompanying Table of any analyses of electrodes which it has furnished for arc welding. This is for the reason that material has been supplied to a large number of users, and varies considerably in analyses in accordance with the ideas of the purchasers. There does not as yet seem to be an agreement as to the most advantageous chemical composition for electrodes, and the company is not prepared either from observations of the results obtained by its customers or from its own experimental work to make a definite recommendation.

The Welding Committee has issued the following Specification for Electrode Wire for Electric Welding. The specification was prepared under the immediate direction of Mr. Herman Lemp.

SPECIFICATION FOR ELECTRODE WIRE FOR METAL-ARC WELDING IN CONNECTION WITH MILD STEEL

Welding Committee Emergency Fleet Corporation

REVISED TO DECEMBER 20, 1918

(Note:—This wire may or may not be covered)

1. *Chemical Composition*

Carbon........................	Not over 0.18 per cent.
Manganese........................	" " 0.55 " "
Phosphorus........................	" " 0.05 " "
Sulphur........................	" " 0.05 " "
Silicon........................	" " 0.08 " "

2. *Sizes and Weights*

Diameter, in mils	Diameter, in fraction of an inch	Pounds per 100 ft.	Feet per 100 lb.
125	1/8	4.16	2400
156	5/32	6.51	1535
188	3/16	9.37	1066

(Allowable tolerance 6 mils plus or minus)

3. *Material.* The material from which the wire is manufactured shall be made by any approved process. Material made by *puddling process not permitted.*

4. *Physical Properties.* Wire to be of uniform homogeneous structure, free from oxides, pipes, seams, etc., as proved by photomicrographs.

5. *Workmanship and Finish.* (*a*) Electric welding wire shall be of the quality and finish known as the "Bright Hard" or "Bright Soft" finish—"Black Annealed" or "Bright Annealed" wire shall not be supplied.

(*b*) The surface shall be free from rust, oil or grease; a slight amount due to lubrication during last drawing is permissible.

6. *Tests.* Electrodes must, before shipment or after delivery, show good commercial weldability when tested by an experienced arc welder. The electrode material shall flow smoothly in relatively small particles through the arc without any detrimental phenomena.

Note: If electrodes to above specifications sputter or flow unevenly, they may be dipped in milk of lime (whitewash) before welding. This dipping may be done in quantity on stock on hand and allowed to dry, or welder may keep a pot of solution on hand into which the electrode is dipped immediately before welding.

7. *Delivery, Packing and Shipping.* Electrodes shall be furnished in straight lengths of either 14 inches or 28 inches, put up in bundles of 50 pounds or 100 pounds as ordered. Each bundle shall be wrapped in heavy paper securely wired and marked on one end showing diameter in mils, trade name and grade of wire.

5. COMPOSITION OF METAL DEPOSITED IN WELD

A few analyses have been made of chemical compositions of the metal deposited in the weld. Results of the analyses of four sets of electrodes before and after the metal was deposited are quoted below from the Westinghouse chapter in Major Caldwell's report, published by the Emergency Fleet Corporation in 1918. To these results are added analyses of Toncan wire as supplied to the author by Mr. R. E. Wagner.

ANALYSES OF ELECTRODE—PER CENT OF IMPURITIES

	Carbon	Manganese	Phosphorus	Sulphur	Silicon
Roebling	0.16	0.56	0.032	0.024	0.016
Norway	0.049	0.021	0.025	0.007	0.08
C. R. S.	0.11	0.72	0.097	0.123	0.011
H. R. S.	0.13 to 0.17	0.50	0.012	0.045	0.011
Toncan	0.10	0.16	0.010	0.046	trace

ANALYSES OF DEPOSITED METAL—PER CENT OF IMPURITIES

	Carbon	Manganese	Phosphorus	Sulphur	Silicon
Roebling	0.05	0.18	0.031	0.036	0.011
Norway	0.05	0.018	0.020	0.072	0.011
C. R. S.	0.05	0.11	0.086	0.072	0.011
H. R. S.	0.14	0.14	0.012	0.039	0.011
Toncan	0.042	0.081	0.019	0.026	0.000

It is notable that most of the carbon and manganese is burned out in traversing the arc.

6. POLARITY

For carbon-arc welding, the standard practise is to connect the graphite electrode to the negative terminal. Mr. Wagner states as his experience that it is very difficult to weld with the carbon arc when the polarity of the carbon is positive. He states it to be almost impossible to direct the heat to the point desired and the welding qualities of the arc under this condition are very poor. He concludes: "Our experience has taught us that it is next to impossible to weld with a carbon arc unless the work is positive and the electrode negative."

For metal arc welding with bare wire, the electrode is usually connected to the negative terminal, but instances occur of bare welding wire which works best when the opposite polarity is employed. Also for some particular sizes and sorts of welds best results are sometimes obtained by a reversal of the polarity. With electrodes heavily covered with flux, the positive terminal is almost always connected to the electrode. Plenty of more or less plausible reasons for these differences have been offered on various occasions. On careful reflection none of these reasons prove particularly satisfying. Amongst other considerations the fact of the entire practicability of arc welding from an alternating-current circuit and of overhead welding have to be taken into account in judging some of these explanations. As yet, we have no satisfactory hypothesis as to what goes on in the welding arc.

7. DIRECT CURRENT VERSUS ALTERNATING CURRENT FOR ARC WELDING

While up to rather recently it had usually been contended that arc welding required a direct-current supply, there are now many advocates of alternating current.

Mr. E. H. Jones in the course of the discussion of Major James Caldwell's paper entitled "Notes on Welding Systems" read on Jan. 22, 1918, before the Institution of Engineers and Shipbuilders in Scotland, stated: "He would like to take this opportunity of drawing attention to the undoubted merits of alternating current for arc welding. For some reason which he was unable to fathom, the general impression was that direct current was superior to alternating for arc welding, but as a matter of fact he found that alternating current was far superior to direct current, and he would recommend the use of alternating current on every possible occasion. Apart entirely from the capital outlay needed, which was vastly higher in the case of direct current, the control of the current was much easier to effect. . . . He estimated that the amount of current which would be necessary to feed 20 operators with direct current would suffice to feed 28 with alternating current."

Mr. R. E. Wagner's experience is as follows: "Electric welding may be done with alternating current as well as with direct current. It is a little more difficult to hold the arc, but this simply resolves itself into a matter of practise. Men who have been regularly doing arc welding with direct current, very quickly learn how to handle the alternating-current arc."

At present there is no agreement as to the applicability of alternating current to carbon arc welding.

8. PERIODICITY FOR ALTERNATING-CURRENT ARC WELDING

Amongst the advocates of the use of alternating current, there is no agreement with reference to the periodicity. Although it is generally maintained that arc welding is only thoroughly practicable with as high a periodicity as fifty or sixty cycles per second, there is, on the other hand, expression given to the opinion that the use of twenty-five cycles, or less, is equally satisfactory. In October, 1918, Mr. R. E. Wagner reported to the Welding Research Sub-committee, that he had found from his tests that alternating current for arc welding could be used with a frequency as low as 12½ cycles and as high as 500 cycles. Mr. Wagner states that while there is no difficulty at either of these extreme periodicities, the arc is more readily held at 500 cycles than at 12½ cycles.

9. BARE OR COVERED ELECTRODES FOR ALTERNATING-CURRENT ARC WELDING

While some maintain that arc welding with alternating current is only at its best when flux-covered electrodes are used, it appears to have been conclusively demonstrated by others that excellent results are being obtained under commercial conditions with bare electrodes and an alternating-current supply. A novice can more quickly learn to weld from an alternating-current supply if he employs flux-covered electrodes. But if he can ultimately learn to weld just as rapidly and successfully with bare electrodes, the difficulties in the initial stages of his education should not be regarded as being of much consequence. Mr. Wagner finds that when welding with alternating current, "manipulation may be simplified in many cases by treating the electrode with a thin coating of ordinary lime."

10. RELATIVE SPEEDS OF ALTERNATING-CURRENT AND DIRECT-CURRENT ARC WELDING

Some contend that alternating-current welding is slower. As an instance of a diametrically opposite experience, the following recently received record of test of a certain electrode may be quoted:

"Its operation on 140 amperes, 115 volts alternating current is very good. It also works satisfactorily on 130 amperes, 75 volts direct current, but the metal flows more slowly on direct current than on alternating current."

The record concerning another type of electrode tested on the same occasion, reads as follows:

"This electrode was tried on 5/16 in. plate, 120 amperes, direct current, 75 volts. Its operation is satisfactory. It also works satisfactorily at 140 amperes, alternating current, 115 volts, but its operation on alternating current is not quite as good as on direct current."

Mr. R. E. Wagner, who has had much to do with the develop-power factors, combined with the fact that this load would be for short periods at very frequent intervals would make it decidedly undesirable from the central station standpoint. The condition would be much improved at 25 cycles, as the same machine would operate equally as well at 25 cycles as at 60 cycles, with about half the kv-a. and about double the power factor. The intervals of operation would however, be the same as for 60 cycles. If, however, a motor generator set with suitable flywheel attached, be provided for operating these machines, these disadvantages are all practically eliminated, this arrangement being such that the motor stores up energy in the flywheel during the interval of no load, the flywheel supplying a large part of the energy during the period of welding. By this means, for instance, a single-phase load of 900 kv-a. at 0.50 power factor for 30 second periods and with intervals of 1½ minutes between periods would be converted to a practically continuous 3-phase load of approximately 200 kv-a. at about 0.85 power factor."

15. Ductility of Arc Welds

Attention has been pertinaciously drawn to results of a very few tests which have appeared to indicate that metal-arc welds are inherently utterly deficient in ductility, yet the Committee has had also before it the results of many well-authenticated tests of ductile metal-arc welds.

It has been claimed that gas welds are more ductile. On this matter Mr. R. E. Wagner writes:

"At several meetings of the Welding Committee, special stress has been brought to bear on the bending qualities of acetylene and gas welds. We have done some experimenting with average acetylene and arc welders, and our impression is, that the acetylene and arc welds are in the same class with respect to bending. I submit herewith (See Fig. 8) a photograph showing comparative bends in acetylene and arc-welded joints, both welds were taken from half-inch plate and both samples were bent under the same conditions, that is, the sharp edge of an angle iron was placed along the weld and pressure applied to the angle iron to make a sharp bend. These I think are average comparative results. * * * * * * As far as our experiments are concerned, we feel, as regards physical characteristics, that acetylene and arc welds are in the same class."

16. RESPECTIVE FIELDS OF GAS AND ELECTRIC ARC WELDING

On this subject, under date of Oct. 22, 1918, Mr. R. P. Jackson, reports to the Welding Research Sub-Committee as follows:

"With reference to the comparative uses or fields of gas and electric-arc welding which came up at the last meeting, it was thought it might be well for some of us to express our opinions on the matter based on our experience with both kinds of welding. In general, we have found gas welding to be more satisfactory for thin material, say ⅛ inch and under, and for general repair work, particularly where various kinds of steel and cast iron are involved. For example, if repairs have to be made on broken machinery, lugs rebuilt, pieces attached to high-carbon steel and work of this character, then the gas-welding methods are superior and the extra cost not ordinarily prohibitive. When it comes, however, to depositing a large amount of metal and welding up structural steel or plates of ¼ inch thickness and upward, the results obtained by the ordinary direct-current arc with the metal electrode are at least equal to the gas welding work and certainly cheaper. In general, too, the finish of gas welding is more regular and better looking and where that is a consideration it may give a preference to gas. In fact, in the Westinghouse factory at East Pittsburgh, there has been considerably more gas work done than electric, but the electric arc welding is on the increase, not so much in displacing gas as in displacing riveting."

A view taken from a gas welding publication is as follows:

"The arc process is chiefly used for filling up blow holes in large steel or iron castings and building up worn surfaces which have not to be machined. With this process the results obtained are somewhat uncertain, and it is generally conceded, apart from the vital question of cost, that fusion produced by the burning of gases is to be preferred to the electric process. Welds made by the electric process are sometimes rough, hard, brittle and unworkable—in most cases this is highly objectionable, but not always so. With any fusion method of welding, annealing of the metal adjacent to the weld is desirable. It is impossible to do this annealing with an electric welder, but with gas welding the blow-pipe flame can be used for heating up the metal surrounding the welded part, and also for heating metal away from the weld, so as to counteract any strains that may be set up in the piece as the weld cools off. There are certain classes of work for which electric welding is the most suitable system, and, on the other hand, there are many classes of work where it would be most impractical, and which can be done satisfactorily only with gas welding. For general workshop use, a gas welding outfit is far better, not only because of its greater economy in installation and operation, but also because of its wider range of usefulness."

In the absence of any experience to the contrary this latter view appears fairly plausible, and it is natural that it should have received wide acceptance. But an enormous volume of experience in arc welding has gradually accumulated and it controverts the correctness of the view. Unfortunately the experimental data available on the subject of *Gas Welding* is surprisingly meagre. The Welding Research Sub-Committee has concluded thatthere are practically no test data from which it can draw any safe generalizations as to the mechanical characteristics of *gas welds*, and that it will be necessary to embark upon its own investigations to obtain suitable data.

Gas welding was an established art before there was any large amount of electric welding. This was still the state of affairs in England until shortly before the author was there in the autumn of 1917. But the war conditions had occasioned in England such a shortage of supplies of oxygen and carbide that the Government, as a war measure, practically forced the wide substitution of arc welding for gas welding. The British Government, in entering upon this policy, had relatively little concern as to the comparative merits of the two methods except in so far as that any merit or advantages found to be associated with arc welding would naturally assist in bringing about its use in place of gas welding.

It was, however, with considerable surprise that it was ascertained that the true economic field for arc welding as compared with gas welding was a very wide one, and that, simply due to inertia and tradition, engineers had been continuing in the contrary belief. Major James Caldwell, of the Admiralty Controller's Department, had wide responsibilities in this task of substituting arc welding as rapidly and generally as possible. Major Caldwell provided the author with the results of his investigations into the relative cost of gas and metal-arc welding. These results, which correspond to conditions in December, 1917, are set forth in Table II.

From Table II it is seen that metal-arc welding was found

TABLE II.—COMPARATIVE COSTS OF WELDING BY OXYACETYLENE AND METAL-ARC WELDING

Thickness of metal, inches	Oxyacetylene					Metal-Arc					Oxyacetylene		Metal-Arc	
	Gas per hour		Cost for gas per foot run pence	Iron wire for filling, pence	Labor per foot run	Power per foot run			Labor per foot run, pence	Cost of electrodes per foot run, pence	Feet run per hour	Total cost per foot run, pence	Feet run per hour	Total cost per foot run, pence
	Oxygen, cubic feet	Acetylene, cubic feet				Volts	Amp.	Kw.						
1/16	3	2.0	0.116	0.131	0.40	100	30	0.075	0.3	12 in. No. 12 = 1.2	30	0.647	40.0	1.575
1/8	9	6.3	0.77	0.196	0.856	100	75	0.312	0.5	18 in. No. 12 = 1.8	14	1.822	24.0	2.612
3/16	13	9.0	1.72	0.262	1.33	100	100	0.50	0.6	24 in. No. 10 = 2.64	9	3.312	20.0	3.74
1/4	17	13.0	2.70	0.250	1.50	100	140	0.70	0.6	22 in No. 8 = 3.6	8	4.45	20.0	4.90
5/16	27	16.0	4.21	0.327	1.714	100	110	1.10	1.2	12 in. No. 10 18 in. No. 8 = 4.02	7	6.25	10.0	6.32
3/8	34	24.0	6.82	0.458	2.00	100	120	1.61	1.61	18 in. No. 10 24 in. No. 8 = 5.58	6	9.278	7.4	8.802
7/16	41	29.0	9.90	0.655	2.40	100	120	2.00	2.0	24 in. No. 10 36 in. No. 8 = 8.04	5	12.955	6.0	12.04
1/2	48	34.0	14.50	0.786	3.00	100	120	2.40	2.4	30 in No 10 42 in. No. 8 = 9.6	4	18.286	5.0	14.40

Labor taken at 1 shilling per hour.
Oxygen taken at ½ pence per cu. ft.
Acetylene taken at 1 pence per cu. ft.
Iron for filling taken at 0.131 pence per ft.
The above figures are based on the British Oxygen Co. standards.

Labor taken at 1 shilling per hour.
Current taken at 1 pence per B.O.T. unit.
Electrodes, No. 10, 1.32 pence per ft.; No. 8, 1.8 pence per ft.; No. 12, 1.2 pence per ft.

to be a faster process for all thicknesses of steel. The British Admiralty results furthermore indicate the economic field for the two methods. The verdict from the data in the Table is in favor of gas welding for thin plates and of electric-arc welding for thick plates. But the comparison is based on the very high cost of electrodes set forth below:

Standard wire gauge	Cost in cents per foot	Ft. per pound of contained iron wire	Cost in cents per pound
Number 8	3.6	15	54
Number 10	2.6	23	60
Number 12	2.4	35	84

By substituting a typical American price for labor and substituting the cost of bare electrodes, such as are used with entire success in America, in place of the cost of flux-covered electrodes of the expensive type employed in arriving at the results set forth in the Table, the revised results show a lower cost for arc welding than for gas welding for all thicknesses above 1\16 inch. The question of the quality of the weld is another matter, but judging from the general reputation of the work of all sorts done by gas welding and by metal-arc welding, they are both thoroughly reliable. No more exact comparison can be made till we have carried through to completion, really elaborate tests of gas welds in order to permit of making a sound comparison with the large amount of research data already obtained with metal-arc welds.

In response to a request for his opinion as to the respective fields for gas and metal-arc welding, Mr. R. E. Wagner, writes as follows: "The present well-tried field for metal-arc welding is confined entirely to welding plates and forms, and a great deal of work has been done on plates varying in thickness from 1/16 in. to ¾ in. Up to 1/8 in. plates, the cost of gas and electric welding is about the same. Beyond this, the cost is in favor of the electric process. No difficulty is experienced in machining electric welds made with the metallic electrode. While it is recognized that the electric-welded-in material will not stand bending equal to that of the plate in which it is deposited, it is on the average equal to gas-deposited material in this respect."

17. RELATIVE DUCTILITY OF ARC WELDS MADE RESPECTIVELY WITH BARE AND COVERED ELECTRODES

By some authorities, ductility is believed to be most readily obtained by employing flux-covered electrodes. On the other hand, the Committee has knowledge of several kinds of bare electrodes of various compositions which, in competent hands, make reasonably ductile welds.

18. SPEED OF ARC WELDING

All sorts of values are given for the speed, in feet per hour, with which various types of joints can be welded. Operators making equally good welds have widely varying degrees of proficiency as regards speed. Any quantitative statement must consequently be of so guarded a character as to be of relatively small use. In general, and within reasonable limits, the speed of welding will increase considerably when larger currents are employed. It appears reasonable to estimate that this increase in speed will probably be about 25 to 35 per cent, for high values of current. This increase is not directly proportional to the current employed because a greater proportion of time is taken to insert new electrodes and the operator is working under more strenuous conditions. Incidentally, the operator who employs the larger current will not only weld quicker but the weld will have also better strength and ductility.

On this point Mr. Wagner writes as follows: "I would not say that speed in arc welding was proportional to the current used. Up to a certain point ductility and strength improve with increased current, but when these conditions are met, we do not obtain the best speed due to increased heating zone and size of weld puddle. Speed may fall off when current is carried beyond certain points."

In a research made by Mr. William Spraragen for the Welding Research Sub-Committee on several tons of half-inch-thick (12.7 mm.) ship plate, the average rate of welding was only two feet (0.6 m.) per hour. Highly skilled welders were employed but they were required to do the best possible work, and the kinds of joints and the particular matters under comparison were very varied and often novel.

However, in the researches carried on by Mr. Spraragen it was found that about 1.9 pounds (0.8 kg.) of metal were deposited per hour when using a 5/32 in. (3.9 mm.) bare electrode and with the plates in a flat position. The amount of electrodes used up was about 2.7 pounds per hour, of which approximately 16.5 per cent was wasted as short ends and 13 per cent burnt or vaporized, the remainder being deposited at the speed of 1.9 pounds per hour mentioned above.

For a 12-foot-cube tank of half-inch-thick steel welded at Pittsfield, the speed of welding was 3-feet per hour. The weight of the steel in this tank was 16,000 lb. and the weight of electrode used up was 334 lb. of which 299 lb. was deposited in the welds. The total welding time was 165 hours corresponding to using up electrodes at the rate of just 2 lb. per hour. The total length of weld was 501 feet, the weight of electrode used up per foot of weld thus being 0.60 lb. The design of this tank comprised eighteen different types of welded joint. Several different operators worked on this job and the average current per operator was 150 amperes.

For the British 125-foot-long Cross-Channel Barge for which the shell plating was composed of ¼-in. and 5/16-in. thick plates, in Mr. H. Jasper Cox's paper read before the Society of Naval Architects on Nov. 15, 1918, and entitled "The Application of Electric Welding to Ship Construction" it is stated that: "After a few initial difficulties had been overcome, an average speed of welding of seven feet per hour was maintained including overhead work which averaged from three to six feet per hour."

In a report appearing on page 67 of the Minutes and Records of the Welding Research Sub-Committee for June 28, 1918, Mr. O. A. Payne, of the British Admiralty, states: "A good welder could weld on about one pound of metal in one hour with the No. 10 Quasi-Arc electrode, using direct current at 100 volts. An electrode containing about 1½ ounces of metal is used up in about three minutes, but this rate cannot be kept up continuously."

The makers of the Quasi-Arc electrode publish the following data for the speed of arc welding in flat position with butt joints, a 60 degree angle and a free distance of ⅛ inch.

Thickness of Plates	Speed in Feet per Hour
⅛ in.	30
¼ in.	18
½ in.	6
1 in.	1.3

I cannot, however, reconcile the high speed of welding ½-inch plate published in this report as 6 ft. per hour, with the report given above by the British Admiralty that a good welder deposits one pound of metal per hour with Quasi-Arc electrode. If the rate given by the manufacturer is correct, it would mean that about four pounds of metal were deposited per hour. On this basis the rate must have been computed on the time taken to melt a single electrode and not the rate at which a welder could operate continuously, allowing for his endurance and for the time taken to insert fresh electrodes in the electrode holder and the time taken for cleaning the surface of each layer before commencing the next layer.

From his observations the author is of the opinion that a representative rate for a good welder lies about midway between these values given respectively by Mr. Payne and by the makers of the Quasi-Arc electrode, say for half-inch plates some two pounds per hour. This, it will be observed, agrees with Mr. Spraragen's experience in welding up some six tons of half-inch ship plate with a dozen or more varieties of butt joint and Mr. Wagner's results with the eight-ton tank. Even this rate of two pounds per hour is only the actual time of the welding operator after his plates are clamped in position. This preliminary work and the preparation of the edges which is quite an undertaking, and requires other kinds of artisans, accounts for a large amount of time and should not be underestimated.

The practise heretofore customary of stating the speed of welding in feet per hour has led to endless confusion as it depends on type of joint, height of weld and various details. A much better basis is to express the speed of welding in pounds of metal deposited per hour. Data for the pounds of metal deposited per hour are gradually becoming quite definite. The pounds of metal per foot of weld required to be deposited can be readily calculated from the drawings or specifications. With the further available knowledge of the average waste in electrode ends and from other causes, the required amount of electrode material for a given job can be estimated.

19. SUITABLE CURRENT FOR GIVEN CASES

For a given type of weld, for example, a double Vee weld in a one-half-inch-thick ship plate, it was found that in the summer of 1918, while some operators employed as low as 100 amperes, others worked with over 150 amperes. Some, in making such a weld, employed electrodes of only 1/8 in. diameter and others preferred electrodes of twice as great cross-section. For the particular size and design of weld above mentioned, the Welding Research Sub-Committee has since had welds made with from 200 to 300 amperes. The conclusion appears justified that the preferable current for such a weld is at least 200 amperes. If the weld of the half-inch-thick plate is of the double-bevel type, some 50 amperes less current should be used for the bottom layer than is used for the second layer, if two layers are used. For three-quarter-inch-thick plates, the most suitable welding current is some 300 amperes. This is of the order of twice the current heretofore usually employed for such a weld.

Mr. Wagner writes: "We have made a number of tests to determine the effect of varying current on the strength of the weld. Tests were made on a ½-in. plate with current values as follows: 80, 125, 150, 180, 220, 275, and 300 amperes. These tests show improvement in the tensile strength and bending qualities of welds as the current increases. The speed of welding increases up to a certain point and then decreases."

20. EFFECT ON ARC WELDING OF VOLTAGE EMPLOYED

Mr. Wagner reports as follows: "We have made a number of tests to determine the influence of variable voltages on the strength and character of electric welds. The experiments were made welding ½ in. plate with 150 amperes held constant and voltage varying as follows: 40, 75, 100, 125, 150, 200 and 225 volts.

"This test demonstrates that there is no material difference in the tensile strength, bending qualities or the appearance of the welded-in material. There is this advantage, however, in the higher voltage, that variations in the strength of the arc do not materially affect the value of the current.

"A curve-drawing ammeter was installed on the welding circuit which showed variations in current at 75 volts, but at 150 volts the current curve was practically a straight line."

21. PREFERABLE SIZE OF ELECTRODE

On certain railways, a single diameter of electrode is employed independently of the size or shape of the plates or parts being welded. The experience of other people leads them to make use of several different sizes of electrodes according to the size of the job and the type of joint. Present British practise appears to be to use such a size of electrode as to have a current density of some 4000 to 6000 amperes per square inch. The investigations of the Welding Research Sub-Committee indicate that at least 10,000 to 12,000 amperes per square inch is suitable for electrodes of ⅛ in. and 5/32 in. diameter and well up toward 10,000 amperes per square inch for electrodes of 3/16 in. and ¾ in. diameter.

22. AUTOMATIC MACHINERY FOR ARC WELDING

Several firms are developing machinery for feeding the electrode automatically. Such machinery appears to be capable of making excellent welds at higher speeds than are attainable by hand feeding.

23. CARBON-ARC WELDING

With the advent of metal-arc welding there has been a tendency to neglect the carbon-arc method. It is quite possible that this attitude is not justified for not only is there now a definite field where the carbon-arc method is advantageous but developments in the art may greatly extend its application.

It is generally agreed that the carbon-arc method is not applicable to vertical and overhead welding, which is, of course, a serious handicap in ship-hull work. The majority opinion of competent observers (with, however, some emphatic dissenting views) appears to indicate that carbon-arc welding is not as reliable as metal-arc welding in ordinary welding, because:

a. Carbon is carried into the deposited material thus reducing its ductility.

b. It is more difficult to obtain good fusion on account of overlapping of deposited metal on the original metal.

c. It is more difficult to manipulate and thus requires greater skill.

d. It is a much hotter arc which means greater discomfort to the operator and therefore lower efficiency.

e. Greater cooling stresses are developed because larger areas of adjacent metal are heated.

On the other hand, it is contended by some that carbon-arc welding can be developed to the point where these objections will no longer exist and thus gain the advantages of this method, the principals of which are:

a. No preparation of the abutting edges is necessary.

b. Greater rate of deposition of metal and therefore greater speed of welding, particularly in heavy work.

c. Probable greater adaptability to automatic welding.

It should be stated that there is very general agreement as to the superiority of the carbon arc over the metal arc for heavy work where strength is not so important, especially cast-iron welding and the filling of holes in iron and steel castings.

24. PREPARATION OF WELDING EDGES

British practise permits the use of smaller angles when the edges of the plates are Veed, than accords with American traditions. If the smaller angles give welds which are equally satisfactory in all respects, the decreased amount of electrode material required, the decreased consumption of electricity, and the increased speed are advantages not to be overlooked; but obviously the matter requires careful investigation. American practise which, up to recently, has been with a very wide angle, appears to have required the consumption of about twice as great a weight of electrode as British practise with the smaller angle. The urgent importance of determining whether the use of the smaller angle involves any sacrifice in quality is evident. There is already considerable basis for the belief that actually better results attend the employment of a smaller angle of bevel when a suitably large current is used. A shoulder in place of the heretofore commonly used sharp bottom edge of the bevel, also constitutes a material gain not only in the saving in welding material, but also in the quality of the weld.

TABLE III.—TIME, METAL, AND CURRENT USED WITH WELDS OF DIFFERENT BEVELS

	Angle of Bevel Used, in Degrees			
	15	30	45	60
Amperes	160.	145.	118.	125.
Weight of electrode used up (lb.)	2.56	3.83	4.63	6.63
Weight of metal deposited (lb.)	1.70	2.55	3.65	5.08
Weight of metal wasted (lb.)	0.86	1.28	0.98	1.55
Pounds deposited per hour	1.82	1.61	1.82	1.81
Feet welded per hour	3.22	1.90	1.50	1.07
Circuit kilowatts	9.91	9.00	7.68	8.25
Kilowatt-hours per foot of weld	3.10	4.70	5.10	7.70

Mr. Wagner states that at Pittsfield they have long adopted the practise of using a 30-degree bevel for plate edgings and that they find it satisfactory for all thicknesses up to ¾ in. He states that this angle gives sufficient room for depositing the metal, reduces the time to weld and the amount of metal deposited.

In one of Mr. Spraragen's researches, various angles of bevel were used. Although the physical tests have not yet been made we can gain from Table III valuable lessons on the time, amount of metal, and electricity consumed for these different angles of bevel. The "free distance" in each case was ⅛ in. and the welding was done in a flat position with 5/32-in. bare electrodes. In each case the weld had a length of three feet.

25. Quality of Overhead Arc Welding

The British Admiralty regards overhead welding as too inferior and too expensive to be employed when it can possibly be avoided. In America a large amount of overhead welding is done in railway shops and it is claimed that it is simply a matter of training operators to the required degree of proficiency.

26. Number of Layers to be Employed

Good progress is being made in obtaining knowledge of the relative characteristics of welds made with different number of layers and of the most suitable current and the most suitable size and type of electrodes to employ for each layer. The tendency is toward the use of at least two layers for half-inch-thick plates, and three layers for three-quarter-inch-thick plate.

27. Rigid vs. Non-rigid Methods of Welding

On this question it is more a matter of determining the conditions essential to obtaining good results with whichever of the two methods is most appropriate for each particular purpose.

The term *rigid* is applied to the process of arc welding, in which the two parts to be joined by welding are, prior to welding, held rigidly by bolting or clamping or by a series of preliminary tack-welds distributed at various points. The rigid plan is the most obvious for welding the hull plates of ships but its critics claim that the resultant joints are deficient in ductility due to the presence of internal stresses. It is considered that by suitably arranging the order of welding it is practicable to so distribute the heat as to avoid these stresses. At any rate, there are many alternative orders of procedure in making welds by the rigid method and elaborate researches should be made to ascertain the procedure which will yield the best result.

The non-rigid method consists in placing at a slight angle to each other the two plates to be welded. As the welding operation progresses along the seam the angle gradually closes and when the weld is completed the width of the welded seam is equal throughout its extent. Such welds are generally considered to be very free from internal stresses, and hence more ductile.

28. CONSEQUENCES OF DIFFERENT LENGTHS OF ARC

The metal arc is much shorter than the carbon arc. As a result the metal arc weld has the advantage that there is less opportunity for oxygen and nitrogen to gain access to the weld and so far as relates to this feature the metal arc weld should be better. But with the carbon arc the added metal does not traverse the arc, the tip of the welding rod being held down close to the surface on which it is to be deposited. This may render the deposited material less subject to contamination in carbon arc welding than in metal arc welding since it has not traversed the arc.

Coming to the exclusive consideration of metal-arc welding, the greater the welding current the less is the area represented by the cylindrical surface of the arc per pound of metal traversing the arc, and consequently the less should be the contamination by oxygen and nitrogen from the surrounding air. So far as this circumstance is concerned, the greater the welding current, for a given case, the greater should be the ductility of the joint. On the other hand, it seems probable that even the most skilful operators will be unable to hold quite so short an arc with the larger current.

29. SPOT AND ARC WELDING

A good deal of progress is being made in America in the use of spot welding for the joining of thick plates. It is believed that spot welding has a great future as applied to shipbuilding and several large spot welders have been built for shipyards.

In some of its applications, spot welding affords a method of preliminarily joining the hull plates, after which the required additional strength is provided by arc welding. The Welding Research Sub-Committee has already made some progress in comparing combined spot and arc welds and combined rivet and arc welds with riveted, spot-welded, and arc-welded joints.

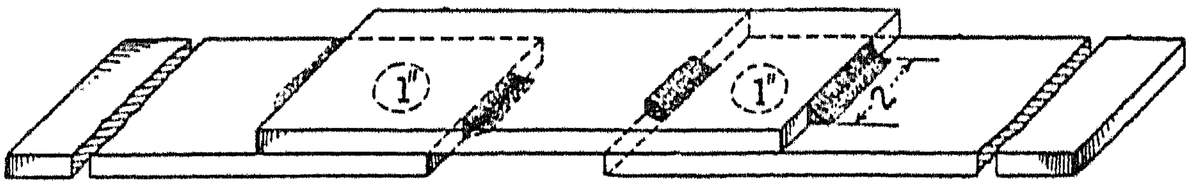


FIG. 9—FILLET AND SPOT WELDED

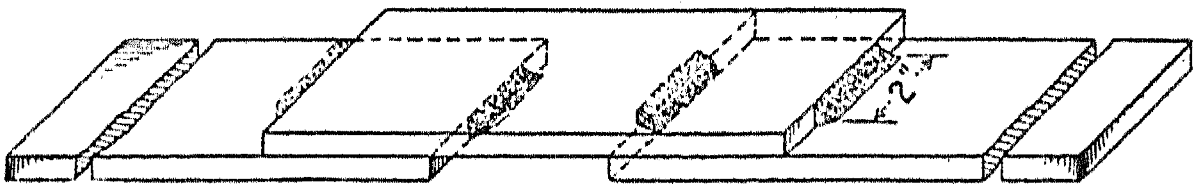


FIG. 10—FILLET WELDED

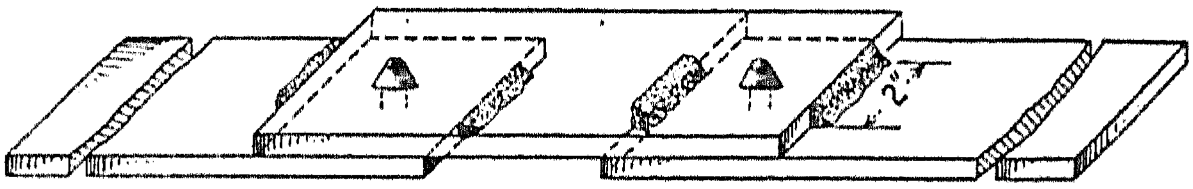


FIG. 11—RIVETED AND FILLET WELDED

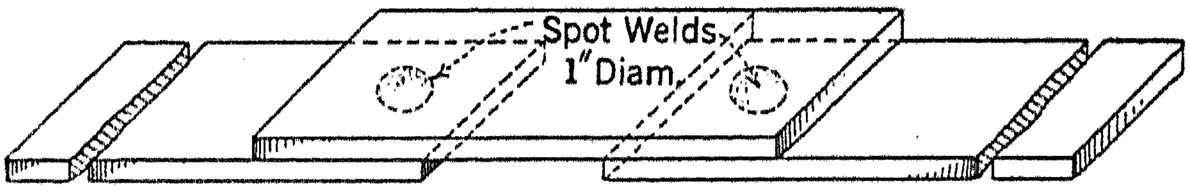


FIG. 12—SPOT WELDED

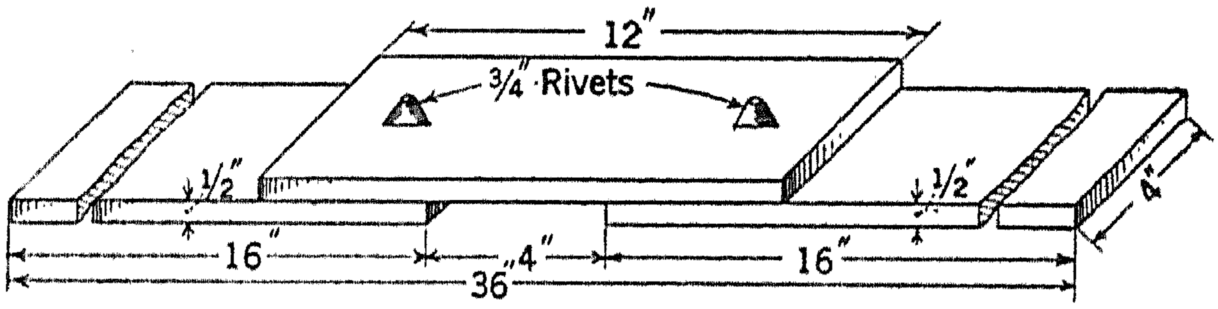


FIG. 13—RIVETED JOINT

It is not a question in such an investigation of spot *versus* arc welding, but of spot *and* arc welding.

In the tests mentioned, the specimens were made up of the following combination:

1. Spot and fillet welded Fig. 9 (2 samples made).
2. Fillet welded—made by welding fillets about two inches in length at the ends of the plates Fig. 10 (2 samples made).
3. Riveted and fillet welded Fig. 11 (1 sample made.)

4. Spot welded—made by welding two spots approximately one inch in diameter, on the plates Fig. 12 (2 samples made).

5. Riveted joint, made by riveting a ½-inch by 4-inch by 12-inch plate with two plates ½-inch by 4-inch by 16-inch, using two ¾-inch rivets and a 4-inch lap. (One sample made).

The results of the test show the comparative strength of the joints as follows:

Spot and fillet welded—	ultimate	load. . .	*50,350 lb.
Fillet welded—	"	" . . .	*37,000 "
Riveted and fillet welded—	"	" . . .	†35,000 "
Spot welded—	"	" . . .	*28,000 "
Riveted joint—	"	" . . .	†13,000 "

Spot welds, as compared with arc-welded butt joints, have the disadvantage of the increased weight corresponding to the overlap.

30. CONDITION OF SURFACES TO BE WELDED

While for spot welding the surfaces may sometimes be *too* clean to obtain the best weld, this cannot be the case with fusion welding. The question of the extent to which it is practicable to go in freeing the surfaces from impurities prior to making the fusion weld is entirely a commercial one. The cleaner the surface, the better the weld. In spot welding it is desirable to have clean surfaces under the electrodes, but scale *between* the two plates is a positive advantage.

31. PRE-HEATING AND HEAT TREATMENT AND HAMMERING WHILE COOLING

Pre-heating, heat treatment and hammering, as applied to fusion welding (both gas and electric) have been the subjects of research, but as yet nothing adequately comprehensive has been planned. It is very important that these deficiencies should be recognized and remedied.

32. TIME REQUIRED TO TRAIN WELDERS

As to the training of welders, there are on the one hand those who are so ill-informed as to assert that good welding can be done after a very few weeks of training, while on the other hand, others assert that an operator can hardly acquire any considerable degree of skill in much less than a year. Still others advocate the substitution of automatic feeds and the complete elimination of dependence upon manual dexterity. For arc welding work requiring more than 100 amperes this is, in the author's opinion, not a trade for women when the arc is manipulated by hand as distinguished from the use of automatic or semi-automatic machinery.

---

*Average of tests on two samples.

†Only one sample made.

33. QUESTION OF NEED FOR SPECIAL MACHINES FOR WELDING

A great variety of machinery for supplying and controlling the current for welding is on the market. Some of this machinery comprises elaborate mechanisms in virtue of which it is claimed that it would be very difficult for even a novice to make a bad weld. Some advocate the use of simple resistance to be inserted in series with the arc on any available circuit, and claim that any additional machinery is superfluous. The capital outlay for the equipment of a welder (at the point of consumption) when the first kind of equipment is used, may be a matter of over $1000.00, while in the second case, well below half of that sum is sufficient.

34. TECHNIQUE OF TESTING WELDS

The ideal weld should presumably be at least as strong and as durable as the metal of the members joined together. In other words, the section containing the weld should have the same chemical and physical characteristics as adjacent sections in the original metal. A weld is therefore measured by the degree of approximation to this condition as dertermined by mechanical, chemical and metallurgical tests of:

a. The parent metal
b. The welded joint
c. The deposited material in the weld.

While during the last year the Welding Research Sub-Committee has made a great deal of progress in establishing standard procedures for the mechanical testing of welds, much still remains to be done. Obviously, the procedure for testing the original metal should follow standard practise as recommended by the American Society for Testing Materials, but there is considerable difference of opinion and uncertainty as to just how and what mechanical tests should be made of the welded joint and of the deposited metal. For instance

a. Should all the usual observations be taken when making a tensile test of a welded joint? Obviously the strength of the union between the two pieces of metal should be determined but in view of the non-homogeneity of the specimen, does not a very different significance attach to yield point, elongation and reduction of area? Where a series of welds having the same ratio of deposited material to original metal is concerned, such data are undoubtedly important for comparison purposes but for evaluating a weld in terms of the original metal, questions are repeatedly being raised as to just what extent these data have value.

b. Would not more reliable information as to the ductility of the weld be obtained if elongation and reduction of area measurements were made on specimens prepared from the deposited metal or from specimens cut lengthwise of the weld instead of crosswise?

c. Similarly with the bending test, which is a test for ductility. There are some (including the author) who would make the bending with the axis of the mandrel *normal* to the weld instead of parallel thereto, which latter position is the one usually employed. It may be that both tests should be made; the normal position as test of the ductility of the deposited material and the parallel position as an additional test of the union between the deposited material and the original metal.

d. How important are torsion tests and impact or shock tests in measuring welded joints?

e. Fatigue tests of welded joints are generally conceded to be vital and the importance of obtaining reliable information as to how this test should be made probably transcends (at present at least) that attached to any other research in the field of fusion welding. The researches should be made:

1. With the Moore bending fatigue machine
2. With rod samples rotated at high speed, as employed by Lloyd's Register in England
3. With the Strohmenger torsion-fatigue machine
4. With the Cammell-Laird bending fatigue apparatus
5. By the Upton-Lewis test.

After the necessary research work has been done to solve these and other similar questions pertaining to the testing of welds, standard specifications for the testing procedure can be prepared which will be properly balanced between the cost of making the tests and the amount of testing necessary to insure a reliable estimate of the weld.

CONCLUSION

The extent of the field of application for fusion welding and spot welding is but little appreciated by engineers other than those who have been directly connected with welding developments. It is evident that this field is an enormous one, including as it does all structures where steel is employed, such as bridges, building structures, tanks of all types and kinds, railway rolling stock, and ships, in addition to numberless miscellaneous applications in industry in general.

However, engineers associated with welding research should be on their guard that their enthusiasm over this great field of application shall not lead them into prematurely endorsing the use of fusion welding or spot welding in constructions where the consequences of failure involve serious menace to life and property, as may often be the case. For example, a particularly important case is that of pressure vessels and especially large high-pressure containers. The success in one hundred installations will not excuse failure (accompanied possibly by fatalities), in the one hundred and first installation. It is the opinion amongst the best informed engineers that before fusion welding can advisedly be employed for large high-pressure vessels, much vigorous and elaborate research work should be carried out on the fatigue characteristics of fusion welds of long seams, and that this research work must comprise full-sized structures since the conditions cannot be reproduced in test samples.

In fact, if the general acceptance of welding, particularly by Inspection boards, underwriters, and classification societies, is to be accomplished in a reasonably short time, such extensive research work on a large scale is absolutely essential in order to demonstrate conclusively that welded joints are equal to or better than joints made by other methods. Obviously the development of the art could proceed along the lines of the usual order of evolution as in the cases of previous arts, but this would, as in those cases, involve the lapse of years.

For structures subjected to less extreme stresses, such as the hulls of ships, the adequacy of fusion welding as a substitute for riveting is in process of being thoroughly demonstrated in actual practise in Great Britain. It is recognized that the hulls of ocean-going ships are exposed to very great stresses, nevertheless there is a clear distinction between the magnitude of those stresses and the stresses to which many large, high-pressure containers are subjected.

The author hopes this paper will aid in focusing attention on the vast importance of the welding art, particularly by occasioning discussion of the many problems in welding research some of which have been mentioned in the paper.

The author cannot undertake to give adequate acknowledgment of his indebtedness to his many associates in the preparation of this paper. The most generous assistance has been given him on every hand. Mr. William Spraragen has extended much assistance in preparing data and in many useful ways. Mr. F. M. Farmer, chief engineer of the Electrical Testing Laboratories, has given very generously of his time in advising the author in detail about many points which arose in the course of the preparation of this paper.

Four Appendices accompany this paper:

Appendix A: Standard Procedure for Testing Welding Electrodes.

Appendix B: The Wirt-Jones Tests of Arc Welded Half-Inch Ship Plates.

Appendix C: Bending Tests of Gas Welds.

Appendix D: Arc-Welded Steel Box.

Appendix A

STANDARD PROCEDURE FOR TESTING WELDING ELECTRODES.
WELDING COMMITTEE, EMERGENCY FLEET CORPORATION
NOVEMBER, 1918

1. *Purposes.* The purpose of this specification is to provide a standard procedure for testing welding electrodes for metal-arc welding which are submitted for the information of, and with the view to the approval by, the Welding Committee of the Emergency Fleet Corporation.

Note.—This specification describes a test *of electrodes* and not a combination of an electrode and of an apparatus. The fact that the applicant is given the option of selecting the system with which the test is made, does not make it a test of that system, any more than the employment of a particular welding operator could be said to make it a test of a welding operator. The system used in making these tests may or may not prove to be of importance. It is sought to minimize the influence of the individuality of the operator by requiring the test to include welds made by at least two operators.

The Welding Committee has two other Sub-Committees into whose province falls the approving and certifying of operators and approving and certifying of systems. The admission that these three tasks overlap does not involve any reason for delaying the important task of proceeding at once with the standardizing of the testing *of electrodes.*

2. *General Conditions.* a. Each applicant shall provide at his factory or elsewhere as he may elect, the necessary facilities and at least two operators for making the test welds prescribed by these specifications. Except when otherwise indicated in these specifications, the applicant shall select the apparatus and other conditions for making test welds which he considers most suitable for his electrodes.

b. The test welds shall be made in the presence of an *authorized representative* of the Welding Committee of the Emergency Fleet Corporation who will be empowered to certify as to the compliance with the conditions prescribed in the specifications. Also representatives of Lloyd's Register of Shipping and the American Bureau of Shipping shall have the opportunity to witness the making of the test welds.

c. Until otherwise declared, the *authorized representative* of the Welding Committee shall be the Electrical Testing Laboratories, 80th Street and East End Avenue, New York, N. Y.

d. The Welding Committee shall assume charge of the completed test welds and have them tested by the *authorized representative* in accordance with these specifications.

e. The *authorized representative* shall render to the Welding Committee a complete, detailed report of each test of an electrode.

f. The cost of carrying out the prescribed tests shall be borne by the applicant.

3. *Sample Electrodes.* About 100 pounds of each electrode to be tested, which will be known as the *sample,* shall be furnished without charge. It shall be accompanied by an affidavit giving the following information:

a. The *trade name* under which the electrode is marketed together with certification that all electrodes bearing this trade name will be substantially the same as the sample submitted.

b. The manufacturer of the complete electrode.

c. The location of the factory in which the electrode was made.

d. The manufacturer of the welding wire.

e. The location of the mill in which the welding wire was produced.

f. The chemical analysis of the welding wire.

The *authorized representative* shall retain the remainder of the sample for reference purposes.

4. *Plate Material.* For these tests one-half inch ship plate shall be used. This material shall comply with the "*standard specification for structural steel for ships*" as adopted by the American Society for Testing Materials, serial designation A 12–16 (see page 98, A. S. T. M. Standards, 1918).

An abstract of this specification is as follows:

1. Open hearth steel.

2. Phosphorous $\begin{cases} \text{acid steel, not over 0.06 per cent.} \\ \text{basic steel, not over 0.04 per cent.} \end{cases}$

Sulphur, not over 0.05 per cent.

5. Tensile strength, pounds per square inch, 58000–68000

Yield point, minimum pounds per square inch, 0.5 tensile strength.

Elongation in 8 inches, minimum per cent., $\frac{1{,}500{,}000}{\text{tensile strength}}$

6. Yield point by drop of beam method.

7. Cold bend test; no cracking on outside of bent portion when bent 180 degrees around a pin the diameter of which is equal to the thickness of the plate.

5. *Cutting of Plates.* a. The method of cutting a plate 5 feet by 20 feet into pieces for welding is shown in the accompanying reproduction of drawing No. 102 of the Bureau of Standards.

b. Each plate shall be given a distinguishing letter, A. B. etc. This letter shall appear upon each piece cut from it.

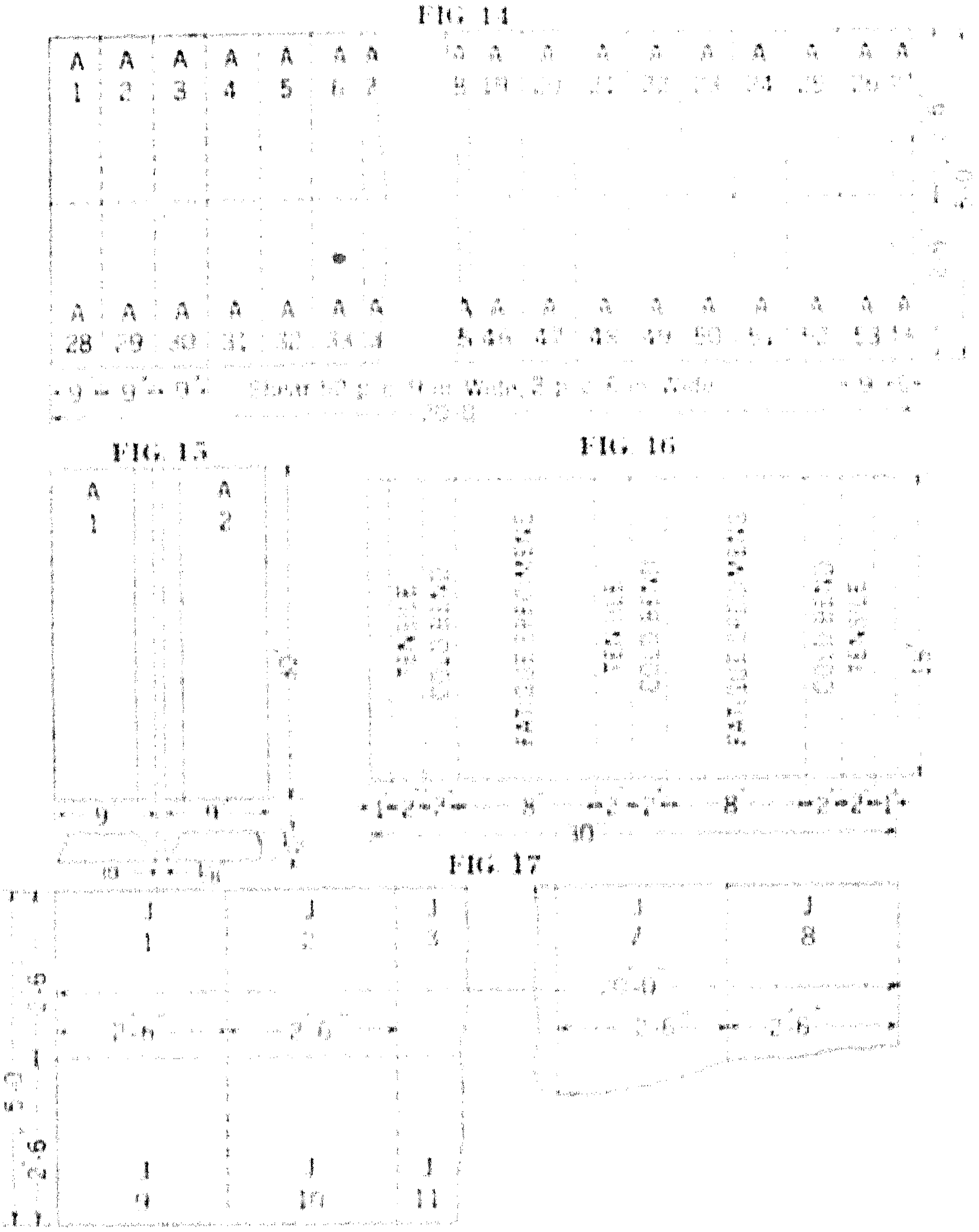
c. Pieces shall be plainly marked with the letter and number arranged as shown in the drawing so that the location of the piece in the plate may be determined.

d. The plates from which tensile, cold bend and fatigue specimens are to be made, shall be cut into pieces 9 inches by 30 inches (see Fig. 14.)

e. The plates from which impact specimens are to be made shall be cut into pieces 30 inches by 30 inches (see Fig. 17).

Notes.—A power shear may be employed for cutting these plates if they are left flat. Pieces whose dimensions are within ¼ inch of the required size will be satisfactory.

6. *Arrangement of Piece for Welding.* a. All welds are to be made across the grain, that is, the welded joint is to be at right angles to the direction of rolling. Fig. 15 shows the correct arrangement of the pieces for the test welds to be used



SPECIMENS FOR TESTS OF WELDING ELECTRODES. WELDING RESEARCH SUB-COMMITTEE EMERGENCY FLEET CORP. BUREAU OF STANDARDS, WASH., D. C.
No. 102 VII-1 Nov. 9, 1918
CHANGED NOV. 25, 1918

for the physical tests, but in Fig. 17, which shows the arrangement of pieces for the impact test, pieces such as J 1 and J 2 should be taken for a test weld instead of J 1 and J 9.

b. Preparatory to making a weld, one 30-inch edge of each piece will be beveled at the angle preferred by the applicant.

If he has no preference, the angle of the bevel shall be 30 degrees as shown in Fig. 15. The bevel is not to be carried through to the bottom edge but only to within 1/16 inch of the bottom edge. In other words, the ends of the pieces to be welded are not to be beveled to a sharp point but 1/16 inch of the original square edge will be left at the bottom of the Vee.

c. For welding, the pieces shall be placed horizontally with their under surfaces raised approximately ⅛ inch above the supporting surface. The beveled edges shall be placed parallel and separated ⅛ inch.

d. The weld shall in all cases be of the type known as a double bevel.

7. *Welding of Pieces.* a. The pieces, which shall be dry and free from rust and foreign substance, shall be "tacked" at each end and in the middle of the joint by welding the plates at each of these places for about one-half inch.

b. After "tacking" the weld shall be completed by working continuously from one end of the joint to the other. The welding material shall be added in at least two layers and when completed the surface of the weld shall be at least flush with and not more than ⅛ inch above the upper surface of the plate. The width of the weld at the top shall not be greater than 1¼ inches. All welding shall be done from the open side of the Vee.

c. After each layer of a weld is completed, it shall be inspected by the *authorized representative.* If unsatisfactory, a new test weld shall be substituted and a report made of reasons for rejection.

8. *Number of Test Welds.* a. The welds will be made by three skilled operators, two of whom will be furnished by the applicant and one by the Welding Committee.

b. Each operator shall make one 30-inch weld for the tensile, cold bend and fatigue tests (see Figure 15) using direct current and a similar weld using alternating current. For these tests, therefore, six (6) test welds will be required for each sample electrode.

c. Each operator shall also make one 30-inch test weld for the impact test (see Fig. 17) using direct current and a similar weld using alternating current. For impact tests, therefore, six (6) test welds will be required for each sample electrode.

d. If, however, the applicant intends his electrode for use with only one kind of current, that kind only shall be used in making the welds. In that case only three (3) test welds will be required for (b) and for (c) instead of six (6).

9. *Arrangement of Conductors.* a. One of the conductors shall be attached to a clamp which will be fastened to both pieces at the end of the joint opposite that from which work is started.

b. For direct-current work, the electrode shall have the polarity desired by the applicant. In the absence of any preference on his part, the instructions of the *authorized representative* shall be followed.

10. *Welding Data.* The following information and observations shall be reported by the *authorized representative* during the welding operations.

a. Trade name or other identification mark of the electrodes.

b. Complete description of the electrode.

c. Sufficient description to identify the welding apparatus or system employed.

d. Identification marks on the pieces being welded.

e. Name of operator.

f Kind of current (*i. e.*, direct current or alternating current); if direct current, polarity and if alternating current, frequency.

g. Electrical quantities as follows: arc volts (both open and closed circuit), arc current and arc watts.

h. Room temperature and humidity.

i. The opinion of the *authorized representative* of the working quality of the electrode. This statement to include a description of any sputtering, boiling or other noticeable peculiarities.

j Elapsed time per weld.

k. Weight of electrode consumed per weld.

l. Any other information which will assist in determining the performance of the electrode, such as a photograph of at least one weld, etc.

11. *Preparation of Welded Plate and of Specimens for Physical Tests* (see Fig. 16).

a. Each test weld shall be machined down on both sides to about the surface of the plate.

b. Specimens shall be cut from *each* test weld reserved for physical tests as follows:

1. *Three Tensile Specimens.* These shall be machined to a uniform width of 1.5 inches unless a weld of great

strength makes it necessary to leave shoulders at the ends, in which case the standard A. S. T. M. test specimens for sheet iron and steel shall be prepared.

2. *Three Cold Bend Specimens.* These shall be machined to a uniform width of 1.5 inches.

3. *Six Fatigue Specimens.*—These shall be machined to about ½ inch diameter and 10 inches long. (The exact dimensions are to be determined by experiment.)

12. *Physical Tests.* a. *Tensile Strength.* Each of the three specimens shall be tested in accordance with the practise recommended by the A. S. T. M. and shall include the determination of the tensile strength, yield point (by drop-of-beam), method), reduction of area and total elongation after rupture in two inches and in eight inches.

b. *Cold-bend Test.* This test shall be made by placing the specimen on two rollers with the apex of the Vee upward and midway between the rollers and loaded at the center of the span thus formed by a cylindrical surface having a diameter of one-half inch. This surface shall bend the specimen downward between the rollers until a fracture appears on the lower side of the specimen when loading shall be stopped and the angle noted through which the specimen has been bent.

c. *Fatigue Test.* Each of the six specimens shall be tested in a special rotating type of machine similar to that used by Lloyd's Register of shipping. (Exact details to be determined by experiment.)

d. *Impact Test.* Each impact-test specimen shall be placed on supports 18 inches high and 4½ feet apart. A spherical weight of 500 pounds shall be allowed to fall freely through a distance of 10 feet striking the weld which shall be at the center of the span. The apex of the Vee shall be upward.

e. *Test of Original Plate.* In order to establish the physical properties of the unwelded plate, tensile, cold-bend and fatigue tests shall be made on a sample selected at random from the pieces used for the test welds but before such welds are made.

OPTIONAL TESTS

The following tests are optional with the applicant but the welding Committee considers that they give information of great importance and recommends that they be made in order to make the report on the electrode more complete.

13. *Chemical Analysis.* A chemical analysis shall be made of:

a. The original plate in one test weld selected at random.

b. The metal at the center of one test weld selected at random. If test welds are made with both direct current and alternating current, an analysis shall be made of one weld of each set.

14. *Photomicrographs.* Photomicrographs shall be made of one specimen weld selected at random as follows: (If test welds are made with both direct current and alternating current, photomicrographs shall be made of one weld of each set.)

a. At center of weld.
b. At juncture of weld and original metal.
c. In adjacent original metal.
d. Cross section of electrode.
e. Longitudinal section of electrode.

Appendix B

WIRT-JONES TESTS OF ARC-WELDED HALF-INCH SHIP PLATE

In Tables IV and V are set forth the results of the tests carried out in 1918 on arc-welded half-inch ship plate by the Welding Research Sub-Committee. The plates with which the welds were made had their edges prepared with a 45-degree double-Vee. The welds were made by a dozen different concerns. In each instance the operator employed whatever conditions were considered most suitable. Several types and sizes of electrodes, including both bare and covered, were used. While in the majority of cases direct current was employed, alternating currents of 25 and 60 cycles are also represented in the series. The number of layers and the current employed and, indeed, all the conditions, were left to the discretion of the individual operator. The length of the weld was eight inches. After the welds were made, the plates were cut up and tests were made at the Bureau of Standards under the direction of Prof. H. L. Whittemore. The tensile tests were made on specimens with a cross-section of one in. by one-half in. Specimens with the weld machined have the projecting metal planed off so that the welded portion is smooth and of approximately the same cross-section as the remainder of the specimen. For the specimens with the weld not machined, no record was made of the added cross-section due to the projecting material.

The cold bend specimens were one in. by one-half in. and were bent around a mandrel of 1.5 in. diameter. The results

TABLE IV
WIRT-JONES TESTS OF ONE-HALF-INCH ARC-WELDED SHIP PLATES

(Machined Samples)

Test number	Current in amperes		Diam. of elec. in in.	Current density, amp. per sq. in.	Tensile tests		Bending	Electrode		
	a-c.	d-c.			Ult. lb. per sq. in.	Per cent. elon. in 2 in.	Angle at which crack starts, degrees	Cov.	Bare	Make
20	175	...	0.125	14,227	62,700	9.0	34	C	..	E.A.C.& W.Co.
1	...	175	0.166	9,118	59,800	9.0	42	..	B	Roebling
22	160	...	0.125	13,008	50,000	6.0	78	C	..	E.A.C.& W.Co.
2	...	155	0.156	8,075	62,600	11.5	44	..	B	Roebling
19	150	...	0.125	12,195	62,800	12.0	32	C	..	Quasi
31	...	150	0.166	6,930	60,900	8.0	..	..	B	Roebling
29	...	150	0.156	7,815	56,400	6.5	..	..	B	W.W.& M.Co.
38	150	...	0.156	7,815	54,500	7.0	45	C	..	E.A.C.& W.Co.
29	...	150	0.156	7,815	54,400	6.0	47	..	B	W.W.& M.Co.
9	...	150	0.156	7,815	44,200	4.0	26	..	B	Toncan
9A	150	...	0.187	5,430	42,500	4.0	30	..	B	Armco
10	...	150	0.156	7,815	41,900	4.5	25	..	B	Toncan
10	...	150	0.156	7,815	41,300	4.5	24	..	B	Toncan
9	...	150	0.156	7,815	38,100	4.0	15	..	B	Toncan
9A	150	...	0.187	5,430	36,300	4.0	..	..	B	Armco
5	...	145	0.156	7,554	61,100	8.5	55	..	B	Am.S.& W.Co.
25	136	...	0.156	7,085	43,700	3.5	26	..	B	Roebling
30	...	135	0.156	7,033	53,200	11.0	..	..	B	W.W.& M.Co.
30	...	135	0.156	7,033	52,000	10.5	50		B	W.W.& M.Co.
37	125	...	0.125	10,162	56,400	7.0	42	C	...	E.A.C.& W.Co.
7	...	120	0.125	9,756	57,600	8.5	50	..	B	Roebling
7	...	120	0.125	9,756	53,700	7.0	45	..	B	Roebling
8	...	115	0.125	9,349	59,400	13.5	60	..	B	Roebling
8	...	115	0.125	9,349	58,200	14.0	..	..	B	Roebling
3	...	115	0.156	5,891	50,600	5.0	34	..	B	Am.S.& W.Co.
16	...	110	0.134	7,799	50,250	7.0	35	C	..	Quasi
27	...	110	0.125	8,943	45,800	8.0	42	..	B	Roebling
21	105	...	0.134	7,444	42,200	5.0	42	C	..	E.A.C.& W.Co.
26	94	...	0.125	7,642	34,200	3.0	15	..	B	Roebling
18	90	...	0.134	6,381	54,000	6.0	65	C	..	Quasi
15	...	90	0.134	6,381	39,400	4.0	30	C	..	Quasi
24		86	0.125	7,154	45,400	6.0	15	..	B	Roebling
23	...	85	0.125	6,930	51,900	4.0	..	..	B	Roebling

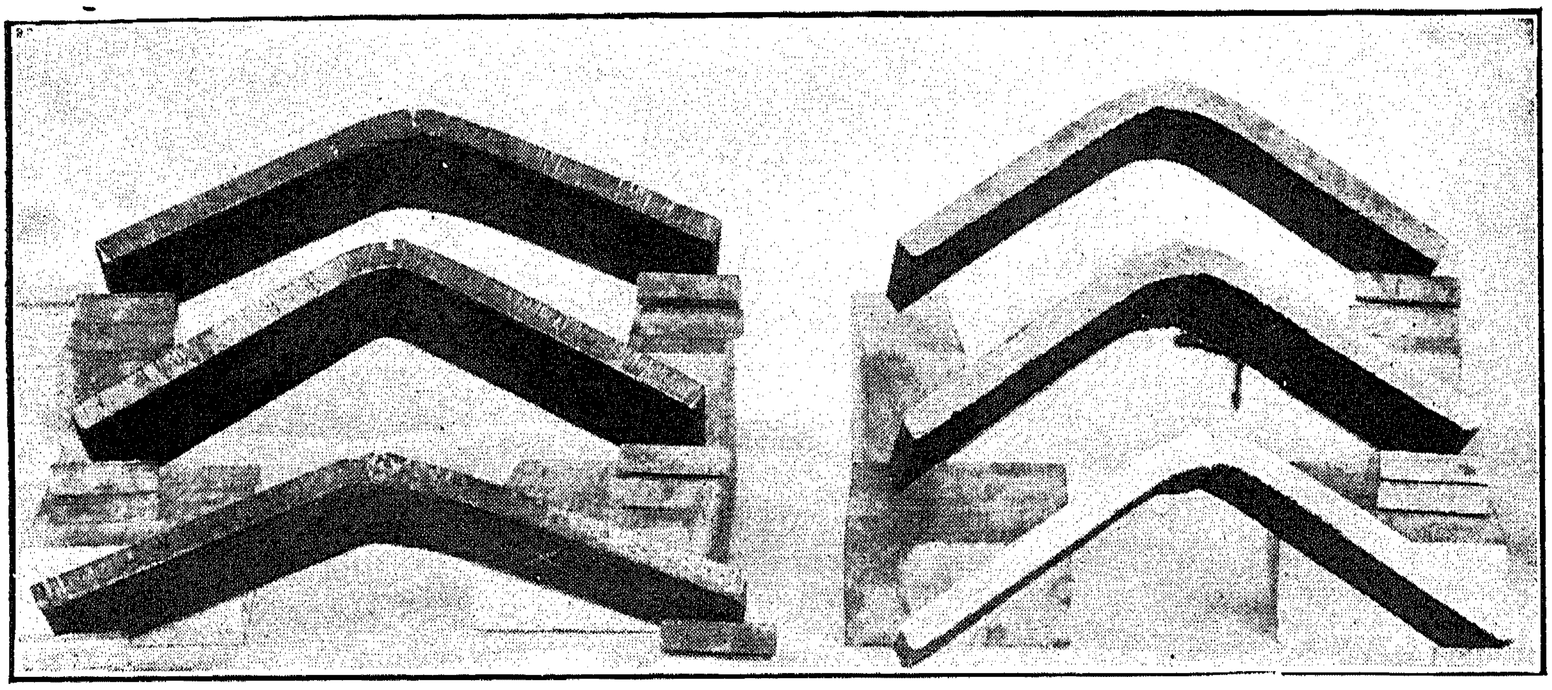


FIG. 8—GAS AND ELECTRIC BENT SAMPLES (GAS WELDS AT LEFT AND ELECTRIC WELDS AT RIGHT)

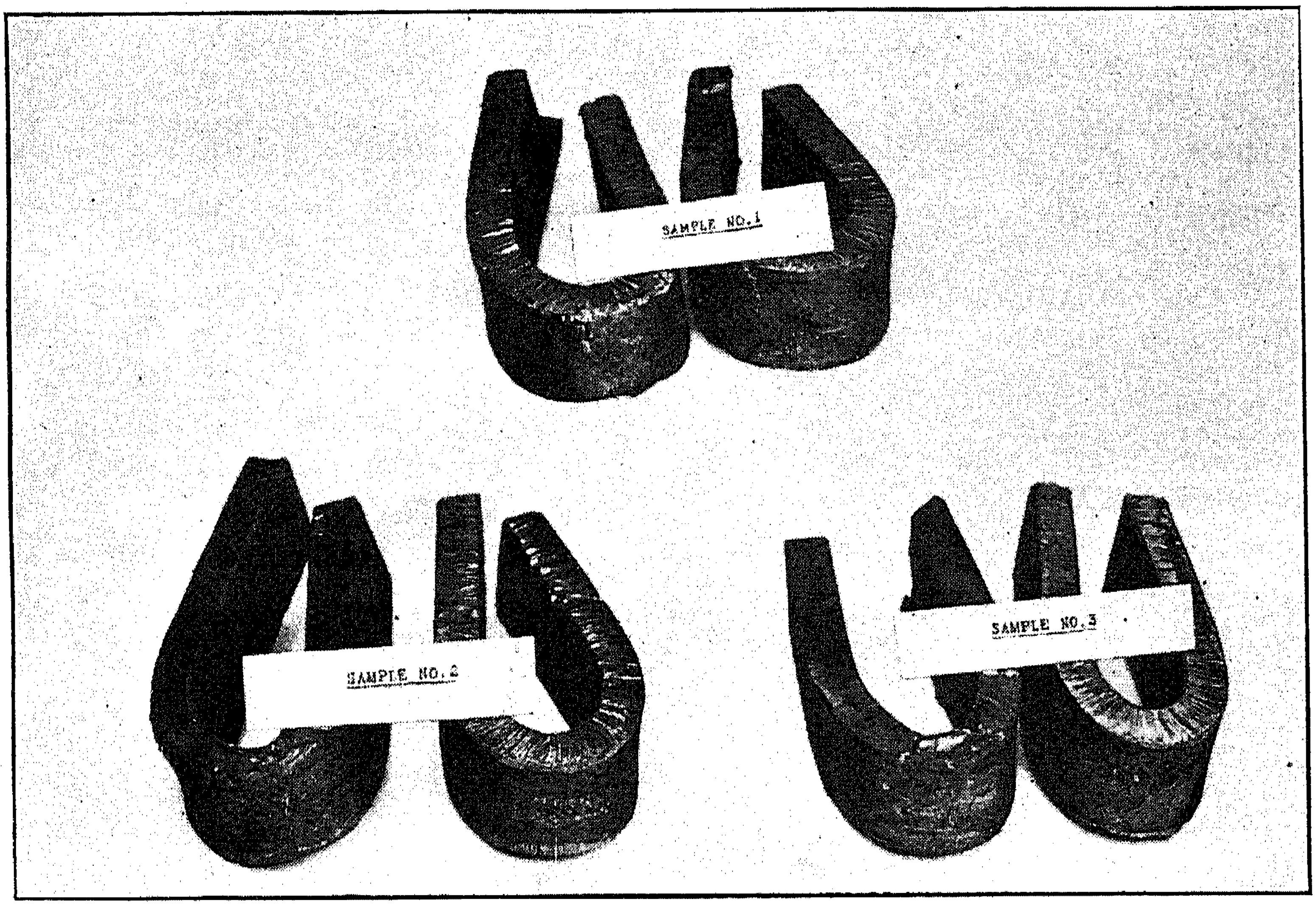


FIG. 18

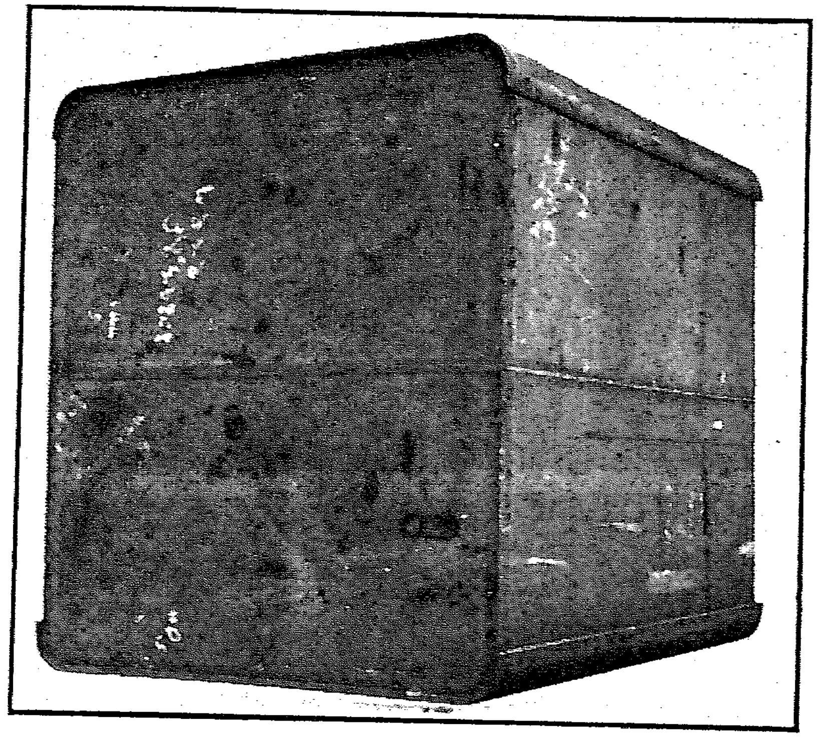
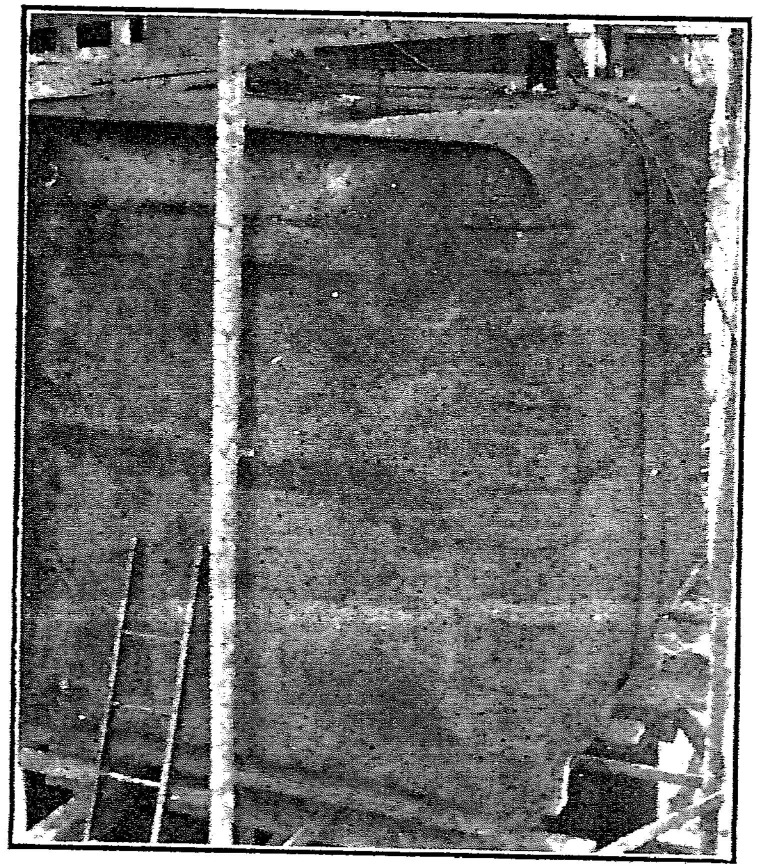
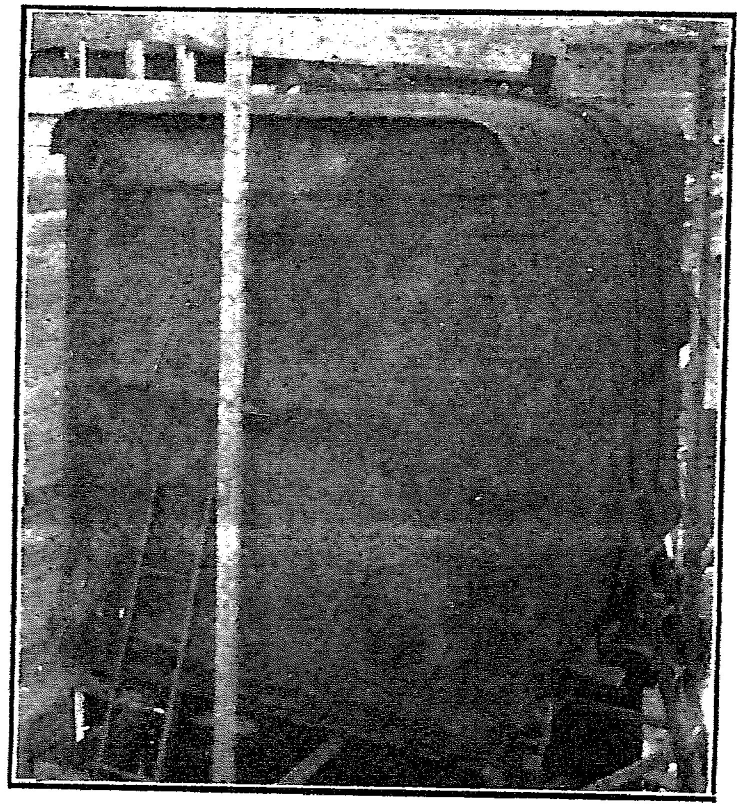


FIG. 20—ELECTRIC-ARC WELDED STEEL BOX OF ONE-HALF INCH TANK STEEL



FIG, 21—ELECTRIC-ARC WELDED BOX UNDER 22-INCH VACUUM—SHOWING TOP SIDE DEPRESSED



[HOBART]

FIG. 22—ELECTRIC-ARC WELDED STEEL BOX UNDER 15 LB. HYDROSTATIC PRESSURE—ONE PHASE OF THE BREATHING TEST

of the tensile and bending tests for the machined samples are given in Table IV. Those for the samples which were not machined are given in Table V.

TABLE V.—WIRT-JONES TESTS OF ONE-HALF INCH ARC-WELDED SHIP PLATES

(Samples Not Machined)

Test No.	Current in amperes		Diam. of elec. in inches	Current density, amp. per sq. in.	Tensile tests		Electrode		
	A. C.	D. C.			Ult. T. S., per sq. in.	Per cent elong. in 2 in.	Coat.	Bare	Make of Electrode
20	175	. . .	0.125	14,275	66,480	11.0	C	. .	E. A. C. & W. Co.
22	160	. . .	0.125	13,008	66,400	9.0	C	. .	E. A. C. & W. Co.
19	150	. . .	0.125	12,195	65,400	6.0	C		Quasi
5	. . .	145	0.156	7,554	52,280	7.0		B	Am. S. & W. Co.
25	136	. . .	0.156	7,085	47,800	4.0		B	Roebling
3	. . .	115	0.156	5,891	65,470	12.0		B	Roebling
27	. . .	110	0.125	8,943	48,500	7.0		B	Am. S. & W. Co.
16	. . .	90	0.134	6,381	58,740	5.2	C	. .	Quasi
18	90	. . .	0.134	6,381	61,760	11.0	C	. .	Quasi
15	. .	90	0.134	6,381	57,540	8.0	C		Quasi
23	. . .	85	0.125	6,910	57,000	5.0		B	Roebling

Appendix C

BENDING TESTS OF GAS WELDS

Too late for inclusion in the body of this paper there has been brought to the author's attention some gas welds made under the direction of Mr. H. I. Walsh, which have withstood very severe bending tests before failing. The specimens are shown in the illustration, Fig. 18. Three pairs of specimens welded with three kinds of electrodes are shown. These specimens show much greater ductility than those shown in Fig. 8.

Appendix D

ARC-WELDED STEEL BOX

Mr. R. E. Wagner has prepared the following description of a large steel box, arc welded with bare electrodes at Pittsfield. The box was built and tested in the latter half of 1918. It was made of half-inch-thick tank steel and its dimensions were 12 ft. by 10 ft. by 9 ft. The total weight was 16,000 lb. The welding was done with 75-volt direct current. This box contained eleven different varieties of welds. Its general dimensions and the types of welds used are shown in Fig. 19. No rivets were used in its construction.

The principal objects to be ascertained in the construction of this box were:

1. Could a structure of this character be made close to drawing dimensions and without excessive distortions and warping of plates and parts?
2. Would the structure be strong and capable of withstanding severe shocks and distortions without serious ruptures?
3. What are the detail costs and time required to build such a structure?

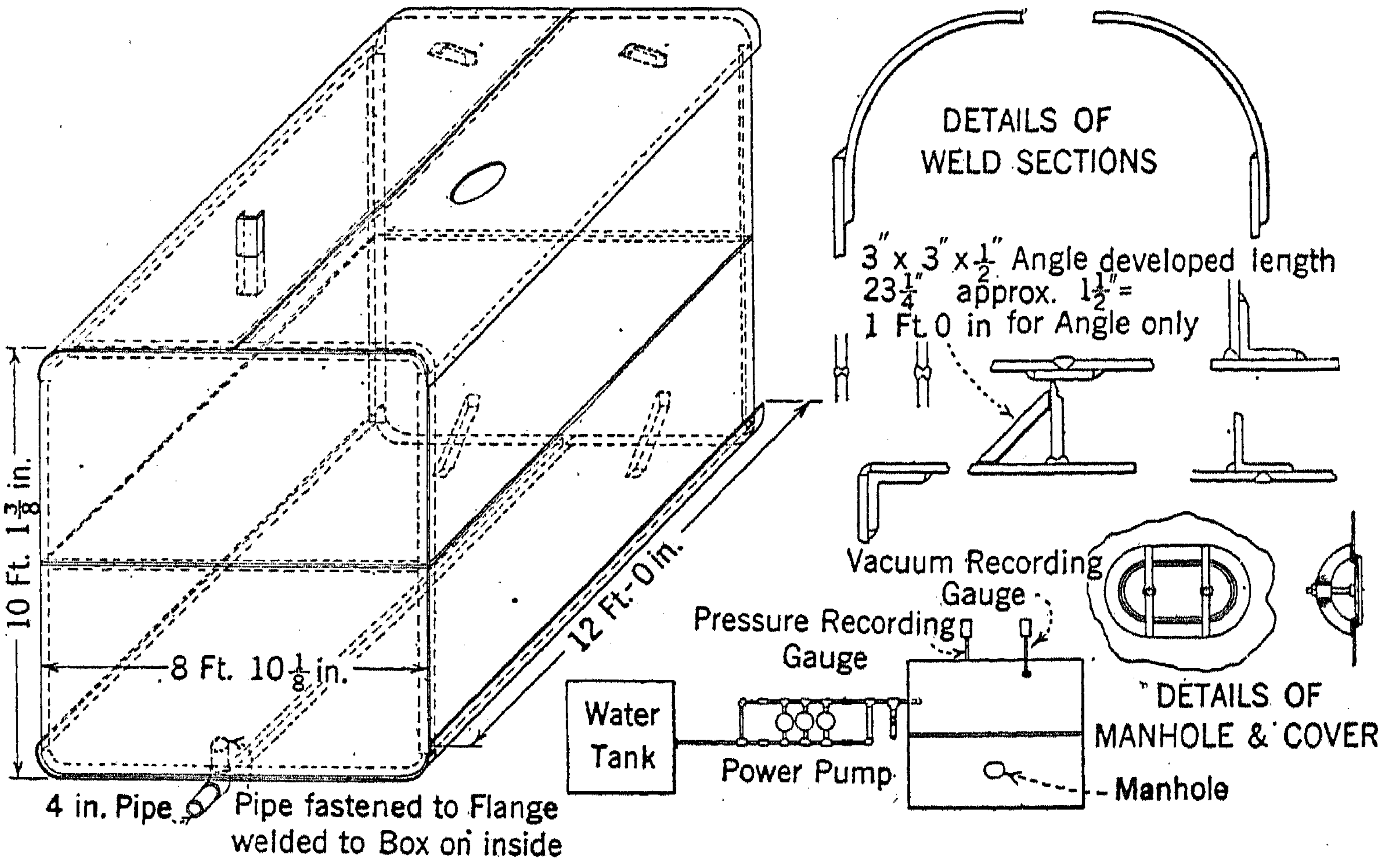

DETAILS OF WELD SECTIONS
3" x 3" x 1/2" Angle developed length 23 1/4" approx. 1 1/2" = 1 Ft. 0 in for Angle only
10 Ft. 1 3/8 in.
8 Ft. 10 1/8 in.
12 Ft.-0 in.
4 in. Pipe
Pipe fastened to Flange welded to Box on inside
Water Tank
Power Pump
Pressure Recording Gauge
Vacuum Recording Gauge
Manhole
DETAILS OF MANHOLE & COVER

FIG. 19—OUTLINE AND DIMENSIONS OF ARC-WELDED STEEL BOX

It was found that if proper allowances are made for contraction and expansion, especially where plates are butted and welded, the resulting plate will show very little warping. The maximum variation of the sides from a plane was 1.5 in. The general appearance of the box when completed is shown in Fig. 20.

The box, when finished, was strong and well built. All joints had a smooth workmanlike finish. A number of incipient leaks developed when the box was first filled with water, but these were closed by hammering. The box was subjected to a hydrostatic pressure test of 43 lb. per square inch, and, after it had been severely stressed and distorted, it ruptured at a corner, the break starting at the top and opening the entire corner. This corner was afterward repaired (as shown in Fig. 21), and the box was subjected to a "breathing" test consisting of alternate applications of 15 lb. pressure and 22 in. of vacuum.

During each cycle of this "breathing" test, the top of the tank had a maximum movement of 8 in. from bulge to depression. At the sides the movement was 6 in. Every time that the side, bottom and top plates moved from bulge to depression, there was a sharp snap of the plates as they moved from "arch in" to "arch out." These sudden changes in form subjected the welded joints not only to bending stresses but to severe snapping stresses. At the end of the twelfth cycle a break occurred in a double-bevel weld. At one end the break left the weld and went into the solid plate. This break was patched and the "breathing" tests were resumed. The next break which occurred after the 235th cycle of operation extended 24 in. into the solid end plate as well as 17 in. along a weld. This break will be repaired and the tests will be resumed.

The following cost data are of interest:

Welding	$ 151.00
Fabricating and assembling	134.00
Blacksmithing	24.00
Testing and supervision	50.00
Welding wire	30.00
Steel plate	651.00
Total cost	$1040.00

Total pounds of wire used	334
Total pounds of wire deposited and lost	299
Total scrap ends of wire	35
Total feet of welding	501
Total hours of welding	165
Total kilowatt hours	1011
Per cent scrap ends of wire	11
Wire deposited per foot (pounds)	0.6
Kilowatt hours per pound of wire deposited	3.4
Average feet welded per hour	3.0
Average kilowatt hours per foot of welding	2.0
Per cent. of time welder is welding	60
Diameter of welding wire, inch	$^3/_{16}$

Six different welders obtained the results noted, which are representative for a job of this character.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 19, 1919.

Copyright 1918. By A. I. E. E.

WELDING AS A PROCESS IN SHIP CONSTRUCTION

BY S. V. GOODALL

ABSTRACT OF PAPER

The paper points to the necessity for a reduction in the cost of shipbuilding, and as riveting is one of the most expensive items of construction the substitution of welding for riveting would decrease the cost of construction considerably. A brief review is given of what has been done in substituting welding for riveting, and to the limited extent to which electric welding has been tried it has been found successful. Lloyd's Register is prepared to classify electrically-welded vessels subject to certain provisions, but shipbuilders have not as yet adopted welding to a large extent for the reason, in the author's opinion, that they know that welds are lacking in uniformity and it is impossible to tell when a joint is good or bad. The author believes that the correct method of approaching the problem of electric welding is first to obtain workmen who are thoroughly trained and skilled and the suggestion is made that only certified men should be employed at this work. From the welding of minor parts the process could be advanced to more important members until eventually a ship would be produced in which both the processes of welding and riveting would be utilized in whatever proportions would attain maximum economy of construction.

DURING the war, economy of time in the production of all that was needed to terminate hostilities successfully was of paramount importance. Economy in first cost and maintenance was relatively unimportant. If the production of a ship, an aeroplane, a tank or any implement of war could hasten the end by a single day it was more economical, in the long run, to place that implement on service today instead of tomorrow, even at a somewhat higher cost, for the shortening of the struggle by that single day meant a saving of many lives and millions of dollars. But besides being called upon to save time the engineer was forced to economize in labor and to use unskilled workmen. During the war gas welding and electric welding have been enormously developed as processes in the manufacture of munitions of all kinds. It goes without saying, therefore, that in these processes engineers found a means of saving time and labor.

Now that hostilities have ceased, economy in first cost and maintenance becomes once again of primary importance, economy of time and labor being of value only in so far as it results in cheaper production. This is particularly true of shipbuilding. Every shipbuilder should now carefully review the various items of cost in the production of a ship, and while he may devote his energies to a reduction all round, he will, if he be wise, give greatest attention to a reduction in the cost of the most expensive of those items. Riveting is this most expensive item. It has been estimated that the labor cost of riveting is about 40 per cent of the total labor cost of building the structure of a steel cargo carrier, while the labor cost of shopwork on structural material is about 15 to 20 per cent. During the past few years considerable attention has been given to improvements in shopwork and it is unlikely that much economy can be effected in this direction. Can anything be done to lower the cost of riveting? The advocates of welding reply "Yes, by a wide substitution of welding for riveting." This is a claim that should not be lightly placed on one side but carefully and critically examined by all shipbuilders. To offer some slight assistance in that examination this paper has been prepared.

Gas welding has been used in shipyards for many years, resulting in economy in the production of staple angles and similar smith work. It has found a wide field in the construction of light fittings, being more particularly of value for thin work than for heavy plating. As it may be considered that gas welding has now found its level as a shipbuilding process, this subject is not enlarged upon; the little that is said must not be taken as a measure of its importance.

Electric welding as a science and art has been dealt with at this meeting. As a process in shipbuilding much literature on the subject already exists, the most comprehensive papers known to the author being that by Mr. W. S. Abell, Chief Ship Surveyor of Lloyd's Register, read before the British North East Coast Institution of Shipbuilders and Engineers at Newcastle in November, 1918, and that by Mr. H. Jasper Cox, read before the Society of Naval Architects and Marine Engineers at Philadelphia in the same month. The author makes no attempt, even if he were capable of doing so, of giving such detailed particulars of the subjects as can be found in these and other papers, but he does attempt to speak, as a shipbuilder to shipbuilders, plainly and impartially, without extravagant claims, and with some little knowledge of experience so far obtained, on the present position of the process and the policy which should be adopted, looking towards the future.

It is very well known that electric welding has been of great value on repair work, particularly during the war. Ships that would otherwise have been laid up for weeks while new castings were obtained, old plates removed and replaced by new, &c., &c., have been rapidly repaired and returned to service. If the ship repairer finds a process of such value the shipbuilder should at least inquire whether it would not be of some value to him.

The Classification Societies have sanctioned the use of electric welding for a considerable amount of work of little importance so far as structural strength is concerned. The Welding Committee of the Emergency Fleet Corporation has found by actual experience that a labor saving of at least 60 per cent results from the substitution of arc welding for riveting on these minor parts of a ship; and Mr. J. H. Anderton has estimated that a saving in time of 70 per cent was effected at Hog Island.

Lloyd's Register of Shipbuilding is prepared to classify an electrically welded vessel, subject to the notations, "Experimental" and "Electrically Welded," and provided the builders conform to certain rules. This decision was reached after an extensive and exhaustive series of trials. Before any electric welding system can be employed on a vessel which Lloyd's will classify, specimens welded according to that system must satisfy tests, which, at first sight, appear almost prohibitive in their severity, but an electrically welded ship is now under construction at Cammell Laird's, England, to be classified by Lloyd's, who have approved the employment of the Quasi-Arc Company's system in its construction. This is a small vessel about 150 feet long, with plating generally 3/8 to 1/2 inch in thickness.

Electric welding work carried out by the British Admiralty has been described in a paper read by the author before the Engineers' Club of Philadelphia, in July, 1918. Since that time valuable experience with this work on service has been obtained. The arc welded barge has been satisfactory. The cost of material, electric current and labor for this vessel was £301 ($1463) as compared with £389 ($1892) for the cost of riveting, caulking and drilling on a similar barge constructed at the same yard. The vessel was designed to be rivetless as a demonstration of arc welding; it was found that greater economy would result if certain parts such as beams to frames, floor plates to angles, etc., were riveted.

As the plating of this barge is thin and the stresses to which it is subjected are not comparable to those which a large seagoing ship is called upon to withstand, greater interest attaches to experience with other British ships in which arc welding has been employed by the Admiralty, particularly to those where the work has been comparatively heavy. It must be said at once that failures of welded joints have been reported, although the number of such failures has not been large considering the amount of work that has been done, the lack of experience, the dearth of skilled operators, and the radical change in method. The Admiralty authorities, however, consider it wise, pending further experience, to defer the adoption of electric welding for those parts of the structure subjected to high complex stresses unless the work can be so tested as to demonstrate exactly that such stresses can be safely carried.

Lap welding has been more satisfactory than butt welding and the latter has been, for the present, discarded for all important work where the plating is 3/8-inch or more in thickness. It is hoped that this situation will not last long, for it is in butt welding that the greatest gain from the use of this process may be expected, and lap joints necessitate a certain amount of overhead welding which is difficult and unreliable.

While the above briefly sums up the present position of electric welding as a process in shipbuilding it may be fairly asked why—if the Classification Societies permit the process on minor parts; if Lloyd's are prepared, subject to certain conditions to classify an electrically welded ship; if the British Admiralty has employed the process to a considerable extent; if American railroads have used welding extensively on locomotives—why has not electric welding been adopted in American shipyards to a greater extent than it has been up to the present? There is no doubt that during the war, shipbuilders in the United States were too hard pressed to be able to give much time and thought to the development of an entirely new process in ship construction. That is one reason. Inate conservatism is another. But the real reason, in the author's opinion, is that shipbuilders know that, at present, welds are lacking in uniformity, and it is not yet possible to tell when the welded joint is good or bad. Hence they hang back and the author commends their wisdom.

It is far better that the first ocean-going electrically welded ship should be on service three years hence with every joint sufficiently sound—universally accepted as a thorough success—than that it should be at the bottom of the sea tomorrow because, while 999 joints were sound, one went astray.

How can this certainty that every joint will be efficient be ensured? While the electrical engineer and the metallurgist can assist the shipbuilder to solve this problem, there is much that he himself can do.

In the first place he must realize the importance of employing only thoroughly skilled and experienced workmen and he should call in the manufacturers of electric welding apparatus to assist him. The latter, for their own protection and the welfare of the industry on which they rely, should see to it that no ship should be allowed to be built with some of the thoroughly rotten work done by absolutely unskilled welders that the author has seen in this country. It is strongly urged that steps should be taken at once to establish welding as a skilled trade and only certified men should be allowed to be employed. All the reports that have been seen on good and bad welds lay stress upon the fact that, in general, defects are mechanical and not an inherent feature of the process. Skilled men working under an experienced foreman would know, for example, that clean surfaces are necessary for good work, they would quickly learn what types of joint are made with ease, in what portions of a ship welding is cheap or where it is unreliable, and from their ranks should be drawn the draughtsmen who will design in detail the welded vessel of the future. The Electric Welding Committee of the Emergency Fleet Corporation has spent much time and thought on the attempt to discover a practical non-destructive method of testing welds. While such a method of testing would be very valuable, it is the author's opinion, that when only welders of proved skill are employed under experienced supervision, such tests would be no more necessary than a shear test on every rivet of a riveted ship; but, so long as such welders are not available the shipbuilder will look askance at a process that may be good generally but may have weak spots of which he is ignorant.

Secondly, after he has trained a reliable staff of welders on work of minor importance the shipbuilder must devote his attention to the best method of assembly of large parts, so that he may feel reasonably sure that every joint of the finished structure possesses the qualities which his welders can with certainty produce in test pieces. From the very outset it has been realized that this is a difficult problem which can be solved only by patient trial and experience. Present opinion at the British Admiralty is that the only satisfactory method is by the use of closing bolts as for riveted joints. In the case of the Richborough barge a certain amount of buckling resulted on the outboard edge of the garboard strakes after these plates had been welded to the keel plates, but this was removed when the bilge plates were put into position and bolted up. In the heavier work it may be that, buckling of plates being impossible, the internal stresses set up in welding were the causes of the failures that occurred. It is understood that this method is being adopted at Cammell Laird's but, again the plating of this vessel is not very heavy and it is definitely known that this feature received most careful consideration from the very beginning. In the author's opinion this difficulty is by no means insuperable. Its existence should spur on the progressive shipbuilder, for when he has solved the problem, by that much will he be ahead of his competitors.

In the third place it appears to be the fact that shipbuilding steel in Great Britain is more adapted to welding than shipbuilding steel in this country, and the former is also more regular in its qualities. If shipbuilders are satisfied that such is the case and are also convinced that electric welding has a great future in ship construction, they should take concerted action to ensure that steel producers supply the material best suited to their purpose.

So far the author has urged a careful survey of the possibilities of welding as a substitute for riveting only on the ground that the latter is the most costly item of labor on structural work and it behooves the shipbuilder to reduce that cost. But there are other advantages incidental to welding that must not be overlooked. A sound arc-welded joint is necessarily watertight and once such a joint with known efficiency can be made with certainty caulking is eliminated and the work of water-testing made easier. It must be admitted that, so far, the advantages anticipated in this direction have not been fully realized, but that they will be realized is only a matter of time, and it has already been fully demonstrated that welded joints, being stiffer, stand up better under test and there is no breaking away of the caulk such as is sometimes experienced with riveted connections under water pressure. Another advantage is a saving in steel due to the absence of rivet heads and the smaller width of laps and butt straps. Some advocates of welding claim that the labor of marking off, punching and assembling will be much reduced, more particularly by the adoption of spot-welding, but the author is bound to say that at present he is not a convert to the adoption of spot-welding for heavy ship work, it is his opinion that there are many difficulties ahead for those shipbuilders who attempt this form of welding and until those difficulties have been fairly and squarely met he hesitates to affirm that spot-welding will be found a valuable process in ship construction.

In conclusion, it is desired once again to place the subject before the shipbuilder in this light. The cost of ship construction must be reduced. A reduction in riveting is the most fruitful field for economy. Electric welding as a substitute has been tried to a limited extent and found successful. Classification Societies are prepared to accept the process, though at present the only vessels built or under construction are small. There are difficulties ahead but they do not appear insuperable. The best line of approach appears to be to build up a staff of thoroughly skilled welders, to gain experience by the adoption of welding on minor parts, to proceed cautiously in the extension of the process to more important members, not to expect a great saving in time and cost immediately, but to persevere, not necessarily towards the rivetless ship but towards a vessel in which the employment of both processes of riveting and welding is so adjusted that more riveting or more welding could only be done at greater cost.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 19, 1919.

Copyright 1919. By A. I. E. E.

FUSION IN ARC WELDING

BY O. H. ESCHHOLZ

Abstract of Paper

Although welding literature is replete with discussions of applications, little information is available descriptive either of non-destructive inspection methods or of the process of metallic electrode arc welding. This paper calls attention to such characteristics as penetration and overlap, peculiar to this process, which facilitate visual inspection and discusses briefly the effect of arc length, welding procedure, electrode material, arc current and electrode diameter upon these characteristics.

THE physical characteristics of a weld are admittedly determined chiefly by the degree of fusion secured between adjacent metals. In the production of welds by the metallic electrode process, a fair estimate of the fusion may be obtained by an examination of the surface of the deposited metal and the depth of the arc crater formed when welding at normal speed. These observations give an adequate indication of the two prime characteristics of fusion—penetration and over-lap—which, however, are definitely revealed only on exposing a section through the zone fusion.

Fig. 1 illustrates the union between a layer of metal deposited from a low carbon steel electrode upon a low carbon steel plate. The line of demarcation between the plate and deposit indicates the zone of fusion, the polished and etched surface of the deposited material presenting a different appearance, due to its crystalline structure, from that of the plate material. Two characteristics are at once apparent; a penetration of the deposited metal into the plate metal over almost the entire width of the layer, and a slight overlapping upon the plates without fusion of the extreme edges of the deposit. The penetration is due to liquefying of the plate metal by the energy absorbed from the arc as well as that absorbed from the molten deposit.

At the center of the deposit, where both effects are present, the penetration is greatest; at the extreme edges, however, where the deposit has overflowed the area included by the arc crater, the energy liberated on cooling is insufficient to melt the plate metal and the deposit congeals without fusing to it. It is typical of such overlapping that the edges of the deposit are well rounded, a distinct gap often existing between the plate and edge of deposit. The convex surface of the deposit thus forms a re-entrant angle with the plate edge. To insure satisfactory fusion a proper degree of penetration should be obtained with no overlap.

PENETRATION

Fig. 2 illustrates a section through the crater obtained on breaking an arc during the process of depositing a layer of metal on a steel plate. It may be observed that the depth of the crater depression is comparable to the depth of penetration of the deposited metal below the plate surface. A simple visual indication is, therefore, available during the welding operation for gaging the penetration.

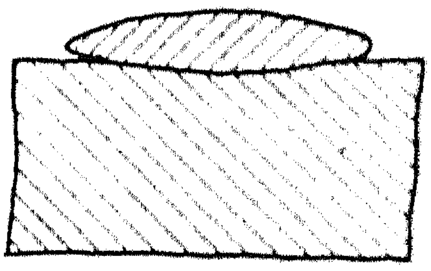


FIG. 4 — EXTREME OVERLAP OBTAINED ON WELDING WITH AN ELECTRODE OF LOWER MELTING POINT THAN THAT OF THE PLATE METAL

OVERLAP

The convex edges of the deposit shown in Fig. 2, particularly evident at the right of each view, would lead one to expect an appreciable overlap. This is disclosed on a careful inspection of the sections. Fig. 3 illustrates a surface contour with no overlap. This condition is usually typified by the absence of even the smallest re-entrant angle, an intimate contact between plate and deposit being secured, the junction between the two metals often forming a concave surface.

A condition of extreme overlap is shown in Fig. 4. This is typical of a low melting point electrode or of too great speed of arc travel. The convex deposit edge and the re-entrant angle between it and the plate are pronounced.

ARC LENGTH

The effect on fusion of arc length has been discussed at considerable length by welding engineers. Some assert that better fusion is secured for a given arc current with a long arc than with a short arc, due to the greater amount of energy

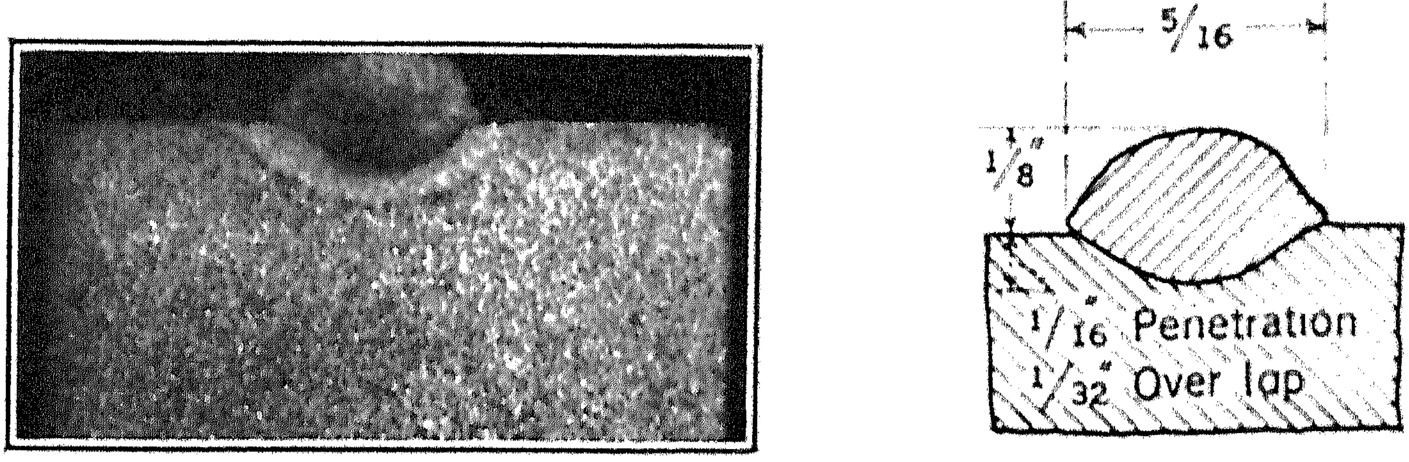


Fig. 1—Section Through A Layer of Electrode Material Deposited on a Mild Steel Plate When Using a Short Arc and a Proper Welding Current—Showing Degree of Penetration and Extent of Overlap

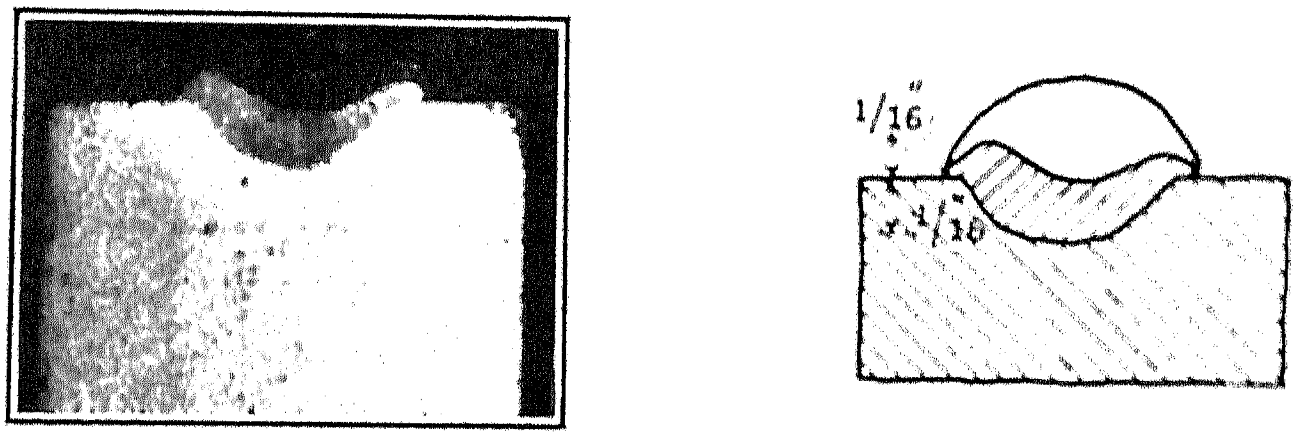


Fig. 2—Section Through the Crater Remaining on Interrupting Arc Current Showing the Comparable Depth of Crater Depression and Deposit Penetration

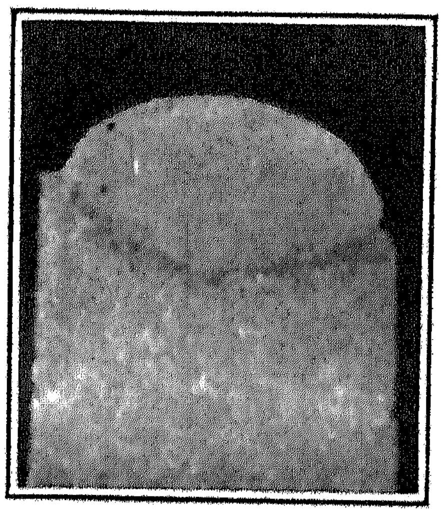


Fig. 3—Absence of Overlap When Depositing Metal of a High Melting Point on a Plate of Low Melting Point

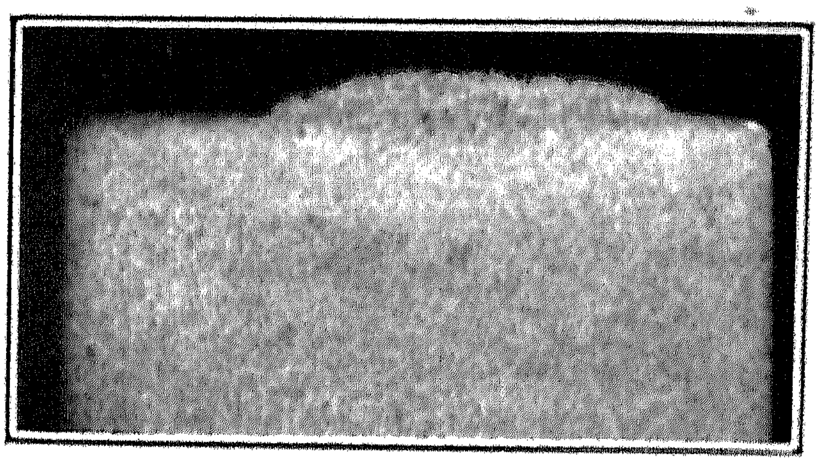


Fig. 5—Characteristic Section of Deposits Obtained with a Long Arc—Indicating Excessive Overlap with Practically no Penetration

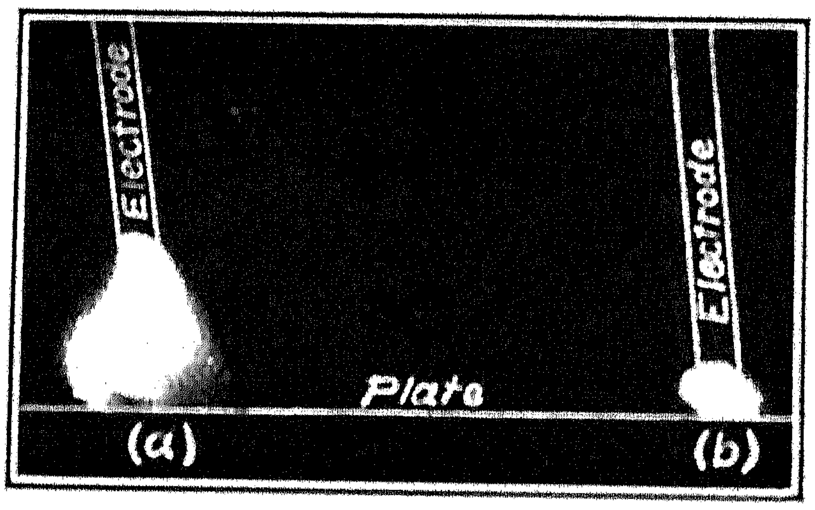
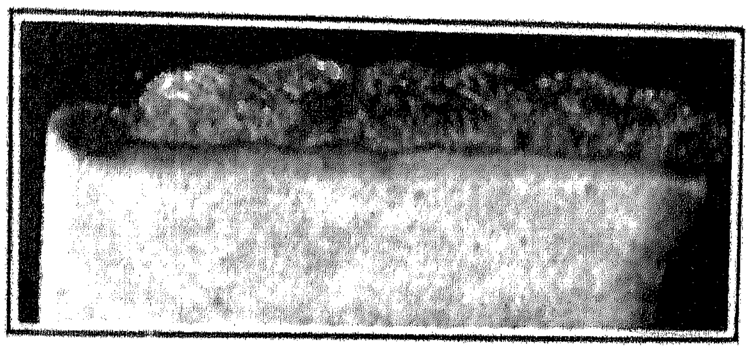


Fig. 6

A—Illustrating the Exposure of New Metal Resulting on Holding a Long Arc and the Protection Obtained

B—From the Enveloping Gases with a Short Arc



[Eschholz]

Fig. 9—Section through the Center of a Deposited Layer in the Direction of Forming Deposit—Showing Degree of Penetration with no Unfused Areas

absorbed by the arc; others have stated that the degree of fusion is dependent solely on the arc current and, of course, the thermal characteristics of the metal welded. For a given arc current it has been the author's experience that fusion is not improved with increase in arc length, that the best weld characteristics are obtained when holding a short arc, not longer than 1/8 in. and preferably about 1/16 in. With a given arc length the ease of securing complete fusion increases with arc current until a maximum value is reached beyond which further increase results in a weakened weld structure.

Figs. 1 and 5 are typical of sections through deposits formed with a short and a long arc, respectively, the arc current being approximately the same in each instance. With a short arc the plate metal is liquefied by the energy required to maintain the arc crater, by that conducted from the arc flame in constant intimate contact with the plate and by the energy radiated from the arc stream and electrode terminals, as well as by that absorbed from the molten electrode deposit. When the molten material passes through a short arc it is quite completely enveloped by hot gases and suffers but a slight loss in temperature before being deposited. Upon drawing a long arc the air currents in its vicinity blow the arc stream and arc flame about violently, so that it is difficult to maintain the plate terminal of the arc in a given position long enough to effect sufficient penetration. Moreover, the energy absorbed from the arc stream and electrode terminal becomes practically negligible, due to the greater diffusion of the radiation in the region of the arc plate terminal, while the energy absorbed from the molten metal is decreased because of the loss in temperature the deposit has sustained in transition. Some conception of the appearance of short and long arc lengths may be obtained from Fig. 6.

WELDING PROCEDURE

The extreme condition of no overlap is difficult to secure when depositing a single layer of electrode metal upon the usual low-carbon steel plate. For most purposes a slight overlap, such as is indicated in Fig. 1, is permissible in single layers as a simple procedure of depositing the subsequent layers will fuse the overlapped areas.

Fig. 7 illustrates the characteristic penetration and overlap when parallel abutting layers are formed. The minimum overlap for this condition will approach 1/16 in. per layer. If the width of a layer is increased by weaving the electrode back and forth the percentage overlap will be decreased. However, it may be eliminated by the simple expedient of always overlapping adjacent layers as shown in Fig. 8. On forming the second tier of deposits the same precaution should be taken to prevent unfused zones. Fig. 9 shows a typical section through the center of a layer. It may be noted that although the depth of penetration varies somewhat, due to the speed of arc travel and unavoidable variation in arc length, good fusion has been secured between the plate and deposit along the entire length of the deposit.

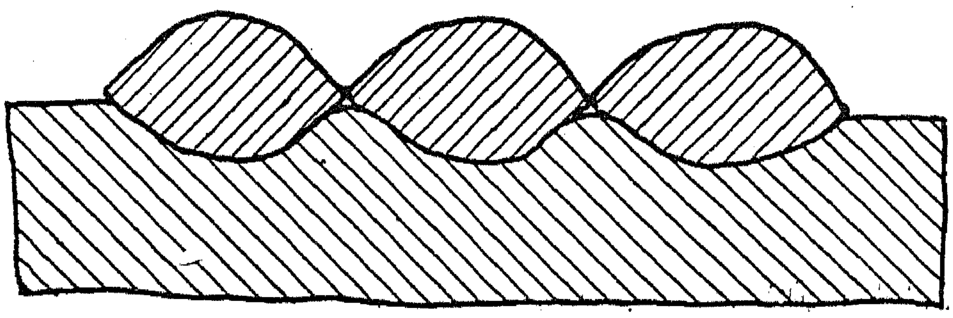


Fig. 7—Fusion Obtained on Abutting Parallel Deposits

Since the choice of electrode material, arc current and electrode diameter largely determine, with a given manipulative skill and welding procedure, the depth of arc crater and deposit contour, a brief discussion follows of such characteristics which affect fusion between the deposit and plate metals.

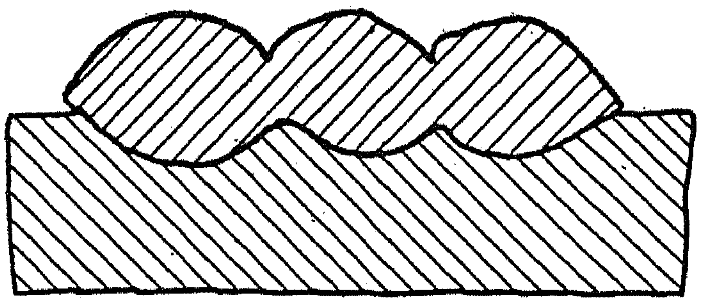


Fig. 8—Fusion Secured when Parallel Deposits are Overlapped

Electrode Material

To minimize the occurrence of unfused areas, the melting point of the electrode metal should exceed that of the plate metal. Fig. 1 illustrates the fusion obtained with the melting points approximately equal, (0.18 per cent carbon, 1510 deg. cent. approx.), Fig. 3 that secured with a greater melting point for the electrode (0.18 per cent carbon, 1510 deg. cent. approx.) than for the plate (3 per cent carbon, 1280 deg. cent. approx.)

and Fig. 4 that obtained with a lower electrode melting point (1.1 per cent carbon, 1440 deg. cent. approx.) The melting point of most weldable steels may be gaged with sufficient accuracy for welding purposes by the carbon content as shown in Fig. 10. The usual additions of other alloying ingredients reduce the melting point a negligible amount.

While pure iron possesses the highest melting point, its use is not advisable since excessive oxidation of the vaporized metal occurs. However, a commercially pure iron alloyed with about 0.18 per cent carbon and 0.5 per cent manganese greatly reduces the oxidation, and improves the ease of manipulation without an appreciable sacrifice in melting point. For most purposes the use of this electrode gives a reasonable minimum overlap.

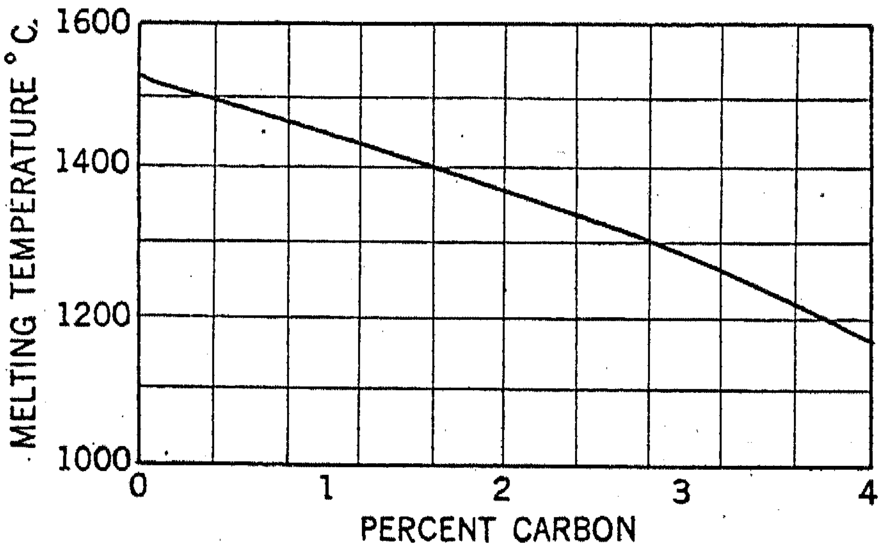


Fig. 10—Change in Melting Point of Iron Alloyed with Various Amounts of Carbon

Under exceptional conditions such as high thermal capacity of weld, or overhead welding with a low current, the overlap may be excessive even when using this electrode, it is customary then to preheat the object to be welded or to adopt a welding procedure which utilizes the thermal energy from preliminary welding operations to raise the temperature of the object welded.

Arc Current

For a given plate thickness the welding current will vary with the type of weld scarf, cleanliness of surface, thermal capacity, and conductivity of the metal, and position of plate with respect to the welder. The values shown in Fig. 11 apply only to clean, butt-welded plates, single V, scarfed to 90 deg., supported in a horizontal position, below welder, in poor thermal contact with any metal of high thermal capacity. The fusion obtained with too low and too high arc currents is shown in Fig. 12. In Fig. 12A the overlap is excessive and the penetration unsatisfactory because of the small amount of energy applied at the arc crater. In Fig. 12B the excessive energy consumption at the plate arc terminal produces excessive penetration, a further increase in current permitting cutting through the plate.

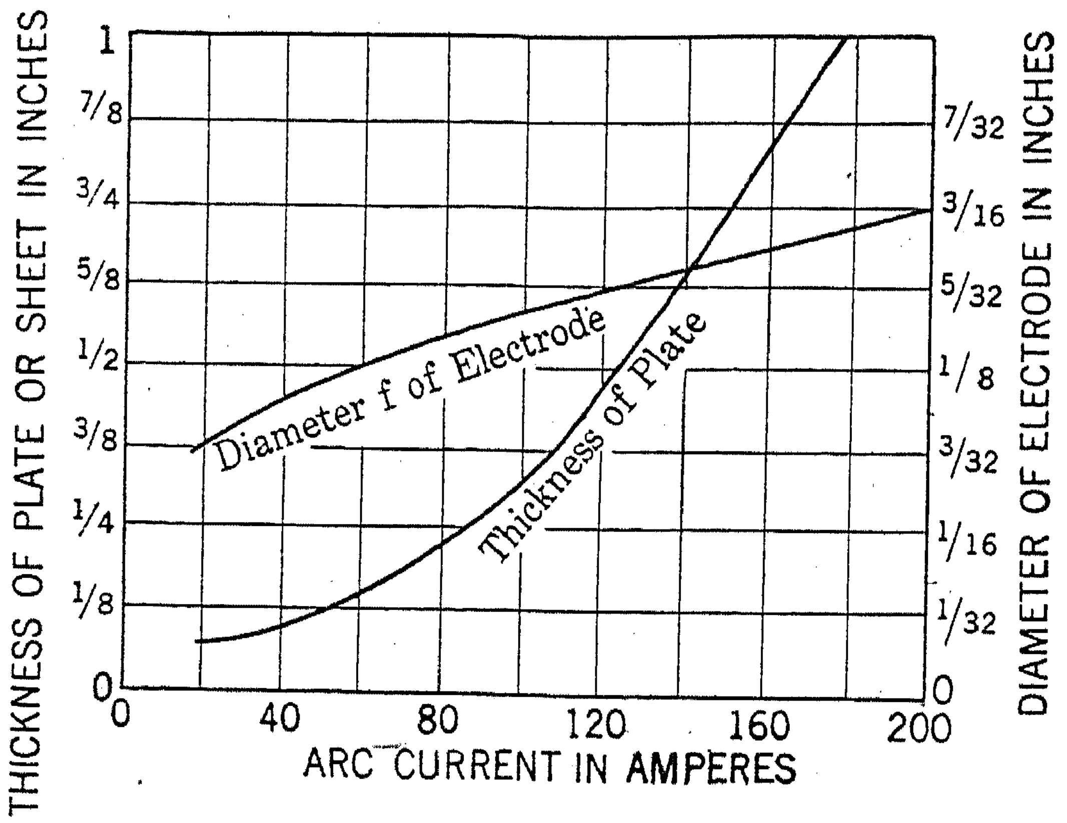
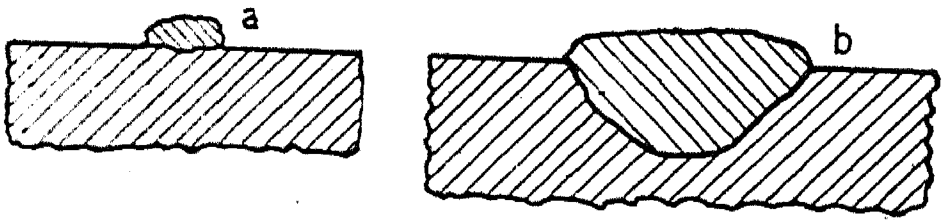


Fig. 11—Approximate Values of Arc Current on Welding Under Conditions of Low Thermal Capacity for Weld

Fig. 13 indicates approximately the effect of variation of arc current on fusion and therefore on weld strength for (a) skilled and (b) unskilled welders. Weld structures are occasionally encountered in which variations in thermal capacity and plate temperature necessitate changes in arc current if complete fusion is to be secured. Observations of the deposit contour and crater depth usually serve to indicate to the operator when such changes are advisable.



A–Fusion Secured with too Low an Arc Current–Excessive Overlap
B–With too High an Arc Current—Excessive Penetration
Fig. 12

Electrode Diameter

A close adjustment of electrode diameter to arc current is not essential. For miscellaneous direct-current welding the densities usually fall within a range of 7000 to 10,000 amperes per square inch, if an 0.18 per cent carbon steel electrode is used. However, whenever practicable, the author prefers a closer range of 8000 to 9000 amperes per square inch. With too great an electrode diameter for a given current, the arc becomes more unstable, increasing the manipulative skill required and decreasing the rate of depositing electrode material. Too small a diameter causes a deposition of electrode material before the plate has been properly liquefied; it also results in overheating and softening of electrode which increases the skill required to prevent the electrode from "freezing" to the plate. With the correct diameter the electrode material is sprayed in a regular stream at a rate which will insure maximum welding speed consistent with good fusion.

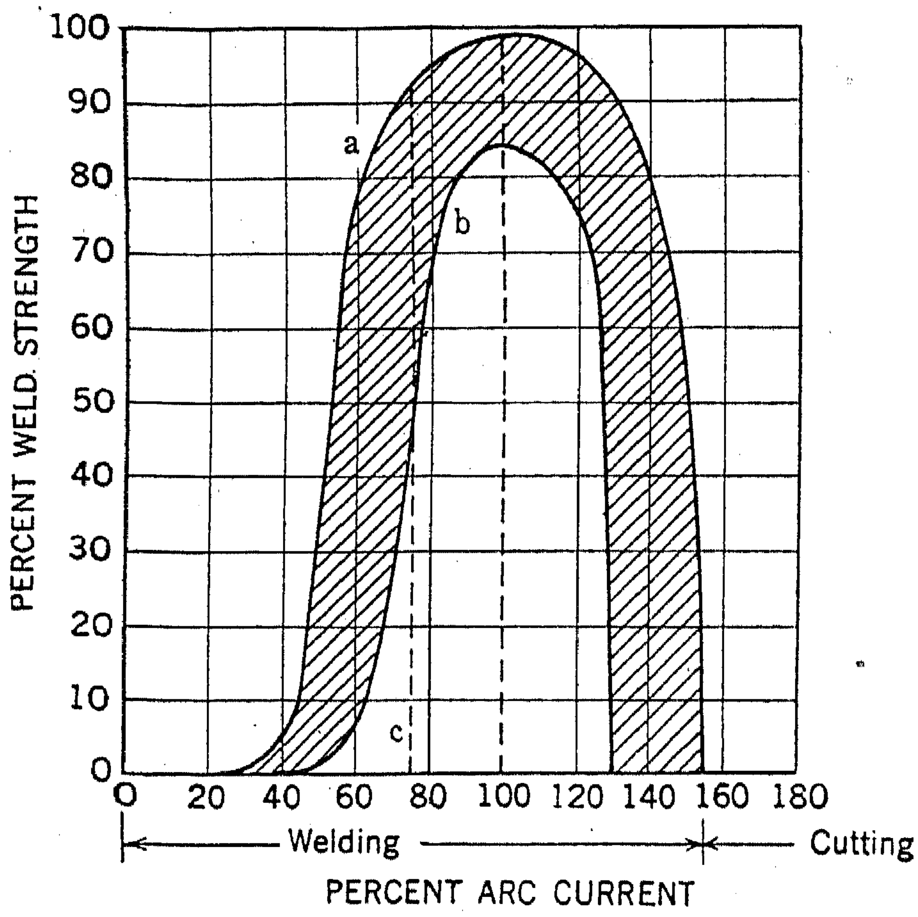


FIG. 13—APPROXIMATE CHANGE IN WELD STRENGTH WITH CHANGE IN ARC CURRENT, FOR
A—SKILLED, AND B—UNSKILLED WELDERS

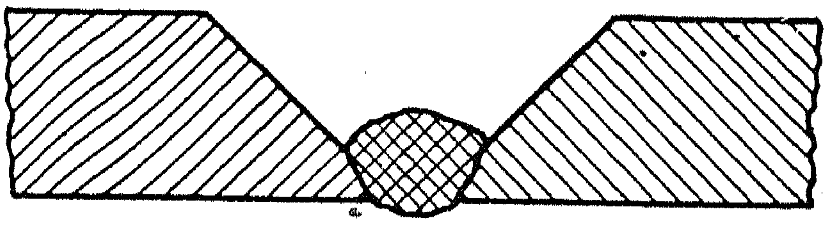


FIG. 14—SECTION THROUGH BUTT WELDED PLATE SHOWING GOOD FUSION AT 150 ARC AMPERES

Example. The characteristics of fusion and the effect of a control of the conditions which govern them may be summarized in comparing the procedure for making butt and lap welds.

In butt welding ½-in. low-carbon steel plates, scarfed as shown in Fig. 14, a $^5/_{32}$-in. diameter welding electrode containing approximately 0.18 per cent carbon, 0.5 per cent manganese, would preferably be used with an arc current of approximately 150 amperes. With moderate skill, the strength of weld should approximate 90 per cent that of the plate. If the same current and electrode diameter are used in welding the lap joint formed from ½ in. plates shown in Fig. 15, poor fusion will be obtained owing to the increased thermal capacity of the abutting sections. As shown in Fig. 15 the penetration will be too slight and the overlap excessive, similar to the case shown in Fig. 12. In fact, sections through completed lap joints most invariably show that in forming the first deposit, which is the most important, even expert welders tend to penetrate only one side of the weld scarf with resultant overlap on the other. Breakdown tests of lap joints have plainly shown failure due to this cause.

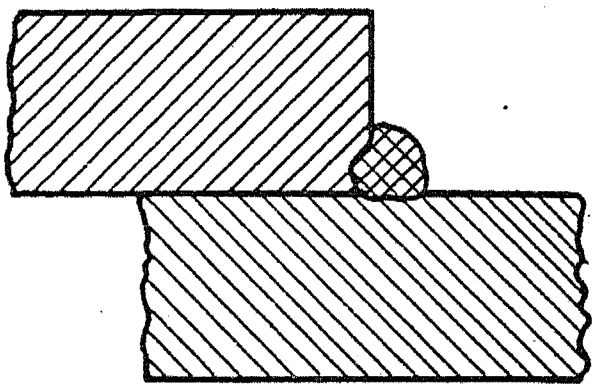


FIG. 15—SECTION THROUGH LAP WELD—PLATE, ARC CURRENT, ELECTRODE DIAMETER SAME AS FOR FIG. 11—SHOWING POOR FUSION DUE TO INCREASED THERMAL CAPACITY

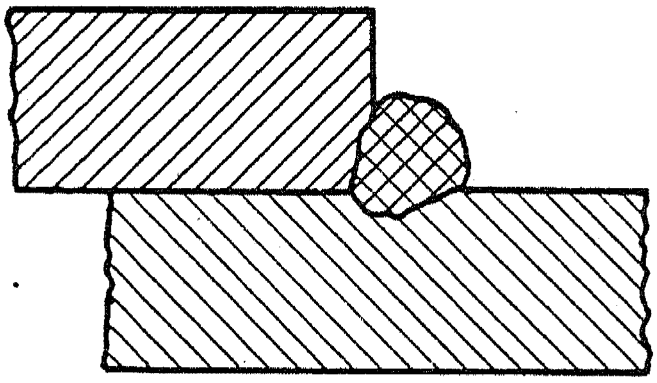


FIG. 16—GOOD FUSION IN LAP WELD OBTAINED ON INCREASING ARC CURRENT TO 225 AMPERES

To secure complete fusion on both faces of the joint it is necessary to increase the arc current and with it the electrode diameter. For the condition shown in Fig. 16 an arc current of 225 amperes and an electrode diameter of $\frac{3}{16}$ in. was found to produce complete fusion, facilitate electrode manipulation and increase rate of electrode deposition.

CONCLUSION

The fusion obtained in arc-welded joints between the plate and deposit metals is determined by the penetration of the deposit metal into the plate metal and the overlap of the edge of the deposit layer on the plate. The penetration is comparable to the depth of arc crater as observed by the welder when moving the arc terminal over the plate with uniform velocity, and the overlap is indicated by the contour of the surface of the congealed deposit in the region adjacent to the material welded. For most welds a crater depression of $\frac{1}{16}$ in. will give the desired penetration while a contour which does not show a re-entrant angle between plate and deposit indicates an overlap per layer of approximately $^1/_{32}$ in. or less.

DISCUSSION ON "WELDING MILD STEEL" (HOBART), "ELECTRIC WELDING IN SHIPBUILDING" (GOODALL) AND "FUSION IN ARC WELDING" (ESCHHOLZ), NEW YORK, N. Y., FEBRUARY 19, 1919.

W. H. Hill: We have fully sympathized with Commander Goodall in his conception of the dangers of using welding in making the joints on ships; and the picture that he gives, with the possibility of one weld in a thousand failing in a stormy sea, perhaps isn't a very encouraging one.

We have been developing another form of ship, in which we do not have to use welding for the tensile strength joints, but which is a welded ship nevertheless. The ship is built in cast studs. It is a system of ship building which is even more rapid than that of the roll plate ship, and it has joints which upon investigation seem perfect. The joints are of that kind which are necessary for ships, before welded ships will be generally satisfactory for carrying.

There is one other point I would like to bring to your attention in connection with the strength of welds, particularly with regard to the filament in crystals. This point has been developed in connection with electric furnace work. In making our castings for ship plates I have been fortunate to come in contact with Mr. Snyder, and he has developed a furnace during the last year or two which melts steel in the absence of oxygen; that is, the furnace is so tight, the air has no chance of getting in. The pressure inside is slightly in excess of that outside, so that the gases come out rather than go in.

Now, he found something very interesting in testing the castings made from such steel. The tensile strength is increased, the ductility is increased and the elastic limit, and his explanation of it is the fact that the film is reduced in thickness and thereby permits a stronger adhesion of one grain to the other.

I am in harmony with Mr. Eschholz' suggestion of less contact with the air. It is in that direction that we have to look for greater strength in welds.

H. A. Hornor: Commander Goodall was extremely wise in bringing out the requirements for a skilled welder. After months of investigation of arc welding, we have found undoubtedly that the crux of the situation rests in the skilled welder. How are you going to get the skilled welder? It might be of interest to know what has been the experience of the Fleet Corporation regarding this question.

The system introduced by the Fleet Corporation permitted the shipbuilder to select the men whom he wished trained as skilled welders. You can see, therefore, that we met with a miscellaneous type.

Now, I wish to put myself on record as saying that anyone who undertakes to drill or to use the laboring type of man on arc welding might just as well forget electric welding. I am merely giving you this as a belief from an experience of eight months in this connection.

The Fleet Corporation gave out as an award a certificate as soon as a man had completed his course, and I think the proportion of certificates that were issued, Mr. President, were very, very few. It was utterly impossible to drill the majority of the men who were sent to the school for training.

To be sure, our training had to be intensive. The shipbuilder would naturally complain if we couldn't return a man to him within a reasonable course of time. We, therefore, cut our courses as short as possible. We didn't put limits, however, upon the course. There were some remarkable records made, but those remarkable records were never made by the laboring type of man, and the laboring type of man, I am sure, returned from the school as inadequate to do arc welding as when he came in.

I believe that a great many men have blamed the arc welding process quite improperly, due to the fact that we were endeavoring to make the application in a most improper way. If the conditions are not fitted for a special job, a skilled operator will stop and I doubt very much whether you will ever find a skilled operator who can make a perfect arc welded joint when he is being annoyed by things flying over in his vicinity, etc. I assure you a skilled arc welder has to concentrate with all his senses absolutely on the weld. Give him what he requires and he will return to you a proper weld, one that is good and sure. In the majority of cases where a skilled operator can make a good weld, I think the researches of this Committee will prove that when you test that weld, you will have a difficult time to break it.

C. A. Adams: It may interest you to know that there is pretty good evidence that the prejudice of riveters against arc welding, a very natural one, has caused intentional disturbance of the arc welders to the extent of throwing hot things in their vicinity.

Captain Corbett: Why cannot the ship men and the steel manufacturers get together and devise some type of plate with an interlocking shape, which will give them a much higher factor of safety?

W. H. Hill: That is exactly what the cast steel ship is doing.

Captain Corbett: My suggestion differs from that of Mr. Hill in this respect: If I understand him correctly, his plan consists in having cast steel plates with interlocking edges. My suggestion was to use our ordinary rolled steel plates with the ends cut in such a manner that they would dovetail together. Then by means of buck welding, the stresses will be distributed more efficiently by being in different lines.

A. M. Candy: According to the tables at the end of Section 5 of Mr. Hobart's paper the deposit obtained with

Roebling cold-rolled steel, and Toncan electrodes contained considerably less carbon and manganese than the original electrodes; whereas very little carbon is lost in the case of the hot-rolled steel electrode. When using the hot-rolled steel electrodes, the metal apparently has a tendency, probably due to greater surface tension, to pass through the arc in larger particles than the other electrode materials. This affords greater protection against oxidation, and thereby prevents much loss of carbon. The larger globules tend to increase the arc fluctuations due to greater partial short-circuiting of the arc. Actual oscillographic records show that with the hot-rolled steel electrodes the current peaks were 70 per cent above normal whereas the peaks for the other electrodes were only about 25 per cent above normal. Probably due to the larger globules, a greater proportion of slag in the electrode was carried into the weld. Some four or five test pieces made with the hot-rolled steel electrodes were all considerably below the tensile strength of the samples welded with the other electrodes. These facts indicate that an electrode may not be satisfactory even though of approximately the proper chemical composition. For example, the hot-rolled steel and Roebling electrodes are quite similar in chemical analysis.

SPEED OBTAINED IN ARC-WELDING SHIP PLATES

Test No.	Speed, in feet per hour	Current used	
		Amperes	Cycle
1	3.33	170–180	..
2	2.86	150–160	..
19	1.90	150	60
20	1.48	175	60
21	1.59	105	25
22	0.97	160	25

In Section 10, the relative speeds of alternating and direct-current arc welding are mentioned. By referring to Table IV we find that tests 1 and 2 were made with direct current, that tests 19, 20, 21 and 22 were made with alternating current. The speeds of welding are shown in the accompanying table. Of course these are isolated cases, but it is interesting to see what is obtained from actual tests.

In Section 19, Mr. Hobart has brought out the question of suitable current values. This is probably one of the most important and least understood problems in the entire arc-welding category. In fact, it is the problem of proper fusion in arc welding, which can only be secured by the use of the proper current volumes, depending on the characteristics and temperature of the material to be welded, and by the use of the proper diameter of electrodes for the current. A visual inspection of the deposited metal will almost unfailingly indicate when the desired effects are obtained. The method of determination is very ably and clearly pointed out in Mr. Eschholz's paper; namely, judging the penetration metal by the depth of the arc craters, measured from a straight line across the crater top to the bottom of the crater, and an examination of the edge of the deposits for re-entrant angles, to see how much of an angle there is.

Mr. Hobart also made reference to the effect on arc welding of voltage employed. I do not believe that the question of voltage is quite as simple as inferred, unless coated or wrapped metal electrodes or carbon electrodes are used. We will grant that for most open work an experienced conscientious welder can produce results with circuit potentials as high as 250 volts; however, an inexperienced or a careless operator will follow the course of least effort and maintain a long arc, resulting in an unconcentrated porous deposit, which will be poorly fused and does not penetrate into the shank metal. An inexperienced operator certainly ought not be to trusted with such high open-circuit voltages, as he is sure to produce unsatisfactory welds.

C. J. Holslag: Speed in welding may be advantageous or disadvantageous according as to whether the metal is deposited faster than the operator can make sure it is being deposited in the crater made fluid by the arc, or whether some of the metal is getting ahead or outside of the crater. Speed at which good welding can be done is an advantage and alternating current provides a greater speed than direct-current welding with the electrode negative, because with the electrode negative approximately 60 per cent of the heat is in the work, and with alternating current the heat is naturally evenly divided between the electrode and work.

Direct current with the electrode positive provides faster welding, but generally too fast to be of any use, as mentioned, unless the arc is closed in by a very heavy coating so that the metal is enclosed in a viscous sleeve, or where the work is very rough, such as filling in with a large electrode on a casting where strength is not required. In a test recently made with direct-current and alternating-current machines, the rate of melting was taken; that is, the number of pounds per hour that could be melted with no stops was determined. In each case under the same conditions, *i. e.*, different currents and voltages were taken, but comparisons made only when they were the same, alternating-current melting was from 20 to 30 per cent faster. For instance, at 150 amperes and 25 volts, 4 lb. (1.8 kg.) an hr. were deposited with alternating current and 3 lb. (1.3 kg.) an hr. with direct current; and at 175 amp. and 25 volts, 5 lb. (2.2 kg.) an hr. with alternating current and 4 lb. (1.8 kg.) an hr. with direct current. With a 5/32-in. (3.9 mm.) electrode, from 4 to 5 lb. an hr. is the maximum amount that can be deposited in electric arc welding, for with higher current densities good welding is lost.

Speed also depends on the length of the arc because the rate of melting depends on the rate of heating and the rate of heating naturally depends on the voltage across the arc, as well as the current through it. With a long arc, this rate can be raised, and hence the welding speeded up; but as pointed out in Mr. Eschholz's paper, everything is in favor of a short arc for good welding, *i. e.*, less air radiation, less contact with the air for oxidation and forming of nitrides, greater chance that metal of the electrode will deposit in the crater and nowhere else. With alternating-current apparatus, the greater possibilities for automatically holding a short arc, and constant rate of heating are generally taken advantage of. Hence any test of speed must be accompanied by voltage across the arc and amperes through it, which means by definition, the same rate of heating; and with the same rate of heating, the only factor left is the disposition of the heat at the positive and negative electrodes, for as explained before, the amount of heat at the electrode end determines the rate of its melting.

W. L. Merrill: Mr. Hornor touched on the training of arc welders. Perhaps when you realize that all of the Liberty motors had some kind of welding, either arc or acetylene, that for about twelve years locomotive frames have been repaired by arc welding, and that the majority of these operators have been workmen, you will recognize that his statement should have been qualified.

These men haven't been particularly chosen, but were men who have been picked for that trade and trained for it, and I wish to record the statement that with suitable training and instruction, it is my belief that any good, live, honest working man, be he a boiler maker, or a mechanic of any kind, can be trained to be a successful welder. The evidence of this is noted above. Also, the particular man who is doing the welding, and who is the most insistent welder that this Research Committee has ever found, is a working man.

Until about two years ago, spot welding was limited to very thin sections, say up to ¼ or ⅜ in. (6.3 or 9.5 mm.) thick; beyond that the field had not been thoroughly explored. It was thought that it might be advantageous to use this method in the construction of ships, so tests were made and a machine built, the results of which have been published in Capt. Caldwell's report and in one issue of the *General Electric Review*, which has been referred to by Mr. Hobart. As a result of those tests, it was found possible economically to build machinery capable of welding two or more pieces of structural steel up to a total of 3 in. (7.6 cm.) that being the maximum thickness we tried to weld.

Unlike arc welding, spot welding has very few variables; and a surface inspection in practically all cases is all that is necessary to determine a 100 per cent weld. Also, like the arc weld that we have been hearing about today, spot welding can be done to more than 100 per cent strength of the plate. As an example, Section 29 of Mr. Hobart's paper contains some test samples that were made up of different kinds of welding. The two-spot weldings, which would be used for these thicknesses of plates, gave a breaking strength of 28,000 lb. (12,700 kg.) as against 13,000 lb. (5896 kg.) for the standard riveting for that same plate.

As a result of experiments carried on in the large machine, which by the way was 2000-kv-a. capacity, although we did not reach the capacity of the machine, three separate and distinct machines were designed for use in fabricating of ships; that is work to be done in the shop, not on the ships. Pictures and explanations of these are given in Mr. Hobart's paper. These machines have been completed, and two of them are in the possession of the Emergency Fleet Corporation at the present time, working out the practicability of adapting them to the fabrication of ship parts in place of laying out, drilling, and riveting. Only experience will tell what saving or gain, if any, there will be in the use of this type of machinery. One thing, however, we know can be gained, and that is for a given strength of a given member, the joints will not be limiting features, and if 60 per cent is figured for riveted joints, the thickness of the plate can be reduced proportionately because you are always sure of 100 or more per cent strength if spot welding is done with proper care.

D. C. Alexander, Jr.: In mining work, manganese steel is used for rails, crossovers, and, I believe in some cases, wheels on mining cars. Now, for that sort of work it has been customary, when the manganese steel parts wore out, to scrap them because there seemed to be no way of repairing them. A covered electrode, if it is properly covered, will deposit the slag on the molten metal so that the constituents of the original metal can be retained in the positive metal. So by starting with a high manganese-steel wire and putting the suitable covering on that wire, it is possible to deposit on a plate, or manganese specimen of some sort, 12 per cent manganese steel. That in itself justifies the covered electrode for that particular kind of work. I imagine mining engineers may have occasion to build up worn parts composed of high-carbon steel. The same effect can be produced by using a 0.7 per cent carbon steel wire, and depositing 0.5 per cent carbon steel, to build up carbon steel parts.

The remarks made this afternoon about covered electrodes have been a little bit vague in some cases, and possibly some of you have the idea that any kind of covering on an electrode produces the same result. I notice in Mr. Hobart's paper that, in some instances where it did not give satisfactory results, a great improvement could be made by dipping the wire into the melted iron. That may be correct, and some people may think they will get a big improvement in that way. But when you think about it, you must realize that in steel-furnace practise you must have suitable slide, you can not have any kind of slide; similarly the material used as electrodes must not be selected because it is cheap or common, but because it is the only one found to contain the proper constituents. What we use is blue asbestos, which is not like white asbestos at all. The coating fuses with the steel and covers the molten steel with a thin slag, which keeps out the oxygen and prevents the formation of oxide. It also maintains in the molten metal the constituents of the steel that we want to keep there.

W. Spraragen: The angle referred to in Table III of Mr. Hobart's paper is the angle on each plate, therefore the total opening in each weld will be just double that given. For instance, the total opening for the 15 deg. bevel listed in this table would be 30 deg. The statement is in connection with Section 24, and Mr. Hobart states that no physical test has been made on these samples. I have, however, received from the Electric Testing Laboratories some results of tensile and bending tests on these specimens; each result given below is an average of three test specimens.

Angle of bevel on plates, in degrees	Total opening, in degrees	Ultimate strength, in lb. per sq. in.	Angle of bevel at which crack starts, in degrees
15	30	44,000	9
30	60	51,000	15
45	90	45,300	13
60	120	48,800	14

These results seem to indicate that the advantages claimed by the American traditions for the very large opening are rather questionable if we also take into account the time, electrode material, and power saved by the smaller angle of bevel, as shown in Table III. Obviously one could have too small an opening so that the operator would not have sufficient room in which to properly manipulate his electrode. The results of the above test would indicate that this point was reached when the total opening was 30 deg.

J. C. Armor: (communicated after adjournment). Mr. Hobart has presented a comprehensive list of points that are of vital interest in fusion welding. In connection with his 4th point of the summary directly preceding Section I, "Preferable Composition for Bare Welding Wires," would say that in alternating-current arc welding of steel plates with bare electrodes good sound welds can be produced with wire that is composed of practically pure iron alloyed with from 0.02 to 0.65 per cent manganese. The brands of welding wire that contain 0.30 to 0.60 per cent manganese will generally produce the strongest welds, while those carrying a very small amount of manganese will, in general, produce the softest welds. I have not had much success with wires carrying more than 0.65 per cent manganese or with alloying elements other than manganese.

In connection with the 7th, 8th, 10th, and 11th points, would say that there is very little, if any, difference in the quality of work done with alternating current and direct current, provided, of course, that proper conditions are maintained in both cases. It is as easy to work with alternating current as with direct current, provided the alternating-current "striking" voltage is sufficiently high. Any commercial frequency is suitable for arc welding if sufficient open-circuit voltage is employed. The lower frequencies require somewhat higher voltages. My experience indicates that the speed of welding is about the same with alternating current and direct current. An operator who is experienced in direct-current welding usually adjusts his alternating current to a value that allows him to manipulate the fused metal in the way he has always done with direct current, with the result that he melts in about the same quantity in a given time.

In connection with Mr. Hobart's 20th point," Effect on Arc Welding of Voltage Employed," it is, of course, a well-known fact that the alternating-current arc is much more easily extinguished than the direct-current arc and also that the lower the frequency the more easily is the arc interrupted. These facts must be taken into account in welding with alternating current. The general tendency has been to employ alternating potentials of too low a value for effective work, with the result that the operator cannot manipulate his arc properly and therefore has difficulty in producing good work. This question of open-circuit potential is extremely important; and if it is desired to be free to use all welding wires now on the market, the "striking" voltage for 60 cycles should be at least 150 volts. Good welds can be made with less voltage when using certain brands of wire but other brands will require 150 volts. When the voltage is too low, there is a very strong tendency to hold the arc between the electrode and the top, or outside, of the weld, with the result that the metal at the bottom of the weld is not fused in as well as it should be. Arc welding, even under the best possible conditions, is very trying to a conscientious operator and for that reason the work should be done in the easiest practicable manner, as it is impossible for an operator to do uniformly good work if he is under a continuous strain. While an expert may make a few very good test welds, using a potential so low that he has the greatest difficulty in working, he will not be willing to work that way day after day; and if required to do so he will tend to become discontented and careless. For the reason just given, the alternating-current potential employed for most commercial wires now on the market should, in the writer's opinion, be about 140 or 150 volts.

In regard to Mr. Hobart's 28th point, "Consequences of Different Lengths of Arc," the writer's experience has been that the shortest practicable arc produces the best welds. A long arc, on the other hand, deposits inferior and weak metal in the weld. No doubt this is due to the absorption of larger amounts of oxygen and nitrogen from the air. The question of arc length is certainly as important as the electrode composition and the secret of good welding with most of the commercial wires now on the market is found in the maintenance of a very short arc manipulated so that the deposited metal fuses into the plate over the whole surface to be welded.

S. V. Goodall: (by letter). Mr. Hobart omits to refer to the investigations carried out by the Welding Research Sub-committee to ascertain whether any trouble is to be expected from excessive corrosion of a welded joint exposed to sea water. The point was raised, early in 1918, that on account of the difference in chemical composition and electrolytic characteristics of mild-steel ship plate and the metal laid on at the weld, it is possible that excessive corrosion would set in at the joint if the protective and anti-fouling compositions were accidentally removed or otherwise rendered ineffective, and the joint might fail in consequence. As welding had already been employed for effecting repairs to under-water parts of ships, the Welding Research Sub-committee instituted inquiries to ascertain whether excessive corrosion had been experienced in these cases; and while few replies were received, owing to the fact that the history of the weld after the repair was effected had not been kept, no actual cases were cited where excessive corrosion had occurred.

The British Admiralty and the U. S. Navy Department are carefully watching the effect of sea water on welded joints, and under the superintendence of Commander H. G. Knox of the U. S. Construction Corps, a series of tests on the corrosion of welded joints made with various types of electrodes is being carried out at the Norfolk Navy Yard. The Bureau of Standards is also investigating this matter. So far the experience at present available and the opinions obtained lead to the conclusion that no danger need be feared from excessive corrosion of a weld on service; further experience, however, is considered necessary before this question can be regarded as definitely settled.

With regard to the application of spot welding to shipbuilding, Mr. Hobart states that it is planned that bulkheads, frames, floors, and other parts shall be spot-welded and then transported to their places in the ship. This sounds attractive; but the shipbuilder will at once ask how the cost of such spot welding compares with that of riveting. The riveting of parts such as those mentioned can be done in shops or sheds, and is comparatively inexpensive. If it is to be displaced by spot welding, this latter process must be considerably cheaper than I understand it to be; and if Mr. Hobart could add particulars as to the relative cost of spot welding and riveting such members, it would add to the value of this part of the paper. With regard to the adoption of spot welding for work at the ship, there are many difficulties ahead for those shipbuilders who attempt this work; this opinion is based on experience with portable hydraulic riveters in ship construction and is altogether apart from the practicability, or otherwise, of efficiently connecting two heavy plates by spot welding. Even in this latter respect it must be remembered that there is considerable difference between the spot welding of two small thick plane plates in a shop and the the spot welding of two large nonplanar plates forming the complex structure of a ship. Moreover, spot welding, compared with line welding, possesses the disadvantage that economy of steel is not likely to be so great.

Mr. Hobart, in presenting the paper, rather ineffectively trailed the coat-tails of the bare-wire advocates in the face of those who believe in covered electrodes. In Great Britain, shipbuilders and engineers are not paying the additional price of covered electrodes for philanthropic reasons; as soon as they are convinced that the bare electrode will give as good and as uniform results as the more costly covered wire, Mr. Hobart may rest assured that the bare electrode will be freely used in that country. But I am bound to say that present experience in Great Britain indicates that the covered electrode gives superior results and gives such results more uniformly, the superiority being particularly indicated in fatigue tests, which is an extremely important matter if welding is to be employed on parts of ships contributing to general structural strength. Mr. Hobart states that one type of electrode must yield results uniformly superior to those obtained with another type in order to afford economic justification for a five times greater price. It is not considered that this matter can be summed up in this manner in regard to shipbuilding if electric welding is employed on such parts that the resulting joint is of importance in the general structural strength. If, by using a high-priced electrode, the desired minimum standard of strength can be always attained with certainty, while the lower priced electrode can not be relied on to attain such a standard at all times, the shipbuilder would not be justified in using the cheaper article, unless it were possible to subject all the joints to some test that would imitate in quantity and quality the forces that will be brought upon the joints on service. The cost of such tests might offset the lower cost of bare electrodes. Moreover, if two different types of electrodes were used on a ship—the cheaper where strength is not of much importance and the more expensive where structural strength is vital—extremely careful supervision would be necessary, and it would probably be more practicable for the shipbuilder to adopt the higher priced article throughout.

It is desired to emphasize point 30 of Mr. Hobart's paper, on the necessity of a clean surface for fusion welding. Some of the unsatisfactory work that I have seen is due to the attempt to weld together plates, etc., having surfaces that are oily or are otherwise insufficiently clean.

R. P. Jackson (by letter): Both the varied and somewhat conflicting information set forth in Mr. Hobart's paper and that obtained by observation and experience indicate that in a large measure electric welding is an art rather than a science. While it is obvious that a great deal can be done in the way of eliminating the useless and fake elements, after all that a scientific investigation can contribute has been taken into account, the actual production of safe welds is essentially a personal problem. A skillful welder will do some very creditable work with a variety of materials and equipment and an unskillful one will do poor and unreliable work with the most perfect equipment.

It is this feature of the essential dependence on the skill of the individual welder that has been such a handicap to the more extended use of welding. The difficulty is that a designing engineer laying out his computations and drawings cannot know with any degree of certainty what he can depend on in the way of a joint as compared with riveted work put up with a very moderate degree of skill but capable of being checked by inspection.

While this comment does not disparage the necessity of research work and particularly of a greater degree of publicity as to the actual facts, its purpose is to accentuate the necessity of training welders and inspectors. The writer feels that no extensive undertaking like shipbuilding can be worked out except as a result of the building up of a welding organization, comprising a superintendent, inspectors and welders who are conscientious and know their job. Without such an organization, no cautious engineer would specify the use of welding for the obvious reason that he could not be sure what he would get. With such an organization, on the other hand, there is almost no engineering structure that could not be undertaken, with the only limitation as to the amount of welding to be the accessibility of the assembled structure for the proper welding operation.

F. M. Farmer (by letter): Mr. Hobart's paper presents many phases of the welding art concerning which there are very divergent views. In many cases, the wide differences of opinion are based on the results of mechanical tests, but an examination of the mechanical test data discloses a wide variation in character of the tests made and in the details of procedure, so that it is frequently not only difficult to analyze and check the basis of the conclusions but impossible to correlate various groups of test results. It would seem, therefore, that the technique of testing welds is one of the phases of the welding art that should be studied and standardized in order that the results of mechanical tests made by different people may be strictly comparable and have the same significance to all.

The vital difference between testing a specimen of steel that includes a welded joint and testing an ordinary specimen is the non-homogeneity of the weld specimen. The weld specimen has at its center a section composed of material that usually has physical, chemical, and metallurgical characteristics distinctly different from the adjoining metal. Furthermore, the section of added metal is more or less irregular and the ratio of added metal to original metal in the section is quite variable. Consequently the details of procedure and of recording data should be more minutely prescribed than in the testing of ordinary specimens.

A conspicuous example of a mechanical test that requires standardization is the cold bending test, a standart test for mild steel. With the usual method, the specimen is bent around a pin of prescribed diameter and, as all specifications usually require bending to 180 deg. without fracture, the manner of applying the load is immaterial because the specimen is ultimately completely wrapped around the pin. But a specimen containing a welded joint at the center will usually not withstand bending to 180 deg. because of the much lower ductility of the deposited metal in the weld. It is therefore obvious that the farther apart the points of support are, the greater will be the angle attained before cracking occurs, simply because most of the bending will take place in the original metal. It is therefore highly desirable that there be prescribed a standard test that fixes exactly the distance between the supports, the curvature of the surface of supports, and the relation between this curvature, that of the loading surface, and the thickness of the specimen.

The Research Sub-Committee of the Welding Committee of the National Research Council is using, in connection with some of its research work, a bending test that tests a specimen 1½ in. (38.1 mm.) wide as a simple beam with two supports and with the load applied at the center of the weld and midway between the supports. The load is applied to a cylindrical surface, about which the specimen is bent, that has a diameter equal to the thickness of the specimen, which corresponds to the specifications of the American Society for Testing Materials for cold bending tests of steel plate. The supporting surfaces are placed as close together as practicable so that the deposited metal in the weld will occupy the largest possible proportion of the beam. This minimum distance is obviously that which will permit the bending of a specimen to 180 deg. The corners of the supports are machined and polished to a radius equal to the diameter of the loading surface and to the thickness of the specimen. The initial distance between the points of support is then the sum of twice the radius of the corners of the supports, the diameter of the loading surface, twice the thickness of the specimen and ⅛ in. (3.1 mm.) which gives 1/16 in. (1.5 mm.) clearance on each side. Such a test is very easily made in a standard testing machine by attaching a suitably prepared die to the moving head of the machine and setting two blocks with rounded corners on the bed of the machine with the proper space between them. The first crack on the under side of the specimen is readily detected with the aid of a mirror.

In a considerable number of tests of weld specimens made by this method, the maximum angle obtained was of the order of 15 deg. while the angles usually reported in the literature are of the order of 30 deg. to 60 deg. It seems very certain that the relatively small angle found is due to the greater severity of the test and not to any inferiority of the welds. It should be stated that the parent metal can be bent to 180 deg. by this method without a sign of a crack.

Similar standardization is required in tensile tests, particularly the gage length. With long gage lengths, the elongation in the weld itself is obscured by the stretch in the parent metal.

Mr. Hobart referred to the feeling that has been frequently expressed that more information in regard to the durability of the welded joints under the action of repeated loading is needed before entire confidence in welding in certain fields can be obtained. A special fatigue testing machine was developed in connection with the work of the Research Sub-Committee of the Welding Committee for testing specimens of welds in ½-in. (12.7 mm.) plate. The machine employs the rotating-beam principle, the beam having two free supports at the ends and two loads at the middle. The arrangement is such that the stress is absolutely uniform throughout the length of the specimen occupied by the weld and the exact maximum fibre stress can be calculated. The principle is not at all new but the particular form of apparatus developed is very small, simple, and inexpensive. The results of tests recently made indicate that where standard ship-plate steel will withstand 4,000,000 or 5,000,000 reversals of stress at 25,000 lb. per sq. in. before failure, electrically welded joints will withstand of the order of 700,000 or 800,000 repetitions of the stress. All the failures occur in the deposited metal, indicating that there is no difficulty in getting a thorough bond between the added metal and the original metal, but that the added metal has not the fatigue strength of the original metal. This simply confirms the general view that greater ductility in the deposited metal of electric welds must be had before the process can be applied to a maximum extent in those very large fields where this characteristic is absolutely essential.

W. S. Andrews (by letter): Mr. Hobart has omitted to make any mention of colored glasses to be used by welders, in connection with goggles, shields, and masks for protecting their eyes and skin from the dangerous radiations produced by the electric arc. The iron arc is especially prolific in generating ultra-violet radiation, which will induce painful and dangerous inflammation in the unprotected eye and a powerful arc will produce a sunburn effect on the bare skin in a very short time. Leather gloves on the hands and shields or masks of light material, such as thin wood or vulcanized fibre, afford a perfect protection to the face and neck from these harmful rays. The protection of the eyes from these rays and the intense glare of the arc, however, is not so easy, as it involves the necessity of preserving clear vision for the operator.

It is well known that the human eye is more sensitive to yellow and green light than it is to red and blue light; so, in order to secure the clearest vision with the least amount of glare, densely colored glasses should be used. These should exclude most of the red rays and practically all blue and violet rays, while the orange, yellow, and green rays should be toned down to the lowest degree consistent with clear observation of the work in hand. Owing to individual difference in visual acuteness, no absolute rule can be given as to the exact amount of obscuration necessary for various operations. This may be decided best by each operator for himself, for one man may be able to see clearly through a glass that would obscure the vision of another. On the other hand, the fundamental color transmission of all safety glasses should be determined by an expert in accordance with spectroscopic analysis. There are many safety glasses on the market that fully conform with these requirements. Glasses dense enough to tone down the intense glare of the electric arc to an extent comfortable to the eye will always afford sure protection against ultra-violet radiation.

Infra-red radiation or heat rays naturally produce a drying effect on the skin; but as the latter is sensitive to them, they give their own danger signal. The shields and masks worn as a protection against the ultra-violet rays will usually be found a sufficient guard against heat rays. Most of the reliable dense-colored safety glasses on the market are also good shields against heat rays, so there is very little danger, if any, to be apprehended from this source. All harmful effects to the eye from the radiations of the iron arc may indeed be entirely avoided by the exercise of a little common sense and intelligent caution.

J. C. Lincoln (by letter): The general problem of electric welding, as it presents itself to the writer, is the problem of substituting electric welding for riveting. This substitution should take place in the construction of ships, boilers, bridges, and in the industrial arts. It is a large problem but a solution of the electric welding problem will eventually very largely replace the art of riveting for patching together of steel members. In the construction of a ship's hull, smooth steel plates from 3/8 in. (9.5 mm.) to 3/4 in. thick, 6 ft. (1.8 m.) wide, and from 15 ft. (4.5 m.) to 30 ft. long must be joined to one another and to the framework. The plates must be welded to one another in such a way that the builder can be certain that the joint is equal to or similar in strength and ductility to the present riveted construction. Experience has shown that the ordinary electric weld, using the metallic electrode process, for samples 2 in. (5 cm.) or 3 in. (7.6 cm.) wide and from 3/8 to 3/4 in. thick will produce a weld that, when made with sufficient care and tested in the pulling machine is very much stronger than the best riveted joint. The writer, however, does not think that this statement can be made when large sheets are welded together, as in the construction of a ship's hull. The heating, which is necessarily a part of the electric welding process, introduces contraction stresses between the plates that are welded, and these internal stresses, in some cases, are practically equal to the strength of the weld. It is impossible by any means now known to determine the amount of these stresses and, consequently, the use of the riveting process for welding the plates, which is much more expensive and not so strong, is universally used because it eliminates these stresses. In some cases these internal stresses can be relieved by annealing the whole sheet, including the joints, after it is welded. This process has been used with entire success for welding some tanks for holding ammonia under a pressure of 200 lb. per sq. in. It is obvious that in the construction of a ship's hull such an annealing process is out of the question. The conclusion, therefore, is that if the construction of ship's hulls by electric or gas welding is to be successful, the weld must be of such a nature that the welded material can stretch and relieve the internal or contraction stresses and this stretching must not crack the joint at any time nor weaken the strength of the welded joint.

One or two hundred tests on metallic welded joints with the best electrode that could be secured convinced the writer that the metallic electrode welding process will not produce a joint that will fulfill the above specifications; consequently, his attention has been directed toward the use of the carbon arc. He has developed the practise to a point where it is possible to say that the metal in the weld fulfills the specifications necessary to take care of the internal stresses outlined above. In other words, the practise of carbon arc welding has been advanced to the point where it is easy to make joints that are past the elastic limit of ordinary steel plate. The metal in these welds at the same time is ductile so that a test piece of metal cut from the weld itself and pulled in a testing machine will show satisfactory elongation and reduction of area. The writer hopes within a short time to be able to present the details of this process.

Herman Lemp (by letter): This paper is a very good resumé of the art of fusion and spot welding of mild steel, but it has entirely overlooked the well-known process of resistance welding, originated by Prof. Elihu Thomson. The Thomson process, the sire of spot welding, is so well established in our industries and has been so thoroughly discussed in the technical press and the records of the A. I. E. E. that it has almost passed the stage of debate, but may well be called to the attention of the younger engineers·

When one considers the vast quantities of mild-steel products daily welded in the United States alone by that process, and this, with unskilled persons operating automatic and semi-automatic machinery, with a speed unexcelled by the fusion process, the writer may be pardoned for drawing attention to one characteristic feature of the process; namely, that parts to be welded are substantially protected against contact with the oxygen of the air, and what few oxides may be formed near the edges are effectively squeezed out of the joint by the pressure exerted on completing the weld. In addition, all kinds of metals and alloys, some of which are termed unweldable, are successfully welded thereby. Crane chains, harness rings, automobile rims, bicycle tubing, wire fences, carriage tires, metal wheels, etc., are welded daily in such quantities as would astonish any one when confronted with actual figures and in the welding of valve flanges to cylinders of Liberty aviation motors at Detroit, the Thomson process easily surpassed the gas and electric-arc processes, as regards both speed and excellence of finished product.

Spot welding was the first process considered by the Welding Committee created by the A. I. E. E. in the summer of 1917, and its adaptability to ship-plate welding was clearly recognized at that time in respect to fabricated parts that could be brought to a stationary spot welder in place of having the spot welder brought to the work. A portable spot welder was designed for ½ in. (12.7 mm.) ship plates. The long secondary leads required for spanning the gap, and consequently reduced output by reactance, failed to give satisfactory performance with the transformers originally designed for ½ in. plates. The increased capacity of the transformer required to overcome the effects of reactance has always been a bugaboo in connection with spot welders or electric welders of large dimensions, and the method of overcoming this reactance by making two welds in series by two transformers placed on opposite sides of the plates suggested itself to the writer in August of 1917, and was communicated to Mr. Hobart at that time. This statement in no way is meant to reflect upon the able work done by Mr. Weed, who independently developed and is responsible for the practical and successful design of the Duplex spot welder referred to in Mr. Hobart's paper. As the current flowing through both welds is substantially the same, the heating effect will depend entirely on the local resistance, which, in turn, depends on contact pressure. It is this difficulty that has been overcome in so satisfactory a manner by Mr. Weed by utilizing independent contact pressure for the two welds made simultaneously.

Spot welding, as the name implies, may be carried out in two ways. The electrodes through which the current is passed to the plates may be of so small a section that only the metal directly under the contacts or in its immediate vicinity becomes heated; or the plates to be welded together may be provided with raised surfaces, or teats, which form a restricted contact with each other and are heated to welding temperature. In the latter case, the electrodes supplying the current need not be so small in area as in the former modification of the process. As the thickness of the plates to be welded is increased, both the current density and the pressures are materially increased, and the problem of producing a satisfactory electrode with good wearing qualities becomes more difficult.

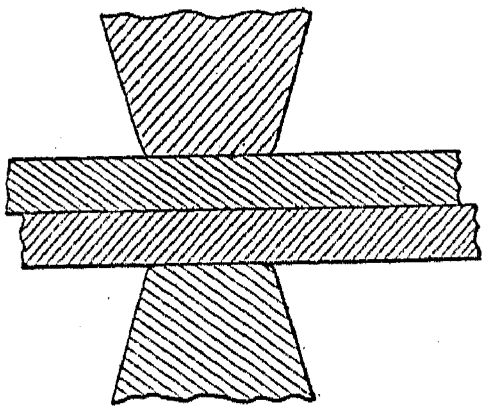


FIG. 1

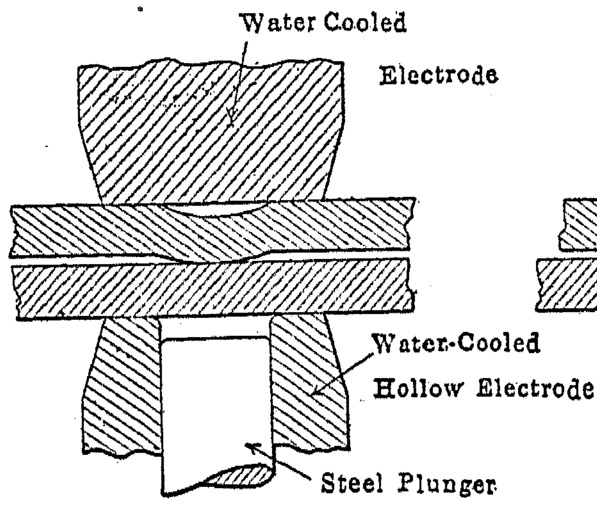


FIG. 2

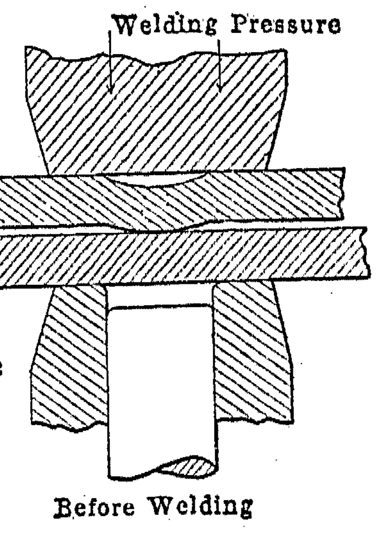


FIG. 3

From his experience with an electric armor-plate annealer, which he designed in 1894, the writer has learned that tremendous quantities of current can be passed through relatively small contact surfaces of copper, provided they are backed up by an increasing mass of copper properly water-cooled; 50,000 to 60,000 amperes were taken through contacts 1 in. (25.4 mm.) square, rounded off so as to make almost a point contact at times, and these contacts would last almost indefinitely, but in that particular application the pressure on the contacts was relatively light. I have been assured by Mr. Merrill, who was connected with the heavy spot-welding work referred to in Mr. Hobart's paper, that the problem of contacts is not as serious as one may think. It may be well, however, to consider the modification shown in the accompanying sketches, when large energies for spot welding are not available.

Fig. 1 shows the present method, in which two plates are placed in contact with each other and current passed through them through contact surfaces of restricted area. In this case the pressures on the contact surfaces have to be considerable and the surfaces themselves are relatively small.

Fig. 2 shows one of the plates provided with a series of teats produced by a punch press, similar to the one used for punching holes for riveting but in which the punch is rounded and is allowed to enter a small distance into the metal. Such a plate, when placed in contact with a water-cooled electrode of liberal dimensions, will make contact only on a ring surface, not in the middle. The other plate, which is without teats, rests on a hollow electrode, also of liberal dimensions, as far as contact surface is concerned. The teat in the upper plate insures a contact with the other plate at the apex of the teat and will be heated at the point of contact with the other plate.

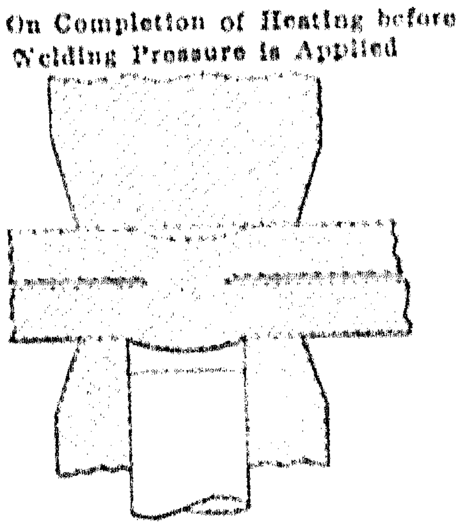


FIG. 4

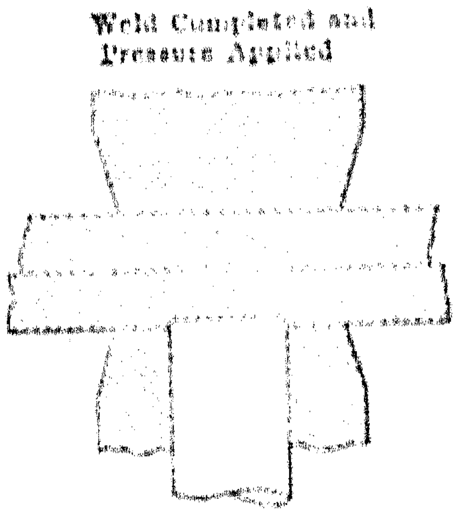


FIG. 5

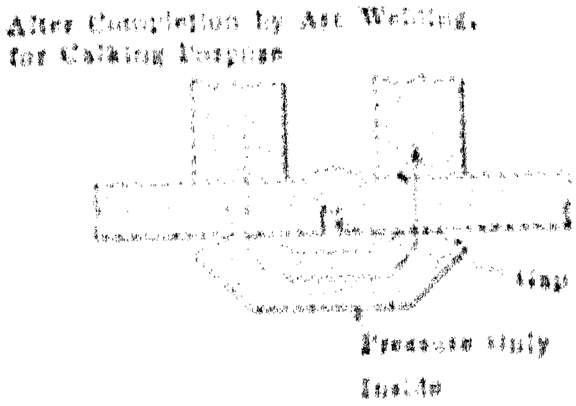


FIG. 6

Owing to the fact that the lower plate rests on a circular electrode unsupported and uncooled in the center, the center will be heated evenly with the teat in the upper plate. In the center of the lower electrode, but normally not in contact therewith, is a steel plunger, which may be separately operated by either hydraulic or pneumatic pressure.

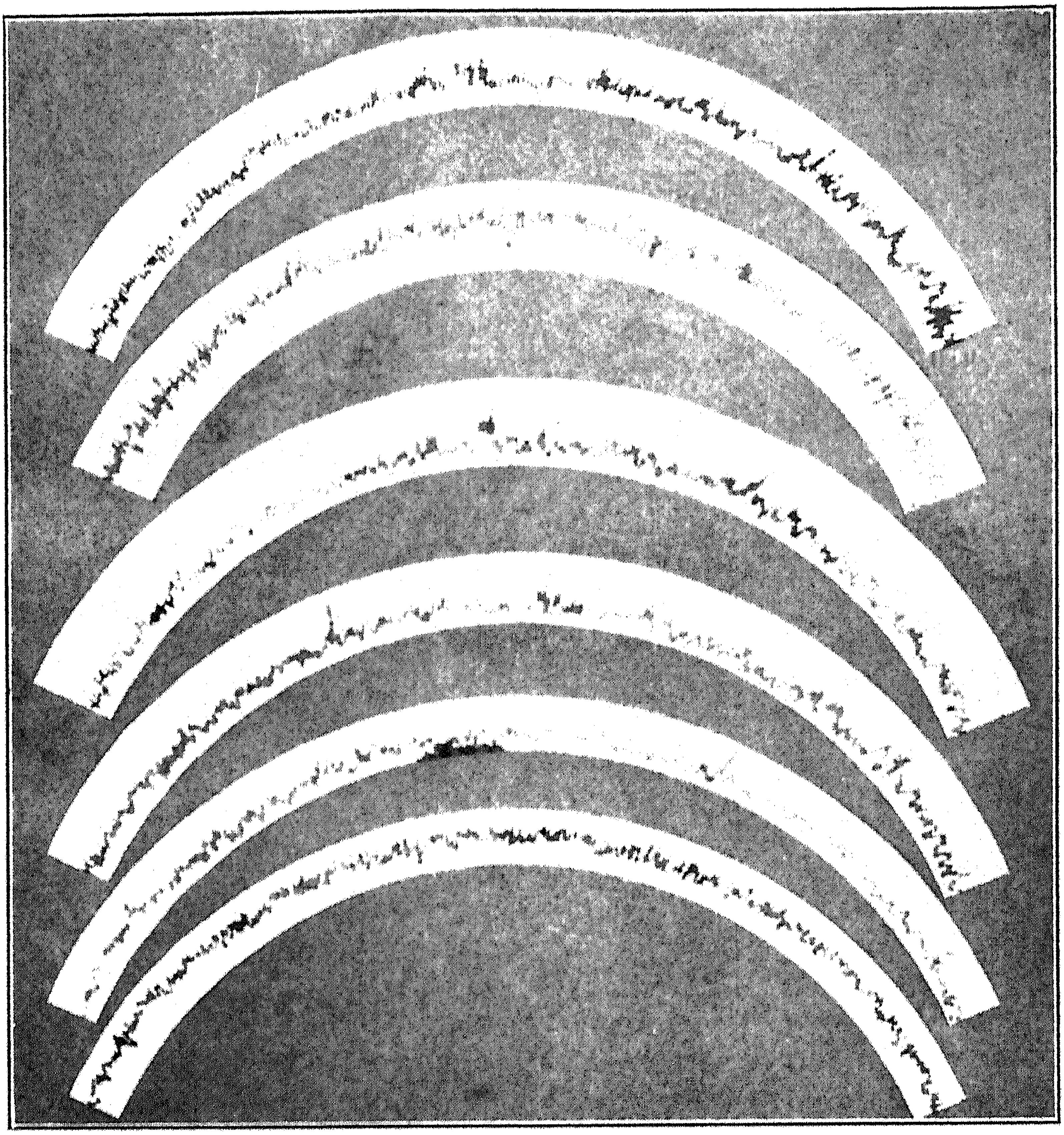
Fig. 3 shows the plates in contact under ordinary pressure suitable for proper heating at the beginning of the weld. Fig. 4 shows the plates on the completion of welding; they are still under contact pressure but the hydraulic pressure has not been applied. It will be noted that the plates have been pushed together and a cup-shaped depression formed at the bottom of the lower plate that fits into the space in the hollow electrode. It will also be noted that the hollow depression in the top of the upper plate, formed at the time when the teat was made, still exists. Fig. 5 shows the last part of the process. After cessation of current, pressure on the central plunger is applied, leaving the upper plate restored to its original form with a spot weld in the center and a cup-shaped depression in the bottom.

By this method we have, first, the certainty of concentrating the heat always at the teat, even if the plates should be slightly bent; second, lasting copper contacts; and, third, the heated metal is worked, reducing its loose structure to a denser fibre. The drawbacks of the method are that one of the plates, at least, will require an additional mechanical operation to provide the teats, and once these teats are formed, welds can be made at no other place unless loose metal buttons are inserted.

Welding Ship Plates with One Transformer. A method of welding ship plates by spot welding, which requires only one transformer in close contact with the plates, and hence a relatively low resistance, is shown in Fig. 6. The two plates to be welded are butted together without actually touching, leaving a narrow gap. A butt strap, or individual strap with teats, is then pressed against the inner side of the plates. The transformer is on the outside of the hull of the ship, so the current passes from plate to plate through the strap, and pressure is applied from the inside. After the welds are completed, the gap is closed by arc welding for calking purposes only. The welder and pressure devices, both inside and outside of the plates, are to be pulled together by direct-current electromagnets, which on the basis of 100 lb. per sq. in. can be made of sufficient size to give the necessary pressure.

Polarity. For carbon arc welding, the carbon electrode should be connected to the negative terminal; if connected to the positive, there is danger of making a hard weld, which cannot be properly machined. The positive pole is always hotter than the negative, hence, when welding with metal electrodes, it is best to connect the positive to the plates as they are usually of greater volume than the electrode and should be heated sufficiently to produce fusion. Under certain conditions only as when very thin plates are to be welded with the metallic arc, it might be well to connect the positive to the electrode. When welding with alternating current, graphite electrodes may be used when the plates to be welded have turned up edges to be united together and where no machining is required after welding. On general principles, metallic electrodes are more suitable for alternating currents.

Harry D. Morton (by letter): *Theory of Electric Arc.* The generally accepted theory of the electric arc is that part of the electrode material is vaporized, and that this vaporous tube or column forms a path for the electric current. As a result of the vaporous character of the current path, all arcs are inherently unstable; and the maximum of instability is no doubt found in that form of arc employed for metallic-electrode welding purposes. We here have, in conjunction with the natural instability characteristic of all arcs, rapidly fusing electrode materials and the disturbing



[MORTON]

FIG. 7—TYPICAL AMMETER CHARTS OF OPERATION OF AUTOMATIC METALLIC-ELECTRODE ARC-WELDING MACHINE. AVERAGE TIME ABOUT 1 MIN. 45 SEC.

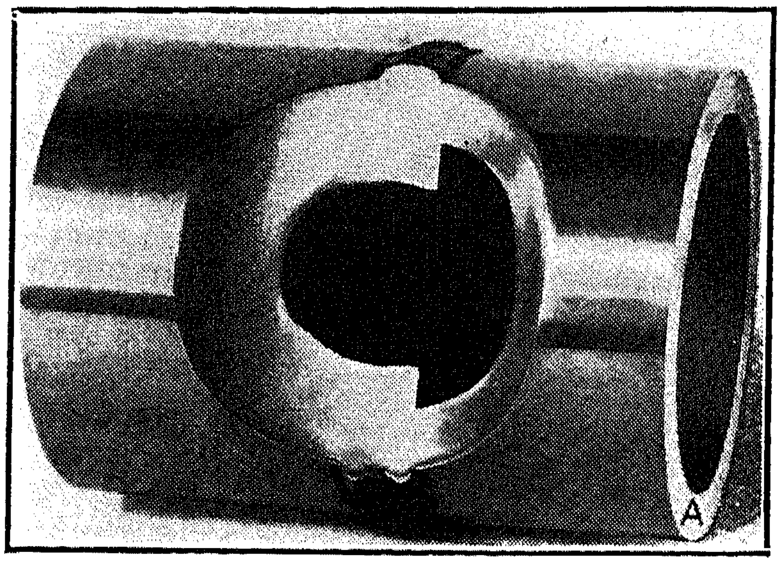


Fig. 8—Piloted Cup Automatically Welded by Metallic-Electrode Arc Process to Tube to Form 75 mm. Shrapnel Shell

Analysis of Electrode Material: Silicon, 0.02 per cent; Sulphur, 0.013 per cent; Phosphorous, 0.07 per cent; Manganese, Trace; Carbon, 0.07 per cent; Aluminum, 0.038 per cent.

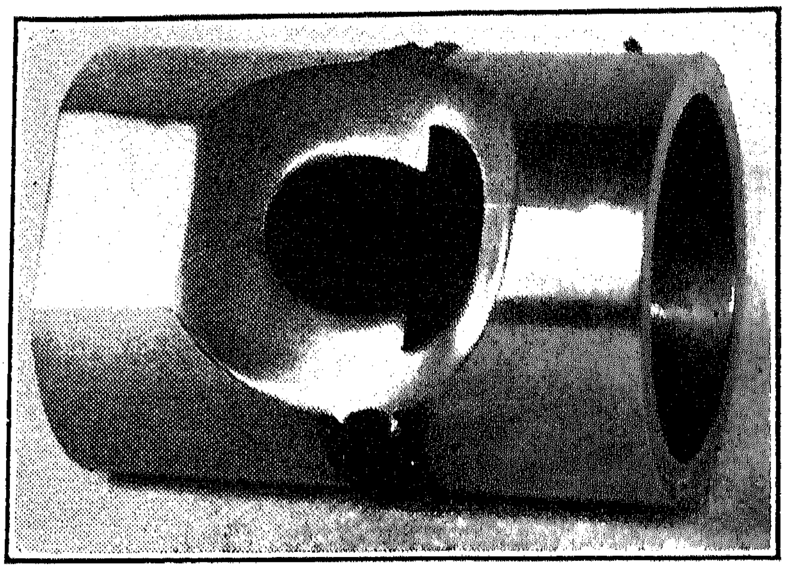


Fig. 9—Piloted Cup Automatically Welded by Metallic-Electrode Arc Process to Tube to Form 75 mm. Shrapnel Shell

Analysis of Electrode Material: Silicon, 0.03 per cent; Sulphur, 0.049 per cent; Phosphorous, 0.008 per cent; Manganese, 0.31 per cent; Carbon, 0.28 per cent.

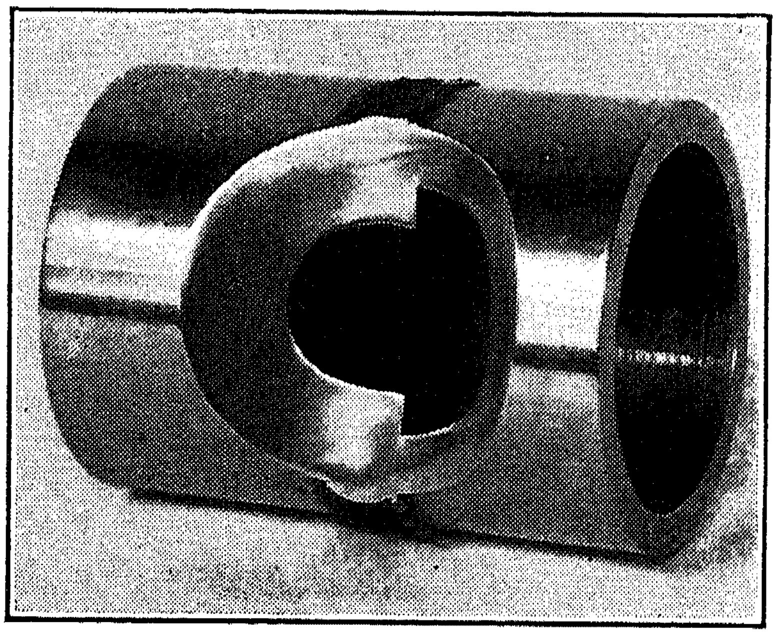


Fig. 10—Piloted Cup Automatically Welded by Metallic-Electrode Arc Process to Tube to Form 75 mm. Shrapnel Shell

Analysis of Electrode Material: Silicon, 0.02 per cent; Sulphur, 0.032 per cent, Phosphorous, 0.008 per cent; Manganese, 0.20 per cent, Carbon, 0.18 per cent.

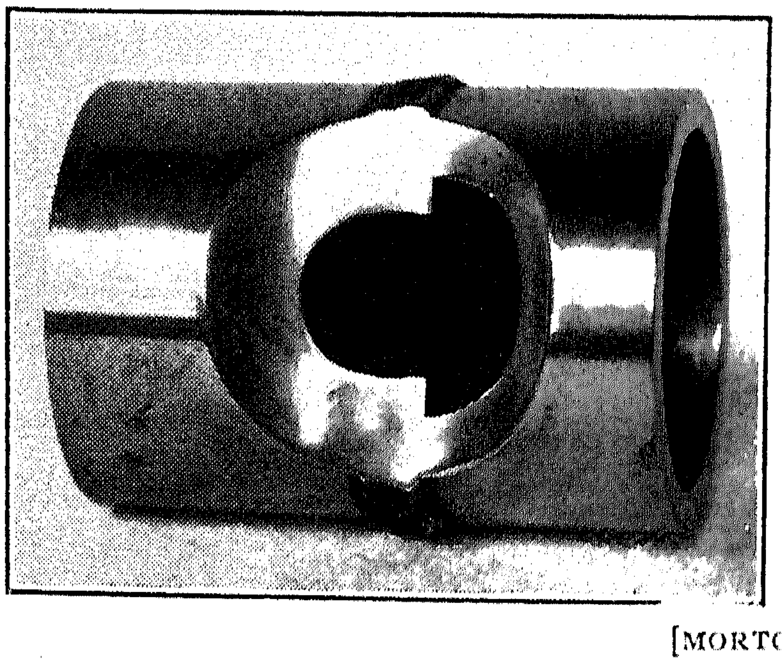


Fig. 11—Piloted Cup Automatically Welded by Metallic-Electrode Arc Process to Tube to Form 75 mm. Shrapnel Shell

Analysis of Electrode Material: Silicon, 0.04 per cent; Sulphur, 0.016 per cent; Phosphorous, 0.058 per cent; Manganese, None; Carbon, 0.24 per cent.

effect of the constant passage through the arc of a large quantity of molten metal to form the weld. This molten metal must pass through the arc so rapidly that it will not be injured or materially contaminated, otherwise the weld will be useless. Prima facie, the combination of these unfavorable conditions would seem to justify fully the skepticism of most electrical engineers as to the possibility of effecting such control of the metallic arc as to permit of uniformity and continuity in welding results. In addition, there is another and more important factor, and one that seriously mitigates against this desired uniformity and continuity, namely, the personal equation of the operator. The consensus of opinion, so far as is known to the writer, seems to be that about 95 per cent of the welding result is dependent on the skill of the operator and that at least six months' practise is necessary to acquire reasonably satisfactory proficiency.

As the result of thousands of observations of welds produced automatically (wherein the personal equation is entirely eliminated), the writer inclines toward the theory that the molten electrode material passes through the arc in the form of globules; and that where ⅛-in. electrode material is employed with a current of about 150 amperes these globules are deposited at the rate of approximately two per second. The passage through the arc of each globule apparently constitutes a specific cause of instability in addition to those existent with slowly consumed electrodes. This hypothesis seems to be borne out by ammeter records (typical specimens of which appear in Fig. 7), together with the fact that the electrode fuses at the rate of 0.2 in. (5.08 mm.) per sec. Moreover, the globules appear to be approximately equal in volume to a piece of wire 0.125 in. (3.07 mm.) in diameter and 0.1 in. (2.54 mm.) long.

Essentials for Maintaining Uniform Length of Arc. Assuming this theory to be correct, to maintain a uniform arc length in manual welding, the operator must feed the electrode toward the work at the rate of 0.1 in. upon the deposition of each globule, in other words, 0.1 in. twice per second, a synchronism beyond human attainment. Simultaneously with such feeding, the arc must be moved over the work to melt the work material, distribute the molten electrode material, and form the weld. Inasmuch as the effect of the arc is highly localized, it is reasonable to suppose that different parts of the welding area present relatively wide variations in respect to temperature, fluidity, and conductivity of the molten mass—controlling factors not within the ken of the human mind. The situation is further complicated by the facts that neither the welding wire nor the work material is uniform in fusibility or in conductivity, and that the contour of the work varies continually as its surface is fused and the molten metal is caused to flow. The belief is general that a very short arc is productive of the best welding results; but it is an arc of this character that makes the greatest demands on the skill of the operator, for there is always the danger that the electrode will actually contact with the work and destroy the arc.

Effect of Fluctuations of Arc Length on Fusion Energy. As the fusing energy of the arc varies widely with fluctuations in the arc length and as the uniformity of the weld depends on the constancy and correctness of this fusing energy, it seems remarkable that operators are able ever to acquire such a degree of skill as to enable them to produce welds that are even commercially satisfactory. Further, so far as the writer is informed, there is no means, other than such as would be destructive, for determining whether a completed weld is good or bad. The logical solution appeared, to the writer, to be the elimination of the personal equation and the substitution therefor of means whereby tendencies toward variations in the arc would be caused automatically to correct themselves, just as a steam engine, through the action of its governor, is caused to control its own speed.

Methods of Mechanically Stabilizing and Controlling the Arc. The efforts of the writer, for a number of years, have been directed toward stabilizing and controlling the metallic arc, and applying such stabilizing and controlling means to two general lines of welding machinery: (1) machines wherein the feeding of the electrode wire, with reference to the work, and the producing simultaneously therewith of relative movement between the wire and the work are both automatically effected; and (2) what, for lack of a better term, might be called a semi-automatic machine, in which the feeding of the electrode and the control of the arc are accomplished automatically but the traversing of the electrode with reference to the work is manually effected by the operator, permitting him the exercise of judgment with reference to the quantity of metal to be deposited in various parts of the groove. The automatic machine has been in successful operation for a long period and the semi-automatic machine for about five months. While the goal was not attained without many difficulties and a great expenditure of time and money, the results have been surprisingly successful.

Because of the lack of any definite data as to what actually occurs in this form of arc, or why it occurs, due, no doubt, to the impossibility of differentiating between phenomena that are characteristic of the arc and phenomena due to the personal equation of the welder, it seemed logical that the initial step should be to so environ the arc that it would not be subject to erratic extraneous influences, to the end that reasonably definite determinations might be substituted for scientific speculation. In the design and construction of the machines, great care was exercised to minimize the possibility of mechanical defects that might lead to erroneous conclusions. Starting with the assumption that the work could only be based on open-minded observation of the behavior of the arc under machine control, an automatic welding machine was built in which was incorporated the greatest possible number of adjustable features, in order that, if necessary, it might be possible to wander far afield in the investigations. This adjustability has proved invaluable in that it has permitted logical, consistent, and consequential experimenting over a very wide range of conditions. Working under these favorable circumstances, there were soon segregated a few clearly demonstrable facts to serve as a foundation for the structure, which has since been added to, brick by brick, as it were.

Efforts have been directed toward the practical rather than the scientific aspect of the subject. The operation of the automatic machines has brought to light many curious and interesting phenomena, some of which appear to negative conclusions heretofore formed which have been predicated upon observations made in connection with manual welding. It is hoped that these and other phenomena, which can thus be identified as purely arc characteristics, will be the subject of profitable scientific investigation when time is available for this purpose.

Development of Automatic Machines. In the five forms of machines made in the course of the development, the welding wire is automatically fed to the arc; and, in the first four machines, the relative movement between the work and the welding wire is automatically and simultaneously effected. Early in his investigations, the writer concluded that a substantial equilibrium must be maintained between the fusing energy of the arc and the feeding rate of the welding strip; and it soon became evident that if the welding strip is mechanically fed forward at a uniform rate equal to the average rate of consumption with the selected arc energy, this equilibrium is actually maintained by the arc itself, which seems to have, within certain circumscribed limits, a compensatory action as follows: When the arc shortens, the resistance decreases and the current rises. This rise in current causes the welding strip to fuse more rapidly than it is fed, thereby causing the arc to lengthen. Conversely, when the arc lengthens, the resistance increases, the current falls, the welding strip is fused more slowly than it is fed, and the moving strip restores the arc to its normal length.

While this compensatory action of the arc will maintain the necessary equilibrium between the fusing energy and the feeding rate under very carefully adjusted conditions, this takes place only within relatively narrow limits. It was very apparent that, due to variations in the contour of the work, and, perhaps, to differences in the fusibility or conductivity of the welding strip or of the work, the range of this self-compensatory action of the arc was frequently insufficient to prevent either contacting of the welding strip with the work or a rupture of the arc due to its becoming too long. The problem that arose was to devise means whereby the natural self-compensatory action of the arc could be so greatly accentuated as to preclude, within wide limits, the occurrence of marked arc abnormalities. There was ultimately evolved, by experiment, such a relation between the fusing energy of the arc and the feeding rate of the welding strip as to give the desired arc length under normal conditions; and tendencies toward abnormalities in arc conditions, no matter how produced, were caused to bring into operation compensatory means for automatically, progressively, and correctively varying this relation between fusing energy and feeding rate, such compensatory means being under the control of a dominant characteristic of the arc. In their ultimate forms, the devices for effecting the control of the arc are simple and entirely positive in action, making discrepancies between fusing energy and feeding rate self-compensatory throughout widely varying welding conditions. For instance, the shrapnel shell shown in Fig. 8 was automatically welded with wire differing greatly in chemical constitution from that used on the shell shown in Fig. 9 (see analyses), yet no change was made in either the mechanical or the electrical adjustments. The radically different welding conditions were compensated for solely by the operation of the automatic control. The electrode materials used for the shells shown in Figs. 10 and 11 differed so greatly from those employed respectively in welding the shells shown in Figs. 8 and 9 that a change in the relation between fusing energy and feeding rate had to be made manually. After this adjustment was made, the shells were welded with their respective electrodes, which varied widely in their chemical constitution, without further manually changing either the mechanical or the electrical conditions.

In a recent test of the semi-automatic machine, shown in Fig. 12, successful welds were made under the condition that the impressed voltage of the welding generator was changed throughout a range of from 50 to 65 volts, without necessitating any manual adjustment. The only observable effects of the wide variations in the supply voltage were slight differences in the arc length.

In short, the compensatory action of the control has proved effective over so wide a range of welding conditions, not only as to the electrical supply and chemical constitution of both electrode and work materials, but as to extensive variations in the contour of the work and in many other particulars, as to make it seem apparent that the machines do not represent merely successful laboratory experiments but are suited to the requirements of actual commercial welding.

Angle of Electrode to Work on Insertion of Extra Resistance. One particularly interesting observation resulting from the experiments is that the angle of inclination of the electrode with reference to the work is very important. An angular variation of 5 deg. will sometimes determine the difference between

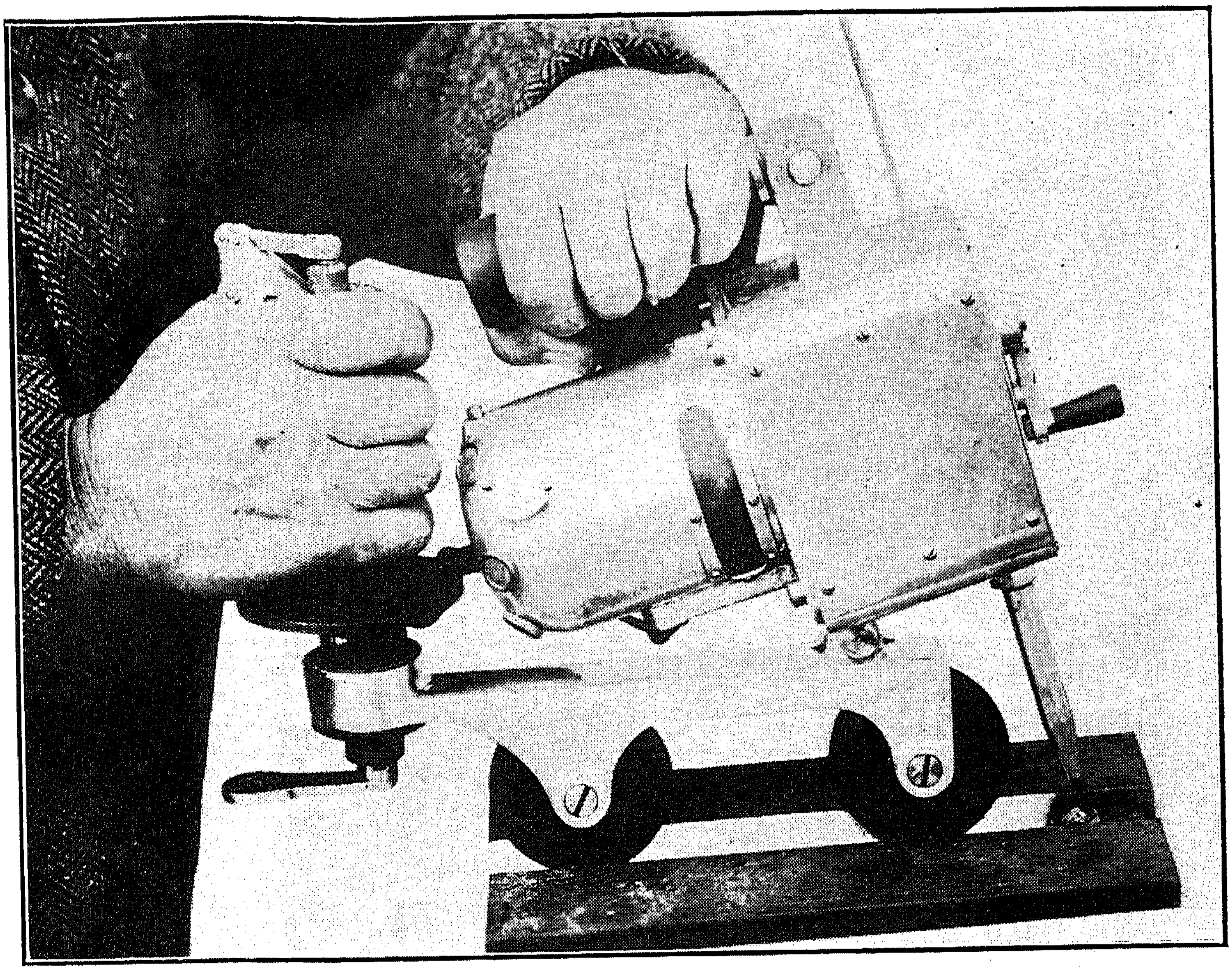


FIG. 12—SEMI-AUTOMATIC METALLIC-ELECTRODE ARC-WELDING MACHINE

The electrode is automatically fed to the arc, which is automatically maintained, while the machine is manually moved along the grove to be welded.

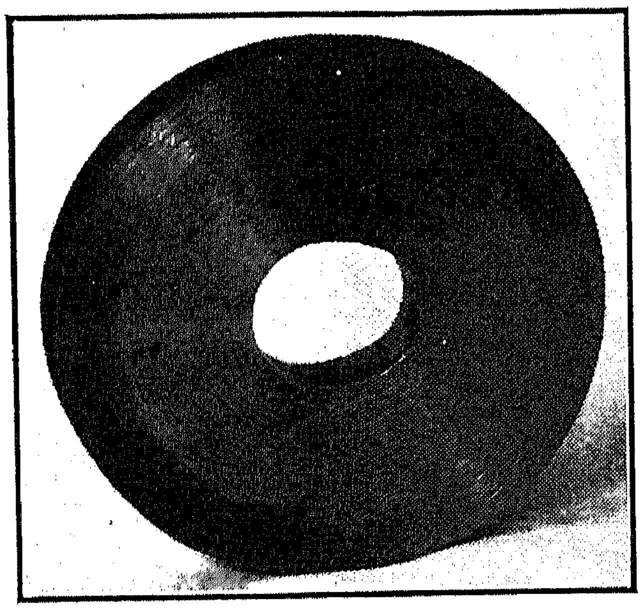
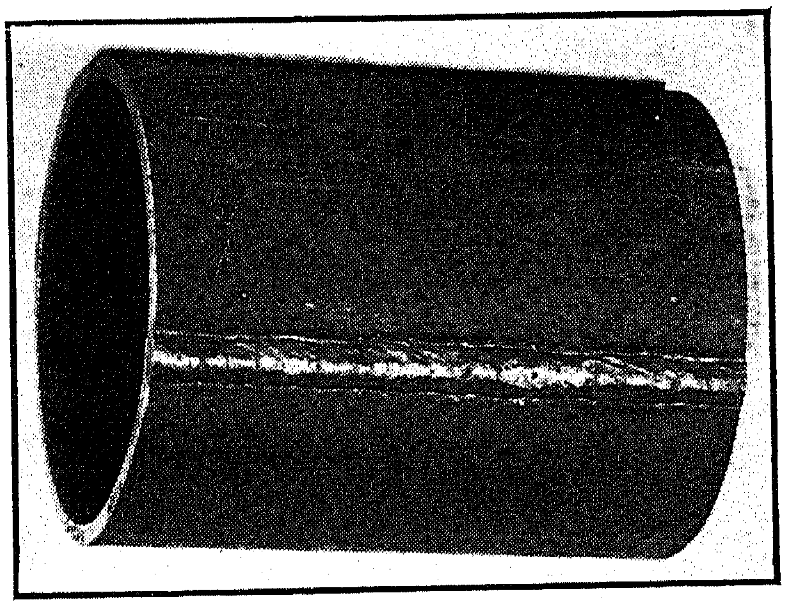


FIG. 13—TUNGSTEN HIGH-SPEED RING AUTOMATICALLY WELDED BY METALLIC-ELECTRODE ARC PROCESS TO COLD-ROLLED CORE TO FORM MILLING-CUTTER BLANK



[MORTON]

FIG. 14—NO. 11 GAGE STEEL TUBING AUTOMATICALLY WELDED BY METALLIC-ELECTRODE ARC PROCESS AT THE RATE OF 1 FOOT (30.4 CM.) PER MIN.

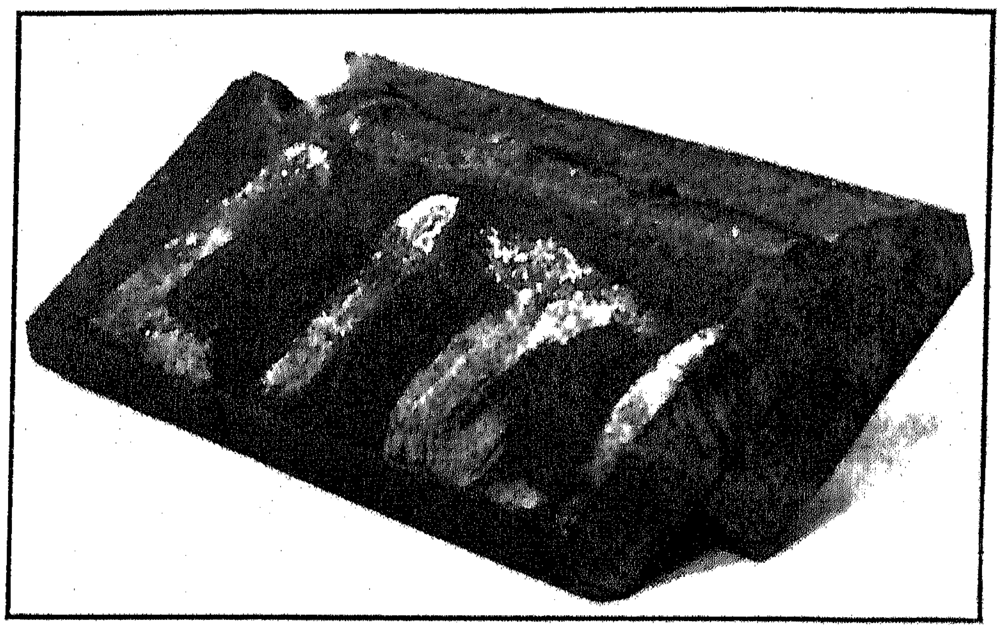


FIG. 15—TWO ½-IN. SHIP PLATES AUTOMATICALLY WELDED BY METALLIC-ELECTRODE ARC PROCESS TO FORM LAP JOINT

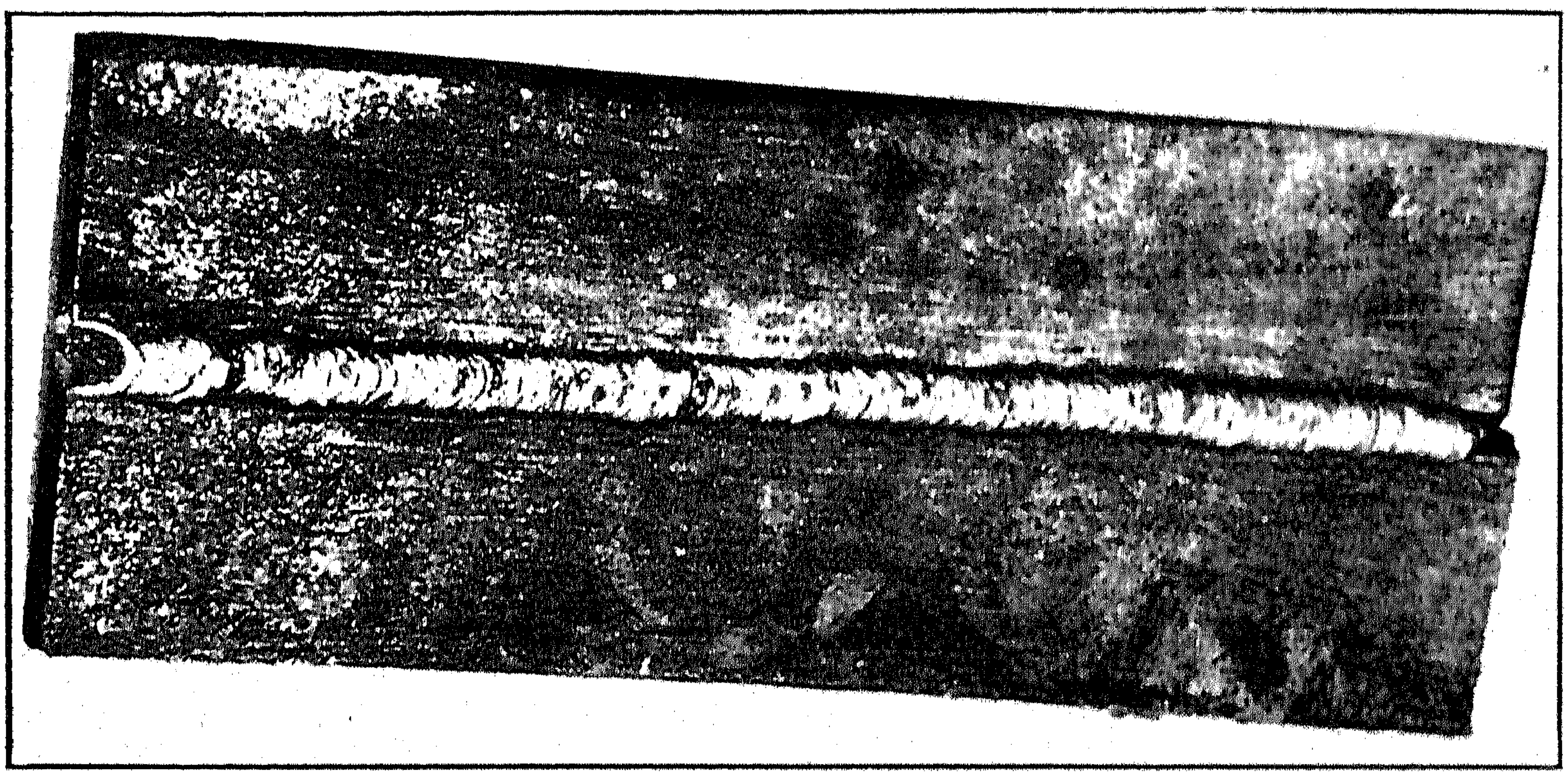
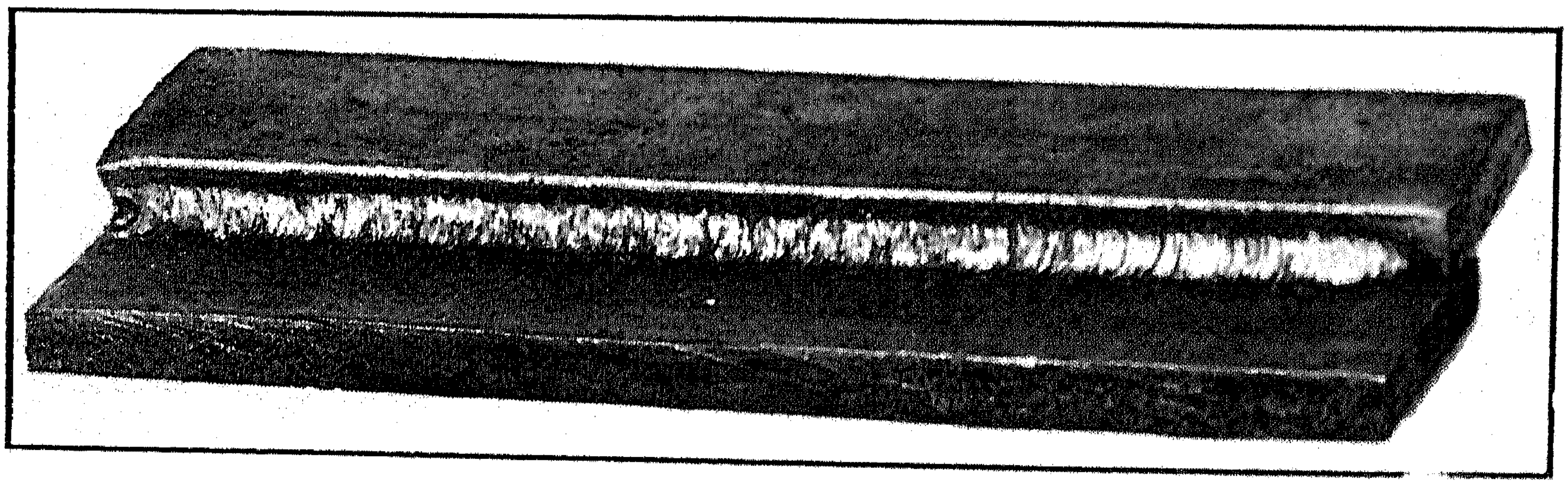


FIG. 16—TWO ½-IN. SHIP PLATES AUTOMATICALLY WELDED BY METALLIC-ELECTRODE ARC PROCESS TO FORM BUTT JOINT



[MORTON]

FIG. 17—TWO ½-IN. SHIP PLATES AUTOMATICALLY WELDED BY METALLIC-ELECTRODE ARC PROCESS, SHOWING FIRST OF THREE LAYERS TO FORM LAP JOINT

ss and failure in a weld. About 15 deg. from the per-
cular works well in many cases. In welding some mater-
he electrode should drag, that is, point toward the part
ly welded rather than toward the unwelded part of the

ile it has been customary in some welding systems to
de means whereby extra resistance is inserted in series
the arc at the instant of the initial contact which starts
ow of current, which resistance is automatically cut out
the striking of the arc, experience with the automatic
ines indicates that this is quite unnecessary.

lation of Electrode to Work Materials. Early in the ex-
ents, it was noted that in many cases there was a de-
ly marked affinity between particular electrode materials
particular work materials. A slight change in either ele-
affects the degree of this affinity. While it has invariably
possible to control and maintain the arc and weld con-
usly, in some instances incompatibility between electrode
rial and work material has been productive of interesting
omena. For instance, the combination of work material
l of about 0.45 per cent carbon content) and the particular
rode material shown in Fig. 8 produced an arc that was
rkably quiet and free from sputtering. Throughout the
, this arc was suggestive of the quiet flame of a candle or
, the erratic behavior that we are accustomed to associate
the ordinary metallic arc being absent. The effect is
cted in the uniform deposition of the welding material.
n some classes of work material Bessemer wire, which some
orities claim cannot be used in metallic-electrode arc
ing, produces an arc and a weld very satisfactory in
earance. On other work material, the Bessemer wire arc
iolently explosive. These explosions are accompanied by
e sharp reports and the scattering over some considerable
ance of globules of molten metal frequently 3/32 in. (2.38
.) or more in diameter. Under certain other conditions,
arently growing out of incompatibility between the work
erial and the electrode material, the oxygen flame accom-
ying the arc gyrates very rapidly about the arc, produc-
an effect suggestive of the "whirling dervish."

rom both the practical and the scientific points of view,
writer has experimented quite extensively with varying
binations of work material and electrode material.
oughout all the differences in arc conditions, many of which
ably accentuate the natural inclination toward instability,
control has so operated as to justify the expression "the
persists".

enerally speaking, the Swedish and Norway iron wires seem
produce more quiet arcs and, possibly, a more uniform de-
ition of electrode material, than do wires of other classes.
ese welds may perhaps be found to be slightly more ductile

than those made with wires of other chemical composition. On the other hand, these soft wires, although undoubtedly of relatively high fusibility, do not, for some reason, seem to produce an arc that cuts into some work material as deeply as might be desired, nor as deeply as do the arcs formed with certain other kinds of wire. Considered from every angle, the writer is disposed to regard the Roebling welding wire as the best he has thus far tested for use on mild steel. This wire produces a reasonably quiet arc which seems to cut into the work to more than ordinary depth, while, at the same time, the electrode material is fused with more than average rapidity—thus increasing the welding rate.

While scientists will no doubt ultimately arrive at the correct hypothesis for solving the problem of why one combination of electrode material and work material is productive of better results than can be obtained with another combination, the writer's conclusion is that, with the data at present available, the determinations must be made by actual experimenting—having in mind the qualities desired in the particular weld, such as ductility, tensile strength, elongation, and elastic limit. Inasmuch as it is possible, with the automatic machine, to maintain arc uniformity with practically any kind of electrode material and to produce welds which, under low magnification, at least, appear to be perfect, and which respond favorably to ordinary tests such as bending, cutting and filing, it is reasonable to conclude that proper selection of electrode material will be productive of perfect welds on any kind of work material. To date, no steel has been tested on which apparently satisfactory welds could not be made. High-speed tungsten steel has been successfully welded to cold-rolled shafting, using Bessemer wire as electrode material, as is shown in Fig. 13 Ordinary steels varying in carbon content from perhaps 0.10 to 0.55 per cent have been welded with entire success.

Condition of Metal in Crater. Because of the fact that the complete welding operation has been automatic and may be continued for a considerable length of time, say 5 min., an exceptional opportunity has been afforded for close concentration upon and study of the appearance of the arc. What seems to occur is that the molten metal in the crater is in a state of violent surging, suggestive of a small lake lashed by a terrific storm. The waves are dashed against the sides of the crater, where the molten metal of which they are composed quickly solidifies. The surgings do not seem to synchronize with nor to be caused by the falling of the globules of molten metal into the crater, but seem rather to be continuous. They give the impression that the molten metal is subjected to an action arising from the disturbance of some powerful force associated with the arc—such, for instance, as might result from the violent distortion of a strong magnetic field. Altogether, the crater phenomena are very impressive; and the writer hopes ere long to be able to have motion pictures made which, when enlarged, should not only afford material for most fascinating study, but also throw light upon some of the mysterious happenings in the arc.

Electrodes and Currents Used. So far, electrode wires 1/8 in. (3.17 mm.) in diameter have been chiefly used in the machines. Successful welds have been made with current values ranging from below 90 to above 200 amperes at impressed voltages of 40, 45, 50, 55, 60, 65, and 80. Under these varying conditions, the voltage across the arc has been roughly from 16 to 22. The machines have thus far been run only on direct current. Inasmuch as it is possible, by electrical and mechanical adjustments, to establish nearly any arc length that may be found to be most desirable for a particular class of work, and as the control system will maintain substantially that arc length indefinitely, the fully automatic type of machine is nearly as certain in operation as a lathe, drill press, or other machine tool.

Automatic and Semi-automatic Welding Machine. Upon the writer's appointment to membership on the Welding Committee of the Emergency Fleet Corpn., Prof. Adams and other members of the Committee urged upon him the advisability, for purposes of ship welding, of incorporating in a small and readily portable welding tool the features of automatic wire feed and arc control. The tool designed is shown in Fig. 12. In welding with this so-called semi-automatic machine, which weighs about 10¼ lb. (4.6 kg.), the operator draws the tool along the groove to be welded at such a rate as will result in the deposition of the quantity of metal required to satisfactorily effect the weld. This tool is intended for use in the many restricted spaces encountered in ship welding, which would be relatively inacessible to a fully automatic machine. In its use, the skill required by the operator is reduced to a minimum. After having practised with the welding tool for not more than two hours, the writer demonstrated it to Mr. E. L. Hirt, of Bethlehem Shipbuilding Corpn. Mr. Hirt expressed it as his opinion that it would require six months to train a welder to such a degree of proficiency as to enable him to make a weld equally good in appearance.

Mr. Hobart, under "Fusion Welding," says "There is always a matter of 0.1 in. (2.5 mm.) or more between the end of the welding rod and the work." While undoubtedly it is difficult, if not impossible, to maintain in manual welding an arc shorter than this, the writer has frequently, with the automatic machines, made continuous and strikingly good welds with arcs of much less length. In fact, in some cases there has been continuously maintained an arc so short that there hardly seemed to be any actual separation. The writer has even wondered whether, under these conditions, there was not a close approach to casting with a continuous stream of fluid metal acting as the current conveyor in lieu of or in parallel with the usual assumed vapor path. The work that has been done indicates that under automatic control much shorter arcs can be utilized than have hitherto been deemed possible, and with probable marked gain in quality of work in some instances; also, that there is much to be learned as to the mode of current action and current conduction in such an arc.

Results Obtained with Automatic Welding Machine. With the automatic machine, black drawing steel 0.109 in. (2.76 mm.) thick has been welded at the rate of 22 in. (55.8 cm.) per min. A Detroit manufacturer welded manually with oxyacetylene at the rate of four per hour a large number of mine floats 10 in. (254 mm.) in diameter, made of this material. The automatic machine made the welds at the rate of forty per hour. Liberty motor valve cages 2¾ in. (69.8 mm.) in diameter have been welded to cylinders in 36 sec., as against about 5 min. required for manual welding. No. 11 gage steel tubing, shown in Fig. 14, has been welded, with an unnecessarily heavy deposit of metal, at the rate of 1 ft. (30.4 cm.) per min. The productive capacity of the machines so far made has been from three to ten times that of manual welding methods, depending on the thickness of the work material; the difference in favor of automatic welding varies inversely as such thickness. The writer is now designing an improved type of machine for use especially on heavy work, with which machine it is expected to be able automatically to lapweld ½ in. (12.7 mm.) ship plates, in the manner shown in Fig. 15, at the rate of 15 ft. (4.5 m.) per hr. One of the largest shipbuilding concerns in the United States reports that the general average of all its manual welders on this class of work is from 1 ft. (30.4 cm.) to 18 in. (45.7 cm.) per hr. Other specimens of automatic welding on ship plates are shown in Figs. 16 and 17.

Bare wire only has been used in the automatic machines; and the results obtained seem to indicate that the covering of the electrodes is an expensive superfluity. If the chief advantage of the covered electrode lies in the ability of the operator to maintain a very short arc, an arc equally short and possibly shorter can be continuously maintained by the automatic machine using bare electrodes.

No attempt has thus far been made to use the automatic machines on over head work. The welds made with the fully automatic machine have been of three kinds: the usual longitudinal form, annular about a horizontal axis, and annular about a vertical axis. The experience gained indicates that there is no difficulty in dealing with any ordinary contour.

As far as the maintenance of arc uniformity and the apparent character of the welds are concerned, they do not seem to be greatly influenced by any of the considerations referred to in the specifications appearing in Section 4 of Mr. Hobart's paper. The writer has repeatedly welded with wire showing evidence of pipes and seams, as well as with rusty wire and with wire covered with dirt and grease. In this connection it may be said that no pains is ever taken to remove rust, scale, or slag from the work material—even where welds are superimposed. Apparently under uniform conditions of work traverse, arc length, and electrode angle of inclination, such as are possible in the automatic machine, impurities vanish before the portion of the work on which they occur reaches the welding area of the arc.

Referring to Section 15 entitled, "Ductility of Arc Welds:" the writer is fully convinced that with the use of the automatic machine, ductility, like other physical properties in the weld, can be controlled by proper selection of electrode wire, in conjunction with electrical and mechanical adjustments best suited to the particular purpose in view. Automatic welds have repeatedly been made on $^5/_{16}$ in. (7.93 mm.) mild steel which, when subjected to a 90 deg. bend, showed a marked extrusion of the welded material but no sign of fracture. When the welded pieces are cut with a hack saw, it is very unusual to be able to note any difference in cutting qualities between the unwelded and the welded parts.

Welding Thin Material. While the automatic machine has not been used on metal less than 0.109 in. thick, it is fair to presume that, with proper adjustments, entirely satisfactory results can be obtained on much thinner work—particularly if the nature of the work is such as to permit of the use of a chill. The best method in welding very light metal seems to be to use a small electrode, a relatively low current, and a high rate of work traverse. In this way welding conditions may be controlled to almost any desired extent, because the heating action of the arc can be modified, its effect intensely localized, and the edges to be welded subjected to the fusing action for as brief a time as might be found necessary to prevent burning of the metal. These conditions, which seem to the writer to be requisite in order to successfully weld very thin material, cannot be met by the manual welder. It is here that the deficiencies incident to the personal equation become most apparent. A very slight variation in arc length or the least hesitancy in moving the arc over the work will almost certainly result in its being burned through. In short, this class of welding calls for a coordination of faculties and a delicacy of manipulation beyond the capabilities of the most skillful manual electric welder. Therefore this work is usually done with the oxy-acetylene flame, wherein fusing conditions are far more easily controlled than is possible in manual metallic-electrode arc welding.

R. E. Wagner: (by letter): The practise of arc welding, as it exists today, consists largely of the results of experiments and practical experience; the study of the underlying scientific principles has not kept pace with its practical application. This situation, however, does not differ materially from that of many other arts where practise leads the way to the laws and principles that govern their operations.

Arc welding has been widely used for a number of years and, where properly employed, is a reliable and economical process by which steel parts may be fused together. Excellent welding is being done due largely to the skill of the operator.

Many systems have been developed, all based on some particular feature that the inventor has considered as being important. Very little was done to bring out the advantages of these systems until the Welding Committee of the Emergency Fleet Corpn. began its work of research; then it was found that few of the fundamental principles of the art were known. It was generally admitted that a short arc should be used, that the size of electrodes and amount of current required would change for different thicknesses of pieces to be welded, that pieces would have different preparations for welding depending on their thickness and strength of weld desired, etc. But the best values in any of these cases could not be definitely specified. It is, therefore, necessary that a great deal of research work be done in order to establish the fundamentals of the art.

Among the fundamentals to be investigated, for example, are: (1) How is the character of the weld affected by varying values of current with a constant voltage supply? (2) How is the character of the weld affected by varying values of supply voltage at constant current? In determining these conditions we must assume a thickness and kind of steel plate, size and kind of electrode, a preparation of plate edges for welding, a method of welding, a good operator, and a source of current.

In an effort to answer these questions, tests were made with ½-in. tank-steel plates that contained 0.25 per cent carbon, 0.40 per cent manganese, 0.025 per cent phosphorus, 0.000 per cent silicon, and 0.028 per cent sulphur. The electrodes used contained 0.185 per cent carbon, 0.561 per cent manganese, 0.037 per cent phosphorus, a trace of silicon, and 0.038 per cent sulphur. The plate edges were prepared with a double *V* and had 45 deg. and 30 deg. bevels; they were 10 in. (254 mm.) by 4 in. (101.6 mm.) in size and the length of the weld was 10 in. Two sizes of Roebling electrodes were used, the 5/32-in. (3.96 mm.) and the 3/32 in. (4.76 mm.). All the welds were made by the same operator, who was a good welder; direct current was used. Later, the raised part of the welds was ground off even with the plate surface and the welded plates were cut into strips 1 in. (25.4 mm.) wide for the tensile and bending tests.

With varying direct current, and a constant 75-volts, the following average values were obtained:

Current amperes	Tensile strength, lb. per sq. in.	Brinnell hardness value	
		Plate	Weld
80	37,000	106	114
125	50,000	114	114
150	57,000	122	122
180	63,000	122	114
220	66,000	114	114
275	62,000	114	114
300	58,000	122	114

The curve given in Fig. 18 shows the plotting of values obtained in the test. The 5/32-in. electrode was used up to 220 amperes where it became so difficult to manipulate that a change was made to 3/16-in. The tests at 180 and 220 amperes were repeated with the 3/16-in. electrode in the place of the 5/32-in. so that the observations would overlap and then the test was carried on to 300 amperes. This curve indicates that the highest average tensile strength, in a weld of this particular kind and under the conditions specified, will be obtained with a 5/32- or 3/16-in. electrode at 220 amperes. Fig. 19 shows the broken ends of the same welds covered by the curve. Better fusion conditions may be readily observed by the gradual disappearance of the vertex of the *V*. At 220 amperes the line of the *V* has disappeared. Reduction in tensile strength beyond 220 amperes is due apparently to the increased porosity of the deposited metal and difficulty in manipulation.

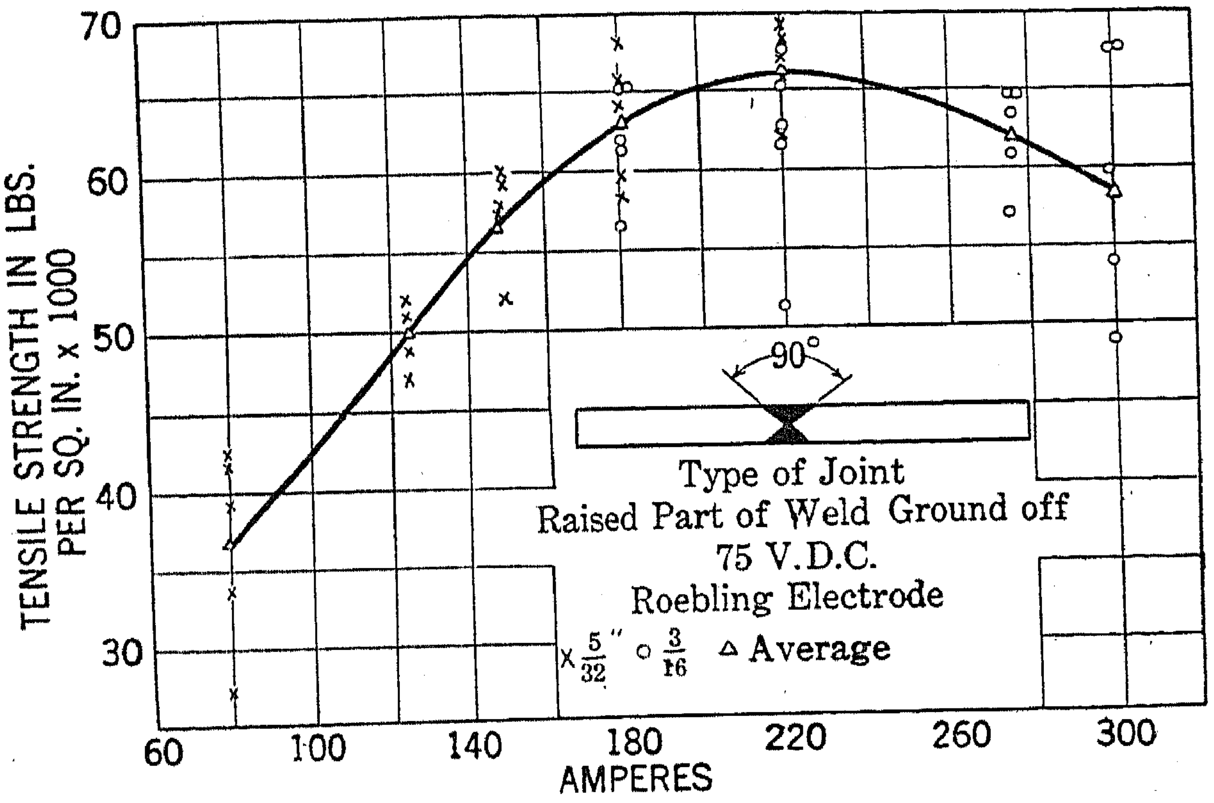


FIG. 18—VARIATIONS IN TENSILE STRENGTH WITH VARYING CURRENT

The results of tests of welds made at a constant direct current of 150 amperes and voltage varying from 40 to 225 are as follows:

Volts	Tensile strength, lb. per sq. in.	Volts	Tensile strength, lb. per sq. in.
40	50,000	150	57,000
75	57,000	200	60,000
100	54,000	225	58,000
125	57,000		

The curve given in Fig. 20 shows that the tensile strength is little affected by the change in voltage. The only advantage of a higher voltage is to produce a more steady current, as slight variations in the length of the arc produced by the operator will affect the total resistance of the circuit less as the resistance of the circuit increases with the higher voltage.

Fig. 21 shows the variations in current at low and high voltages during welding operations. The higher voltage gives the more steady current. Various systems have been invented to maintain a constant current in the arc with a low supply voltage, but a good operator will not produce sufficient variations in his current to materially affect the strength of his weld at any voltage down to 40 volts direct current. These observations indicate that for any particular condition, the selection of the proper current is important if the strongest weld is to be obtained; and that if the current is right, the voltage of the supply system, if it is between 40 and 225, may be disregarded.

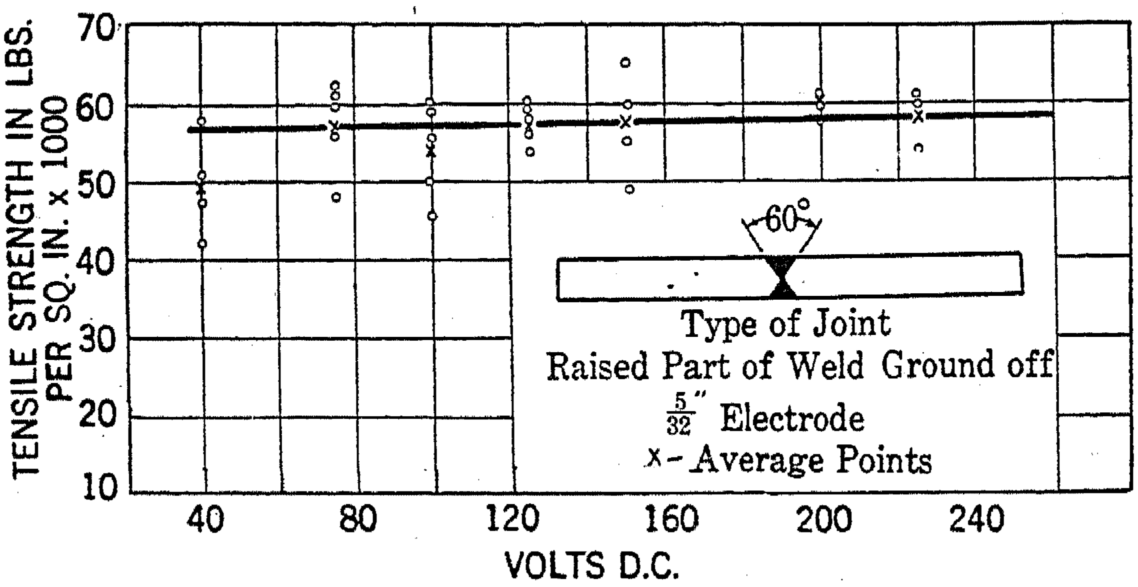
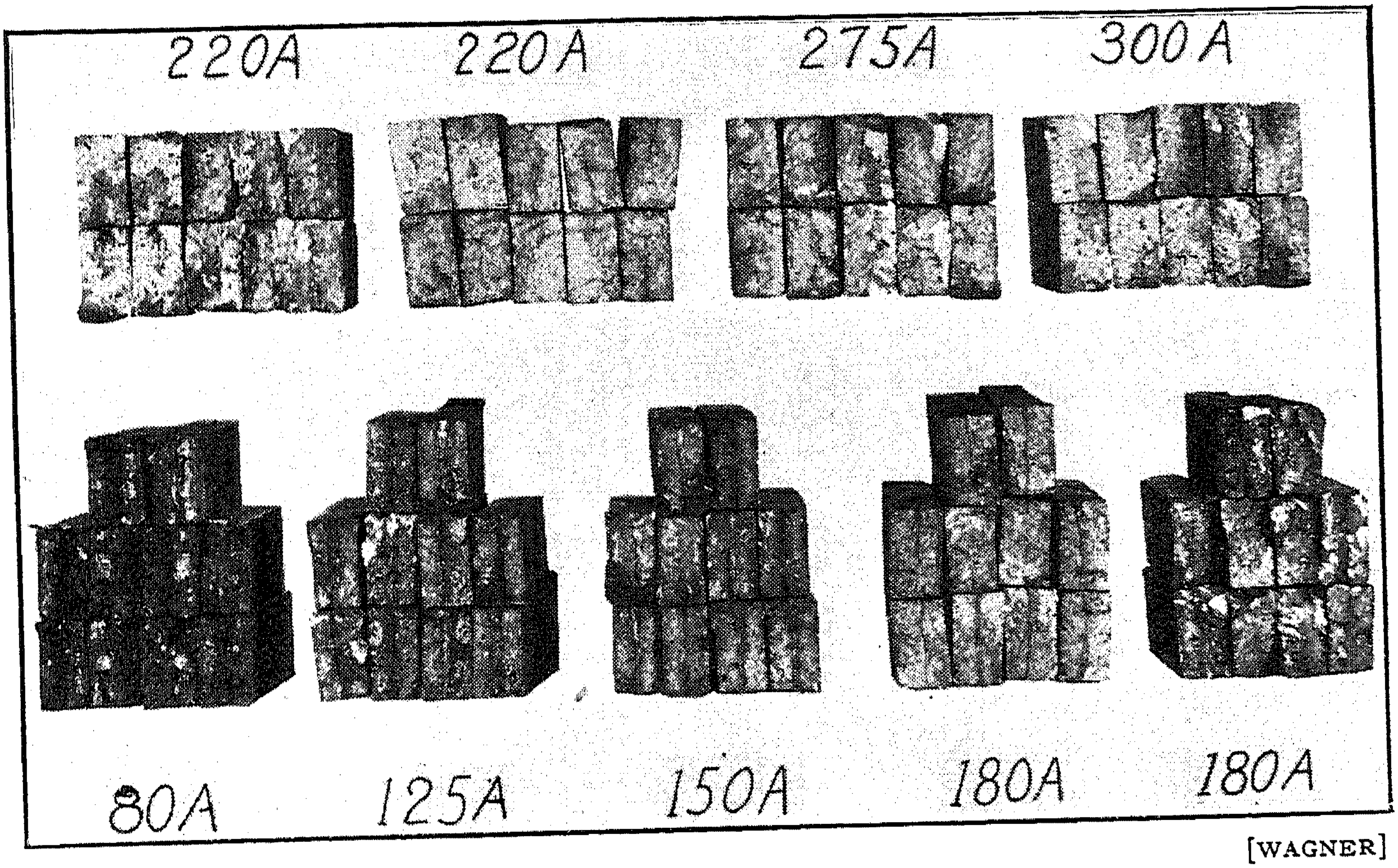


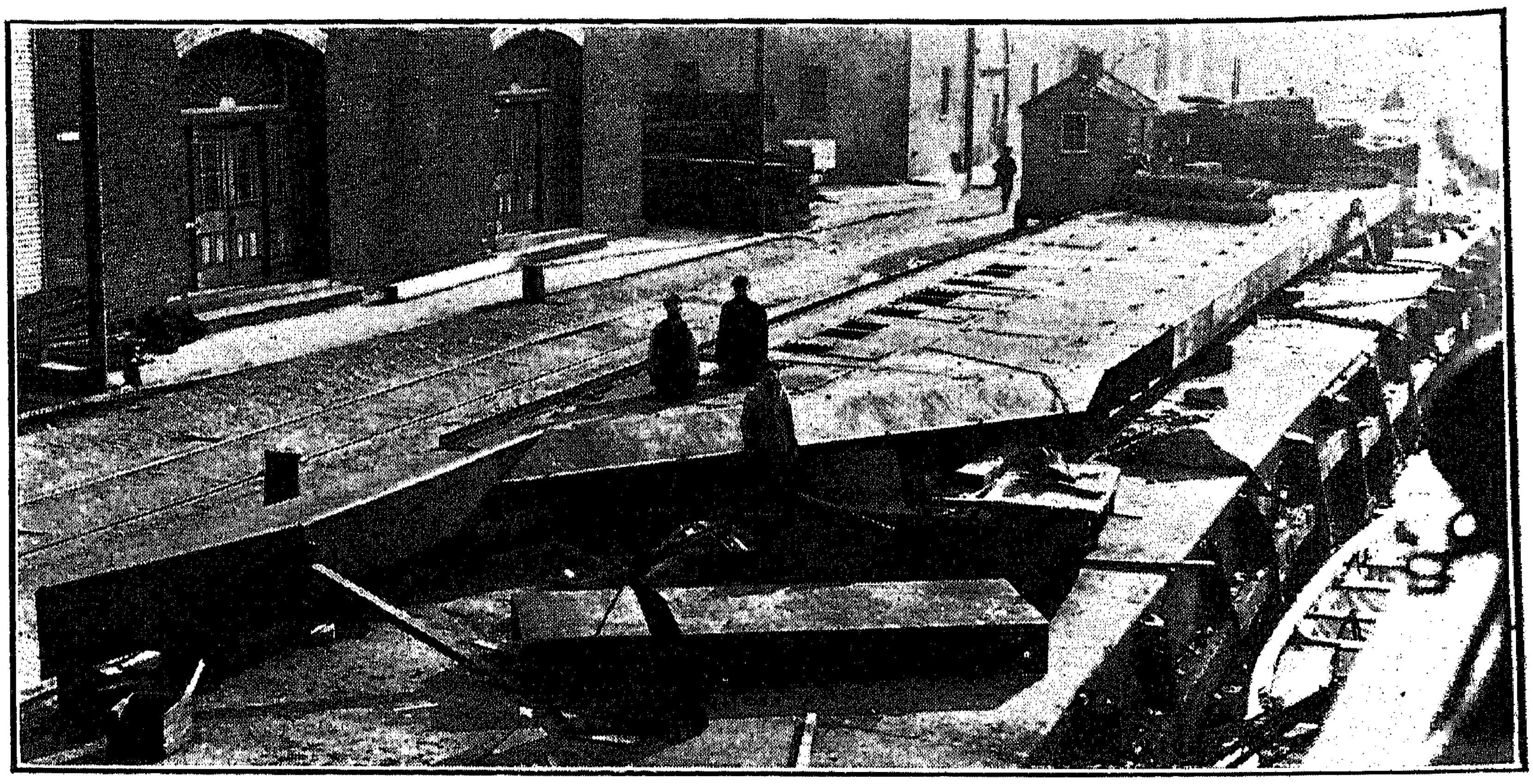
Fig. 20—Variations in Tensile Strength with Varying Voltage, Amperes Constant at 150

Bending tests were made for each condition of current and voltage, but it was not observed that the bending characteristics of the welds were in any way altered by the variations in current and voltage noted.



[WAGNER]

FIG. 19—APPEARANCE OF WELDS WHEN TEST PIECES WERE BROKEN



[KNOX]

FIG. 27—ARC-WELDED BATTLE TOWING TARGET UNDER CONSTRUCTION AT NORFOLK NAVY YARD

These observations are only a very small part of the broad field that is open for investigation and research. It is hoped that many investigators will submit data bearing on these two and other essential features of welding.

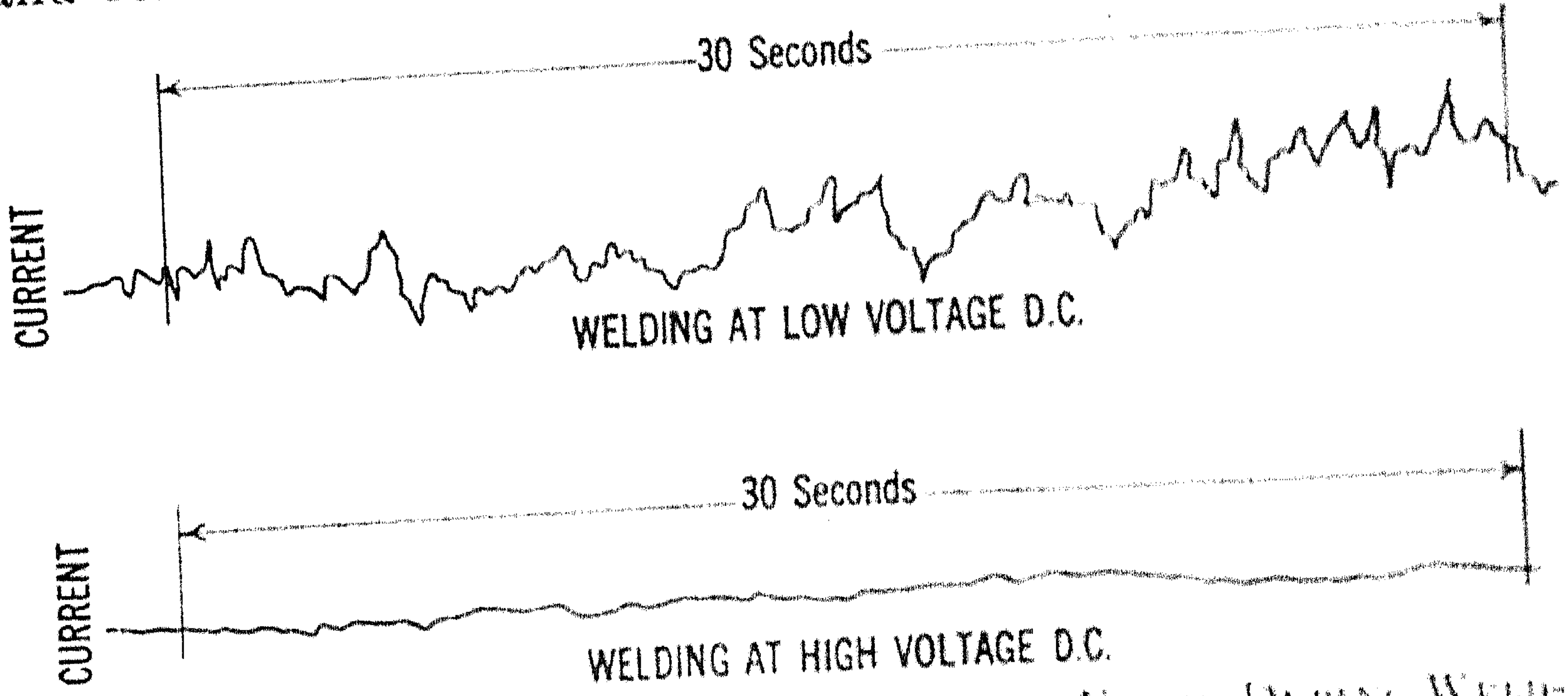


FIG. 21—COMPARATIVE STABILITIES OF CURRENT VALUE DURING WELDING AT LOW AND HIGH VOLTAGES DIRECT CURRENT

T. T. Heaton (by letter): For the past 27 years the writer has been using electric arc welding with a carbon electrode. Fig. 2 of Mr. Hobart's paper shows the operator holding the welding rod in his left hand. This is not our practise. If metal is to be added to the weld, it is laid over the seam and fused by the direct action of the arc. But in many cases no metal is added, the parts to be united are simply joined together by fusing. A good deal of importance must be attached to the design of the parts to be welded so as to get good results in the welds and to avoid, as far as possible, any distortion due to the effect of contraction.

In Section 6 of Mr. Hobart's papers Mr. Wagner states, from his experience, that it is very difficult to weld with a carbon arc when the polarity of the carbon is positive. In welding mild steel or iron, the writer has found this to be quite true. For various reasons it is best to weld with the work positive and the carbon negative. Any deposit from one pole to the other takes place from the positive to the negative; therefore it is not possible to carry any foreign matter into the weld, so no carbon from the electrode can be mixed with the weld. Besides, the positive pole is considerably hotter than the negative, which is an advantage. It was stated recently that the temperature of the electric arc is approximately 3650 deg. or more cent., and that the positive pole is about 1000 deg. in excess of the negative.

In Section 23, certain criticisms are given: "(*a*) Carbon is carried into the deposited material thus reducing its ductility." With the arrangement of poles in the manner adopted by us this does not occur. Analyses show that the result of the welding is to remove more or less all impurities from mild steel leaving a very considerably increased proportion of pure iron, as compared with the metal before welding. This applies to welds made in sheet steel where added metal was simply scrap of the same material.

"(*b*) It is more difficult to obtain good fusion." I do not find that this is so.

"(*c*) It is more difficult to manipulate" than the metallic-electrode system. This is true to a certain extent, but so far as our experiences go, the carbon-arc welds show no disadvantage in comparison. Probably this is due to the fact that the operator with the carbon are possessed, in the tests made, greater ability through long practise. The shorter training of many operators with the metallic arc may account for this. But I consider that there is better fusion with the carbon electrode as the operator can distribute the arc as long as he requires to produce a good weld and to direct it in any way that may be suitable, whereas with the metal electrode, he is limited to the time required to melt it.

With regard to criticism (*d*) we do not find that there is any discomfort to the operator.

"(*e*) Greater cooling stresses are developed because larger areas of adjacent metal are heated." For some classes of joints it is an advantage to spread the heat over a fairly wide area, as the metal surrounding the weld is improved thereby.

At the end of Section 23 certain counter-statements are made: "(*a*) No preparation of the abutting edges is necessary." I scarcely agree with this unless the metal is thin.

"(*b*) Greater rate of deposition of metal and greater speed of welding" is possible. This again depends on the thickness of metal.

"(c) Probable greater adaptability to automatic welding." So far as my experience goes, attempts at automatic welding with the carbon electrode have not been successful and in any case can only be limited to very special examples.

There is no doubt whatever that electric welding by this method is exceedingly successful, both technically and commercially, but our experience is that it is best to specialize the operators in certain kinds of work, as it is very difficult to find welders capable of doing a variety of work with success. To do this requires not only practise but a high order of intelligence in the workman.

Some time ago, we made some experiments in the speed of welding by this system with the following results:

A straight weld uniting two sheets 3½ mm. thick took 2¾ min. per ft. run to make a perfectly sound hammered weld. A stamped steel boss was welded into a mild steel sheet, also 3½ mm. thick. The circumference of weld, 17¼ in. (43.7 cm.) was made first by welding one side and turning over and welding the opposite side. The outside weld occupied 2½ min.

and the inside 2 min. These are, of course, good times and the welds were made by a practised workman.

The writer has followed the work done by the Welding Research Sub-committee in America with very great interest and is of the opinion that its labor will confer a benefit to engineering science.

Chas. R. Darling (by letter): It may be of interest, in connection with Mr. Hobart's paper, to indicate the present position of electric welding in Great Britain. In general, the progress of spot welding is somewhat slow, but the development of arc welding has been rapid during the past two years. Among the larger structures to which arc welding has been applied are: a rivetless barge with a displacement of 225 tons that has given excellent service during the last nine months, pontoons, and experimental ships under construction. Details of the last, with respect to cost of production and seaworthiness, are awaited with interest. In shipyard and general engineering repairs, arc welding is growing rapidly in popularity. Mention may be made of the successful deposition of manganese steel on mild steel on the rims of dredger buckets and in other cases where a hard wearing surface is required.

All who have experimented, in this country, are agreed that a thorough investigation of the destructive iron arc is necessary in order to arrive at the best method of working. At present, it is not possible to insure that two consecutive welds, made under apparently identical conditions, will give even approximately similar results under test. An inspection of the Wirt-Jones tests, Table IV, makes this clear; in my own experience it has been quite common to obtain results varying between 62,000 (28,132 kg.) and 52,000 lb. (23,586 kg.) in ultimate tensile strength, and 8.0 to 4.5 per cent. of elongation, without any obvious reason for the difference. Until we know what is going on in the arc during the process of welding, it is unlikely that a standard method of working can be formulated.

One notable difference in the practise in America and in England is the general adoption of the coated, or fluxed, electrode by British welders, in contrast with the wide use of bare electrodes in America. Workmen claim that it is much easier to work with a fluxed electrode, as it is less difficult to maintain the arc; also that a better lateral spreading of the deposit is secured by the use of flux. It is difficult to overcome the prejudice of the welders, so most employers use fluxed electrodes in spite of the greatly increased cost. My impression is that much of the trouble experienced with bare electrodes has been due to improper chemical composition, and might be overcome by the use of rods of uniform and correct chemical content. There is no doubt that the general use of costly fluxed electrodes will seriously impede the extension of arc welding in many directions.

Direct current is generally preferred to alternating owing to the greater ease of striking and maintaining the arc, and also to the fact that the welders' hand soon becomes unsteady when alternating current is used. Good welders prefer direct current for overhead work. These factors have combined to make the use of direct current general in British practise. Good work has been accomplished with alternating current, but it is not recommended by makers of equipment.

The results of experiments made by members of the British Research Committee regarding the effect of voltage agree with those of Mr. Wagner. Some earlier outfits were constructed on the assumption that a minimum of 100 volts was essential; but it is now customary to employ 60 to 70 volts at the generator. The question of the effect of current is more debatable. From an analysis of the Wirt-Jones tests, the conclusion has been drawn that "the stronger the current the better the weld." This interpretation does not appear entirely justified, in view of the discrepancies presented by the results. For example, No. 31, with 150 amperes direct current, shows an ultimate tensile strength of 60,900 lb. (27, 623 kg.), while No. 9, also with 150 amperes direct current, shows only 38,100 lb. (17;281 kg.) Until greater uniformity is obtained, it does not appear possible to generalize on this point. In some recent work, Dr. J. H. Paterson of the British Committee found that the effective maximum deposition was obtained with 140 amperes when 20 gr. per min. were deposited. Beyond this strength, owing to splashing, the effective deposit diminished. Moreover, analysis showed that the minimum amount of oxygen entered the deposit at 140 amperes. This point is urgently in need of settlement, with a view to guiding the design of equipment; and a series of tests under identical conditions as far as procurable, should be conducted to decide the question. At least six welds, using identical plates and electrodes, should be made at each current strength to enable a correct decision to be arrived at.

With regard to the speed of arc welding, the maximum, according to Dr. Patterson's figures, would be 1200 gr. or about 2.6 lb. per hr., not allowing for intervals. Experiments confirming this would be useful, so as to settle this point definitely.

J. Churchward (by letter): Mr. Hobart states that various kinds of electrodes are advocated, running from the cheapest fence wire at 10 cents per lb. up to covered electrode costing five times that price. The importance of the electrode question has not been covered extensively enough. It is of the utmost importance to have an electrode that will be absolutely uniform in its welding qualities; that the deposited metal be as near the same chemical composition of the parent metal as it is possible to have in electric arc welding. If these things are accomplished, the price, within reasonable limits, should be disregarded.

It has been suggested that, when welding wire shows bad welding properties, it be dipped into whitewash to insure its uniformity of flow. The welder must take extra precautions to see that the slag introduced by this coating is puddled out of the weld. It is better to buy a good bare electrode than to coat a bad one, as with the latter there is always the danger of introducing slag into the weld.

The specifications for electrodes from the Welding Committee of the Emergency Fleet Corpn. are: Carbon not over 0.18 per cent, manganese not over 0.55 per cent, phosphorus and sulphur under 0.05 per cent, silicon under 0.08 per cent. The high limits are covered by patents in which an excess of manganese was allowed. The average manganese content for old welding wire was not over 0.35 per cent. If I understand these specifications correctly, they do not demand the rejection of any wire that would give satisfactory results even though not of this chemical composition.

Ductility of Welds Made with Covered and Bare Electrodes. Relative to the ductility of welds made with covered and bare electrode welds, our data show that normally coated electrodes produce less ductile welds than bare electrodes due to the slag inclusions made with the former.

Effect of Voltage on Arc Welding. Increased voltage, say from 40 volts up, enables the inexperienced operator to weld with greater ease on account of the length of the arc this voltage allows; but it sacrifices the strength of the weld. Experienced operators will not vary the length of the arc more than 3 or 4 volts, regardless of the open-circuit voltage of the machine. The voltage used across the arc by expert welders varies from 16 to 19 volts. With an inexperienced welder or learner, records show that a high open-circuit voltage has a tendency to make the operator careless and thereby cause bad welds.

Overhead Welding. The quality of overhead arc welding depends entirely on the experience of the operator, the quality of the wire, and the characteristics of system used. With excellent wire, an expert operator, and a machine of the proper characteristics, overhead welding may be very easily as strong if not stronger than flat welding.

Different Lengths of Arc. It is essential in metallic arc welding that the arc be kept as short as possible. This introduces two variables, the quality of the wire and the expertness of the welder. An expert welder with a good wire will maintain a 16-volt arc; with a poor wire he will try to maintain the same length of arc with many interruptions due to the breaking of the arc. An inexperienced welder will maintain the length of arc that will enable him to deposit the metal with the least possible exertion. If the electrode is good, the arc will be short. If the electrode is bad the arc will be long, if the design of the machine so allows. With a long arc, there is less heat per unit area in the parent metal and therefore less concentration of welding heat than with a short arc. If the voltage is so high that the operator is enabled to draw a long arc, although it may be easier for him, the weld will suffer. One would draw the conclusion from the above that with the personal element, the current and type of electrode constant, the quality of the weld depends on the length of the arc.

Charles H. Kicklighter (by letter): The disk-depression method of electric welding was developed primarily as a simple and effective means for forming a highly efficient joint or seam between heavy commercial plates. After the preparation of the parts, by making flat-bottom depressions in the surfaces of the plates and introducing therein thin flat disks, the parts are brought into the proper position and subjected to pressure and heating electric current between the terminals of an electric welding machine. The intermediate bodies, or disks, are welded to both plates and are softened and forced into the depressions so as to permit the faces of the plates to come into immediate contact.

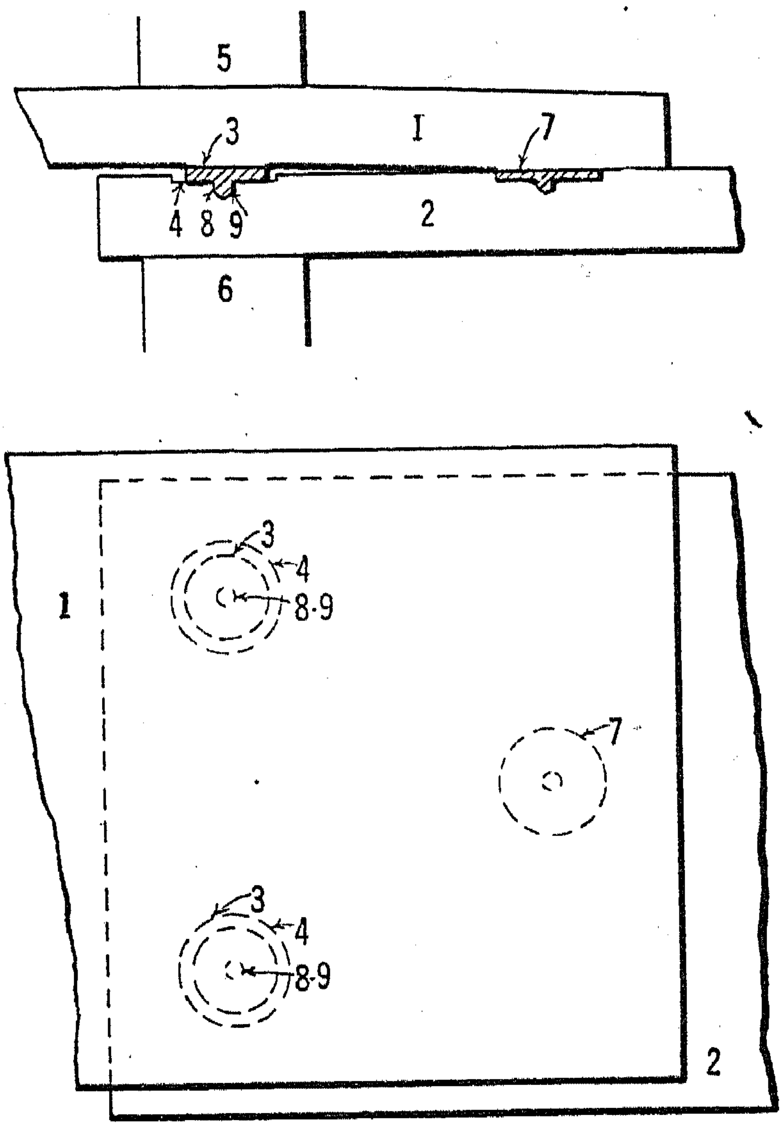


Fig. 22—Disk-Depression Welding

The diameter of the disk is slightly less than the diameter of the depression, but the thickness of the disk is greater than the depth of the depression, so as to hold the plates out of initial contact. Flat disks are used as the best results are obtained by having the areas of initial contact substantially the same as the areas of the desired weld and apply the pressure normal to these surfaces. The surfaces of contact are thereby brought to a welding heat at all points at about the same instant and a thorough fusion of the metals at all points is obtained by the time the intermediate metal is softened sufficiently to be forced into conformity with the depression. A plurality of such welds, properly staggered, makes an efficient joint.

Description of Process. Simple flat disks may be placed loose in flat bottom depressions, as in Fig. 22. Here disks *3*, in depressions *4*, between plates *1* and *2* are subjected to pressure and the heating electric current between terminals *5* and *6* of an electric welding machine. In order to hold the disks at the centers of the depressions during the progress of the work, small register projections *8*, on the disks fit snugly into register holes *9* in the bottoms of the depressions. A special drill for cutting the depressions and register holes is provided and a bank of, say, six, eight, or twelve such drills, operating simultaneously, will cut the depressions rapidly and economically. A special attachment limits the depth of the cut.

The shifting of the disks may be prevented by sticking them to the bottoms of the depressions; by fusing them at one or more small projecting points of the contacting surfaces. Depressions may be cut in each plate near the edge and the plates brought into position with the depressions overlapping or falling beyond each other, so that, in the welded joint, the full stresses can never come on a plate where it has been weakened in the least by such depression.

Disk-depression Welding versus Riveting. The advantages of the disk-depression method of electric welding over riveting are: a joint of greater efficiency and greater economy in production. A joint of very high efficiency is obtained, for the plates are not weakened by the cutting away of material in punching rivet holes. The small amount of metal removed in making the depressions is negligible and even this is restored by the disk material, which is fused into and completely fills the depressions. By cutting depressions in both plates near their edges and overlapping these, the full stress never comes on a plate where depressions have been cut. The only reason why the joint may not be made as strong if not stronger than the solid plate is a slight tendency of the metal of the plates to crystallize when heated, but due to the small region heated and the low temperature attained in the greater part of this region, this weakening effect is very slight.

With proper means of bracing or mechanical framing, the seam can be completed for the present cost of rivets alone. The amount of heating current is relatively small and the heating is limited to the immediate position of the weld. As the welds can be made rapidly, the conduction of heat by the plates causes very little heating of other parts.

Disk-depression Welding versus Arc and Autogenous Welding. The advantages of the disk-depression method of electric welding over arc or autogenous welding are: greater efficiency of joint; greater saving in time, labor, and electric power; and elimination of distortion troubles during heating and resulting stresses after cooling. Arc-welded or autogenous-welded joints are never as strong as the solid plate unless they are built up of considerably greater thickness than the plate. The great saving in time, labor, and power will be appreciated when one considers the length of time, labor, and the amount of heat necessary to weld a seam between thick plates by the arc or autogenous method. With the disk-depression method, a few seconds per weld is sufficient.

Disk-depression Welding versus Spot Welding. The advantage of the disk-depression method of electric welding over present methods of spot welding are: greater efficiency of joint; greater economy of welding current; smaller capacity and less cost of welding machines; practically no trouble because of distortion of work during operation due to heat; greater thicknesses of plates that can be welded; and simplicity and ease of preparing work prior to welding operation.

A high efficiency of the joint is insured by having clean surfaces free from scale. The operator is then certain that a perfect weld has been made when the disk becomes sufficiently fused to let the plates come into immediate contact. If the contacting surfaces are covered with scale, he cannot be absolutely sure of a perfect weld. The disks are of sufficient thickness to hold the plates out of immediate contact before the welding operation and thus serve to narrow the path of the welding current and concentrate its heating effect. The resistance to the welding current is small, thus requiring a minimum amount of electric power and a welding machine of relatively small capacity. With clean surfaces, the mechanical pressure does not have to be so great as where there is scale or the plates have to be forced into contact about an intermediate body with no depression in connection therewith. For this reason, a welding machine of lighter construction and smaller cost will perform the work.

The greater part of the resistance to the welding current is at the surfaces where the disk contacts with the plates. Here the heat is concentrated and in a few seconds it becomes so intense as to fuse the thin disk to both plates before the surrounding parts of the plates become heated by the conduction of heat. The seam does not become nearly so hot as in other methods of welding, the distortion of the work due to expansion is not nearly so great, and the stresses set up in the plates due to cooling and unequal contraction are small. As most of the heat is developed and concentrated at the contacting surfaces of the disk, plates of considerable thickness may be easily welded. As to the thickness of the material, there does not appear to be other drawback than the matter of forcing the plates through a very small distance into close contact. As the region of intense heat is so small and the weld is performed before the heat has spread much by conduction, the region affected by any tendency to crystallize is very limited and the strength of the joint is not appreciably affected thereby.

The slight increase in cost and the slight loss of simplicity occasioned by cutting the depressions is far outweighed by such considerations as the insuring of a perfect weld, a highly efficient joint, a large saving in electric power, the use of lighter and less expensive welding machines, rapidity of the welding operation, freedom from heat-distortion and stresses set up by the subsequent cooling and contraction, less crystallization of the metal due to heating, etc. It is preferable to cut the depressions before the plates are bent into shape.

Theory of Method. Fig. 23 illustrates the contracted path of the heating electric current as it passes between the broad flat-end terminals *5* and *6* through plates *1* and *2*. Most of the resistance to this current is along the surfaces of contact *aa* and *bb*, and most of the heat is developed in the disk *3* and in plates *1* and *2* in immediate contact therewith. Fig. 24 shows how it is often attempted to concentrate the pressure by means of conical terminals *5* and *6* in order to obtain a weld between plates *1* and *2*. Due to the rigidity of the thick plates, this pressure is distributed over a wide range between the plates *1* and *2*. The heating current also spreads considerably at *cc* where plates *1* and *2* are in contact. The path of the current is contracted most at *dd* and *ee*, causing considerable loss of heat, marring of the surfaces of plates *1* and *2*, and damage to terminals *5* and *6*.

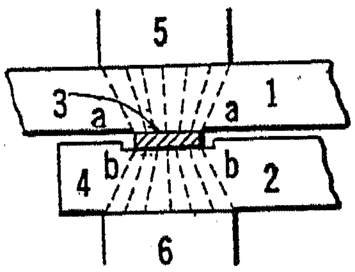


FIG. 23

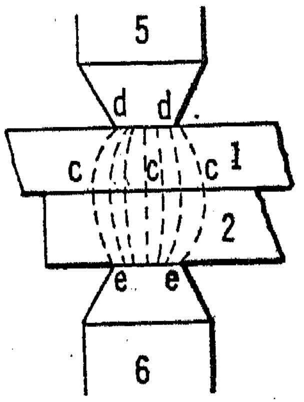


FIG. 24

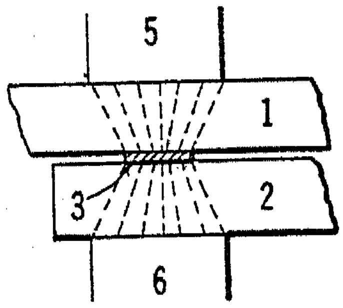


FIG. 25

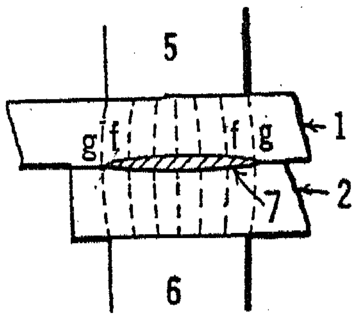


FIG. 26

Fig. 25 illustrates a disk *3* placed between two plates *1* and *2* and welded thereto by current passing between terminals *5* and *6*; if it is now attempted to force the plates into immediate contact, as in Fig. 26, the disk flattens and spreads out as at *7*. The area *ff* may be welded if the surfaces are clean, but the areas *fg* where the molten metal has been squeezed out against the cold plate surfaces is not welded; there is also some difficulty in forcing the plates into immediate contact around the weld. There is considerable widening of the path *gg* of the current between the plates; this causes considerable heat in the joint, resulting in heat-distortion and subsequent cooling stresses. The region of partial crystallization is greatly enlarged.

Tests. In order to determine the efficiency of the disk-depression method of electric welding, commercial bars 3/8 in. (9.5 mm.) thick and 1 in. (25.4 mm.) wide were lapped and welded, some in two places and others in three. Depressions were cut in one of the bars 7/16 in. (11.1 mm.) in diameter and 3/64 in. (1.1 mm.) deep. Disks 1/16 in. (1.5 mm.) thick cut from a 3/8 in. commercial round bar were placed between blunt flat welding terminals of an ordinary spot-welding machine. The capacity of this machine in ordinary spot-welding work was 1/8 in. (3.1 mm.) plates or less, but we were enabled to get splendid welds on stock 3/8 in. and 1/2 in. thick by the disk depression method. Some samples were grasped in the jaws of a vise and hammered over until they tore apart, revealing a perfect weld and a fibrous structure. Others were tested in a tensile testing machine until they broke. Bars with two welds broke at 13,090 lb. (5,937 kg.) and those with three welds at 19,405 lb. (8,801 kg.). The solid stock broke at 19,275 lb. and had an elastic limit of 13,125 lb.

Zay Jeffries (by letter): The following is quoted from a letter, dated Sept. 9, 1918, to Mr. Hobart.

"In reference to the improvement of electric welds by heat treatment, I have observed that the steels higher in carbon are helped more by heat treatment than the low-carbon steels. High-carbon steels are often the cause of a martensitic structure in the weld and, of course, the higher the carbon the more brittle will the martensite be, and the more will ductility be restored by heating above the critical temperature and cooling slowly. I have observed well-defined martensite in welds made with piano wire and with a nickel steel containing about 0.2 per cent carbon and 3.5 per cent nickel. As the carbon decreases, the martensitic needles become farther apart and the steel becomes more ductile.

"In addition to the martensitic needles, there are other needles to which Mr. Ruder has referred, which he seems to have proved to be nitride of iron, or at least the presence of the needles if they are not iron nitride seems to be due to the presence of nitrogen. We must not conclude prematurely, however, that the needles themselves are composed of nitride of iron because Timofeeff shows the development of the same microstructure by straining commercially pure zinc at room temperature. There is no sign of this structure in the annealed zinc so that the needles must be a direct result of the straining. At the same time, Howe has produced needles of a similar nature in manganese steel and there is no reason to suppose that the composition of the needles is any different from that of the remaining mass of the metal.

"But it has not been demonstrated that welds containing needles are always brittle and that welds not containing these needles are always ductile. In fact, I have examined many welds under the microscope that did not show the needles, but which were brittle.

"Recently J. C. Lincoln gave me four samples of electrically welded steel for microscopic examination. These were made with a modified carbon-arc process. Two steel plates ⅜ in. thick were placed edge to edge on a flat surface and the first side welded, then the plates were turned over and the other side welded. Bending tests were made in such a manner that the two plates bent parallel with the weld. The bending was stopped when the first crack appeared in the weld. Weld No. 1 was made in an atmosphere of carbon dioxide and a crack appeared after bending 90 deg. Weld No. 2 was made in an atmosphere consisting of about 90 per cent nitrogen and 10 per cent carbon dioxide and bent 60 deg. before cracking. Weld No. 3 was made under flux and it bent 50 deg. before cracking. Weld No. 4 was made in the air and a wide crack opened up after bending 45 deg. Welds Nos. 1 and 2 are unusually ductile for electric welds and they also contain the needles. Weld No. 3 contains the least number of needles, but it is not the most ductile. Weld No. 4 contains needles and it is the most brittle of the four. This weld, by the way, is an unusually good air weld.

"Having found, therefore, that electric welds containing needles may be ductile and electric welds containing no needles may be brittle, it seems evident that the needles are not the main cause of brittle welds. In this connection I should like to present J. C. Lincoln's opinion, namely, that oxygen is the "embrittling element" in electric welds. He has reached this conclusion after many and varied tests. The same idea is strongly suggested by the most interesting results reported by Mr. Wagner with magnesium and boron suboxide. Both of these would be oxygen removers. That the magnesium did remove oxygen seems apparent from Mr. Wagner's remark that the liquid metal in the weld "boiled" when the magnesium came in contact with it.

"Iron oxide can be seen in the welds under the microscope in the form of globules. In a microscopic examination we cannot hope to see particles of iron oxide containing less than about a million molecules. Owing to the nature of the transfer of metal in the making of an electric weld and the rapidity of solidification, it would not be surprising if much oxygen was present in the form of iron oxide in particles too small to be seen under the microscope. So far as the microscopic examination is concerned, these particles might actually be in solution. It is, therefore, possible that the "embrittling element," or at least the portion of the element which is the most harmful, is present in such small particles that it cannot be seen under the microscope.

"The weld made by Mr. Wagner with Armco iron weld rod coated with magnesium powder is the most ductile electric weld I have seen.

"It seems to me that this lead is so promising that it should be followed with the utmost speed. If it works out commercially, as well as in the sample submitted, it is my opinion that the matter of the quality of electric welds will be quite satisfactory. If the weld is ductile, it need not have as high tensile strength as when it is brittle."

It seems probable that Mr. Lincoln's experiments in no case entirely excluded nitrogen, but it is probable that there was a smaller percentage of oxygen in the atmosphere surrounding the welds where gases other than air were used than in the weld made in the open air. The weld made in an atmosphere of 90 per cent nitrogen should have less oxide between the grains than that made in air, but there seems to be no reason why it should have less nitride present than the one made in air. The weld made in the 90 per cent nitrogen atmosphere, however, was more ductile than that made in air, which would seem to bear out Mr. Lincoln's conclusion that oxygen is the "embrittling element."

It may be definitely concluded that nitrogen is the element that embrittles the grains. If the grains contained less nitride and hence had lower elastic limits, the grain-boundary strength might remain unchanged (that is the same as now observed) and the weld would show considerable increase in ductility on tensile test, without necessarily decreasing in strength. It would, therefore, seem advisable to keep the nitrogen content low in electric-steel fusion welds.

It also seems that the grain-boundary weakness is due to either oxygen or nitrogen or both. It remains to be decided how much each of these elements contributes to the weakness of the grain boundaries and to eliminate one or both from the welding atmosphere if sufficiently beneficial.

A. S. Kinsey (by letter): In paragraph 4 of Section 16 Mr. Hobart states that "an enormous volume of experience in arc welding has gradually accumulated and it controverts the correctness of the view" of a gas welding publication quoted which shows the advantages of the oxyacetylene over the electric-arc process. I presume the author refers only to the statement of the limits of the arc weld, and that even though there may be an absence of acceptable test data on gas welds on which to base his judgment, he will agree that oxyacetylene welds have, when carefully made, proved highly satisfactory to the best engineering practise, both in ductility and tensile efficiency. Also, there surely can be no doubt in the mind of the author as to the accuracy of the statement of the quoted article, Section 16, as to the value and adaptability of the oxyacetylene torch for annealing a steel weld. The reference to the absence of satisfactory laboratory tests of oxyacetylene welds need bear no undue significance, as the real situation has been that the oxyacetylene welds were so much stronger and more economical than the forged welds that tests of the physical value of gas welds were never necessary. They have stood the test of actual practise. On the other hand, is it not true that in the development of the electric-arc weld for use on steel plates for ships, it has been necessary to know exactly what are the physical limits of the arc weld in order to determine the risk connected with its use on ship hulls? This is what caused laboratory tests to be made of arc welds.

H. G. Knox: (by letter.) There is one subject that Mr. Hobart seems to have inadvertently omitted from his very valuable paper, and that is corrosion. The shipbuilder, perhaps more than anyone else, has constantly to guard against it, our chief reliance being paint and galvanizing, where practicable.

The corrodibility of a welded joint in salt water must be determined before deciding whether this type of construction is to supersede riveting. At the present time, this question cannot be answered, although considerable work has been started in an attempt to arrive at a conclusion. The British Admiralty and several British shipbuilders have studied it extensively. Their experiments and conclusions have been very carefully followed and reported in this country by Naval Constructor Goodall of the British Navy. To date, these show that no undue fear need be felt from corrosion. At the Norfolk Navy Yard a series of plates welded with various electrodes were submerged in dilute sulphuric acid, such as is used in cleaning prior to galvanizing. While this test may not be conclusive in any quantitative way, it does show a marked difference in the corrodibility of welds made with different electrodes under different conditions. To draw any conclusions from this accelerated test would be, however, ill advised.

In riveted joints, the exposed heads and points of the rivets always show the greatest tendency to corrode. This is due to two reasons: first, as they are raised above the surrounding surface the paint is more susceptible to abrasion; and, second, electrolytic action is set up between rivet and plate. These same reasons lead us to expect a tendency of the welded joint to corrode.

Various methods have been suggested for decreasing the probable inherent corrodibility of the weld, in addition to the customary coat of paint. These generally embody the introduction of alloys, principally copper, into the electrode material. Plating the electrode with copper is not effective in that the deposited copper segregates in the weld and actually increases the electrolytic action. As far as is known, no extensive test on the so-called copper-bearing alloy steels, which might be made up into electrodes, has been carried out, but when obtainable, electrodes of this alloy will be subjected to service tests.

About 9 months ago, we started attaching small plates, welded with different manufacturers' electrodes, to the under-water hulls of steel ships placed in dry dock. Up to the present time none of these ships have redocked in the Norfolk Yard, so no results from this source are available. We have also used several makes of electrodes in the construction of an arc-welded battle towing target, which will soon be launched. See Fig. 27.

It can readily be seen from the foregoing that little definite information on the subject of corrosion is available, but for the time being, at least, I do not believe any special precautions need be taken in the protection of welded joints beyond that employed for the surrounding plates.

John Martin (by letter): It was the writer's privilege to represent the American Bureau of Shipping on the Welding Research Sub-committee over which Mr. Hobart presided. The foreword that the paper deals principally with investigations undertaken by the Welding Research Sub-committee relieves the author of any save editorial responsibility for certain irreconcilable statements, some of which might be expected in the experimental research data of any new industry and especially in one like metallic-arc welding, which introduces so many variable factors and conditions essential to the process that a long acquaintance with the technic and data records is required before one can detect and discard an irreconcilable result.

The tables of Section 5 state that a Norway bare electrode wire contains 0.007 per cent sulphur and that the deposited metal of weld contains 0.072 per cent sulphur. As the ½-in. ship plate used for testing electrodes held 0.032 per cent sulphur, where did the increased sulphur come from? The metallic arc is a reducing agent and no flux was used. This illustration is used to show how carefully all test reports should be scrutinized and compared before insertion in finished records and in no sense to detract from the value of the author's compilation.

Composition of Ship-plate Steel. Where ship-hull plates are united metallurgically (welded) instead of physically (riveted), the chemical composition of the plate will become as important as the physical performance of its test specimen prior to installation. A certain amount of slag (reduction of foreign matter, carbon, manganese, phosphorus, sulphur, and silicon) is driven off in all fusions whatsoever the welding process used (smallest, however, in spot welding) and the chemical composition of the ship plate must be such that, after welding, stresses will be transmitted through the joint as efficiently as through the plate, that is, the joint shall be 100 per cent efficient the same as many riveted joints. This condition is impossible of accomplishment by arc welding from our present knowledge of the art as the deposited metal in the weld is always pure iron whatever the chemical composition of the electrode may be. In the spot weld, although the writer has no knowledge of their chemical composition, the "spots" more nearly approach the original composition of plate as the slag driven off (a measure of the chemical change within metal) is inconsiderable compared with that of arc welds.

Fusion Welding and Spot Welding. The author distinguishes between welds as fusion welds and spot welds. Spot welding is fusion welding and to differentiate by grouping under fusion is apt to create a false picture in the uninitiated reader's mind It is suggested, starting with the premise that a weld with heat means fusion, to classify welds processively as spot, gas, carbon-arc, metallic-arc welds in which the physical difference is that of continuity of weld.

Metallic-arc Welding. Inasmuch as overhead vertical and horizontal welds are equally well performed with the metallic arc, it may be assumed that the flow (or bombardment of metallic molecules) of metal from the electrode to the joint follows Newton's law of attraction of matter (directly as the mass and inversely as the square of the distance). Molecular cohesion of the electrode molecules, lessened by the heat of the arc, is overcome by the attraction of like mass in the plates; the result is the passage of electrode molecules to the plate, whence adhesion ensues, which becomes cohesion upon cooling contraction and consequent crystallization of deposited material.

Spot Welding. The author's description of spot welding is satisfactory save that no initial voltage is specified. In one test performed in the writer's presence 32,000 amperes with a voltage of 4 successfully spot-welded pairs of ⅝ in. to 2½ in. (15 to 63 mm.) ship steel plates together, the diameter of the spots being 1 in., the pitch 3 in., and the hydraulic pressure 7½ tons, the time of the current was 15 sec. and 15 ft. (4.5 m.) was welded in 30 min. The spots sheared at 56,000 lb. or partly pulled and sheared at 60,000 lb. Water- and oil-tight joints resulted without calking. For plates of the same pitch calking or arc fillet welds would be required for water-tight joints; though by lessening the pitch and decreasing the diameters of the spots, practically the same water-tight results could be had.

Summarization of Points for Discussion. From the fact that the author has found 34 points, the discussion of which he believes may advance welding technic but only one of which concerns spot welding, leads one to infer that much is to be learned about arc and little about spot welding. The writer believes that the greatest obstacle to the adaptation of spot welding to hull construction is the difficulty of manipulation of spot-welding apparatus in and about the hulls of ships, and that is but a mechanical detail to be worked out. If a year ago it had been realized that there were 33 variables in the arc-welding and but three in the spot-welding problem, the latter would have determined perhaps the more fruitful field for experimental investigation in seeking a satisfactory weld for ship construction.

The writer differs with the author's contentions that the best basis to express the speed of arc welding is in pounds of metal deposited per hour. Feet of joint per hour welded would better assist the ship builder in his estimates and cost sheets and certainly would afford a simpler basis for comparison between welding methods. The writer heartily endorses the author's belief that spot welding has a great future as applied to shipbuilding but does not agree with the statement that spot welding affords a method of preliminarily jointing the hull plates after which the required strength is furnished by arc welding. The writer would have expressed the thought thus: spot welding affords a method of joining the hull plates, after which the required water tightness may be secured by calking or calking arc welding.

The author points to the future application of spot welding to hull construction when he finds that the strength of the spot-welded joint, with calking arc weld, is 50,000 lb. (22,679 kg.), of the arc-welded joint, with calking arc weld, is 37,000 lb. (16,782 kg.); and of the riveted welded joint, with calking arc weld, is 13,000 lb. (5896 kg.). In other words, the joints are as 4 : 3 : 1, respectively. That is, the spot-welded joint with calking arc fillet is one and one-third times as strong as the arc-welded joint and four times as strong as the riveted joint.

Conclusion. The writer agrees with the author in his conclusion that welding should be applied in the general fields where the hazard to life would be minimum before applying it to constructions in which joint failures would jeopardize human lives. But he deprecates the author's under-rating the stresses in ships as being less severe on welded joints than those in high-pressure containers. The stresses in ships are dynamic, tensile and compressive, torsional, alternating, and an admixture of all these, whereas those in a high-pressure container, even in marine boilers, are largely static and it is the alternations of stresses that fatigue ship members and require the ever watchful eye of the classification society to conserve property and preserve life.

O. H. Eschholz: (communicated after adjournment.) In discussing the quality of welds made with a-c. and d-c. arcs, Section 11, a number of authorities are quoted as saying that a-c. welds are better than d-c. welds. As a review of the data given in the Wirts-Jones tests shows about equal maximum strength for d-c. and a-c. welds, I would be interested in knowing what basis exists for this opinion.

In the last paragraph of Section 19 Mr. Wagner is quoted as saying that "Tests were made on a ½ in. plate with current values as follows: 80, 125, 150, 180, 220, 275 and 300 amperes. These tests show improvement in the tensile strength and bending qualities of welds as the current increases." How is this statement reconciled with the information in the following table compiled from the data in the Wirts-Jones tests?

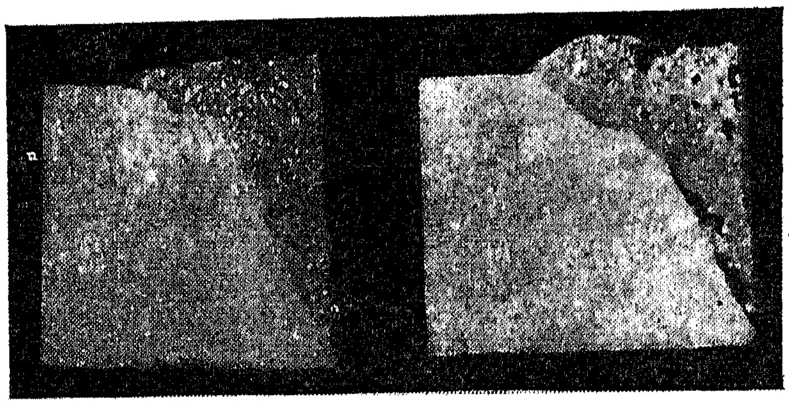
Characteristics of D-C. Arc Welds.

PLATE ULTIMATE TENSILE STRENGTH = 64,700.

Test No.	Weld U. T. S. in % of plate U. T. S.	Current	Amperes per sq. in.	Per cent Elong. in 2 in.	Angle at which break started.
2	97 %	155	8,075	11.5	42°
5	94.5	145	7,554	8.5	55°
31	94.2	150	6,930	8.0	..
1	92.5	175	9,118	9.	42
8	92.0	115	9,756	13.5	60
8	90.	115	9,349	14.	..

These results show that a maximum strength is obtained at 155 amperes with a decrease on either increasing or decreasing the arc current from this value.

In Section 21 the author states that "The investigations of the Welding Research Subcommittee are indicating that at least 10,000 or 12,000 amperes per square inch are suitable for electrodes of 1/8 in. and 5/32 in. diameter................." This conclusion can hardly apply to d-c. arc welding as the Wirts-Jones tests show a falling off in weld strength on increasing or decreasing the current density from 8000 amperes per sq. in., which confirms my experience.



a—(Good fusion) b—(Poor fusion)

FIG. 28

The use of a smaller bevel angle, as discussed in Section 24, undoubtedly leads to welding economies. However, it should be recognized that the use of this refinement is only justified when coupled with a proper "free" distance and a highly skilled operator. Fig. 28a illustrates the possible excellent fusion that may be obtained with 60 deg. scarf, while Fig. 28b illustrates the probable incomplete fusion resulting if the welder is fatigued or indifferent to the factors that produce fusion.

An address presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 20, 1919.

Copyright 1919. By A. I. E. E.

ENGINEERING AND INDUSTRIAL STANDARDIZATION

BY C. A. ADAMS

INTRODUCTION

MOST of us speak of standardization in a vague sort of way, usually having in mind some particular corner of the field, but without sensing the comprehensiveness of the word or the magnitude and importance of the field covered, not realizing that the cooperation without which our modern society would be impossible, would in turn be impossible without a large measure of standardization. It might almost be said that the degree of standardization in any nation is a measure of its civilization, certainly in the material or industrial sense of the word.

Even in the tribe a common language was necessary, namely a standardization of the meaning of words. A dictionary is merely a recorded standardization of the spelling, pronunciation and meaning of words, as determined by usage and common consent. To be sure we still have dialects and colloquialism, but these are gradually decreasing. In civilized intercourse the accurate use of words is absolutely essential to mutual understanding, however rare it may be, and although new words are being slowly added, the meaning of each must be stable, that is standardized, if it is to serve its purpose.

As between nations we still have many different languages, although most of the civilized world now uses substantially the same alphabet and there is a persistent effort to establish a universal language.

After language, followed standardization in coinage, weights and measures. Our present Bureau of Standards is the outgrowth of the old Bureau of Weights and Measures, which was largely the repository for standards of length, weight, volume, etc. Imagine the hopeless confusion in this country if each state had its own language as well as its own system of coinage and of weights and measures. Even as between nations differences in language, coinage, weights and measures, constitute serious barriers to intercourse, social and commercial, which tend to prevent interchange of ideas and commodities and thus to retard that mutual understanding which makes for peace.

But standardization has already extended far beyond these fundamental fields and is now growing with leaps and bounds.

It is my purpose in presenting this subject to discuss chiefly standardization in these less fundamental fields; to point out its importance and its dangers; to indicate the comparative chaos which still reigns in most parts of the field of engineering and industrial standardization, not only in the standards themselves but underlying this in the methods of creating them; and finally to outline the machinery proposed by the American Engineering Standards Committee to remedy this condition.

Before undertaking the general discussion a brief classification and review of each field may be of interest, excluding the alphabet and coinage as outside of the engineering and industrial group.

CLASSIFICATION OF STANDARDS

Language. Definitions of scientific and technical words, terms, phrases, abbreviations, symbols and diagrams.

Standardization of these elements of our technical language is just as fundamental to intelligent intercourse in this field as in the case of ordinary word intercourse; in fact it is more fundamental since we are here dealing mostly with accurately measurable quantities.

Some progress has been made in this field even to the extent of international standardization of terminology, but chiefly in connection with very fundamental matters, there being still in the outlying fields a very wide diversity of usage in the matter of symbols, abbreviations and diagrams, and in a lesser degree in the use of words. There are at least fourteen different organizations in this country alone, each of which has its own system of diagrammatic representation covering electrical wiring and devices. Many other similar illustrations are available of the frightful inefficiency of the present haphazard uncoordinated work in this field.

Standards of Quantity. Length, area, volume, mass, weight density, pressure, heat, light, illumination, electricity, magnetism, radioactivity, physical constants and their relation to each other.

Some of the fundamental standards of this group are of such importance in the every-day life and trade of a nation that they are prescribed by law and each city or county has the necessary machinery for enforcing the law.

Nearly all European nations as well as the Latin-American nations have adopted the rational metric system, whereas the English speaking nations still cling to the English system with its confusion of oddly related weights and measures.

However since the electrical and magnetic units are the same the world over, since they are based upon the metric system, and since practically all scientific work is handled by that system, a vast amount of time is wasted by the English speaking nations not only by the necessity of becoming familiar with both systems, but also in their use due to the frequent necessity of transformations from one to the other. This latter loss is particularly serious in the field of electrical engineering. It is my own conviction that in the case of a real electrical engineer, a full year of his professional life is wasted on this account. This is partly during his educational work and involves an immense loss of efficiency due to the diversion of his attention from the real understanding of the subject.

Standards of Quality. Specifications for materials together with standard methods for testing their qualities.

Certain standards of this type are also of such importance to the welfare of society that they are prescribed by law notably in the food field, and a bill is now before Congress for the standardization of fertilizers.

Standard specifications and tests for the materials of engineering and construction are almost wholly prepared by the American Society for Testing Materials, which society was organized solely for this purpose. In no other field is the work of standardization better organized.

Standards of Performance. Operating characteristics of machines and devices, output, rating, speed, efficiency, durability, etc., as specified in terms susceptible to measurement, together with the methods of making these measurements.

The most notable example of comprehensive work in this field is to be found in the Standardization Rules of this Institute. Another is in the Power Test Code of the A. S. M. E.

Standards of Practise. Regulations or codes dealing with construction, installation and operation, based upon considerations of safety, quality, economy, convenience, etc.

The oldest code in the electrical field is the National Electric Code of the N. F. P. A. first issued in 1897 and since frequently revised. This has been largely copied in the local municipal regulations of most large cities.

The National Electrical Safety Code was issued about three year ago by the Bureau of Standards after long and careful collaboration with the most interested organizations. This covers safety to life rather than to fire.

The Boiler Code covering standards of construction for boilers was issued by the A. S. M. E. in 1914 after some years of laborious work involving cooperation with the interested organizations.

There is now under consideration the preparation of some fifty odd industrial safety codes. This work will probably be carried out according to the procedure prescribed by the American Engineering Standards Committee, outlined below, with the appropriate organizations acting as sponsors. The Bureau of Standards will doubtless serve as sponsor for some of these codes.

Dimensional and Type Standards. In any product the standardization of types, sizes or sets of dimensions in order to reduce the number of types and sizes and hence the cost of production.

A few illustrations will indicate the tremendous scope of this group and the comparative chaos that still reigns.

Among the earliest and most fundamental attempts in this group are:

Screw Threads, Bolts, Nuts, etc.

In 1864 a Committee of the Franklin Institute recommended the adoption of a system devised by William Sellers, now known as U. S. Standard.

In 1884 Charles Bower, of Warder, Bushnell & Glessner, Springfield, Ohio, adopted standard sizes of screw threads for bolts and taps, based upon the U. S. Standard screw thread.

In 1895 the American, Hartford and Worcester Machine Companies issued standards for set-screws and cap-screws, also based upon the U. S. Standard screw thread.

In 1906 the Association of Licensed Automobile Manufacturers adopted standards for automobile screws and nuts.

In 1907 the American Society of Mechanical Engineers accepted the report of its committee on standard proportions for machine screws. This Committee was appointed in 1902, and had devoted five years to the task. As far as the screw threads themselves are concerned, the American Society of Mechanical Engineers standard differs only in very minor details from the Sellers' or U. S. Standard, but the A. S. M. E. report gives standards also for taps, special screws, special taps, and screw heads of various types, together with tolerances.

In 1912, the Society of Automobile Engineers was organized and has adopted fine screw thread standards, as well as many other size standards of automobile materials and parts.

In the fall of 1918 a screw thread commission was appointed by act of congress and is still at work. In this work some consideration is being had of the international situation, and although there are obviously great difficulties in adapting our threads to those of the metric system nations, the differences between the U. S. and the British threads are slight, though sufficient to prevent interchangeability without excessive tolerances.

I have been told on good authority that there are fifteen different types of screw thread standards in use by the War Department alone.

Wire, Drill and Sheet Metal Gages

Of these there are several varieties still in use, although for copper wires, the Brown & Sharpe or American Wire gage is most commonly used for the smaller sizes, and the Edison or circular mil gage for all sizes above 0000 B. & S. All these gages were originally adopted by some corporation such as the twist drill gage of the Morse Twist Drill & Machine Company, the Stubbs steel wire gage, etc.

An act of Congress in 1893, established the Standard Gage for sheet iron and steel. Previous to this time there were many gages in use, and the addition of this one seemed only to add to the confusion.

In 1895 a joint Committee of the A. S. M. E. and the American Railway Master Mechanics Association agreed to recommend the use of the decimal gage,—that is a gage whose number is the thickness in thousandths of an inch, and to recommend the abandonment and disuse of the various other gages then in use.

In 1904 the Westinghouse Electric and Manufacturing Company abandoned the use of gage numbers referring to wire, sheet metal, etc.

There are now at least eight different sheet metal gages in use in this country.

Other fundamental standards in this group are,—pipe threads and flanges; pipe unions; and special threads for fixtures and fittings, *e. g.* incandescent lamp sockets. Consider the enormous saving in expense and the convenience resulting from our almost universal standard lamp socket.

In the less fundamental part of this field, namely the standardization of types and designs, most of the work has been done until comparatively recently, within individual corporations as a matter of production economy. A notable example of this is the Ford automobile.

But within the past few years much has been accomplished in the automobile industry as a whole by the S. A. E.

This also is true in certain parts of the railway field in the standard freight and passenger cars, through the Master Car Builders Association. The variety is however still enormous.

There are at least 500 different types of locomotives, although the U. S. Railroad Administration has recently reduced the standard types of passenger locomotives to twelve.

One of the most striking examples of the possibilities in this field is the attempt only just started by one of our most progressive manufacturers of electrical machinery, a member of this Institute, to develop standard electric motor designs. Nine manufacturers are working together to this end. Consider the immense advantage to the consumer resulting from the general interchangeability of parts, and the reduction in the number of spare parts carried by a user who has motors of several of these cooperating manufacturers. Moreover the designs will contain the best features of all these makes, and there will still remain ample opportunity for competition in production cost, and quality of workmanship.

NEED AND VALUE OF STANDARDIZATION

This would hardly seem to need argument, but a few illustrations may not be out of place.

Not many years ago the average automobile had nearly as many sizes and types of bolts as there were bolts on the car, with a resulting inconvenience almost insufferable. Since that time the Society of Automobile (now Automotive) Engineers has carried on a tremendous campaign of standardization which has not only vastly reduced the annoyance of repair work to the purchaser but has greatly reduced the cost of automobiles. It is also largely responsible for the unquestioned primacy of this country in that field.

Another illustration from the same field was when that same Society started to standardize the sizes of steel tubing. All the manufacturers agreed except one who persisted in his odd sizes. Shortly thereafter this manufacturer, wishing to purchase some steel tubing, asked for prices and shipment. He was told that the price would be about 30 per cent higher than for the standard sizes, and that the shipment would be made in three months as against immediate shipment of the standard sizes from stock. It is needless to say that this manufacturer soon adopted the standard sizes.

An illustration from the electrical field has to do with the rating of electrical machinery. In the early days there was no agreement as to what was meant, *e. g.*, by a ten horse-power motor. As a result fair competition was impossible. The ratings of the several manufacturers varied as much as 30 per cent and the customer was at the mercy of the persuasive talents of the salesman. It took the Standards Committee of the A. I. E. E. five years to revise its old system of rating of electrical machinery in such a way as to be rational, scientific, satisfactory to all concerned, and capable of reasonably accurate checking by commercial tests. The results of this work have proved to be of world-wide value. They have placed competition on a far more equitable and generally satisfactory basis; they have encouraged more careful and thorough designing; they have encouraged a more careful selection of size on the part of the user, and have generally resulted in better machines.

DANGERS OF STANDARDIZATION

In certain fields notably in standardization of types and in dimensional standardization when the latter is carried into complete designs, there is an obvious danger that standardization may develop into crystallization and serve as a brake on progress; but this is merely an obstacle and not an argument, since it is obviously possible to keep the standards abreast of progress. There is certainly no more difficulty in improving

12 standard types of locomotives than in making the same improvements in the 500 types which exist today. If the twelve types are not sufficient to meet effectively the variety of demands of service, make it 20 or 30 or 50; but the maximum real need will certainly be so far below the present number as to constitute an annual saving of many millions.

The more fundamental a field the greater the need of standardization, and the greater the difficulty of making a change.

A notable example of this latter difficulty is in connection with our abominable English system of weights and measures which just "happened" in those early days when the importance of a rational or simply related system was not appreciated. Now each generation is unwilling to shoulder the expense of a change which will be of chief benefit to ensuing generations. But this would hardly constitute a sufficient reason for never adopting a standard system of weights and measures.

Similarly but in quite a different field, it could hardly be claimed by the most pessimistic that standardization in the manufacture of automobiles had been anything but a striking success.

MACHINERY OF STANDARDIZATION

At present the work of standardization is carried on by numerous organizations each in its own field and with its own machinery, usually through a standards committee. But the fields overlap and the machinery differs largely in the different organizations. In many cases the committees are made up of men who do not realize that a standard must be not only rational and technically correct but also acceptable to all those interested in its production and use. The mere promulgation of a standard by any society however powerful, does not make it a real standard.

Moreover the fields of the several organizations overlap in large degree; there is much duplication of effort and confusion of result, and many of the standards promulgated are practically useless.

Cooperation between two or more organizations has been tried, sometimes successfully, but there was no recognized machinery of cooperation, no standard procedure, and confusion still reigns.

To meet this need there has been recently created an American Engineering Standards Committee whose purpose, organization and method of operation are outlined below.

AMERICAN ENGINEERING STANDARDS COMMITTEE*

The American Engineering Standards Committee was formed by joint action of five national engineering societies (American Society of Civil Engineers, American Institute of Mining Engineers, American Society of Mechanical Engineers, American Institute of Electrical Engineers and the American Society for Testing Materials), called the *Founder Societies*, to meet a long felt need of some available and satisfactory machinery for the development of engineering and industrial standards, by the operation of which duplication would be avoided and cooperation between all interested organizations and Government departments secured; so that when a standard or group of standards is developed it will be acceptable to all concerned and therefore a real American standard. Moreover as international standardization develops, there is imperative need for an authoritative national body to deal with the corresponding foreign bodies. National Engineering Standards Committees are already in operation in England, Canada, France and Holland, and others will undoubtedly be organized in the near future.

The A. E. S. Committee machinery now proposed for the development of standards is made up as follows:

a. The Committee proper or the "*Main Committee*" (with three representatives from each of the five "Founder Societies" and of three Government departments), whose functions are chiefly those of an organizing, coordinating and steering Committee.

The War Department, the Navy Department and the Department of Commerce (Bureau of Standards) have already been invited to join in this movement and to appoint representatives on the Main Committee. The War Department has already accepted the invitation and is now appointing representatives. Assurances are at hand that the other departments will also accept.

b. *Sectional Committees* one for each group of standards (with representatives from all organizations or Government departments vitally interested in that particular group of standards) whose function is to prepare the standards in question under the direction of the most vitally interested organization known as the "Sponsor Body".

---

*In discussions with prospective cooperating societies, certain proposals for changes in the method of selecting the members of the Main Committee have been proposed and are under consideration; but these changes will not affect the general procedure or interrupt the continuity of the work of the Committee.

c. The *Sponsor Society* or *body* may be one of the Founder Societies, a Government department, or one of the "Cooperating Societies" or organizations.

d. Under *Cooperating Societies* it is intended to include all organizations interested in the production of standards and willing to cooperate.

PROCEDURE

1. When the development of a particular group of standards is proposed, the Main Committee assigns the work to the appropriate organization as "Sponsor", or, if the situation seems to indicate that more than one organization is equally interested, to these organizations as "Joint Sponsor."

2. The sponsor then appoints the Sectional Committee subject to the approval of the main committee. The purpose of this approval is merely to assure the comprehensive representation of all the interests involved. Complete records of all interested organizations and of their standardization work will be kept on file and properly classified in the office of the Main Committee. The Main Committee or its Secretary will thus be able, either promptly to suggest the proper representation to a sponsor on request, or to approve or amplify the representation as provisionally proposed by the Sponsor.

3. After a group of standards has been prepared and accepted by a Sectional Committee, it is submitted to the Sponsor Body for its approval and then to the Main Committee with a full report of its history. When approved by both the Sponsor Body and the Main Committee, the standards in question become American Standards.

4. When the report of any sectional committee is being considered by the Main Committee, three members of that sectional committee are invited to sit with the Main Committee to report, discuss and vote on the standards in question as if they were regular members of the Committee. Thus each sectional committee, (and therefore usually each Sponsor Body), will be represented on the Main Committee when standards in which they are interested are being discussed.

5. The scrutiny of a standard by the Main Committee is to make sure that the proper procedure was pursued, that it was prepared by a comprehensively representative sectional

committee, that the vote of acceptance was nearly enough unanimous, and that the standard is consistent with other related standards. Consideration is also given to international relations; but the Main Committee is not supposed to pass upon the details as it is obviously impossible for a moderate sized workable committee to cover the details of all parts of the field.

6. After approval by the Main Committee the standard is published by the Sponsor Body with the statement that it has been approved by the A. E. S. Committee, and labelled "American Standard" with the appropriate descriptive title.

Briefly summarized this procedure is as follows:

Standard assigned by Main Committee to Sponsor Body.

Sponsor Body appoints a thoroughly representative Sectional Committee, subject to approval of Main Committee.

Sectional Committee prepares standard and submits to Sponsor Body which then submits the standard with its approval to the Main Committee.

The Standard is then published by the Sponsor Body and labelled "American Standard."

The machinery thus provides for absolutely comprehensive cooperation, eliminates overlapping and duplication of effort, and yet does not undesirably restrict the initiative of the several cooperating societies.

With proper support and the cooperation of all interested organizations, it should contribute largely to the industrial development of the country.

Moreover it should be a potent factor in promoting international standardization and foreign commerce.

In the broad sense international standardization means a common industrial language and the removal of one of those barriers which tend to separate nations and to give rise to misunderstandings.

The more of these barriers we can remove, the more likely we are to attain that lasting peace which the world longs for.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers New York, February 20, 1919.

Copyright 1919. By A. I. E. E.

THE GENERAL EQUATIONS OF THE ELECTRIC CIRCUIT III

Variation of Constants r, L, C, and g, and its Effects

BY CHARLES P. STEINMETZ

ABSTRACT OF PAPER

In the usual theory of transients the assumption is made, that resistance, inductance, capacity and conductance are constant. This however, is not correct, and as the result thereof, it was not possible to theoretically investigate, and numerically calculate the dissipation of high-frequency disturbances, the flattening of the wave fronts of impulses, the rounding off of steep waves, etc., with the time and the distance of travel, and therefrom to determine the distance, to which the danger from such disturbances extends, and to investigate the conditions of line construction, which limit the danger zone of such phenomena to the smallest local extent.

In the following, two of the foremost causes of change of the line constants with the equivalent frequency are investigated, the unequal current distribution in the conductor, and the electric radiation from the conductor, and shown, that within the range of frequencies which may be met in industrial circuits, the effective resistance of the conductor, and its attenuation constant, may increase more than a million fold.

Equations of the line constants as function of the equivalent frequency are derived, and applications thereof made to a few problems:

(1) The laws of conduction of high-frequency currents, such as produced by lightning discharges and similar disturbances, and the conclusions resulting therefrom on the nature of the conductor.

(2) The decay of high-frequency sine waves in transmission lines.

(3) The attenuation of rectangular waves.

(4) The flattening of the wave front of steep impulses.

It is shown that in high-frequency conduction the section of the conductor is of little importance, but the circumference is the determining factor, except at very high frequencies, where size, shape, and material—within certain limits—becomes of secondary importance. The inductance and the capacity of the conductor remain constant up to very high frequencies, but the effective resistance may increase many thousand fold. At high frequencies, not all the energy consumed by the effective resistance of the conductor is converted into heat, but the major part may be radiated into space and thereby affect other circuits.

The conditions, which cause a flattening of steep wave fronts and impulses, and a rounding of irregular waves, are investigated, and it is shown, that within the range of distances in which

danger may result from a steep wave front or high frequency, the steepness of the wave front decreases with the square root of the distance traveled by the wave; that size and material of the conductor have little influence, but the distance between conductor and return conductor is the main factor in flattening the wave front and thereby limiting the danger zone.

For convenience, the theoretical part has been separated and placed in an appendix, giving in the text the discussion, with numerous tables derived from the theoretical equations, and curves illustrating these tables.

The investigation is not general, but rather limited to special applications, as the field is too new to permit fargoing generalization.

I.—General

IN 1908, in a paper under the above title, the general equations of the electric circuit were presented and discussed, and in a second paper in 1916, under the title "Outline of a Theory of Impulse Currents," the application of these general equations to the special cases of direct-current circuits, alternating-current circuits, etc., was discussed, and more particularly a theory of impulse currents outlined, that is, transient currents which are non-periodic in time, and either non-periodic or periodic in space.

In both of these papers however, and in all general theoretical investigations on these subjects, as far as I know, the assumption is made, that the circuit constants; resistance r, inductance L, capacity C and shunted conductance g, are constant. While this is true, with sufficient approximation, for the usual machine frequencies and for moderately high frequencies, experience shows that it is not even approximately true for very high frequencies and for very sudden circuit changes, as steep wave front impulses, etc.

If r, L, C and g are assumed as constant, it follows that the attenuation is independent of the frequency, that is, waves of all frequencies decay at the same rate, and as the result, a complex wave or an impulse traversing a circuit dies out without changing its wave shape or the steepness of its wave front.

Experience, however, has conclusively shown that steep wave fronts are dangerous only near their origin, and rapidly lose their destructiveness by the flattening of the wave front when running along the circuit. Experimentally, small inductances shunted by a spark gap, inserted in transmission lines for testing for high frequencies or steep wave fronts, have shown appreciable spark lengths, that is, high-voltage gradients, only near the origin of the disturbance, but no appreciable spark length beyond a few miles from the origin of the disturbance.

The rectangular wave of starting a transmission line by connecting it to a source of voltage, which is given by the theory under the assumption of constant r, L, C and g, and thus shown in most text books, has never been shown in any of the oscillograms of transmission lines, as far as I know.

If r and L are constant, the power factor of the line conductor, $\frac{r}{\sqrt{r^2 + (2\pi fL)}}$, should with increasing frequency continuously decrease, and reach extremely low values, at very high frequencies, so that at these, an oscillatory disturbance should be sustained over very many cycles, and show with increasing frequency an increasing liability to become a sustained or cumulative oscillation. Experience, however, shows that high-frequency oscillations die out much more rapidly than accounted for by the standard theory, and show at very high frequency practically no tendency to become cumulative.

This does not mean that the present theory of transients, which is based on the assumption of constant r, L, C and g, is useless by not correctly representing the actual phenomena, but it means that it correctly represents the transient only in its initial stage and near its origin, but does not satisfactorily represent its course after its initial stage and at some distance from the origin, at least not for high-frequency transients or steep wave fronts.

As the destructive effect of voltage does not depend only on the value of the voltage, but also on the duration of its application, this is a limitation, though not as serious as it may appear, since it tends to understate the danger of high-frequency transients.

However, it makes it impossible to determine such important questions as; how far the danger zone of a high-frequency disturbance or steep wave front extends; how the destructiveness decreases with the distance; how it depends on line construction, conductor shape and materials, etc., and to approach the problem of so designing the circuit as to limit the danger from any such disturbance to the smallest local extent.

In the following, therefore, the attempt is made to investigate and calculate theoretically some of the factors, which cause a change of the line constants r, L, C, and g, with increasing frequency or steepness of wave front, and to investigate theoretically the effect produced on the course of the transient with regard to duration and wave shape, by the variation of the line constants.

As this field is rather new, no very general treatment is yet possible, but rather, an outline of the problem, and applications to some of the most important phenomena, such as the conduction of high-frequency currents, the changes of wave shape with time and distance of wave travel, and the flattening of the wave front, and its bearing on circuit construction.

The two most important factors in the variation of the circuit constants r, L, C, and g, seem to be the *unequal current distribution in the conductor* and the *finite velocity of the electric field.*

Unequal Current Distribution in the Conductor

The magnetic field of the current surrounds this current and fills all the space outside thereof, up to the return current. Some of the magnetic field due to the current in the interior and in the center of a conductor carrying current, thus is inside of the conductor, while all the magnetic field of the current in the outer layer of the conductor is outside of it. Therefore, more magnetic field surrounds the current in the interior of the conductor than the current in its outer layer, and the inductance therefore increases from the outer layer of the conductor towards its interior, by the "internal magnetic field." In the interior of the conductor the reactance voltage thus is higher than on the outside.

At low frequency, with moderate size of conductor, this difference is inappreciable in its effect. At higher frequencies however, the higher reactance in the interior of the conductor, due to this internal magnetic field, causes the current density to decrease towards the interior of the conductor, and the current to lag, until finally the current flows practically only through a thin layer of the conductor surface.

As the result thereof, the effective resistance of the conductor is increased, due to the uneconomical use of the conductor material caused by the lower current density in the interior, and due to the phase displacement, which results in the sum of the currents in the successive layers of the conductor being larger than the resultant current. Due to this unequal current distribution, the internal reactance of the conductor is decreased, as less current penetrates to the interior of the conductor, and thus produces less magnetic field inside of the conductor.

The derivation of the equations of the effective resistance of unequal current distribution in the conductor, r_1, and of the internal reactance x_1 under these conditions, is given in appendix A. It is interesting to note, that effective resistance and internal reactance, with increasing frequency, approach the same limit, and become proportional to the square root of the frequency, the square root of the permeability, and the square root of the resistivity of the conductor material, while at low frequencies the resistance is independent of the frequency and directly proportional to the resistivity, and the internal reactance is independent of the resistivity and directly proportional to the frequency.

FINITE VELOCITY OF THE ELECTRIC FIELD

In calculating the inductance or the capacity of a conductor, usually the assumption is made, that the electric field is instantaneous, that is, the magnetic field appears and disappears simultaneously with the current, and therefore, is in phase with the current, and the dielectric field appears and disappears simultaneously with the voltage. In reality, however, this is not strictly correct, but the electric field moves with a finite velocity from the conductor outwards. While this velocity is inconceivably high: $S = 3 \times 10^{10}$ cm. per sec., or 300,000 km. per sec., its effect is appreciable at high frequencies. Consider, for instance, a circuit representing average transmission line conditions with 6 ft. = 182 cm., between conductors, traversed by a current of $f = 10^6$, or one million cycles, such as may be produced by a nearby lightning discharge. The wave length of this current and thus of its magnetic field then would be $\frac{S}{f} = \frac{3 \times 10^{10}}{10^6} = 30{,}000$ cm. and the distance of 182 cm. between the line conductors would be $\frac{182}{30{,}000}$ or $\frac{1}{165}$ of a wave length or $\frac{360}{165} = 2.2$ deg. That is, the magnetic field of the current, when it reaches the return conductor, would not be in phase with the current, but $\frac{1}{165}$ of a wave length or 2.2 deg. behind the current. The voltage induced by the magnetic field would not be in quadrature with the current, or wattless, but lag $90 + 2.2 = 92.2$ deg. behind the current, thus have an energy component equal to cos 92.2 deg. = 3.8 per cent, giving rise to an effective resistance r_3, equal to 3.8 per cent of the reactance x. Even if at normal frequencies of 60 cycles the reactance is equal only to the ohmic resistance r_0 — usually it is larger—the reactance x to 10^6 cycles would be $\frac{10^6}{60}$, = 16,700 times the ohmic resistance, and the effective resistance of magnetic radiation, r_3, being 3.8 per cent of this reactance, thus would be 630 times the ohmic resistance: $r_3 = 630\, r_0$. Thus, the ohmic resistance would be entirely negligible compared with the effective resistance resulting from the finite velocity of the magnetic field.

Considering however, a high-frequency oscillation of 10^6 cycles, not between the line conductors, but between line conductor and ground, and assume, under average transmission line conditions, 30 ft. as the average height of the conductor above ground. The magnetic field of the conductor then can be represented as that between the conductor and its image conductor, 30 ft. below ground, and the distance between conductor and return conductor would be 2×30 ft. = 1820 cm. The lag of the magnetic field, due to the finite velocity of propagation, then becomes 22 deg. thus quite appreciable, and the energy component of the voltage induced by the magnetic field is $\cos (90 + 22^0) = 37$ per cent. This would give rise to an effective or radiation resistance $r_3 = 0.37\, x$. As in this case, the 60-cycle reactance usually is much larger than the ohmic resistance, assuming it as twice would make the radiation resistance $r_3 = 12{,}600\, r_0$, or more than ten thousand times the true ohmic resistance. It is true, that at these high frequencies, the ohmic resistance would be very greatly increased by unequal current distribution in the conductor. But the effective resistance of unequal current distribution increases only proportionally to the square root of the frequency, and, assuming for instance conductor No. 00 B. & S. gage, the effective resistance of unequal current distribution, r_1, would at 10^6 cycles be about 36 times the low-frequency ohmic resistance r_0. Thus the effective resistance of magnetic radiation would still be $\frac{12{,}600}{36} = 350$ times the effective resistance of unequal current distribution: $r_3 = 350\ r_1$, and the latter, therefore, is negligible compared with the former.

It is interesting to note, that the effective resistance of radiation, r_3, does not represent energy dissipation in the conductor by conversion into heat, but energy dissipation by radiation into space, and in distinction from the "radiation resistance," which dissipates energy into space, the effective resistance of unequal current distribution may be called a "thermal resistance," as it converts electric energy into heat

Thus in this instance of a 10^6-cycle high-frequency discharge between transmission line conductor and ground, the heating of the conductor would be increased 36 fold over that produced by a low-frequency current of the same amperage, by the increase of resistance by unequal current distribution in the conductor; but the total energy dissipation by the conductor would be increased by magnetic radiation still 350 times more, so that the total energy dissipation by the 10^6-cycle current would be 12,600 times greater than it would be with a low-frequency current of the same value, and the attenuation or rate of decay of the current thus would be increased many thousand times, over that calculated on the assumption of constant resistance at all frequencies.

The derivation of the equations of the effective resistance of magnetic radiation, and in general of the effects of the finite velocity of the electric field on the line constants, are given in appendix B.

The magnetic radiation resistance is proportional to the square of the frequency (except at extremely high frequencies). It therefore, is negligible at low and medium frequencies, but becomes the dominating factor at high frequencies. It is proportional to the distance of the return conductor, but entirely independent of size, shape, or material of the conductor, as is to be expected, since it represents the energy dissipated into space. Only at extremely high frequencies, the rise of radiation resistance becomes less than proportional to the square of the frequency. It becomes practically independent of the distance of the return conductor, when the latter becomes of the magnitude of the quarter wave length.

The same applies to the capacity. Due to the finite velocity of propagation, the dielectric or electrostatic field lags behind the voltage which produces it, by the same angle by which the magnetic field lags behind the current, and the capacity current or charging current thus is not in quadrature with the voltage, or reactive, but displaced in phase by more than 90 deg. thus contains a negative energy component, which gives rise to a shunted conductance of dielectric radiation g. This gives rise to an energy dissipation by the conductor, at high frequencies, by dielectric radiation into space, of the same magnitude as the energy dissipation by magnetic radiation, above considered.

The term "shunted conductance" g has been introduced into the general equations of the electric circuit largely from theoretical reasons, as representing the power consumption proportional to the voltage. Most theoretical investigations of transmission circuits consider only r, L and C as the circuit constants, and omit g, since under average transmission line conditions, at low and moderate frequencies, g usually is negligible. In communication circuits, as telegraph and telephone, there is a "leakage," which would be represented by a shunted conductance, and in underground cables there is a considerable energy consumption by dielectric losses in the insulation, as the investigations of the last years have shown, which gives a shunted conductance. In overhead power lines however, energy losses depending on the voltage,—and leading to a term g—have been known only at those high voltages where corona appears. It is interesting therefore to note, that at high frequencies, "shunted conductance" g may reach very formidable values even in transmission lines, due to electrostatic radiation.

In investigating the effect of the finite velocity of the electric field on the inductance L and the capacity C, in appendix B, it is seen, that the equations of external inductance and of capacity are not affected, but remain the same as the usual values derived by neglecting the velocity of the electric field, except at extremely high frequencies, when the distance of the return conductor approaches quarter wave length.

EQUATIONS OF ELECTRICAL CONSTANTS, AND NUMERICAL VALUES

In appendix C are given, compiled from appendices A and B, the equations of the components of the electrical constants, as functions of the frequency, for conductors with return conductor, and also for conductors without return conductor (as approximated by lightning strokes or wireless antennae):

Resistance: true ohmic resistance or effective resistance of unequal current distribution, and magnetic radiation resistance.

Reactance: low frequency internal reactance or internal reactance of unequal current distribution, and external reactance.

Inductance: low frequency internal inductance or internal inductance of unequal current distribution, and external inductance.

Shunted conductance and capacity are not so satisfactorily represented, and therefore, instead of representing energy storage and power dissipation depending on the voltage by a conductance g and a capacity C or susceptance b, in shunt with each other, it is more convenient to represent them by an effective resistance, the dielectric radiation resistance r_c, and a capacity reactance x_c, in series with each other.

As seen from appendix C, four successive stages may be distinguished in the expressions of the circuit constants as functions of the frequency.

1. Low frequencies, such as the machine frequencies of 25 and 60 cycles. The resistance is the true ohmic resistance, the internal reactance and inductance that corresponding to uniform current density throughout the conductor, with conductors of moderate size, and of non-magnetic material.

2. Medium frequencies, of the magnitude of a thousand to ten thousand cycles. Resistance and internal reactance or inductance are those due to unequal current distribution in the conductor, that is, the resistance is rapidly increasing, and the internal inductance decreasing. The conductance g is still negligible, radiation effects still absent, and all the energy loss that of thermal resistance.

3. High frequencies, of the magnitude of one hundred thousand to one million cycles. The radiation resistance is appreciable and becomes the dominating factor in the energy dissipation. The internal inductance has practically disappeared, due to the current penetrating only a thin surface layer. A considerable shunted conductance exists due to the dielectric radiation.

4. Extremely high frequencies. Of the magnitude of many millions of cycles, when the quarter wave length has become of the same magnitude or less than the distance of the return conductor. Radiation effects entirely dominate, and the usual expressions of inductance and of capacity have ceased to apply.

This last case is of little industrial importance, as such extremely high frequencies propagate only over short distances.

It would come into consideration only in calculating the flattening of the wave front of a rectangular impulse in the immediate neighborhood of its origin, and similar problems.

Thus far, a general investigation does not seem feasible. Substituting the equations of the circuit constants, as functions of the frequency, into the general equations of the electric circuit, leads to expressions too complex for general utility, and the investigation thus must largely be made by numerical calculations.

Only when the frequencies which are of importance in the problem, lie fairly well in one of the four ranges above discussed —as is the case in the investigation of the flattening of a steep wave front in moderate distances from its origin,—a more general theoretical investigation becomes possible.

As instances of industrial importance may be considered in the following:

The conduction of high frequency currents such as produced by lightning and similar disturbances.

The attenuation of waves in overhead transmission circuits.

The rounding of rectangular waves.

The flattening of steep wave fronts.

II.—Lightning Conductors

It has been known for many years that currents of high frequency, very sudden impulses, etc., such as are given by lightning discharges and similar disturbances in electric circuits, do not follow the laws of low-frequency currents. The conduction of such currents however, is of very great industrial importance; in the discharge path of lightning arresters and other protective devices, their ground connections, their connections with the circuit, etc., it is of fundamental importance to design and select the conductor so as to offer minimum impedance to the discharge. Inversely, in the connection of the station to the transmission line, it is of importance to design the conductor for maximum impedance against such currents. The protection of buildings by the lightning rod depends on the design of the conductor system, etc.

However, the high-frequency impedance of the conductor is not the only quantity of importance. At high frequencies, the thermal resistance, which dissipates energy in the conductor by conversion into heat, may be a small part only of the total effective resistance, and the radiation resistance dominates.

That is, most of the power consumed in the conductor is radiated into space. Thus it is not sufficient to merely afford a discharge path for lightning, because while discharging through such path, most of the energy of the lightning may be communicated by radiation to other bodies. This accounts for the power exerted upon bodies near the path of a lightning stroke, the destructive effects of a "side stroke." Hence it is of importance also, to study the extent of the electric field of the high-frequency conductor, to ascertain that nothing is within the electric field, which might be damaged by intercepting the electric field, or may lead the power, communicated to it by the radiation from the discharge path, into other circuits where it may be destructive. Thus a lightning arrester, while functioning perfectly and supposedly giving perfect protection, may be a source of danger, by concentrating the energy of lightning into its discharge path, if this discharge path is carried in close inductive relation to conductors of the circuit which is to be protected.

To show the relative magnitude of the different line constants and their components, in Table I are given their numerical values, in ohms per meter length of conductor, for frequencies from one cycle to one hundred million cycles per second, of a copper wire No. 00 B. & S. gage, for the three conditions:

a. Return conductor at $l' = 6$ ft. $= 182$ cm. corresponding to a discharge between transmission line conductors.

b. With the ground as return conductor, at 30 ft. distance, that is, $l' = 2 \times 30$ ft. $= 1820$ cm. corresponding to a discharge between transmission line and ground.

c. No return conductor, corresponding to the vertical discharge path of a lightning arrester ground, a lightning stroke, etc.

Some of these values are plotted in Fig. 1.

The low frequency values of resistance r_0 and external and internal reactance $x_0 + x_{10}$, have no existence at the higher frequencies. But as they are the values calculated by the usual formulas, they are given in Table I, for comparison with the true effective high-frequency values. The values r_2 and x_2, though given for all frequencies, have a meaning only for the very high frequencies, since at lower frequencies the condition of a conductor without return conductor can hardly be realized, as any conductor within a quarter wave length would act as a more or less effective return conductor.

TABLE I.

CIRCUIT CONSTANTS OF COPPER WIRE NO. 00 B. & S. GAGE PER METER LENGTH OF CONDUCTOR

a. return conductor at 6 ft. = 182 cm. distance.
b. return conductor at 2 × 30 ft. = 1820 cm. distance.
c. no return conductor

conductivity: $\gamma = 6.2 \times 10^5$

		Cycles f =	1	10	10^2	10^3	10^4	10^5	10^6	10^7	10^8
Low Frequency		r_0 =	.000240	.000240	.000240	.000240	.000240	.000240	.000240	.000240	.000240
		x_{10} =	.0000003	.000003	.00003	.0003	.003	.03	.3	3	30
	a.	x_0 =	.0000075	.000075	.00075	.0075	.075	.75	7.5	75	750
	b.		.0000104	.000104	.00104	.0104	.104	1.04	10.4	104	1,040
	a.	z_0 =	.000240	.000252	.000816	.0078	.078	.78	7.8	78	780
	b.		.000240	.000263	.001097	.0107	.107	1.07	10.7	107	1,070
	a.	cos ω_0 =	1.000	.952	.293	.0308	.0031	.00031	.000031	.000003	.0000003
	b.		1.000	.912	.218	.0225	.0022	.00022	.000022	.000002	.0000002
High Frequency		r_1 =	.000240	.000240	.000240	.000273	.000865	.00273	.00865	.0273	.0865
		x_1 =	.0000003	.000003	.000030	.000273	.000865	.00273	.00865	.0273	.0865
	a.	r_3 =	....	....	....	....	.0000048	.00048	.048	4.8	225
	b.		....	....	....	....	.000048	.0048	.48	22.5	194
	c.	r_2 =	.000002	.000020	.000197	.00197	.0197	.197	1.97	19.7	197
	a.	x_3 =	.0000075	.000075	.00075	0.075	.075	.75	7.5	75	490
	b.		.0000104	.000104	.00104	.0104	.104	1.04	10.4	78	505
	c.	x_2 =	.0000285	.000256	.00227	.0198	.169	1.40	11.2	83	515
	a.	z =	.000240	.000252	.000816	.00778	.0759	.753	7.5	75.2	539
	b.		.000240	.000263	.001097	.01068	.1049	1.043	10.4	81.2	541
	c.		.000242	.000367	.00234	.0201	.170	1.42	11.4	85.3	551
	a.	cos ω =	1.000	.952	.293	.0351	.0115	.0043	.0076	.0642	.418
	b.		1.000	.912	.218	.0257	.0087	.0072	.047	.277	.358
	c.		1.000	.710	.186	.112	.121	.141	.173	.232	.358
	a.	$\frac{r_1 + r_3}{r_0}$ =	1.000	1.000	1.000	1.14	3.62	13.4	236	20,100	937,000
	b.		1.000	1.000	1.000	1.14	3.80	31.3	2,030	94,000	808,000
	c.	$\frac{r_1 + r_2}{r_0}$ =	1.008	1.082	1.82	9.35	85.5	832	8,250	82,000	820,000

As seen from the equations, and illustrated in Table I, the thermal resistance of the conductor, r_1, that is, the resistance which converts electrical energy into heat in the conductor, is the true ohmic resistance at low frequencies, but with increasing frequency begins to rise due to unequal current distribution in the conductor (at about 1000 cycles in copper wire No. 00 B. & S. gage) and approaches proportionality with the square root of the frequency, hence reaches values many times the ohmic resistance, at very high frequencies.

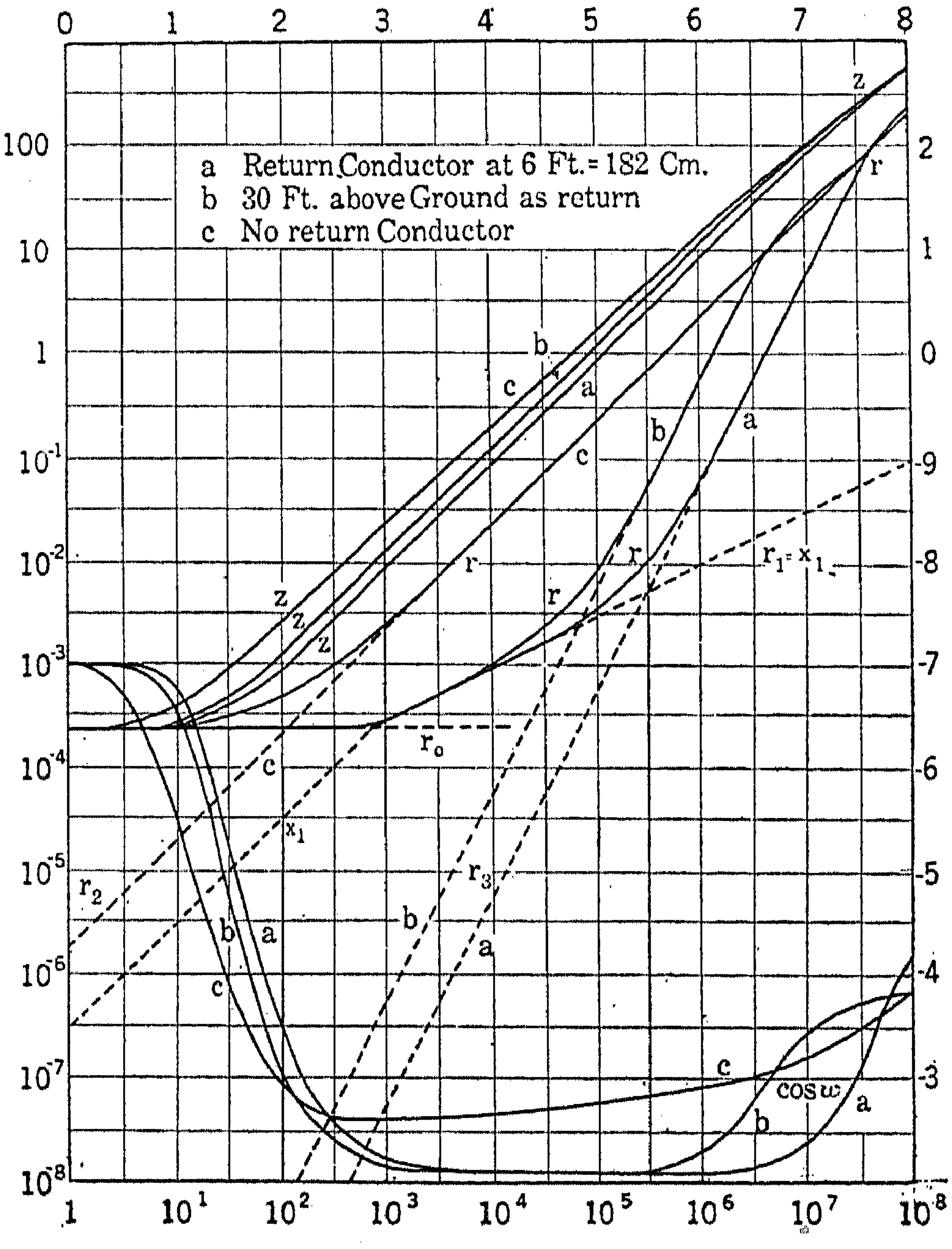


FIG. 1—HIGH-FREQUENCY CONSTANTS OF COPPER WIRE NO. 00 B. &. S GAGE

The radiation resistance of the conductor without return conductor, r_2, is proportional to the frequency, but the radiation resistance of the conductor with return conductor, r_3, is proportional to the square of the frequency, hence very small until high frequencies are reached – 10,000 to 100,000 cycles. The radiation resistance r_3 of the conductor with return conductor then increases very rapidly and reaches values many thousand times the ohmic resistance. At the very highest frequencies, many millions of cycles, its rate of increase becomes less again, and it approaches proportionality with the first power of the frequency, and approaches the value of radiation resistance r_2, of the conductor without return conductor, at frequencies of a wave length comparable with the distance of the return conductor. The radiation resistance r_3 of the conductor with return conductor is the larger, the greater the distance of the return conductor, and is proportional to this distance, within the range of which it is proportional to the square of the frequency. The radiation resistance of the conductor without return conductor, at the very highest frequencies, is the same as that of the conductor with return conductor, but, being proportional to the frequency, with decreasing frequency, it decreases at a lesser rate, and would even at commercial machine frequencies still be appreciable, if at such frequencies the conditions of a conductor without return conductor could be realized.

The total effective resistance of a conductor under transmission line conditions, that is, with return conductor at finite distance, is at low frequencies constant and is the true ohmic resistance. With increasing frequency, it begins to increase first slowly—at about 1000 cycles under transmission line conditions—and approaches proportionality to the square root of the frequency, as the result of the screening effect of the unequal current distribution in the conductor. Then the increase becomes more rapid, due to the appearance of the radiation resistance—at about 100,000 cycles under transmission line conditions—and reaches proportionality with the square of the frequency, at values many thousand times the ohmic resistance. Finally, at the very highest frequencies—10 million cycles—the rate of increase becomes less again, and approaches proportionality with the frequency.

It is interesting to note that the external reactance of the conductor with return conductor, or radiation reactance x_3, has up to very high frequencies, millions of cycles, the same value x_0 as calculated by the low-frequency formula, that is, by neglecting the finite velocity of the field, hence is proportional to the frequency. The internal reactance $x_1 = x_{10}$, and is proportional to the frequency at low frequencies, but drops behind x_{10} due to unequal current distribution in the conductor, and approaches proportionality with the square root of the frequency. As, however, the internal reactance in a small part of the total reactance, it follows, that the total reactance of the conductor and thus also the absolute value of the impedance (for all higher frequencies, where the reactance preponderates) can be calculated by the usual low frequency reactance formula, which neglects the finite velocity of the field. Hence, the inductance L of the conductor can be assumed as approximately constant for all frequencies up to millions of cycles; it decreases only very slowly by the decreasing internal reactance of unequal current distribution. Only at the very highest frequencies, where the wave length is comparable with the distance of the return conductor, the inductance L decreases, and the reactance $x_1 + x_3$ increases less than proportional to the frequency.

In a conductor without return conductor, the reactance at the very highest frequencies is approximately the same as in a conductor with return conductor. With decreasing frequency, however, $x_1 + x_2$ decreases less than proportional to the frequency, that is, the inductance L increases—and becomes infinity for zero frequency, if such were possible.

Without considering unequal current distribution in the conductor and the finite velocity of the electric field, the power factor $\cos \omega$ would steadily decrease, from unity at very low frequencies, to zero at infinite frequency. Due to the increase of the effective resistance, the power factor $\cos \omega$ first decreases, from unity at low frequency, down to a minimum at some high frequency, and then increases again to high values at very high frequencies. The minimum value of the power factor is the lower and occurs at the higher frequencies, the shorter is the distance of the return conductor. Thus with the return conductor at 6 ft. distance, the power factor is 0.43 per cent at 100,000 cycles; with the return conductor at 60 ft. distance, it is 0.72 per cent; in the conductor without return conductor the power factor is 11.2 per cent at 1000 cycles.

It is of interest to determine the effect of size, shape and material on the high-frequency constants of a conductor.

These high-frequency constants are, per unit length of conductor:

Internal constants:

Thermal resistance and internal reactance:

$$r_1 = x_1 = \frac{\pi}{l_1}\sqrt{\frac{0.4\,\mu f}{\gamma}}\,10^{-4} \text{ ohms per cm.} \qquad (8)\ (9)$$

External constants:
Radiation resistance:

$$r_3 = \frac{8\pi^2 f^2 l'}{S} 10^{-9} \text{ ohms per cm.} \qquad (\mathbf{16})$$

External reactance:

$$x_3 = 4\pi f \log \frac{2\pi l'}{l_2} 10^{-9} \text{ ohms per cm.} \qquad (\mathbf{19})$$

These approximations hold for all but the very highest, and very low frequencies, that is, are correct within the frequency range with lower limit of about 1000 cycles, and upper limit of about 10 million cycles. Thus they apply for all those high frequencies which are of importance in the disturbances occuring in industrial circuits with the exception of the lowest harmonics of low-frequency surges.

The constants of the conductor material enter the equations only as the ratio $\frac{\mu}{\gamma}$, permeability to conductivity, in the internal constants r_1 and x_1. Thus higher permeability has the same effect in increasing the thermal resistance as lower conductivity, and for instance, a cast silicon rod of permeability $\mu = 1$, and conductivity $\gamma = 55$, has the same high-frequency resistance and reactance, as an iron rod of the same size, of wrought iron, of permeability $\mu = 2{,}000$ and conductivity $\gamma = 1.1 \times 10^5$, that is, of the same $\frac{\mu}{\gamma} = 0.0182$, though the latter has 2000 times the conductivity of the former.

Provided however, that size of conductor and frequency are such as to fulfill the conditions under which equations (**8**) and (**9**) are applicable, which is, that the conductor is large compared with the depth of penetration of the current into the conductor:

$$l_p = \frac{10^4}{\pi \sqrt{0.4 \; \gamma \mu f}}$$

Thus an iron rod of two inches (five cm.) diameter has at one million cycles the same thermal resistance as a silicon rod of the same size: 0.17 ohms per meter, since the depth of penetration is $l_p = 0.00034$ cm. for iron, 0.68 cm. for silicon, thus in either case small compared with radius of the conductor $l_r = 2.5$ cm.

At 10,000 cycles, however, the iron rod has the thermal resistance and internal reactance $r_1 = x_1 =$ 0.017 ohms per meter, the penetration being $l_p =$ 0.0034 cm. thus small. For the silicon rod, however, at 10,000 cycles the penetration is $l_p =$ 6.8 cm. thus at the radius $l_r =$ 2.5 cm. formulas (8) and (9) do not apply any more, but it is approximately (that is neglecting unequal current distribution): $r_1 =$ 0.093 ohms per meter, or 5.5 times the resistance of the iron rod, while the internal reactance is $x_1 =$ 0.031 ohms per meter, hence 80 per cent higher than that of the iron rod of the same size.

In the equations of the external constants, the radiation resistance and reactance, the material constants of the conductor do not enter, and the radiation resistance and the external reactance thus are independent of the conductor material.

The dimensional constants of the conductor, size and shape, enter the equation only as the circumference of the conductor l_1, l_2, that is, only the circumference of the conductor counts in high-frequency conduction, and all conductors of the same material, regardless of size and shape, have the same high-frequency resistances and reactances as long as they have the same conductor circumference. Thus a solid copper rod or a thin copper cylinder of the same outer diameter as the rod, or a flat copper ribbon of a circumference equal to that of the rod, are equally good high-frequency conductors, though the hollow cylinder or the ribbon may contain only a small part of the material contained in the solid copper rod. Provided, however, that the thickness or depth of the conductor,(the thickness of wall of the hollow cylinder, half the thickness of the copper ribbon) is larger than the depth of penetration of current into the conductor, which is

$$l_p = \frac{10^4}{\pi \sqrt{0.4\ \gamma\ \mu}}$$

In the expression of the radiation resistance, r_3, neither the material nor the dimensions of the conductor enter, that is,the radiation resistance of a conductor is independent of size, shape or material of the conductor and depends only on frequency and distance of the return conductor.

Thus a thin steel wire or a wet string have the same radiation resistance as a large copper bar. Obviously, the thermal resistance of the former is much larger and the total effective resistance thus would be larger except at those very high frequencies, at which the radiation resistance dominates.

As illustration, may be calculated, for frequencies from 10 thousand to 10 million cycles and for 60 cycles, the resistances and reactances and thus the total impedance, the power factor, the voltage drop per meter at 100 amperes, of various conductors, for the three conditions:

a. High frequency between conductors 6 ft. apart:

$$l' = 182 \text{ cm.}$$

b. High frequency between conductor and ground 30 ft. below conductor:

$$l' = 1820 \text{ cm.}$$

c. High-frequency discharge through vertical conductor without return conductor:

$$l' = \infty$$

For the conductors:

(1) Copper wire No. 00 B. & S. gage

$l_r = 0.463$ cm. $\gamma = 6.2 \times 10^5$

(2) Iron wire of the same size as (1):

$\mu = 2{,}000$ $\gamma = 1.1 \times 10^5$

(3) Copper ribbon of thickness equal to twice the depth of penetration at 10,000 cycles, and the same amount of material as (1), that is, 0.128 cm. by 5.52 cm.

(4) Iron ribbon of the same size as (3).

(5) Two-inch iron pipe, ⅛ inch thick,

This gives the depth of penetration at $f = 10^4$ cycles:

$$\text{for copper; } l_p = \frac{6.4}{\sqrt{f}} = 0.064 \text{ cm.}$$

$$\text{for iron: } l_p = \frac{0.34}{\sqrt{f}} = 0.0034 \text{ cm.}$$

Table II gives the values of r_0, x_0, r_1, r_3, r_2, x_1, x_3, x_2, $r_1 + r_3$, $r_1 + r_2$, $x_1 + x_3$, $x_1 + x_2$, z, $\cos \omega$, e and l_p, for $f = 10^6$ cycles.

Table III gives the values of $r_1 + r_3$, or $r_1 + r_2$, z, $\cos \omega$ and e, for $f = 60, 10^4, 10^5, 10^6, 10^7$ cycles for the five kinds of conductors.

It is interesting to compare in Table III, the constants of the first four conductors, as they have the same section, representing about average section of transmission conductors, but represent two shapes, round wire and thin flat ribbon, and two kinds of material, copper—high conductivity and non-magnetic,—and iron—magnetic material of medium conductivity.

As seen, the effect of conductor shape and conductor ma-

TABLE II.

CIRCUIT CONSTANTS FOR $f = 10^6$ = ONE MILLION CYCLES, PER METER LENGTH OF CONDUCTOR.

a. return conductor at 6 ft. = 182 cm. distance.
b. return conductor at 2 × 30 ft. = 1820 cm. distance.
c. no return conductor

		(1) Copper wire:	(2) Iron wire:	(3) Copper ribbon:	(4) Iron ribbon:	(5) Iron pipe:	
$r_0 = \frac{100}{A\ \gamma} =$		0.00024	0.00135	0.00024	0.00135	0.000197	
$x_0 = 0.4\,\pi\,f\left(\log\frac{2\,\pi f l'}{l_2} + \frac{l_0}{l_2}\,\frac{\mu}{4}\right)10^{-6} =$	a.	7.8	635	3.05	173	81	ohms
	b.	10.8	638	5.95	176	84	"
$r_1 = \frac{100\,\pi}{l_1}\sqrt{\frac{0.4\,\mu f}{\gamma}}\,10^{-4} =$		0.00865	0.92	0.00233	0.248	0.167	"
$r_s = \frac{0.8\,\pi^2 f^2 l'}{S}\,10^{-6} =$	a.	0.048	0.048	0.048	0.048	0.048	"
	b.	0.48	0.48	0.48	0.48	0.48	"
$r_2 = 0.2\,\pi^2 f\,10^{-6} =$	c.	1.97	1.97	1.97	1.97	1.97	"
$x_1 = \frac{100\,\pi}{l_1}\sqrt{\frac{0.4\,\mu f}{\gamma}}\,10^{-4} =$		0.00865	0.92	0.00233	0.248	0.167	"
$x_s = 0.4\pi f \log\frac{2\,\pi f l'}{l_2}\,10^{-6} =$	a.	7.5	7.5	2.97	2.97	2.47	"
	b.	10.4	10.4	5.87	5.87	5.37	"
$x_2 = 0.4\,\pi f\left(\log\frac{S}{l_2 f} - 0.5772\right)10^{-6} =$	c.	11.2	11.2	9.5	9.5	8.9	"
$r = r_1 + r_s =$	a.	0.0567	0.97	0.0503	0.296	0.215	ohms
	b.	0.489	1.40	0.482	0.728	0.647	"
$r = r_1 + r_2 =$	c.	1.98	2.89	1.97	2.22	2.14	"
$x = x_1 + x_s =$	a.	7.5	8.42	2.97	3.22	2.64	"
	b.	10.4	11.32	5.87	6.12	5.54	"
$x = x_1 + x_2 =$	c.	11.2	12.12	9.50	9.75	9.07	"
$z = \sqrt{r^2 + x^2} =$	a.	7.5	8.46	2.97	3.23	2.65	"
	b.	10.4	11.4	5.89	6.16	5.58	"
	c.	11.4	12.45	9.7	10.0	9.3	"
$\cos\omega = \frac{r}{z} =$	a.	0.76	11.5	1.7	9.2	8.1	per cent
	b.	4.7	12.3	8.2	11.8	11.6	"
	c.	17.3	23.3	20.3	22.2	23.0	"
$e = 100\,z =$	a.	750	846	296	323	265	volts
	b.	1040	1140	589	616	558	"
	c.	1140	1245	970	1000	930	"
$lp = \frac{10^4}{\pi\sqrt{0.4\,\gamma\,\mu f}} =$		0.0064	0.00034	0.0064	0.00034	0.00034	cm.

TABLE III.

	Conductors 6 ft. apart:					Conductor 30 ft. above Ground:					Conductor without return conductor:				
Frequency, cycles:	60	10^4	10^5	10^6	10^7	60	10^4	10^5	10^6	10^7	60	10^4	10^5	10^6	10^7
Resistance, r_1 in ohms per meter:															
1. Copper wire . . .	0.00024	0.00087	0.0052	0.0567	4.827	0.00024	0.00091	0.0075	0.489	22.5	. . . .	0.0206	.200	1.98	19.7
2. Iron wire	0.0072	0.092	0.292	0.97	7.72	0.0072	0.092	0.297	1.40	25.4	. . . .	0.112	.489	2.89	20.0
3. Copper ribbon .	0.00024	0.00024	0.00122	0.0503	4.81	0.00024	0.00028	0.0055	0.482	22.5	. . . .	0.0199	.198	1.97	19.7
4. Iron ribbon	0.0019	0.0248	0.079	0.296	5.58	0.0019	0.0248	0.083	0.728	23.3	. . . .	0.0435	.265	2.22	20.5
5. Iron pipe	0.0013	0.0167	0.053	0.215	5.33	0.0013	0.0167	0.058	0.647	23.0	. . . .	0.0364	.250	2.14	20.2
Impedance, z, in ohms per meter.															
1. Copper wire . . .	0.00053	0.076	0.753	7.51	75	0.00069	0.105	1.043	10.41	81	. . . .	0.115	1.14	11.4	85
2. Iron wire	0.0105	0.191	1.083	8.46	78	0.0106	0.218	1.37	11.4	85	. . . .	0.233	1.49	12.45	88
3. Copper ribbon .	0.00030	0.030	0.298	2.97	30	0.00043	0.059	0.588	5.89	49	. . . .	0.097	0.97	9.7	72
4. Iron ribbon	0.0028	0.060	0.315	3.23	31	0.0030	0.087	0.67	6.16	51	. . . .	0.127	1.06	10.0	74
5. Iron pipe	0.00195	0.0446	0.305	2.65	26	0.0021	0.072	0.592	5.58	47	. . . .	0.112	0.97	9.3	70
Power Factor cos ω, in per cent:															
1. Copper wire . . .	45.3	1.14	0.69	0.76	6.4	34.8	0.87	0.72	4.7	27.8	. . . .	17.9	17.5	17.3	23.2
2. Iron wire	69.5	48.2	27.0	11.5	9.9	68.0	42.2	21.6	12.3	30.0	. . . .	48.0	32.7	23.3	22.7
3. Copper ribbon .	80.0	0.8	0.41	1.7	16.0	53.5	0.48	0.93	8.2	46.0	. . . .	20.5	20.4	20.3	27.3
4. Iron ribbon	68.0	41.3	25.1	9.2	18.0	63.3	28.6	12.4	11.8	45.8	. . . .	34.3	25.0	22.2	27.7
5. Iron pipe	66.7	37.5	17.4	8.1	20.4	62.0	23.2	9.8	11.6	49.0	. . . .	32.5	25.8	23.0	29.0
Voltage, e, in volts per meter															
1. Copper wire . . .	0.053	7.6	75	751	7500	0.069	10.5	104	1041	8100	. . . .	11.5	114	1140	8500
2. Iron wire	1.05	19.1	108	846	7800	1.06	21.8	137	1140	8500	. . . .	23.3	149	1245	8800
3. Copper ribbon .	0.03	3.0	30	297	3000	0.043	5.9	59	589	4900	. . . .	9.7	97	970	7200
4. Iron ribbon	0.28	6	36	323	3100	0.30	8.7	67	616	5100	. . . .	12.7	106	1000	7400
5. Iron pipe	0.195	4.5	30	265	2600	0.21	7.2	59	558	4700	. . . .	11.2	97	930	7000

terial is very great at machine frequencies, 60 cycles, but becomes small and almost negligible at extremely high frequencies. This is rather against the usual assumption.

The reason is that at machine frequencies, the unequal current distribution, or the screening effect of the internal magnetic field, is still practically absent in copper conductors, even in round wires of medium size, while it is practically complete in iron conductors, even in ribbon of $^1/_{20}$-inch thickness; while at very high frequencies the effect of radiation preponderates, which is independent of the material, and the radiation resistance even independent of the shape of the conductor.

Thus under transmission line conditions, first and second section of Table III, at 60 cycles the impedance, and hence the voltage drop in the iron conductor is from 7 to 20 times that of the copper conductor; at 10,000 cycles the voltage drop in the iron conductor is only from 1.5 to 2.5 times that in the copper conductor; the difference has decreased to from 14 per cent to 44 per cent at 100,000 cycles, 5 per cent to 12.5 per cent at one million cycles, while at 10 million cycles the voltage drop in the iron conductor is only 3 to 5 per cent higher, thus practically the same and the only difference is that due to the conductor shape.

The effective resistance, and thus the power consumption in the iron conductor at 60 cycles is from 8 to 30 times that of the copper conductor, but with increasing frequency the difference in the effective resistance increases to from 88 to 106 times at 10,000 cycles, reaches a maximum, and then decreases again, and is from 15 to 65 times that of the copper conductor at 100,000 cycles, only $1\frac{1}{2}$ to 17 times at a million cycles, while at 10 million cycles and above, all the differences in the effective resistance practically disappear.

As the result, the power factor of the conductor,—being the same, 100 per cent, at extremely low frequency and not much different, and fairly high at machine frequency—decreases with increasing frequency, reaches a minimum and then increases again to considerable values at extremely high frequency,—where the high radiation resistance comes into play. The difference between iron and copper, however, is that the minimum value of the power factor, at medium high frequencies, is very low in copper, a fraction of one per cent, while in the iron conductor the power factor always retains considerable values, the minimum being 10 or more times that of the copper conductor. Thus an oscillation in an iron conductor must die out at a much faster rate then in a copper conductor and the liability of the formation of a continual or cumulative oscillation may exist in copper conductors but hardly in iron conductors.

The effect of the shape of the conductor on the impedance or voltage drop is fairly uniform throughout the entire frequency range, the voltage drop being the smaller, the larger the circumference.

With regards to the effective resistance, however, the effect of the conductor shape is considerable at very low frequencies in iron conductors, but absent with copper conductors, due to the absence of the screening effect in copper at low frequency. With increasing frequency, the difference appears in the effective resistance of the copper conductor also, with the appearance of unequal current distribution, and the ratio of the resistance of the round conductor to that of the flat conductor approaches the same value in copper as in iron. With the approach of very high frequency, however, the difference decreases again, with the appearance of radiation effect, and finally vanishes.

Thus to convey currents of extremely high frequency, an iron conductor is almost as good as a copper conductor of the same shape and cross section. As iron is very much cheaper than copper, it follows that in high-frequency conduction an iron conductor under the conditions of Table III should be better than a copper conductor of the same cost and the same general shape, due to the larger size or rather circumference of the iron conductor.

There is, however, a material advantage at extremely high frequencies as well as at moderately high frequencies, in lower voltage drop at the same current resulting from such a shape conductor as gives maximum circumference, such as ribbon or hollow conductor. This advantage of ribbon or hollow tube, over the solid round conductor, exists also in the resistance and thus power consumption at medium high frequencies, but not at extremely high frequencies, but at the latter, in power consumption all conductors, regardless of size, shape or material, are practically equal.

With the thickness of ribbon conductor considered in Table III, of about $^1/_{20}$ inch, which is about the smallest mechanically permissible under usual conditions, the screening effect even in copper conductors is practically complete at 10,000 cycles, that is, the depth of penetration less than one half the thickness of the conductor. It follows, that in the design of high-frequency conductor, the thickness of the ribbon or hollow cylinder is essentially determined by mechanical and not by electrical considerations; in other words, the thinnest mechanically permissible conductor usually is thicker than necessary for carrying the current. As iron usually cannot be employed in as thin ribbon as copper, due to its rusting, an iron conductor would have a larger section than a copper conductor of the same voltage drop and power consumption. Thereby a part of the advantage gained by the employment of the cheaper material would be lost.

The last section of Table III gives the constants of the conductor without return conductor, such as would be represented by the discharge circuit of a lightning arrester, by a wireless telegraph antenna, etc., while the first two sections correspond to transmission line conditions; high-frequency currents between line conductors and between line and ground.

In the third section of Table III, the 60-cycle values are not given, and the values given for the lower high frequencies, 10^4 and even 10^5 cycles, usually have little meaning, are rarely realizable; they would correspond to a vertical conductor, as lightning arrester ground circuit, under conditions where no other conductor is within quarter wave distance. Even at 10^5 cycles, however, the quarter wave length is still 750 m. Thus there will practically always be other conductors within the field of the discharge conductor, acting as a partial return conductor, and the actual values of impedance and resistance, that is, of voltage drop and power consumption in the conductor, thus will be intermediate between those given in the third section, for a conductor without return conductor, and those given in the first sections, for conductors with return conductors. Except at extremely high frequencies, at which the wave length gets so short that the condition of a conductor without return conductor becomes realizable. It is interesting to note, therefore, that at extremely high frequencies, the constants of the conductor without return conductor approach those of the conductor with return conductor. At lower high frequencies, impedance and resistance, and thus voltage drop and power consumption, of the returnless conductor are much higher than those of the conductor with return, the more so the lower the frequency.

However, while the case of the conductor without return conductor is not realizable at low and medium high frequencies in industrial circuits, it probably is more or less realized by the lightning discharge between ground and cloud, and the constants given in the third section of Table III would probably approximately represent the conditions met in the conductors of lightning rods such as used for the protection of buildings against lightning.

It is interesting to note that with such a conductor without return conductor, the power factor is always fairly high, even with copper as conductor material. The impedance and thus the voltage drop does not differ much from those of the conductor with return conductor. The resistance, however, and thus the power consumption are much higher, sometimes, in copper conductors, more than a thousand times as large, showing the large amount of energy radiated by the conductor—which reappears more or less destructively as "induced lightning stroke" in objects in the neighborhood of the lightning stroke.

III.—Wave Decay in Transmission Lines

From the equations given in Appendix C, numerical values of the line constants are calculated and given in Table IV, for average transmission line conditions, that is, a copper wire, No. 00, with 6 ft. = 182 cm. between the conductors, and an average height of 30 ft. = 910 cm. above ground, for the two conditions:

a. a high-frequency oscillation between two line conductors;

b. a high-frequency oscillation between one line conductor and the ground.

The table gives:

The thermal resistance r_1, the radiation resistance r_3, and the total resistance $r = r_1 + r_3$;

The internal reactance x_1, the external reactance x_3 and the total reactance $x = x_1 + x_3$.

The magnetic attenuation $u_1 = \frac{r}{2L}$, the dielectric attenuation $u_2 = \frac{g}{2C}$, and the total attenuation $u = u_1 + u_2$.

The table also gives the duration of a transient in microseconds t, and in cycles N, that is, the time which a high-frequency oscillation of the frequency *f* would last, if continuing with its initial intensity, and the number of cycles, which it would perform. It also gives the power factor, in per cent, of the series circuit, as determined by resistance and inductance, and of the shunt circuit, as determined by shunted conductance and capacity.

As seen, the attenuation constant *u* is constant up to nearly one thousand cycles. Thus in this range, all the frequencies die out at the same rate. From about one thousand cycles up to about 100,000 cycles, the attenuation constant gradually increases, and thus oscillations die out the more rapidly, the higher the frequency, as seen by the gradual decrease of the duration *t*. However, as the increase of the attenuation constant and thus the increase of the rapidity of the decay of the disturbance, in this range, is smaller than the increase of frequency, the number of cycles performed by the oscillation increases. Thus, at 25 or 60 cycles, the stored energy,which supplies the oscillating power, would be expended in less than one cycle, that is, a real oscillation would hardly materialize (except by other sources of energy, as the stored magnetic energy of a transformer connected to the line). At 1000 cycles, the oscillation would last 9 to 12 cycles, and at still higher frequencies reach a maximum of 41.4 cycles at 20,000 cycles frequency, in the oscillation against ground; 64.9 cycles at 100,000 cycles, in the oscillation between line conductors. This represents a fairly well sustained oscillation, in which the cumulative effect of successive cycles may be considerable. Above 100,000 cycles the attenuation constant begins to rise rapidly, and reaches enormous values, due to the rapidly increasing energy dissipation by radiation. As the result, the duration of the oscillation very rapidly decreases, and the number of cycles performed by the oscillation decreases, until, beyond a million cycles, the energy dissipation is so rapid, that practically no oscillation can occur; the oscillation dying out in a cycle or less, thus being practically harmless.

The attenuation constant is plotted, up to 15,000 cycles, in Fig. 2, with the frequency as abscissas, and is plotted in Fig. 3 in logarithmic scale, as (3).

Noteworthy is the great difference between the oscillation against ground, and the oscillation between line conductors. The oscillation against ground is more persistent at low frequencies, due to the greater amount of stored energy in the electric field of the conductor, which reaches all the distance to the ground. When reaching into very high frequencies however, the energy dissipation by radiation becomes appreciable at lower frequencies in the oscillation against ground, than in the oscillation between line conductors, and reaches much higher values, with the result that the decay of an oscillation between line and ground is much more rapid at high frequencies than the decay of an oscillation between line conductors. For instance, at 100,000 cycles, the latter performs

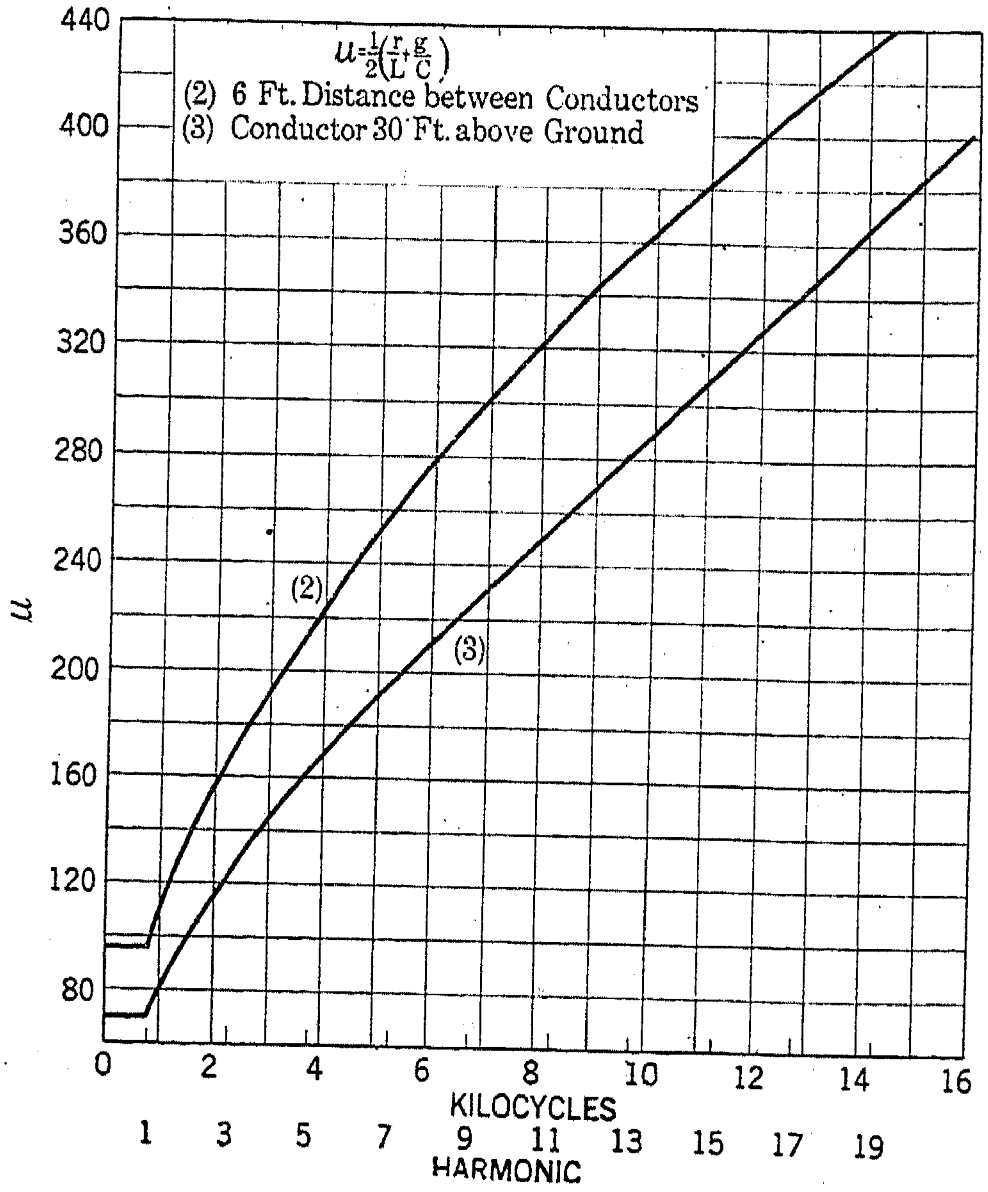


FIG. 2—ATTENUATION CONSTANT OF WIRE No. 00 B. & S. GAGE COPPER $u = 1/2\ (r/L + g/C)$

65 cycles before dying out, while the former has dissipated its energy in 27 cycles, that is, in less than half the time.

To further investigate this, in Tables V and VI the numerical values of effective resistance, power factor, attenuation constant and duration of a transient oscillation, in cycles, are given for six typical conductors and circuits, for frequencies from 10 cycles to five million cycles, and plotted in Fig. 3, 4, and 5, in logarithmic scale.

1, 2, and 3 are lines of high power: copper conductor No. 00 B. & S. gage, in 1 with 18 inches = 45.5 cm. between conductors, corresponding about to average distribution conductors, in 2 with 6 ft. = 182 cm., between conductors, corresponding to about average transmission line conductors with the oscillation between two lines, and in 3 with 60 ft. = 1830 cm. between conductor and return conductor, corresponding to an oscillation between line and ground, under average transmission line conditions, with the conductor 30 ft. above ground. 4, 5 and 6 give the same condition of an oscillation between line and ground, but in 4 an iron wire of the size of No. 00 B. & S.

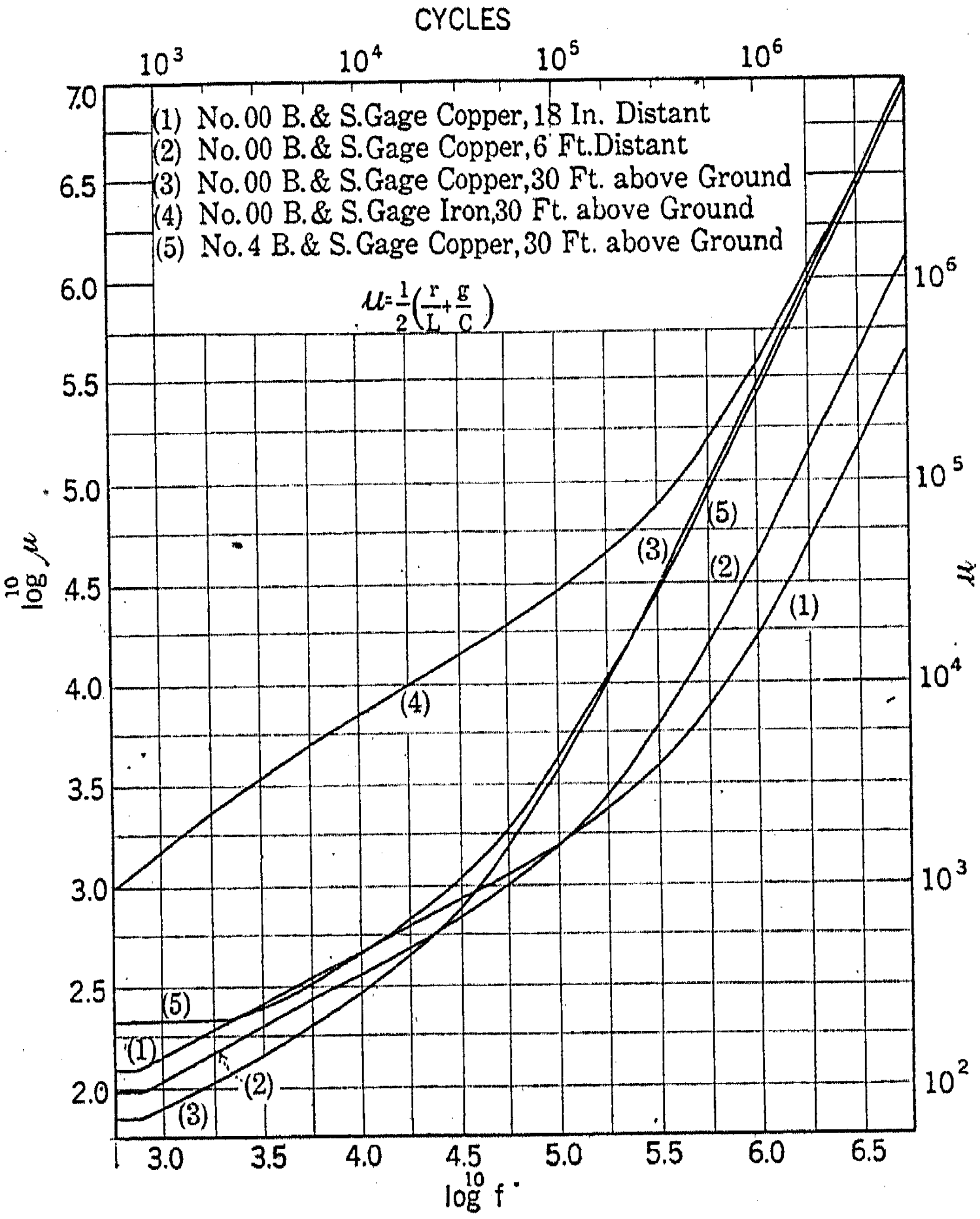


FIG. 3—ATTENUATION CONSTANT $u = 1/2\ (r/L + g/C)$ (TABLE VI)

gage, such as has been proposed for the station end of transmission lines, to oppose the approach of high-frequency disturbances. In 5 a copper wire No. 4 B. & S. gage, that is, a low power transmission line is represented, and in 6 a stranded aluminum conductor of the same conductivity as copper wire No. 00 B. & S. gage.

The equations of the constants for these six circuits are given in Table V. This table also gives the limiting frequencies, between which the various formulas apply with sufficient accuracy for practical purposes, and the lower limits, where the effects become appreciable, in the various conductors.

In Fig. 3 the attenuation constants are plotted, in Fig. 4 the power factors and in Fig. 5 the duration, in cycles.

As such a transient oscillation dies out exponentially, theoretically it has no definite duration, but lasts forever, though practically it may have ceased in a few micro-seconds. Thus as duration is defined the time, or the number of cycles, which the oscillation would last if maintaining its initial intensity. In reality, in this time the duration has decreased to $\frac{1}{\epsilon}$, or 37 per cent of its initial value. Physically, at 37 per cent of its initial value, or $0.37^2 = 0.135$ of its initial energy, it has become practically harmless, so that this measure of duration probably is the most representative.

From Tables V and VI it is seen that there is no marked difference between the stranded aluminum conductor (6), and the solid copper wire of the same conductivity (3), and the values of (6) are not plotted in the figures, but may be represented by (3).

The attenuation constant u, in Fig. 3, is plotted in logarithmic scale, with $\underline{/f}$ as abscissas. In such scale, a difference of one unit means ten times larger or smaller, and a straight line means proportionality to some power of the frequency. This figure well shows the three ranges; the initial horizontal range at low frequency, where the attenuation is constant; the approximately straight moderate slope of medium frequency, where the attenuation constant is proportional to the square root of the frequency, the unequal current distribution in the conductor predominating, and the steep slope at high frequencies, where the radiation resistance predominates, which is proportional to the square of the frequency.

It is interesting to note that at high frequencies the distance of the return conductor is the dominating factor, while the effect of conductor size and material vanishes; in copper wire No. 00 the rate of decay is practically the same as in copper wire No. 4, though the latter has more than three times the resistance, or in the iron wire, which has nearly six times the resistance and 200 times the permeability. The permeability of the iron wire has been assumed as $\mu = 200$, representing load conditions, where by the passage of the low-frequency power current the iron is magnetically near saturation, and its permeability thus lowered. However, the decay of the oscillation between conductor and ground is six to seven times more rapid than that between conductors 6 ft. apart, and that between conductors 18 inches apart about three times less.

This shows, that to produce quicker damping of high-frequency waves, such as are instrumental in steep wave fronts, the most effective way is to separate the conductors as far as possible, perhaps even lead them to the station by separate single-conductor lines; but the use of high-resistance conductors, or of magnetic material, as iron, offers little or practically no advantage in damping very high frequencies or flattening steep wave fronts.

At medium and low frequencies however, the relation reverses, and the decay of the wave is the smaller the greater the distance of the return conductor. The reason is, that in this range the effective resistance is still independent of the conductor distance, while the inductance increases with increasing distance. At medium and low frequencies, the iron conductor offers an enormously increased attenuation—from 10 to 20 times that of non-magnetic conductors.

The power factor of the conductor is plotted in Fig. 4. As seen, it decreases from unity, at very low frequencies, to a minimum, at medium high frequencies, and then increases again to very high values at very high frequencies. The minimum value is a fraction of one per cent except with the iron conductor, where the minimum is very much higher. The power factor is of importance as it indicates the percentage of the oscillating energy, which is dissipated per wave of oscillation. This is represented still better by Fig. 5 the duration of the oscillation in cycles, that is, the number of cycles which an oscillation lasts before dissipating the stored energy which causes it.

At medium high frequencies, the oscillation is the more persistent the lower the ohmic resistance of the conductor and the further away the return conductor, while at very high frequencies the reverse is the case, and the oscillation is the more persistent, the shorter the distance of the return conductor, while the size and material of the conductor ceases to have any effect.

The maximum number of cycles is reached at medium high frequencies, in the range between 20,000 and 100,000 cycles—

depending on conductor size and distance of return conductor. It thus is in this range of frequencies, where an oscillation caused by some disturbance lasts the greatest number of cycles, that the possibility, by some energy supply by means of an arc, etc., to form a stationary oscillation or even a cumulative oscillation, thus to become continuous or "undamped", is greatest.

It would thus appear, that this range of frequencies, of 20,000 to 100,000, represents what may be called the "danger frequencies" of transmission systems. It is interesting to note, that experimental investigations have shown that the natural frequency of oscillation of the high-voltage windings of large power transformers usually is within this range of danger frequencies. The possibility of the formation of destructive cumulative oscillations or stationary waves in the high-voltage windings of large power transformers is therefore greater than probably with any other class of circuits, so that such high-

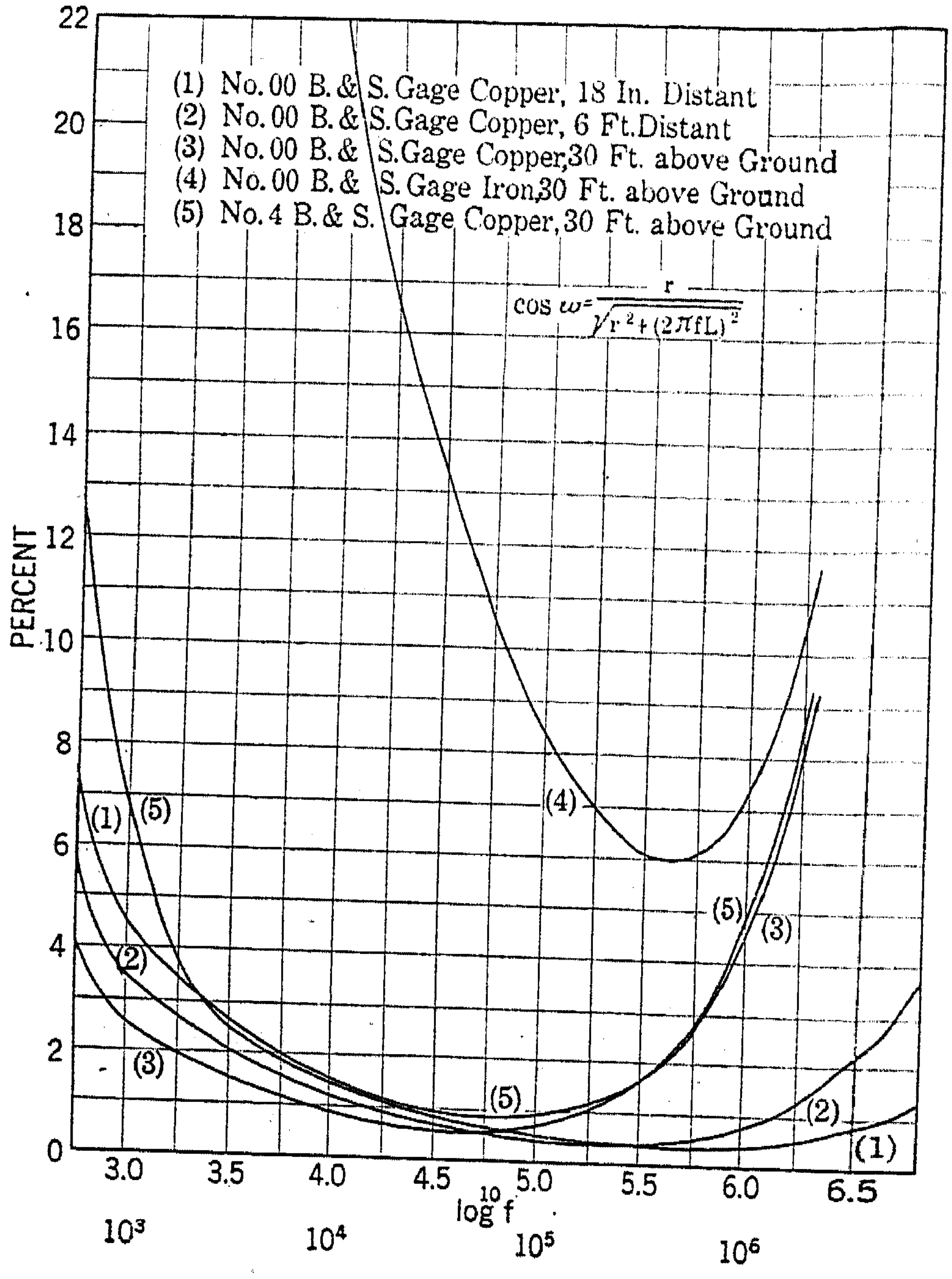


FIG. 4—POWER FACTOR OF SERIES CONSTANT $\cos \omega = \frac{r}{\sqrt{r^2 + (2\pi f L)^2}}$

(TABLE VI)

potential transformer windings require specially high disruptive strength and protection. This accounts for the not infrequent disastrous experience with such transformers, before this matter was realized.

Fig. 5 also shows, that the duration of an oscillation in iron wire, in cycles, is very low at all frequencies. Thus the formation of a stationary oscillation in an iron conductor is practically excluded, but such conductors would act as a dead resistance, damping any oscillation by rapid energy dissipation.

The duration of high-frequency oscillations, in cycles, increases with increasing frequency, to a maximum at medium frequencies. This obviously does not mean that the time during which the oscillation lasts increases; the time, in micro-seconds, naturally decreases with increasing frequency, due to the increasing attenuation constant. Thus in conductor (2) for instance, the oscillation lasts 65 cycles at a frequency of 100,000 cycles, but only 9 cycles at 1000-cycle frequency. However, the 65 cycles are traversed in 650 micro-seconds, while the 9 cycles last 9000 micro-seconds, or 14 times as long. It is not the total time of oscillation, but the cumulative effect due to the numerous and only slowly decreasing successive waves, which increases as represented in Fig. 5.

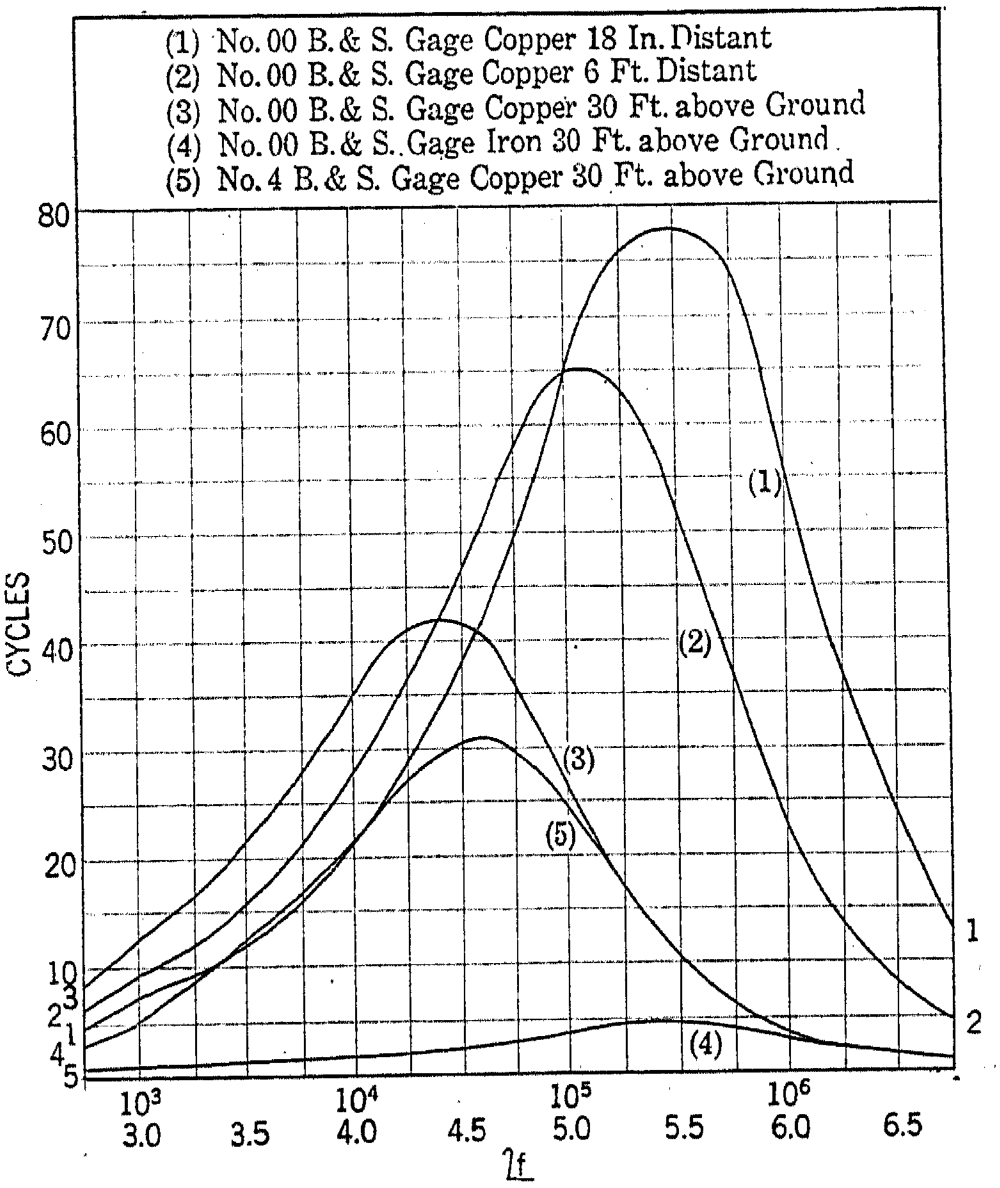


FIG. 5—DURATION OF OSCILLATION IN CYCLES (TABLE VI)

An oscillation between copper wires No. 4 B. & S. gage 6 ft. apart, would give a duration curve, which at moderate, frequencies follows (5) of Fig. 5, but at high frequencies follows (2). Thus the average duration and average rate of decay would be about the same as (3), an oscillation between copper wire No. 00 and ground. However, the oscillation would be more persistent in such a conductor at high, and less persistent at low frequencies. A complex wave, containing all the harmonics from low to very high ones, such as a steep wave front impulse or an approximately rectangular wave, as may be produced by a spark discharge, etc, would have about the same average rate of decay in a copper wire No. 4 with return at 6 ft., as in a copper wire No. 00 with ground return. The wave front would flatten, and the wave round off, approaching more and more a sine shape, due to the more rapid disappearance of the higher frequencies, while at the same time decreasing in amplitude. The wave thus would pass through many intermediate shapes. But these intermediate shapes would be materially different with wire No. 4 and return at 6 ft., as with No. 00 and ground return; in the latter, the flattening of the steep wave front, and rounding of the wave, would be much more rapid at the beginning, due to the shorter duration of the transient, and while such wave would last about the same time, that is, pass over the lines to about the same distance, it would carry steep wave fronts to much shorter distances, that is, its danger zone would be materially less than that of the wave in copper wire No.4 with return at 6 ft.

It therefore, is of great interest to further investigate the effect of the changing attenuation constant on complex waves, and more particularly those with steep wave fronts, as the rectangular waves of starting or disconnecting lines, etc.

IV.—Attenuation of Rectangular Waves

The destructiveness of high frequencies or steep wave fronts in industrial circuits is rarely due to over-voltage between the circuit conductors or between conductor and ground, but is due to the piling up of the voltage locally, in inductive parts of the circuit, such as end turns of transformers or generators, current transformers, potential regulators, etc., or inside of inductive windings as the high-potential coils of power transformers, by the formation of nodes and wave crests.

Such effect may be produced by high-frequency oscillations sustained over a number of cycles, as discussed in III, as oscillations lasting only a very few cycles or a fraction of a cycle, or due to non-oscillating transients, as single impulses, etc. As the high rate of change of voltage with the time, and the correspondingly high voltage gradients along the conductor are the source of danger, to calculate and compare oscillatory and non-oscillatory effects in this respect, it has become customary in the last years to speak of an "equivalent frequency" of impulses, wave fronts or other non-oscillatory transients.

As "effective" or "equivalent" frequency of an impulse, wave front etc., is understood the frequency of an oscillation, which has the same maximum amplitude, e or i, and the same maximum gradient $\frac{de}{dt}$ or $\frac{di}{dt}$. Thus assuming an impulse which reaches a maximum voltage $e = 60{,}000$, and has a maximum rate of increase of voltage of $\frac{de}{dt} = 10^{10}$, that is, a maximum voltage rise at the rate of 10^{10} volts per second, or 10,000 volts per micro-second. As the average voltage rise of a sine wave is $\frac{2}{\pi}$ times the maximum, the average rise of an oscillation of the same maximum gradient as the impulse would be

$$\frac{2}{\pi}\,\frac{de}{dt} = \frac{20{,}000}{\pi}$$

volts per microsecond. The total voltage rise of $e = 60{,}000$ thus would occur in

$$\frac{e}{\frac{2}{\pi}\,\frac{de}{dt}} = \frac{60{,}000\pi}{20{,}000} = 9.4 \text{ micro-seconds.}$$

A complete cycle of this oscillation thus would last $4 \times 9.4 = 37.6$ micro-seconds, and the equivalent frequency of the impulse would be

$$f = \frac{10^6}{37.6} = 26{,}600 \text{ cycles or } 26.6 \text{ kilo cycles.}$$

The equivalent frequency of a perfectly rectangular wave front if such could exist, obviously would be infinity.

A. QUARTER WAVE CHARGING OR DISCHARGING OSCILLATION OF LINE

Considering first the theoretically rectangular wave of connecting a transmission line to a circuit, or disconnecting it from the circuit.

Suppose a transmission line, open at the distant end, is connected to a voltage E. At this moment, the voltage of the line is zero. It should be, in permanent conditions, E. Thus the circuit voltage consists of a permanent voltage E (which is the instantaneous value of the alternating supply voltage at this moment $E_0 \sin \varphi$ and the transient voltage $-E$. We thus have a transient voltage, which uniformly $= -E$ all along the line, except at the switching point $l = 0$, where the transient voltage is zero.

Or, suppose a transmission line, open at the far end, is connected to a source of voltage, and at the moment where this voltage is E, the line short circuits at some point, by a spark discharge, flash-over, etc. Thus at this moment, the voltage $= 0$ at the point of short circuit, and is $= E$ everywhere between this point and the end of the line. Thus we get a line discharge leading to the same transient, a theoretically rectangular wave. In the part of the line between generator and short circuit, we have a different transient, a circuit of voltage $e = 0$ at one end, $e = E$ throughout the entire length at time $t = 0$, and $e = E$ continuously at the other end, where the generator maintains the voltage. However, this again leads to the same transient, of a theoretically rectangular wave.

Assuming thus, as an instance, a transmission line of 100-km. length, of copper wire No. 00 B. & S. gage, 30 ft. above ground, open circuited at the other end $l = 100$ km., and connected to a source of voltage E at the beginning, $l = 0$.

Then the beginning of the line, $l = 0$, is grounded, at the time $t = 0$, thus giving a quarter wave oscillation, with the terminal conditions:

Voltage along the line constant $= E$, at time $t = 0$, except at the beginning of the line, $l = 0$, where the voltage is 0.

Current along the line $= 0$ at $t = 0$, except at $l = 0$, where the current is indefinite.

The equation of the quarter wave oscillation of the line conductor against ground, as usually given[1], then is:

---

[1] See for instance: "Theory and Calculation of Transient Electric Phenomena," Section IV, Chapter VII, equation (57).

$$e = \frac{4\,E}{\pi}\,\epsilon^{-ut}\sum_{0}^{\infty}{}_{n}\,\frac{\sin\,(2\,n+1)\,\tau\,\cos\,(2\,n+1)\,\theta}{n+1} \qquad (\mathbf{1})$$

where:
θ is the time angle of the fundamental wave of oscillation, of frequency:

$$f_0 = \frac{S}{4\,l_0} = \frac{3\times 10^{10}}{4\times 100\times 10^{5}} = 750 \text{ cycles.} \qquad (\mathbf{2})$$

τ is the distance angle, for $l_0 = 100$ km. $= 90° = \pi/2$, that is:

$$\left.\begin{aligned} \theta &= 2\,\pi\,f_0\,t \\ \tau &= \frac{\pi\,l}{2\,l_0} \end{aligned}\right\} \qquad (\mathbf{3})$$

Equation (**1**) however assumes that u, and thus r, L, C and g are constant for all frequencies. As this is not the case, but u is a function of the frequency, and thus of n: ϵ^{-ut} cannot be taken out of the summation sign. Equation (**1**) thus must be written:

$$e = \frac{4\,E}{\pi}\sum_{0}^{\infty}{}_{n}\,\epsilon^{u_n\,t}\,\frac{\sin\,(2\,n+1)\,\tau\,\cos\,(2\,n+1)\,\theta}{2\,n+1} \qquad (\mathbf{4})$$

where u_n is the value of u for the frequency: $f = (2\,n+1)\,f_0$.

From (**4**) follows, as the voltage gradient along the line:

$$\frac{d\,e}{d\,\tau} = \frac{4\,E}{\pi}\sum_{0}^{\infty}{}_{n}\,\epsilon^{-u_n\,t}\cos\,(2\,n+1)\,\tau\,\cos\,(2\,n+1)\,\theta \qquad (\mathbf{5})$$

$$= \frac{2\,E}{\pi}\left\{\sum_{0}^{\infty}{}_{n}\,\epsilon^{-u_n\,t}\cos\,(2\,n+1)\,(\tau+\theta) + \sum_{0}^{\infty}{}_{n}\,\epsilon^{-u_n\,t}\cos\,(2\,n-1)\,(\tau-\theta)\right\} \qquad (\mathbf{6})$$

The maximum voltage gradient occurs at the wave front, that is, for $\theta = \tau$. Substituting this, and substituting further, from (**3**):

$$d\,\tau = \frac{\pi}{2\,l_0}\,d\,l \qquad (\mathbf{7})$$

gives, as the maximum voltage gradient:

$$G = \frac{d\,e}{d\,l} = \frac{E}{l_0} \left\{ \sum_{0}^{\infty}{}_{n}\, \epsilon^{-u_n\,t} \cos{(2\,n + 1)\,2\,\tau} + \sum_{0}^{\infty}{}_{n}\, \epsilon^{-\mu_n t} \right\} \tag{8}$$

It is however:

$$\sum_{0}^{\infty}{}_{n}\, \cos{(2\,n + 1)\,2\,\tau} = 0 \tag{9}$$

for all values of τ except $\tau = 0$ and $\tau = \pi$, that is, the beginning of the line, $l = 0$.

Thus, approximately, as u_n varies gradually:

$$\sum_{0}^{\infty}{}_{n}\, \epsilon^{-u_n\,t} \cos{(2\,\pi + 1)\,2\,\tau} = 0 \tag{10}$$

except for values of $\tau = 0$ or very near thereto.

Substituting (**10**) into (**8**) gives:

$$G = \frac{E}{l_0} \sum_{0}^{\infty}{}_{n}\, \epsilon^{-\mu_n\,t} \tag{11}$$

as the approximate expression of the maximum voltage gradient, that is, the steepness of the wave front, at time t, that is, at distance from the origin of the wave:

$$l = S\,t = 3 \times 10^{10}\,t \tag{12}$$

If θ differs materially from τ, the term with $(\tau - \theta)$ in equation (**6**) also vanishes, and $\frac{d\,e}{d\,\tau} = 0$, that is, there is no voltage gradient except at and near the wave front.

From (**11**) are now calculated numerical values of the steepness of the wavefront G, for various times t after its origin, and thus by (**12**), various distances l from the origin. These numerical values are given in Table VII, for $E = 60{,}000$ volts.

At a fundamental frequency of $f_0 = 750$ cycles, successive harmonics differ from each other by 1500 cycles, and for every value of t, values of $\epsilon^{-u_n\,t}$ thus have to be calculated for the frequencies:

$n =$	0	1	2	3	4	5	6	etc.
$f =$	750	2250	3750	5250	6750	8250	9750	cycles etc.,

until the further terms add no further appreciable amount to $\Sigma\, \epsilon^{-u_n t}$. In calculating, it is found that for instance for $t = 5$ microseconds, or $l = 1.5$ km., this occurs at $f = 5 \times 10^6$ cycle, thus beyond the $\frac{5 \times 10^6}{750} = 6670$th harmonic. Thus 6670 terms of the series would have to be calculated to get this one point of the wave gradient: more terms for shorter, less terms for longer distances l of wave travel. This obviously is impossible, and some simpler approximation, of sufficient accuracy, thus is required.

TABLE VII

ATTENUATION OF WAVE FRONT OF QUARTERWAVE OSCILLATION, OF 100 KM. LINE, 60,000 VOLTS.

Time t, micro-seconds.	Distance l from origin of wave.		f, where $\Sigma\, \epsilon^{-ut}$ vanishes	Voltage $\frac{1}{E}\frac{de}{dt}$	Gradient G, volts per meter.	Wave Front:	
	km:	miles:				Equivalent kilocycles.	Length, meters.
(3) Copper wire No. 00 B. & S. gage 30 ft. = 910 cm. above ground.							
0	0	0	....	∞	60,000	∞	0
.03	.01	.0062	....	5,800	5,500	8,800	6
.575	.1725	.107	10×10^6	1,370	1,290	2,060	73
1.15	.345	.215	8×10^6	1,025	967	1,540	98
2.3	.69	.43	6×10^6	670	630	1,000	150
11.5	3.45	2.15	4×10^6	300	280	450	330
23	6.9	4.3	2×10^6	218	205	330	460
92	27.6	17.2	10^6	104	98	156	960
230	69	43	$.7 \times 10^6$	62	58	92	1,630
2,300	690	430	$.2 \times 10^6$	13	12	19	7,900
23,000	6,900	4,300	50,000	1.3	1.2	1.9	79,000
(1) Copper wire No. 00 B. & S. gage, return conductor at 18 ins. = 45.5 cm distance.							
2.3	.69	.43	20×10^6	3,450	3,240	5,160	29
23	6.9	4.3	10×10^6	1,020	960	1,530	98
230	69	43	4×10^6	250	234	370	400
(5) Copper wire No. 4 B. & S gage, 30 ft. = 910 cm. above ground.							
23	6.9	4.3	2×10^6	227	213	340	440
230	69	43	$.7 \times 10^6$	64	60	94	1,570
(4) Iron wire No. 00 B. & S. gage, 30 ft. = 910 cm. above ground.							
23	6.9	4.3	2×10^6	143	135	215	700
230	69	43	$.3 \times 10^6$	11	10	16	9,400

This may be done as follows:

In the range from 5×10^5 to 10^6 cycles for instance, there are $\frac{10^6 - 5 \times 10^5}{1500} = 333$ harmonics. Instead of calculating u and ϵ^{-ut} for each of these harmonics, calculate ϵ^{-ut} for the average value of these 333 harmonics, and multiply by 333.

Thus, dividing the entire frequency range (beyond the lowest harmonics, which are calculated separately), into groups, and calculating one average value for each group, the calculation of $\Sigma\, \epsilon^{-ut}$ becomes feasible.

Since $\epsilon^{-u\,t}$ is calculated through $\underline{/\epsilon}^{-ut} = -\,u\,t\,\underline{/\epsilon},$ as values of t, multiples of $\frac{1}{\underline{/\epsilon}}$ have been chosen, to still further simplify the calculation, in deriving a curve of gradients G.*

Table VII gives the values of the maximum voltage gradient of the wave front, in volts per meter at 60,000 volts maximum initial line voltage, the equivalent frequency of the wave front, in kilo-cycles, and the length of the wave front, in meters, for various times of wave travel, from 0.03 micro-seconds up, and corresponding distances of wave travel, from 10 meters from the origin as rectangular wave, up to thousands of kilometers, for copper wire No. 00 B. & S. gage, 30 ft. = 910 cm. above ground, with the ground as return.

TABLE VIII.

ATTENUATION OF WAVE FRONT OF QUARTER-WAVE OSCILLATION, OF 100-KM. LINE, 60,000 VOLTS.

Conductor: Size No.: Material:	(3) 00 Copper	(1) 00 Copper	(5) 4 Copper	(4) 00 Iron
Distance of return conductor, cm.: . . .	1820	45.5	1820	1820
After 23 microseconds, 6.9 km.:				
Gradient, volts per meter:	205	960	213	135
Length of wave front, meters:	460	98	440	700
Equivalent kilocycles:	330	1530	340	215
After 230 microseconds, 69 km.:				
Gradient, volts per meter:	58	234	60	10
Length of wave front, meters:	1630	400	1570	9400
Equivalent kilocycles:	92	370	94	16

For comparison are given some data of the same conductor, with the return conductor at 18 inches = 45.5 cm., and also for a copper wire No. 4, and an iron wire No. 00, with the ground as return.

These data are plotted in Fig. 6, showing the wave front, as it gradually flattens out in its travel over the line, from the very steep wave at 172 meters from the origin, to the wave with a front of 1630 meters, 69 km. away.

A comparison of the data of the four circuit conditions is given in Table VIII, and plotted in Fig. 7.

It is interesting to note, that there is practically no difference in the flattening of the wave front on a low resistance conductor, No. 00, and a high resistance conductor, No. 4. There is, however, an enormous difference due to the effect of the close-

---

*Here and in the following, the symbol $\underline{/}$ has been chosen for the common logarithm.

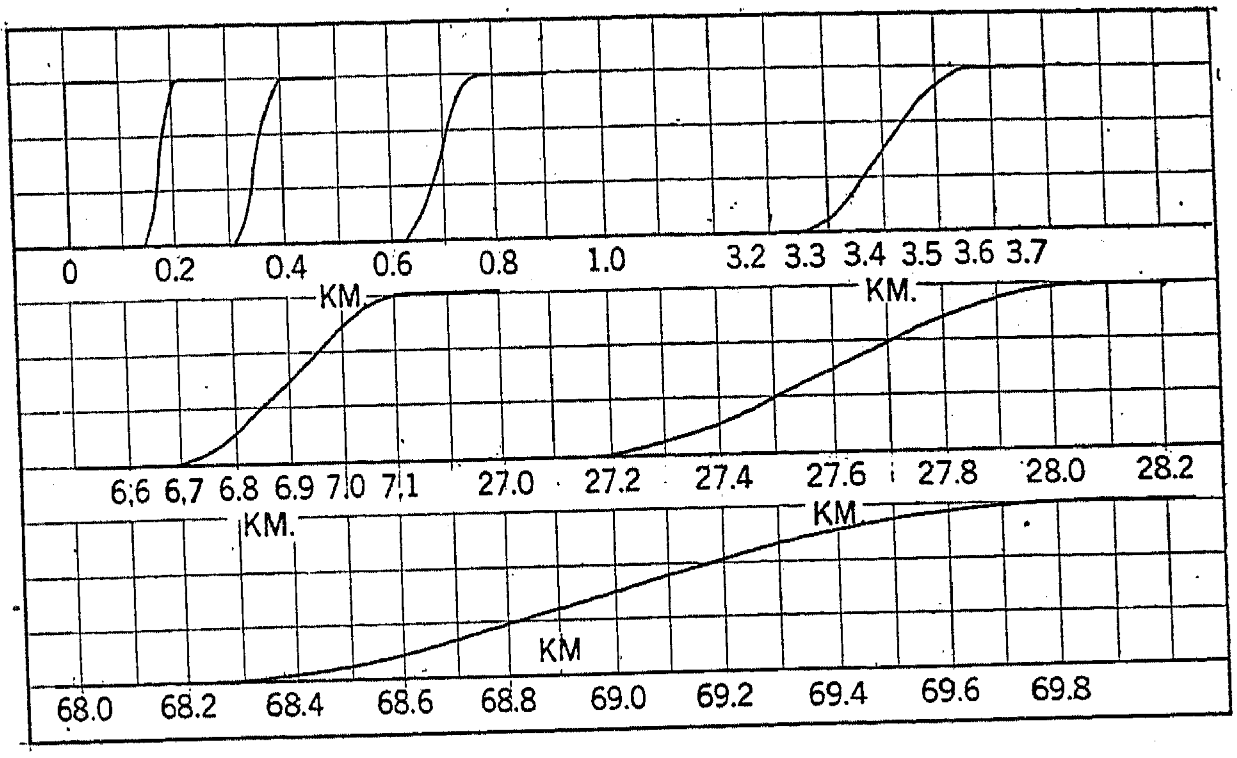


Fig. 6—Flattening of Steep Wave Front in Transmission Line No. 00 B. &. S. Gage Copper 30 ft. Above Ground

ness of the return conductor; with the return conductor at 18 inches distance, the wave front is still materially steeper at 6.9 km. distance, than it is in the conductor with ground return at

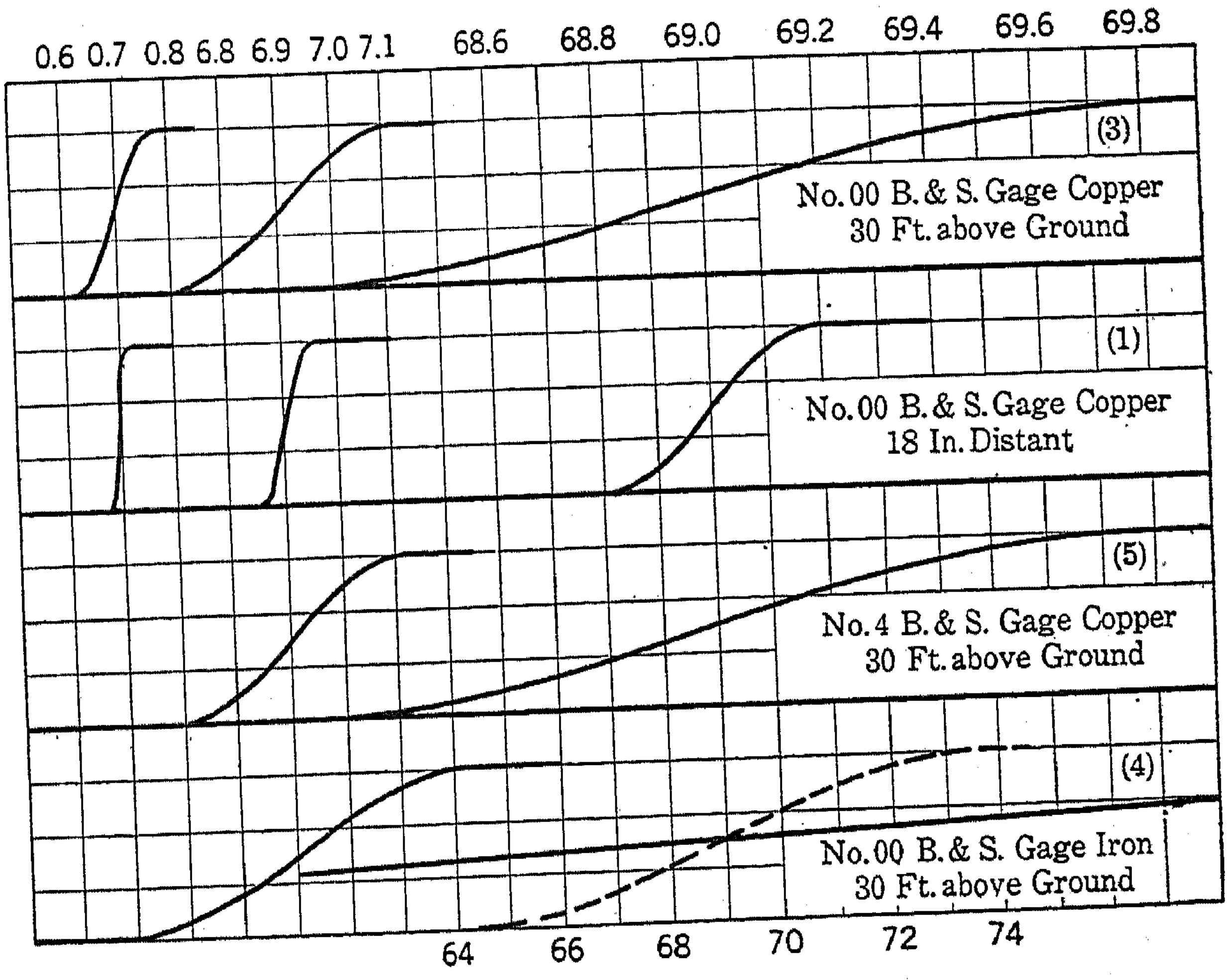


Fig. 7

0.69 km. distance. That is, the flattening of the wave front in the conductor with ground return, is more than ten times as rapid, than in the same conductor with the return conductor closely adjacent. Or in other words, the danger zone of steep wave front, extends in a conductor with the return conductor closely adjacent, to more than ten times the distance than in the conductor with ground return.

This means, where it is desired to transmit a high-frequency impulse or steep wave front to the greatest possible distance, it is essential to arrange conductor and return conductor as closely adjacent as possible. But where it is essential to limit the harmful effect of very high frequency or steep wave front as much as possible to the immediate neighborhood of its origin, the return conductor should be separated as far as possible.

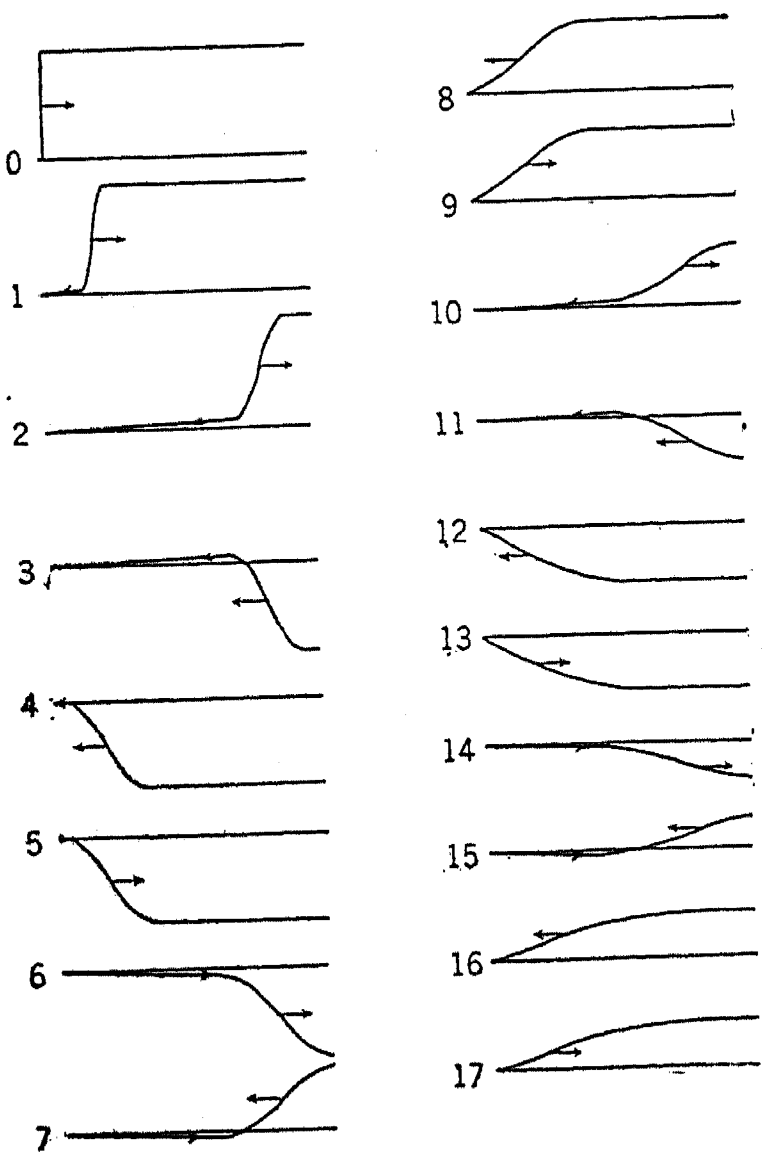


Fig. 8

The data on iron wire are very disappointing; there is an enormous increase in the flattening of the wave front at great distances, by the use of iron as conductor material, so much so that the wave front of the iron conductor at 69 km. distance had to be shown (dotted) at one-tenth the scale of the other conductors, in Fig. 7. But at moderate distances, 6.9 km. from the origin, the flattening of the wave front in the iron conductor is only little greater than in a copper conductor of the same size: 215 kilo-cycles against 330 kilo-cycles. At short distances, the difference almost entirely ceases, and within one km. from the origin, the wave front in an iron conductor is nearly as steep as in a copper conductor of the same size. Thus a short length of iron wire between station and line would exert practically no protection against very high-frequency oscillations or steep wave fronts, such as may be produced by lightning strokes in the neighborhood of the station.

This was to be expected from the shape of the curve of the attenuation constant, shown in Fig. 3.

From the data in Table VII then are constructed and shown in Fig. 8, the successive curves of voltage distribution in the line, as it is discharging (or charging), by the originally rectangular wave running over the line, reflecting at the end of the line and running back, then reflecting again at the beginning of the line and once more traversing it, etc., until gradually the transient energy is dissipated and the line voltage reaches its average, zero in discharge, or the supply voltage in charge. The direction of the wave travel in the successive positions is shown by the arrows in the center of the wave front; the existence and direction of current flow in the line by the arrows in the (nearly) horizontal part of the diagram. As seen, after four to five reflections, the voltage distribution in the line is practically sinusoidal.

However, in these diagrams Figs. 6 to 8, the wave front has been constructed from the maximum voltage gradient derived by the calculation, assuming as approximation the shape of the wave front as a sinusoid. This usually is a sufficient approximation, since the important feature is the maximum gradient, that is, the steepest part of the wave front, which was given by the calculation; but it is not strictly correct, and the wave front differs from sine shape. It thus is of interest to investigate the exact shape of the wave front, in its successive stages of flattening.

RECTANGULAR TRAVELING WAVE

For this purpose we may investigate, as a further example, the gradual destruction of wave shape and decay of a 60,000-cycle rectangular traveling wave, during its passage over a transmission line, changing from the original rectangular wave shape produced at its origin by lightning stroke, spark discharge, etc., into practically a sine wave.

For this purpose, for the elementary symmetrical traveling wave, the equation is:

$$e = \frac{4\,E}{\pi} \sum_{0}^{\infty}{}_{n}\ \epsilon^{-u_n t} \sin\,(2\,n + 1)\,\varphi \qquad (13)$$

where $\varphi = \theta - \tau$ is the running time coordinate, in angular expression.

Values of e are calculated, for various times t, from one micro-second to 360 micro-seconds, for all the angles φ, where e has not yet become constant and equal to E.

In Table IX are given numerical values, plotted in Fig. 9, of

$$e \frac{\pi}{4E} = \sum_{0}^{\infty}{}_{n} \epsilon^{-u_n t} \sin (2n + 1) \varphi,$$

from 0 to 90 deg., and from one to 360 micro-seconds, and derived therefrom, the values of the length of wave front, in degrees and in meters, and the equivalent frequency, in kilo-

TABLE IX.

ATTENUATION OF 60,000-CYCLE RECTANGULAR WAVE, IN LINE OF NO. 00 B. & S. GAGE COPPER, 30 FT. ABOVE GROUND.

Time: t =	0	1	2	5	10	20	40	100	360	ms.
Wave travel; l =	0	.3	.6	1.5	3	6	12	30	108	km.
φ = 0 degrees	0.785	0	0	0	0	0	0	0	0	0
1 "	....	0.250	0.170	0.106	0.074	0.052	0.347	0.022	0.010	
5 "	....	0.785	0.645	0.475						
10 "	....	....	0.785	....	0.606	0.468	0.582	0.217	0.100	
20 "	....	....	....	0.783	0.770	0.707	0.709	0.405	0.193	
30 "	....	....	....	....	0.780	0.762	0.740	0.547	0.282	
40 "	....	....	....	....	....	0.775	0.758	0.639	0.360	
50 "	....	....	....	....	....	....	0.765	0.691	0.426	
60 "	....	....	....	....	....	....	....	0.722	0.476	
70 "	....	....	....	....	....	....	....	0.730	0.513	
80 "	....	....	....	....	....	....	....	....	0.535	
90 "	0.785	0.785	0.785	0.783	0.780	0.775	0.765	0.730	0.540	
Wave Front degrees:	0	10	16	26	44	70	90	140	180	
meters:	0	140	220	340	620	980	1250	1940	2500	
kilo-cycles: max:	∞	1110	750	470	325	230	165	103	63	
avg:	∞	1080	670	420	268	155	120	77	60	
Highest apprec. harmonic	∞	61	45	33	15	9	7	3	1	
Decrease of wave max:	0	....	....	....	0.6	1.3	2.6	7.0	31.2	%
Maximum gradient volts per meter:	∞	690	470	295	205	140	100	65	40	

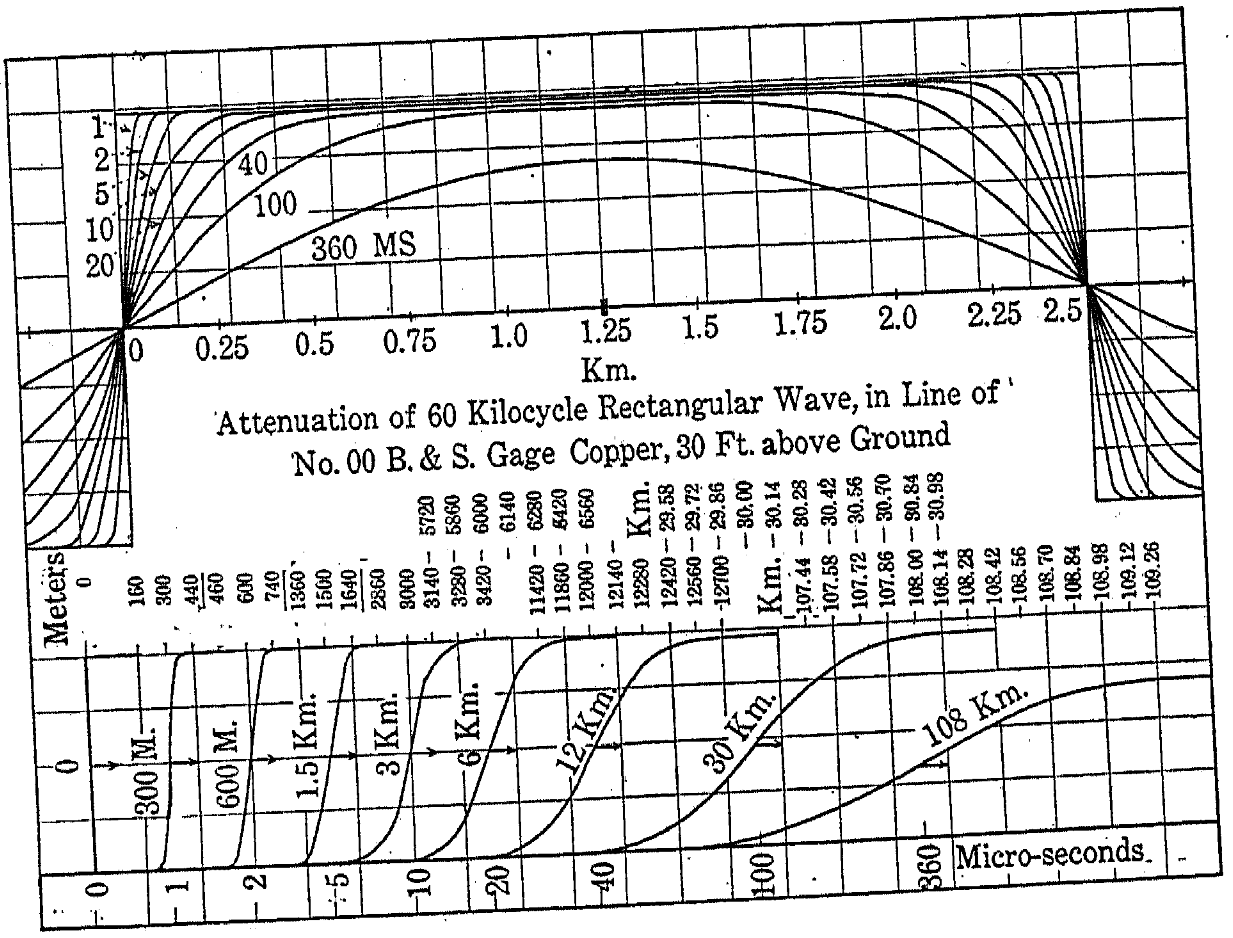
cycles. Two values are given, the maximum kilo-cycles, derived from the steepest part of the wave front, and the average kilo-cycles, derived from the total wave front. The difference indicates the deviation of the shape of the wave front from a sinusoid.

As seen, due to the high fundamental frequency, 60,000 cycles, the number of significant harmonics is very greatly reduced, until frequencies are reached where the attenuation is so enormous, that the destruction of the wave by its energy dissipation occurs in a few meters.

Thus after one micro-second, or 300 meters wave travel, the calculation needs to extend only to the 61st harmonic; after two micro-seconds, to the 45th harmonic, etc., while after 100 micro-seconds, or 30 km. travel, only the third harmonic is still appreciable, and after 360 micro-seconds, or 108 km. travel, even this has disappeared, and the wave is essentially a sine wave.

In 30 km. travel, the wave maximum has decreased 7 per cent; in 108 km. it has decreased 31.2 per cent.

It is important to note however, that waves of relatively high frequency, within the range of the danger frequencies between 20 and 100 kilo-cycles, can travel considerable distances, if no other causes of rapid attenuation are at work but those considered here, and still retain a large part of their energy. Thus the 60,000-cycle wave, after traversing 100 km. of line, still retains about 70 per cent of its amplitude, that is, about 1/2 of its energy. Thus the danger from resonance of power transformer windings with such frequencies is not local but rather extends over a large part of the system.



Attenuation of 60 Kilocycle Rectangular Wave, in Line of No. 00 B. & S. Gage Copper, 30 Ft. above Ground

FIG. 9

The lower part of Fig. 9 shows the shape of the wave front at various distances from the origin; the upper part shows the gradual change of the wave from rectangular to flat top to sine wave.

Plotting for the 750-cycle quarter wave oscillation and the 60,000-cycle traveling wave, the logarithm of the length of the wave front, and of the equivalent frequency of the wave front, against the logarithm of the distance of wave travel, gives practically straight lines, (except for very great distances of wave travel, where the lower harmonics predominate), and from the slope of these lines it follows that: The length of wave front is approximately proportional and the equivalent frequency of the wave front approximately inversely proportional to the square root of the distance of wave travel:

$$l_f = c_1 \sqrt{l} \tag{14}$$

$$f_f = \frac{c_2}{\sqrt{l}} \tag{15}$$

This would give a wave front constant c_1 and an equivalent frequency constant c_2 of the circuit.

V.—Flattening of Steep Wave Fronts

A rectangular wave is represented by the equation

$$e = \frac{4\,E}{\pi} \sum_{0}^{\infty}{}^{n}\, \epsilon^{-ut}\, \frac{\sin\,(2\,n+1)\,\tau\,\cos\,(2\,n+1)\,\theta}{2\,n+1} \tag{1}$$

where

$\theta = 2\,\pi\,f_0\,t =$ time angle, (2)

$\tau = 2\,\pi\,\frac{l}{l_0} =$ distance angle, (3)

$f_0 =$ fundamental frequency,

$l_0 = \frac{S}{f_0} =$ wave length, (4)

$S = 3 \times 10^{10} =$ velocity of propagation,
$E =$ maximum voltage of wave.

The voltage gradient is

$$\frac{d\,e}{d\,l} = \frac{d\,e}{d\,\tau}\,\frac{d\,\tau}{d\,l}$$

$$= \frac{4\,E}{\pi}\,\frac{d\,\tau}{d\,l} \sum_{0}^{\infty}{}^{n}\, \epsilon^{-ut} \cos\,(2\,n+1)\,\tau\,\cos\,(2\,n+1)\,\theta \tag{5}$$

Substituting for $\frac{d\,\tau}{d\,l}$ from (**3**), and resolving the cos-product, gives:

$$\frac{d\,e}{d\,l} = \frac{4\,E}{l_0}\left\{\sum_0^\infty{}_n\ \epsilon^{-ut}\cos\,(2\,n+1)\,(\tau-\theta)\right.$$

$$\left.+\sum_0^\infty{}_n\ \epsilon^{-ut}\cos\,(2\,n+1)\,(\tau+\theta)\right\} \qquad (\mathbf{6})$$

$\sum_0^\infty{}_n\ \epsilon^{-ut}\cos\,(2\,n+1)\,(\tau+\theta)$ approaches zero for $\tau+\theta \neq 0$.

(**7**)

thus:

$$\frac{d\,e}{d\,l} = \frac{4\,E}{l_0}\sum_0^\infty{}_n\ \epsilon^{-ut}\cos\,(2\,n+1)\,(\tau-\theta) \qquad (\mathbf{8})$$

The maximum voltage gradient occurs for $\theta = \tau$, and thus is:

$$G = \frac{d\,e}{d\,l} = \frac{4\,E}{l_0}\sum_0^\infty{}_n\ \epsilon^{-ut} \qquad (\mathbf{9})$$

This can be written in the form:

$$G = \frac{2\,E}{f_0\,l_0}\sum_0^\infty{}_n\ 2\,f_0\,\epsilon^{-ut} \qquad (\mathbf{10})$$

It is however, by (**4**):

$$f_0\,l_0 = S \qquad (\mathbf{11})$$

thus:

$$G = \frac{2\,E}{S}\sum_0^\infty{}_n\ 2\,f_0\,\epsilon^{-ut} \qquad (\mathbf{12})$$

The values of ϵ^{-ut} are taken for all values of frequency differing from each other by $2\,f_0$, and, at and near the wave front, the value of $\sum_0^\infty{}_n\ 2\,f_0\,\epsilon^{-ut}$ thus approaches the value of

$$\int_0^\infty \epsilon^{-ut}\,d\,f$$

Substituting this into (**10**), gives as the equation of the maximum steepness of wave front:

$$G = \frac{d\,e}{d\,l} = \frac{2\,E}{S} \int_0^\infty \epsilon^{-ut}\,d\,f \qquad (\mathbf{12})$$

As seen, in this expression (**12**), wave length and frequency have disappeared. Equation (**12**) thus applies to any wave, whether of finite length or not. That is, it represents broadly (though only approximately) the maximum gradient of any steep wave front or impulse, at the time t after its origin as rectangular wave front or impulse.

TABLE X.

ATTENUATION OF WAVE FRONT OF RECTANGULAR IMPULSE.

Copper Wire No. 00 B. & S. gage, 30 ft. above Ground, Ground Return.

f:	u:	$f\,\epsilon^{-ut}$; $t =$				
		10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
1	70	1	1			
2	"	2	2			
5	"	5	5			
10	"	9	10	88		
20	"	19	20			
50	"	47	50			
10^2	"	93	99	100		
2×10^2	"	186	198	200	8,888	
5×10^2	"	468	496	500		
10^3	80	923	992	1,000		
2×10^3	115	1,760	1,970	2,000		
5×10^3	190	4,150	4,920	5,000		
10^4	287	7,500	9,720	9,999	10,000	88,888
2×10^4	482	10,200	19,000	19,900	20,000	
5×10^4	1,300	10,800	44,000	49,400	50,000	
10^5	3,710	2,450	69,000	96,400	99,500	100,000
2×10^5	12,690	1	56,100	176,000	197,000	200,000
5×10^5	74,200	0	240	190,000	466,000	498,000
10^6	291,500	....	σ	54,300	748,000	970,000
2×10^6	1,160,000	......	......	18	625,000	1,780,000
5×10^6	7,230,000	......	......	σ	3,660	1,860,000
10^7	28,800,000	......	......	......	1	562,000
2×10^7	115,000,000	......	......	......	σ	200
	$\Sigma =$	38,660	206,800	605,000	2,230,000	6,060,000
	$\div 3 \times .4343 =$	29,700	158,700	464,000	1,710,000	4,650,000
	$G = \frac{2\,E}{S} \times ; =$	.119	.635	1.86	6.84	18.6
	[l =	300	30	3	.3	.03 km.]

As u is not a simple function of f, the integration of $\int_0^\infty \epsilon^{-ut}$

df in its general form meets with difficulties, and the integral may be evaluated as follows:

Equation (**12**) can be written in the form

$$G = \frac{2E}{S} \int_{-\infty}^{+\infty} f \epsilon^{-ut} \, d \log f \qquad (\mathbf{13})$$

Plotting then $f \epsilon^{-ut}$ as ordinates, with $\log f$ as abscissas, gives a curve, and the area of this curve gives the integral.

Instead of using $\log f$ as abscissas, it is more convenient to use the common logarithm, $\underline{/f}$, and divide the measured area by $\underline{/\epsilon} = 0.4343$.

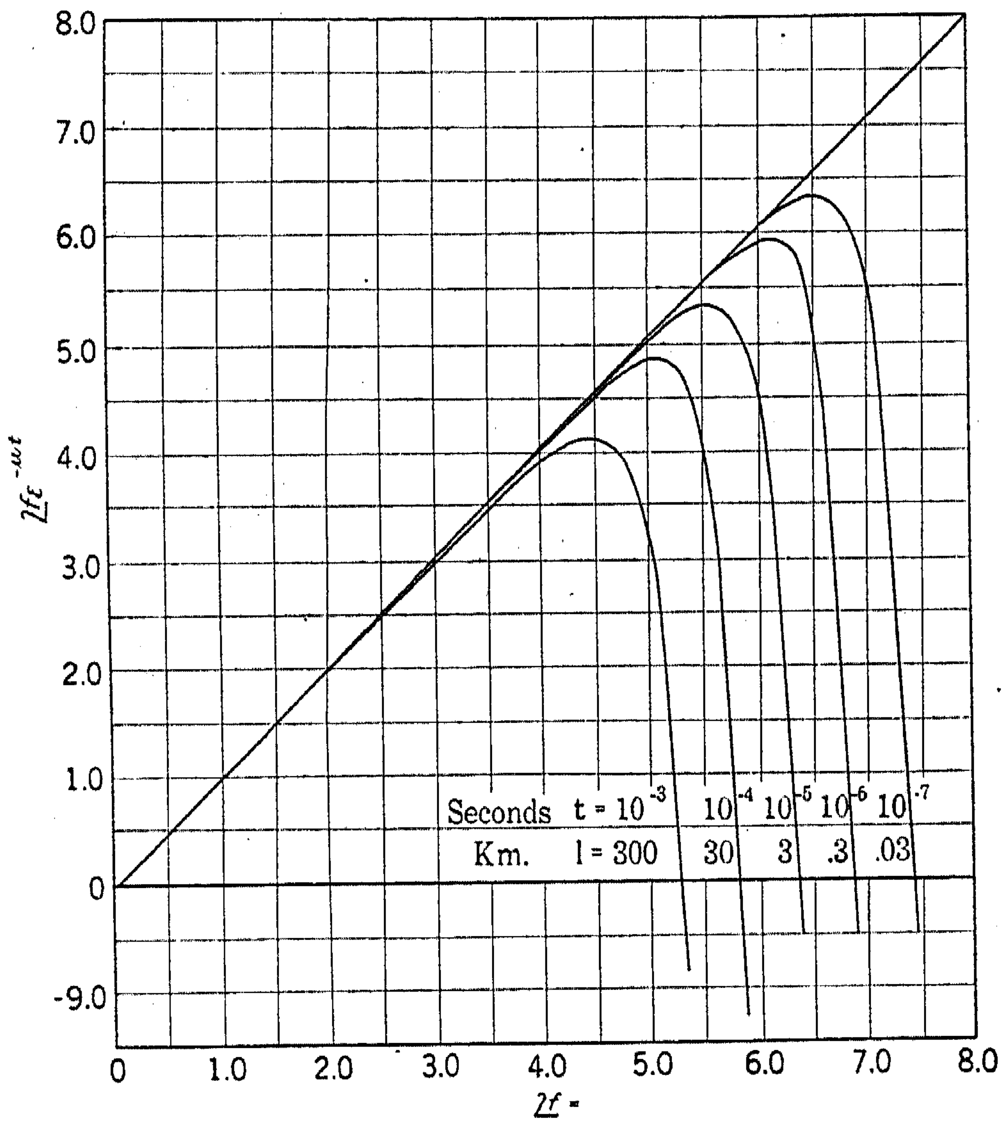


Fig. 10—Attenuation of Rectangular Wave Front of Impulse

Numerically, the integration is done by calculating $f \epsilon^{-ut}$ for (approximately) constant intervals of $\underline{/f}$, adding all the values, and multiplying the sum by the (average) difference between successive $\underline{/f}$.

As an instance is given, in Table X, the calculation for circuit (**3**) of the preceding, that is, copper wire No. 00 B. & S. gage, 30 ft. above ground, with the ground as return conductor, for the times $t = 0.1$, 1, 10, 100 and 1000 micro-seconds, corresponding to the distance of travel of the wave front of 0.03, 0.3, 3, 30 and 300 km. As frequency intervals are selected: 1, 2, 5, 10, 20, 50, 100, 200 etc., giving the logarithms: 0, 0.3, 0.6, 1, 1.3 etc.

These curves are plotted, in logarithmic scale, in Fig. 10.

As seen, all these curves rise as straight lines under 45 degrees, and then very abruptly drop to negligible values.

In Table X, the values of $f\,\epsilon^{-ut}$ are added, then divided by three, since there are three intervals for each unit of $\underline{/f}$, and multiplied by $\underline{/\epsilon}$, to reduce to natural logarithms. Multiplying by E/S then gives the gradient G.

For medium and high frequencies, the attenuation constant u is given by the equations of appendix C as:

$$u = \frac{r_1 + r_3}{2\,(L_1 + L_0)} + \frac{g}{2\,C} \tag{14}$$

Neglecting the internal inductance L_1, as small compared with the external inductance, this gives

$$\left.\begin{aligned} u &= \frac{\frac{\pi}{l_1}\sqrt{2.5\,\frac{\mu}{\gamma}\,10^4}}{\log\frac{l'}{l_r}}\sqrt{f} + \frac{4\,\pi^2\,l'}{S\log\frac{l'}{l_r}}\,f^2 \\ &= m_1\sqrt{f} + m_2\,f^2 \end{aligned}\right\} \tag{15}$$

where

$$m_1 = \frac{\pi\sqrt{2.5\,\frac{\mu}{\gamma}}}{l_1\log\frac{l'}{l_r}}\,10^4 \tag{16}$$

$$m_2 = \frac{4\,\pi^2\,l'}{S\log\frac{l'}{l_r}} \tag{17}$$

For the conductor (3) in Table X, it is:

$$\left.\begin{aligned} m_1 &= 2.6 \\ m_2 &= .29 \times 10^{-6} \end{aligned}\right\} \tag{18}$$

This expression (**15**) of u holds for all frequencies except very low frequencies—below 1000 cycles— and extremely high frequencies—many millions of cycles. The latter are of little importance, as they are wiped out in the immediate neighborhood of the origin of the rectangular impulse. At the low frequencies, the attenuation is so small, within the distances which come into consideration in the wave travel, and these low frequencies give such a small part of the wave front gradient, that the error made by the use of (**15**) is negligible. For instance, in the case of Table X, even at $t =$ 100 microseconds, or 10 km. wave travel, the error made in the voltage gradient by altogether neglecting the attenuation of the frequencies up to 1000 cycles, would be only 0.01 per cent.

Thus the equation (**15**) of the attenuation constant can safely be used for all practical purposes.

As the first term in equation (**15**) is proportional to $\sqrt{f}$, the second term to f^2, the second term is negligible at low and medium frequencies, while the first term is negligible at high frequencies.

Both terms are equal at:

$$f = \left(\frac{m_1}{m_2}\right)^{2/3} \qquad \textbf{(19)}$$

That is, in the above instance, at

$$f = 43{,}000 \text{ cycles.}$$

Thus, for high frequencies, that is, within moderate distances from the origin of the rectangular impulse—up to some kilometers—the first term can be neglected and the attenuation constant expressed by

$$u = m_2 f^2 \qquad \textbf{(20)}$$

The integral in equation (**12**) then becomes,

$$F = \int_0^\infty \epsilon^{-m_2 t f^2} d f \qquad \textbf{(21)}$$

Substituting:

$$x^2 = m_2 t f^2$$

gives:

$$d f = \frac{d x}{\sqrt{m_2 t}}$$

and:

$$F = \frac{1}{\sqrt{m_2 t}} \int_0^\infty \epsilon^{-x^2} d x$$

It is however:

$$\int_0^\infty \epsilon^{-x^2}\,d\,x = 1/2\,\sqrt{\pi} \tag{22}$$

thus:

$$F = \frac{1}{2}\sqrt{\frac{\pi}{m_2\,t}} \tag{23}$$

and:

$$G = \frac{E\sqrt{\pi}}{S\sqrt{m_2\,t}} \tag{24}$$

or, since:

$$l = S\,t \tag{25}$$

$$G = \frac{E\sqrt{\pi}}{\sqrt{m_2\,S\,l}} \tag{26}$$

Substituting (**17**) into (**26**), gives; in cm. and volts per cm.:

$$G = \frac{E}{2}\sqrt{\frac{\log\frac{l'}{l_r}}{\pi\,l'\,l}} \tag{27}$$

$$= 0.282\,E\sqrt{\frac{1}{l'\,l}\log\frac{l'}{l_r}}$$

Thus; approximately, the maximum gradient, or steepness of the wave front of a rectangular impulse, in the neighborhood and at moderate distances from its origin, decreases inversely proportional with the square root of the distance or time of wave travel.

It decreases with increasing distance l' of the return conductor, nearly in inverse proportion to the square root of l'

For the six types of circuits considered in the previous instances, it is:

	$G =$	$/G\sqrt{l} =$
(1) Copper wire 00 B. & S, gage, 18 in. = 45.5 cm. from return con.:	$\frac{1}{\sqrt{l}} \times 1700$	3.230
(2) Copper wire " " " 6 ft. = 182 " " " "	968	2.986
(3) Copper wire " " " 60 ft. = 1820 " " " "	360	2.556
(4) Iron wire " " " 60 " = 1820 " " " "	360	2.556
(5) Copper wire 4 " " 60 " = 1820 " " " "	373	2.572
(6) Aluminum, stranded, same conductivity and arrangement as (3):	353	2.548

where G is given in volts per meter, at $E = 60{,}000$ volts, and l in kilo-meters:

$$\left.\begin{aligned} G &= 89 \times 10^{-3}\, E \sqrt{\frac{1}{l'l} \log \frac{l'}{l_r}} \\ &= 135 \times 10^{-3}\, E \sqrt{\frac{1}{l'l} \underline{/ \frac{l'}{l_r}}} \\ &= 8100 \sqrt{\frac{1}{l'l} \underline{/ \frac{l'}{l_r}}} \end{aligned}\right\} \qquad (28)$$

From Tables X, IX, and VII are collected the values of wave gradients G, and given in Table XI, together with their logarithms, and the $\underline{/ G_0}$, calculated from equation (**28**), for comparison of the different methods of calculation. Table XI then gives the difference Δ, and its value in per cent.

The values of G are plotted in Fig. 11. The drawn line gives the values calculated by equation (**28**); the three cornered stars the values from Table X, the crosses the values from Table IX, and the circles the values from Table VII.

As seen from Table XI and Fig. 11, the agreement of the equations (**27**) and (**28**) is satisfactory with the values of the wave front gradient taken from Tables X and IX.

The values of G from Table X differ erratically. This table was calculated by graphical integration, and it is probable that the intervals have been chosen too large, in that range where the curve drops very abruptly, as seen in Fig. 10.

The agreement with the values from Table IX is very close. In this table, representing the course of a 60,000-cycle rectangular traveling wave, the individual harmonics have been calculated, as they were relatively few, due to the high frequency of the fundamental. This agreement is important as it justifies equation (**27**). In deriving (**27**), we have substituted integration for summation, that is, have replaced the discontinuous values of the individual harmonics by a continuous curve. Any error resulting from this should be greatest where the number of discontinuous harmonics is the least. Table XI and Fig. 11 however, show that the agreement of the gradient of the 60,000 cycle traveling wave with equation (**27**), is good even at $l = 30$ km., where only two significant harmonics are left, the fundamental and the third harmonic.

Thus the method of deriving an equation for G by integration is justified.

Unsatisfactory however, is the agreement of equation (**27**) with the values of the gradient of the quarter wave oscillation, taken from Table VII. These values lay on a straight line in Fig. 11, given as dotted line, showing proportionality with the square root of the distance, but there is a constant error, and the gradients given in Table VII are about 50 per cent greater than those given by equation (**27**).

TABLE XI.

CALCULATION OF WAVE FRONT.

Copper Wire No. 00 B. & S. gage, 30 ft. above Ground.

Dist. l km.	Gradient, volts per meter at E = 60,000 V.				
	G:	/ G:	/ G_0*	Δ	= %
	Impulse, f_0 = 0: (Table X)				
.03	1860	3.269	3.317	+ .048	+ 11.7
.3	684	2.835	2.817	− .018	− 4.3
3	186	2.269	2.317	+ .048	+ 11.7
30	63.5	1.803	1.817	+ .014	+ 3.3
300	11.9	1.075	1.317	(+ .242)	
	Traveling Wave, f_0 = 60,000: (Table IX)				
.3	690	2.839	2.817	− .022	− 5.2
.6	470	2.672	2.667	− .005	− 1.1
1.5	295	2.470	2.468	− .002	− 0.5
3	205	2.312	2.317	+ .005	+ 1.1
6	140	2.146	2.167	+ .021	+ 5.0
12	100	2.000	2.016	+ .016	+ 3.7
30	65	1.813	1.817	+ .004	+ 0.9
108	40	1.602	1.539	(− .063)	
	Quarterwave, f_0 = 750: (Table VII)				
-.01	5500	3.740	3.556	− .184	
.1725	1290	3.111	2.938	− .173	
.345	967	2.985	2.787	− .198	
.69	630	2.799	2.637	− .162	
3.45	280	2.447	2.287	− .160	
6.9	205†	2.312	2.137	− .175	
27.6	98	1.991	1.836	− .155	
69	58	1.763	1.637	(− .125)	
690	12	1.079	1.137	(+ .058)	
6900	1.2	.079	.637	(+ .558)	
	Same, 18 ins. = 45.5 cm. between conductors.				
.69	3240	3.510	3.311	− .199	
6.9	960	2.982	2.811	− .171	
69	234	2.369	2.311	(− .058)	

*Calculated by equation (28).

†131 to 293

The cause of the discrepancy probably is in the method used in calculating the gradients of the quarter wave oscillation given in Table VII. Due to the impossibility of calculating individually thousands of harmonics, the number of harmonics in successive intervals of frequency, has been multiplied with the average attenuation ϵ^{-ut} in this frequency range, and as the average has been used the mean value of the ϵ^{-ut} for the two extremes of the frequency range. However, ϵ^{-ut} drops exponentially and with great rapidity, so that the true average value is much lower than the average of maximum and minimum. Thus for instance in the range from 5×10^5 to 10×10^5 cycles, containing 334 harmonics, ϵ^{-ut} for $f = 5 \times 10^5$ (at $t = 23 \times 10^{-6}$ seconds), is 1.22×10^{-3} and for $f = 10 \times 10^5$, it is 181×10^{-3}, giving an average of 91.1×10^{-3}. However, for $f = 7.5 \times 10^5$, ϵ^{-ut} is 22.5×10^{-3}, thus less than a quarter of the average.

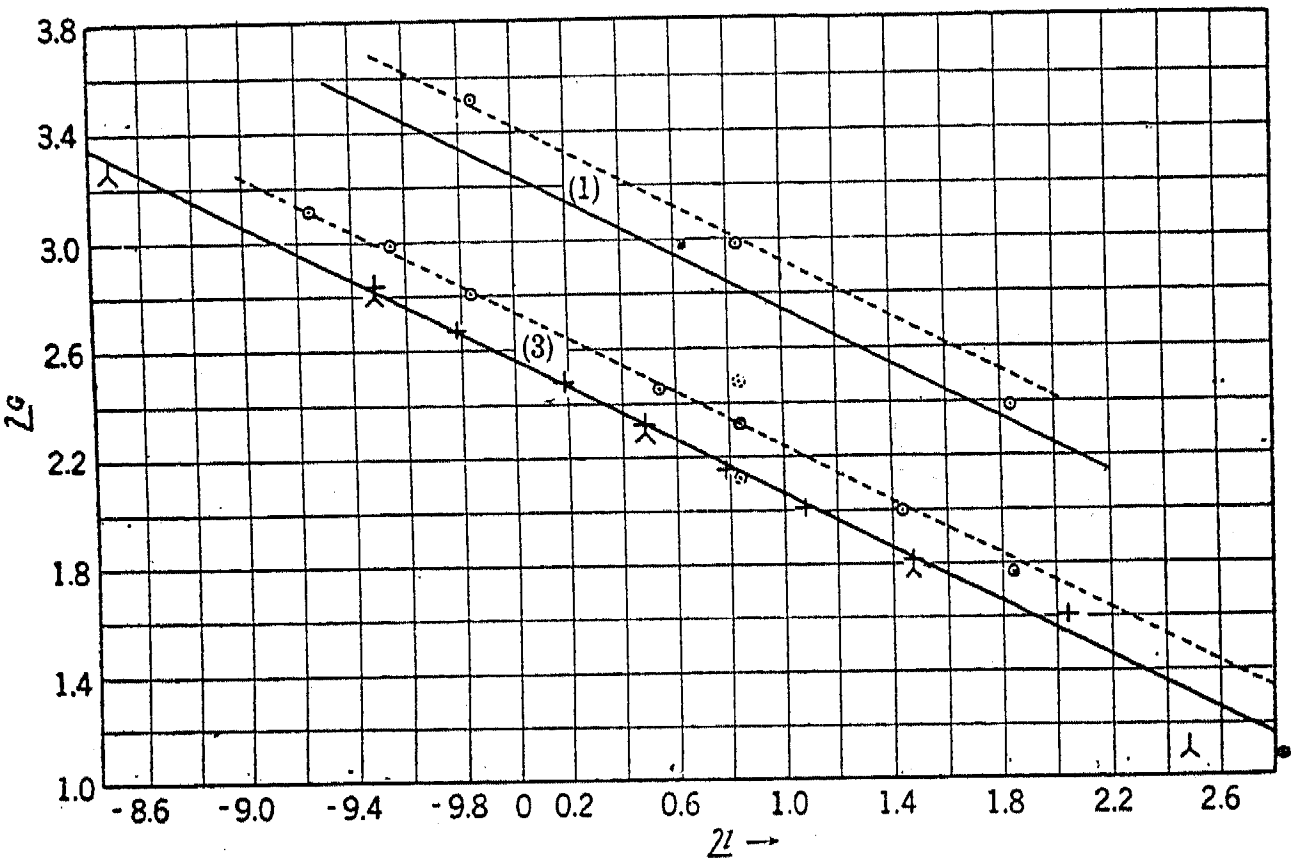


Fig. 11—Wave Front Gradient

To check this, one value, at $t = 23 \times 10^{-6}$ or $l = 6.9$ km., has been re-calculated by using not the average, but the maximum and the minimum of ϵ^{-ut} in each interval, and the two gradients derived therefrom: $G = 293$ and 131, are marked with dotted circles in Fig. 11. As seen, the use of the minimum value of ϵ^{-ut} agrees nearer with equation (**27**), as was to be expected.

It appears probable that the equation (**27**) gives more reliable values of wave front gradient, within the range of its applicability, than the method of calculation used in Table VII. and as it is much simpler, it is preferable.

As seen from Table XI and Fig. 11, the parabolic law of wave front flattening, given by equations (**27**) and (**28**), holds good up to about 30 km. distance of wave travel, and with fair approximation even to 100 km. In the range beyond this—which is of lesser importance, as the flattening of the wave front has greatly reduced its danger at these distances—the values of the gradient G decrease with increasing rapidity, with distance and time, due to the medium high harmonics showing in the attenuation.

At great distances from the origin of the rectangular wave, when the very high harmonics have practically died out and the wave attenuation is determined by the medium-frequency harmonics, the second term of u in equation (**15**) becomes negligible, and u can be approximated by:

$$u = m_1 \sqrt{f} \tag{37}$$

The integral in equation (**12**) then becomes

$$F = \int_0^\infty \epsilon^{-m_1 t \sqrt{f}} \, d f \tag{38}$$

Substituting:

$$m_1 t \sqrt{f} = x$$

gives

$$d f = \frac{2 x \, d x}{m_1^2 t^2}$$

and

$$F = \frac{2}{m_1^2 t^2} \int_0^\infty x \, \epsilon^{-x} \, d x$$

It is however:

$$\int_0^\infty x \, \epsilon^{-x} \, d x = 1 \tag{39}$$

thus

$$F = \frac{2}{m_1^2 t^2} \tag{40}$$

and

$$G = \frac{4 E}{S \, m_1^2 t^2} \tag{41}$$

or:

$$G = \frac{4\,S\,E}{m_1^2\,l^2} \tag{42}$$

Substituting (**16**) gives

$$G = \frac{1.6\,S\,\gamma\,l_1^2\left(\log\frac{l'}{l_r}\right)^2}{\pi^2\,\mu\,l^2}\,E \tag{43}$$

That is, at great distances (or considerable time) from the origin, the flattening of the wave front approaches inverse proportionality with the square of the distance (or time) of the wave travel.

However, this range is of little importance.

APPENDIX A

Unequal Current Distribution in Conductor

It is simple to determine from conductor size and material, and the frequency, whether the unequal current distribution in the conductor is sufficient to appreciably effect the ohmic resistance.[1] Also, if conductor size and frequency are so high that the current penetrates only a thin surface layer of the conductor, the calculation of its effective resistance leads to simple expressions. Only in the intermediary range, where the current distribution throughout the conductor is already very uneven, but appreciable current still penetrates to the center of the conductor, the numerical calculation becomes laborious, involving a slowly convergent series or special functions of which tables are rarely available. However, this range is relatively narrow and can be approximated between the two extremes, with sufficient accuracy for the calculation of transients, while for low-frequency power transmission, conductors of a size reaching into this range would not be used, being uneconomical in conductor material.

As the case of such high frequencies or large conductor sizes, that the current penetrates only a surface layer, has been frequently investigated,[2] the derivation of the equations is unnecessary here. The most satisfactory way of representing it is to calculate the "depth of penetration" of the current into the conductor. The actual or effective resistance of the conductor then is given by circumference of the conductor, times the depth of penetration. This method, of calculating the depth of penetration, has the advantage that it applies to all sizes and shapes of conductors, solid round conductors or hollow tubes, flat ribbon and even such complex shapes as the railway rail, provided only that the depth of penetration is materially less than the depth of the conductor material.

---

[1] "Theory and Calculation of Alternating-Current Phenomena, 5th edition, page 144.

[2] See for instance "Theory and Calculation of TransientPhenomena," Section III, Chapter VII.

The depth of the penetration into the conductor is

$$\left.\begin{aligned} l_p &= \frac{10^4}{\pi\sqrt{0.4\,\gamma\,\mu f}} \\ &= \frac{5030}{\sqrt{\gamma\,\mu f}} \end{aligned}\right\} \qquad (1)$$

where

γ = electrical conductivity of the conductor,
μ = magnetic permeability of the conductor,
f = frequency.

The effective resistance of the conductor then is, per unit length,

$$\left.\begin{aligned} r &= \frac{1}{\gamma\, l_p\, l_1} \\ &= \frac{\pi}{l_1}\sqrt{\frac{0.4\,\mu f}{\gamma}}\,10^{-4} \\ &= \frac{\sqrt{\dfrac{\mu f}{\gamma}}}{5030\, l_1} \text{ ohms per cm.} \end{aligned}\right\} \qquad (2)$$

where

l_1 = circumference of conductor (actual, that is, following all indentations, etc.)

As the true ohmic resistance of the conductor is,

$$r_0 = \frac{1}{\gamma\, s} \text{ ohms per cm.} \qquad (3)$$

where

s = conductor section,

the "resistance ratio", or ratio of the effective resistance of unequal current distribution, to the true ohmic resistance, is:

$$\left.\begin{aligned}\frac{r}{r_0} &= \frac{s\,\pi}{l_1}\sqrt{0.4\,\gamma\,\mu\,f} \\ &= \frac{s}{5030\,l_1}\sqrt{\gamma\,\mu\,f}\end{aligned}\right\} \qquad (4)$$

The internal reactance of unequal current distribution equals the resistance, in the range considered:

$$x_1 = r_1 \qquad (5)$$

while at low frequency, where the current distribution is still uniform, the internal reactance is, per unit length of conductor:

$$\left.\begin{aligned}x_{10} &= \pi\,f\,\mu\,10^{-9} \text{ henry per cm.} \\ &= \pi\,f\,10^{-9} \text{ for air } (\mu = 1)\end{aligned}\right\} \qquad (6)$$

The external inductance of the conductor, that is, the inductance due to the magnetic field outside of the conductor, calculated in the usual manner, that is neglecting the finite velocity of the magnetic field, is, per unit length of conductor,

$$L_0 = 2\log\frac{l'}{l_r}\;10^{-9} \text{ henry per cm.} \qquad (7)$$

where

l_r = radius of conductor

l' = distance of return conductor

or, for conductors of other shape, approximately:

$$L_0 = 2\log\frac{2\,\pi\,l'}{l_2}\;10^{-9} \text{ henry per cm.} \qquad (8)$$

where:

l_2 = shortest circumference of conductor (that is, not including indentations etc.)

Thus, the external reactance, per unit length of conductor is:

$$x_0 = 4\,\pi\,f\log\frac{l'}{l_r}\;10^{-9} \qquad (9)$$

$$= 4\,\pi\,f\log\frac{2\,\pi\,f'}{l_2}\;10^{-9} \text{ ohms per cm.} \qquad (10)$$

As the unequal current distribution in the conductor, and the increase of effective resistance and decrease of internal reactance caused by it, are the result of the magnetic field of the current inside of the conductor, it follows that stranding the conductor does not eliminate this effect, unless the stranding is done in such manner that every strand has the same distance from the conductor surface, that is, interior strands interchange periodically with exterior ones and inversely, and the individual strands are insulated from each other.

Otherwise the stranding has an indirect effect only, in reducing γ and μ in equation (**1**) etc.

Thus, if in the stranded conductor the part p is filled by the conductor material of conductivity γ_0,—the rest $(1 - p)$ being the space between the strands—, then the average conductivity of the conductor is

$$\gamma = p\, \gamma_0$$

In conductors of magnetic material stranding may materially reduce the effective resistance, as the spaces between the strands would greatly reduce the resultant permeability of the conductor, and thereby greatly increase the penetration of the current into the conductor.

APPENDIX B

Radiation Resistance, Inductance and Capacity

Consider a finite section of an infinitely long conductor of radius l_r, with the return conductor at distance l', and assume that l' is small compared with a quarter wave length of the electric field of the conductor.

$$\left.\begin{aligned} \text{Let } i &= I \cos(\theta - \alpha) = \text{current in the conductor} \\ e &= E \cos(\theta - \beta) = \text{voltage of the conductor} \end{aligned}\right\} \quad (\mathbf{1})$$

At a distance l from the conductor, the magnetic field intensity is $0.2/l$ times the current, and lags behind the current by the time required for the magnetic field to travel the distance l, that is, by the angle $a\, l$, where:

$$a = \frac{2\pi f}{S} \quad (\mathbf{2})$$

f = frequency,

$$S = 3 \times 10^{10} = \text{velocity of radiation} \quad (\mathbf{3})$$

The magnetic field intensity thus is given by:

$$H = \frac{0.2\,I \cos(\theta - \alpha - a\,l)}{l} \tag{4}$$

The magnetic component of field, per unit length of conductor (cm.), thus is:

$$\Phi = \int_{l_r}^{l'} H\,d\,l$$

$$= 0.2\,I \int_{l_r}^{l'} \frac{\cos(\theta - \alpha - a\,l)}{l}\,d\,l$$

$$= 0.2\,I \left\{ \cos(\theta - \alpha) \int_{l_r}^{l'} \frac{\cos a\,l}{l}\,d\,l \right.$$

$$\left. + \sin(\theta - \alpha) \int_{l_r}^{l'} \frac{\sin a\,l}{l}\,d\,l \right\} \tag{5}$$

It is however, for values of l' up to about one-half of a quarter wave length of the electric field, with sufficient approximation*:

$$\int_{l_r}^{l'} \frac{\sin a\,l}{l}\,d\,l = a\,l' \tag{6}$$

$$\int_{l_r}^{l'} \frac{\cos a\,l}{l}\,d\,l = \log \frac{l'}{l_r} \tag{7}$$

Thus $$\Phi = 0.2\,I \left\{ \log \frac{l'}{l_r} \cos(\theta - \alpha) \right.$$

$$\left. + a\,l' \sin(\theta - \alpha) \right\} \tag{8}$$

Similar considerations apply to the dielectric field of the conductor, in relation to its voltage.

---

*See "Theory and Calculation of Transient Electric Phenomena," Section III, Chapter VIII: "Velocity of Propagation of Electric Field."

Thus, the two components of the electric field, the magnetic field and the dielectric field, are not in phase with current and voltage, respectively, but lag behind, due to the appearance of the sine component, resulting from the finite velocity of the electric field.

The magnetic field Φ induces a voltage in quadrature with the field Φ:

$$e' = 2\pi f \frac{d\Phi}{d\theta} 10^{-8}$$

$$= 4\pi f I \left\{ \log \frac{l'}{l_r} \sin(\theta - \alpha) + a\,l' \cos(\theta - \alpha) \right\} 10^{-9} \tag{9}$$

where the 10^{-8} reduces from absolute units to volts.

The reactive or sine component of this voltage e' is the e.m.f. of self induction, and gives rise to the reactance,

$$x = 4\pi f \log \frac{l'}{l_r} 10^{-9} \text{ ohms} \tag{10}$$

and the inductance,

$$L = \frac{x}{2\pi f}$$

$$= 2 \log \frac{l'}{l_r} 10^{-9} \text{ henrys} \tag{11}$$

The cosine component of E' is in phase with the current, thus an energy voltage, and gives rise to an effective resistance, the magnetic radiation resistance:

$$r = 4\pi f\, a l' \, 10^{-9} \text{ ohms.} \tag{12}$$

Substituting (**2**) into (**12**) gives:

$$r = \frac{8\pi^2 f^2 l'}{S} 10^{-9} \text{ ohms.} \tag{13}$$

That is, the radiation resistance is proportional to the distance from the return conductor, and to the square of the frequency, but is independent of size, shape or material of the conductor.

From (**11**) and (**13**) follow as the attenuation constant of magnetic radiation,

$$u_1 = \frac{r}{2L} = \frac{4\pi f^2 l'}{2S \log \frac{l'}{l_r}} \tag{14}$$

At a distance l from the conductor, the dielectric or electrostatic field lags behind the voltage (**1**) by angle $a\,l$, thus, per unit length of conductor, is given by,

$$\psi = \Psi \cos (\theta - \beta - \alpha\, l) \tag{15}$$

The voltage gradient is $2/l$ times the dielectric flux, and the total voltage from conductor to return conductor thus is given by,

$$e = \int_{l_r}^{l'} \frac{2\,\psi}{l}\, d\,l$$

or:

$$E \cos (\theta - \beta) = 2\,\Psi \int_{l_r}^{l'} \frac{\cos (\theta - \beta - a\,l)}{l}\, d\,l$$

$$= 2\,\Psi \left\{ \cos (\theta - \beta) \int_{l_r}^{l'} \frac{\cos a\,l}{l}\, d\,l + \sin (\theta - \beta) \int_{l_r}^{l'} \frac{\sin a\,l}{l}\, d\,l \right\} \tag{16}$$

hence, by (**6**) and (**7**):

$$E \cos (\theta - \beta) = 2\,\Psi \left\{ \log \frac{l'}{l_r} \cos (\theta - \beta) + a\,l' \sin (\theta - \beta) \right\} \tag{17}$$

The dielectric field gives rise to a current in quadrature with the field, the capacity or charging current of the conductor:

$$I_1 = 2\,\pi f\,\Psi \tag{18}$$

and, substituting (**18**) into (**17**), gives,

$$E \cos (\theta - \beta) = \frac{I'}{\pi f} \left\{ \log \frac{l'}{l_r} \sin (\theta - \beta) + a\,l' \cos (\theta - \beta) \right\} \tag{19}$$

where the change of the trigonometric functions represents the quadrature relation.

The reactive or sine component of this current I' gives rise to the capacity reactance,

$$x_c = \frac{S^2 \log \frac{l'}{l_r}}{\pi f} 10^{-9} \text{ ohms} \tag{20}$$

and the capacity:

$$C = \frac{1}{2 \pi f x_c}$$

$$= \frac{10^{-9}}{2 S^2 \log \frac{l'}{l_r}} \text{ farads} \tag{21}$$

where the factor S^2 reduces from electrostatic to electromagnetic units.

The cosine component of I' is in phase with the voltage, thus an energy current, and gives rise to an effective resistance, the dielectric radiation resistance.

$$r_c = \frac{a l' S^2}{\pi f} 10^{-9} \text{ ohms,} \tag{22}$$

or, by (**2**):

$$r_c = 2l' \ S \ 10^{-9} \text{ ohms} \tag{23}$$

If the specific capacity of the medium differs from unity, and is κ, then κ enters as factor in the numerator of C and in the denominator of x_c and r_c.

From (**21**) and (**23**) follow as the attenuation constant of dielectric radiation:

$$u_2 = \frac{g}{2 C} = \frac{\pi f r_c}{x_c}$$

$$= \frac{2 \pi^2 f^2 l'}{S^2 \log \frac{l'}{l_r}} \tag{24}$$

$$= u_1$$

that is, the dielectric and the magnetic attenuation constant of magnetic radiation are equal; in other words, by electric radiation, the magnetic and the dielectric energy decay at the same rate, and their ratio thus remains constant.

It is interesting to note, that the expressions of inductance L and capacity C, (**11**) and (**21**), are the same as the values of

L and C, calculated without considering the finite velocity of the electric field, that is, are the "low-frequency values" of external inductance and capacity. Thus:

As long as the distance of the return conductor is small compared with a quarter wave length, electric radiation due to the finite velocity of the electric field, does not affect or change the values of external inductance L and of capacity C, but causes an energy dissipation by electromagnetic and by dielectric or electrostatic radiation, which is represented by effective resistances. The electromagnetic radiation resistance is a series resistance, proportional to the square of the frequency; the electrostatic resistance is a shunted resistance in series with the capacity. It is independent of the frequency and the power consumed by either resistance thus is proportional to the square of the frequency.

Equations (**6**) and (**7**), and thus the equations (**10**), (**11**), (**13**); (**20**), (**21**), (**23**), of the circuit constants x, L, r; x_c, C, r_c, apply only as long as the distance l' of the return conductor is small compared with a quarter wave length, and with sufficient approximation up to l' = one half of a quarter wave length.

With the return conductor at 6 ft. = 182 cm., the minimum wave length would be 8 × 182 = 1456 cm., and the maximum frequency thus:

$$f_0 = \frac{3 \times 10^{10}}{1456} = 20 \text{ million cycles, approx.}$$

With the ground as return, and the conductor 30 ft. above ground, thus the image conductor as equivalent return conductor at 2 × 30 ft. = 1820 cm. distance, the maximum frequency is:

$$f_0 = \frac{3 \times 10^{10}}{8 \times 1820} = 2 \text{ million cycles, approx.}$$

As seen, the approximations (**6**) and (**7**) apply to practically all industrially important transients, with rare exceptions.

For such high frequencies however, where the distance of the return conductor, l', approaches or exceeds a quarter wave length, or in the case of a conductor without return conductor ($l' = \infty$), such as is approximated by the wireless antenna, or by a lightning discharge to ground, the integrals (**6**) and (**7**) have to be evaluated.

They define new functions:*

$$\int_{l_r}^{l'} \frac{\sin a\,l}{l}\,d\,l = \operatorname{col} a\,l_r - \operatorname{col} a\,l' \qquad (\mathbf{25})$$

$$\int_{l_r}^{l'} \frac{\cos a\,l}{l}\,d\,l = \operatorname{sil} a\,l_r - \operatorname{sil} a\,l' \qquad (\mathbf{26})$$

and, since l_r is always small compared with a quarter wave length, it is:

$$\operatorname{col} a\,l_r = \frac{\pi}{2} \qquad (\mathbf{27})$$

$$\operatorname{sil} a\,l_r = \log \frac{1}{a\,l_r} - 0.5772 \qquad (\mathbf{28})$$

Substituting (**25**), (**26**), (**27**) and (**28**) instead of (**6**) and (**7**), into the previous discussion, gives the final complete equations:

$$(10)\quad x = 4\,\pi f \left\{ \log \frac{1}{a\,l_r} - 0.5772 - \operatorname{sil} a\,l' \right\} 10^{-9}$$

$$= 4\,\pi f \left\{ \log \frac{S}{2\,\pi f\,l_r} - 0.5772 - \operatorname{sil} \frac{2\,\pi f\,l'}{S} \right\} 10^{-9} \text{ ohms} \qquad (\mathbf{29})$$

$$(11)\quad L = 2 \left\{ \log \frac{1}{a\,l_r} - 0.5772 - \operatorname{sil} a\,l' \right\} 10^{-9}$$

$$= 2 \left\{ \log \frac{S}{2\,\pi f\,l_r} - 0.5772 - \operatorname{sil} \frac{2\,\pi f\,l'}{S} \right\} 10^{-9} \text{ henrys} \qquad (\mathbf{30})$$

$$(13)\quad r = 4\,\pi f \left\{ \frac{\pi}{2} - \operatorname{col} a\,l' \right\} 10^{-9}$$

$$= 4\,\pi f \left\{ \frac{\pi}{2} - \operatorname{col} \frac{2\,\pi f\,l'}{S} \right\} 10^{-9} \text{ ohms} \qquad (\mathbf{31})$$

---

*Tables of the numerical values of these functions are given in the Appendix to the later editions of "Theory and Calculation of Transient Electric Phenomena."

$$(20)\quad x_c = \frac{S^2}{\pi f}\left\{\log\frac{1}{a\,l_r} - 0.5772 - \operatorname{sil} a\,l'\right\}10^{-9}$$

$$= \frac{S^2}{\pi f}\left\{\log\frac{S}{2\,\pi\,f\,l_r} - 0.5772 - \operatorname{sil}\frac{2\,\pi\,f\,l'}{S}\right\}10^{-9}\text{ ohms} \qquad (\mathbf{32})$$

$$(21)\quad C = \frac{10^{-9}}{2\,S^2\left\{\log\frac{1}{a\,l_r} - 0.5772 - \operatorname{sil} a\,l'\right\}}$$

$$= \frac{10^{-9}}{2\,S^2\left\{\log\frac{S}{2\,\pi\,f\,l_r} - 0.5772 - \operatorname{sil}\frac{2\,\pi\,f\,l'}{S}\right\}}\text{ farads} \qquad (\mathbf{33})$$

$$(23)\quad r_c = \frac{S^2}{\pi f}\left\{\frac{\pi}{2} - \operatorname{col} a\,l'\right\}10^{-9}$$

$$= \frac{S^2}{\pi f}\left\{\frac{\pi}{2} - \operatorname{col}\frac{2\,\pi\,f\,l'}{S}\right\}10^{-9}\text{ ohms} \qquad (\mathbf{34})$$

As seen from the tables of col x, the radiation resistances, and thus the energy dissipation by magnetic and dielectric radiation, increase with increasing distance of the return conductor, reach a maximum when the distance of the return conductor is one-half wave length, and then decrease again.

The maximum radiation resistances, at l' = one-half wave length, are:

$$r = 7.41\,\pi f\,10^{-9}$$

$$= 23.27\,f\,10^{-9}\text{ ohms.} \qquad (\mathbf{35})$$

$$r_c = \frac{1.852\,S^2}{\pi f}\,10^{-9}$$

$$= \frac{0.531}{f}\,10^{12}\text{ ohms.} \qquad (\mathbf{36})$$

Substituting $l' = \infty$ into equations (**29**) to (**34**) gives the radiation constants of the conductor without return conductor.

$l = \infty$ gives:

$$\operatorname{sil} a\,l' = 0 = \operatorname{col} a\,l' \qquad (\mathbf{37})$$

thus:

$$(29)\quad x = 4\,\pi f \left\{ \log \frac{S}{2\,\pi f\, l_r} - 0.5772 \right\} 10^{-9} \text{ ohms} \quad \textbf{(38)}$$

$$(30)\quad L = 2 \left\{ \log \frac{S}{2\,\pi f\, l_r} - 0.5772 \right\} 10^{-9} \text{ henrys} \quad \textbf{(39)}$$

$$(31)\quad r = 2\,\pi^2 f\, 10^{-9} \text{ ohms} \quad \textbf{(40)}$$

$$(32)\quad x_c = \frac{S^2}{\pi f} \left\{ \log \frac{S}{2\,\pi f\, l_r} - 0.5772 \right\} 10^{-9} \text{ ohms.} \quad \textbf{(41)}$$

$$(33)\quad C = \frac{10^{-9}}{2\,S^2 \left\{ \log \frac{S}{2\,\pi f\, l_r} - 0.5772 \right\}} \text{ farads} \quad \textbf{(42)}$$

$$(34)\quad r_c = \frac{S^2}{2 f}\, 10^{-9} \text{ ohms} \quad \textbf{(43)}$$

APPENDIX C

Equations of Electrical Constants

Let: l = length of conductor, cm.
l_r = radius of conductor, cm.
l_1 = circumference of conductor, cm.
l_2 = shortest circumference of conductor, cm.
l' = distance of return conductor
f = frequency, cycles per second
γ = electrical conductivity, mhos per cm^3.
μ = magnetic permeability
$S = 3 \times 10^{10}$ = velocity of radiation in empty space,
it is then, log denoting the natural logarithm:

resistances:

true ohmic resistance (thermal):

$$r_0 = \frac{l}{\gamma\, \pi\, l^2{}_r} \text{ ohms.}$$

effective resistance of unequal current distribution (thermal):

$$r_1 = \frac{\pi\, l}{l_1} \sqrt{\frac{0.4\, \mu f'}{\gamma}}\, 10^{-4} \text{ ohms.}$$

effective magnetic radiation resistance:

(a) return conductor at distance l':

$$r_3 = \frac{8\,\pi^2 f^2 l'\, l}{S}\, 10^{-9} \text{ ohms}$$

at extremely high frequencies:

$$r_4 = 4\,\pi f l \left\{ \frac{\pi}{2} - \mathrm{col}\,\frac{2\,\pi f l'}{S} \right\} 10^{-9}\text{ ohms.}$$

(b) conductor without return conductor:

$$r_2 = 2\,\pi^2 f l\, 10^{-9}\text{ ohms.}$$

effective dielectric radiation (shunted) resistance:[1]

(a) return conductor at distance l':

$$r_c = \frac{2\,l' S}{l}\, 10^{-9}\text{ ohms}$$

at extremely high frequencies:

$$r_c = \frac{S^2}{\pi f l} \left\{ \frac{\pi}{2} - \mathrm{col}\,\frac{2\,\pi f l'}{S} \right\} 10^{-9}\text{ ohms}$$

(b) conductor without return conductor:

$$r_c = \frac{S}{2 f l}\, 10^{-9}\text{ ohms}$$

reactances:

low-frequency internal reactance:

$$x_{10} = \pi f l \mu\, 10^{-9}\text{ ohms.}$$

internal reactance of unequal current distribution:

$$x_1 = \frac{\pi l}{l_1} \sqrt{\frac{0.4\,\mu f}{\gamma}}\, 10^{-4}\text{ ohms,} = r_1$$

external reactance:

(a) return conductor at distance l':

$$x_0 = 4\,\pi f l \log \frac{l'}{l_r}\; 10^{-9} = 4\,\pi f l \log \frac{2\,\pi l'}{l_2}\; 10^{-9}\text{ ohms}$$

at extremely high frequencies:

$$x_4 = 4\,\pi f l \left\{ \log \frac{S}{2\,\pi f l_r} - 0.5772 - \mathrm{sil}\,\frac{2\,\pi f l'}{S} \right\} 10^{-9}\text{ ohms}$$

---

*As shunted resistance and reactance, r_c and x_c are inverse proportional to the length of the conductor l that is, the longer the conductor, the more current is shunted across, and the lower therefor are r_c and x_c. For this reason, the shunt constants usually are given as conductance g and susceptance b. In the present case however, r and x give simpler expressions.

(b) conductor without return conductor:

$$x_2 = 4\,\pi f l \left\{ \log \frac{S}{2\,\pi f l_r} - 0.5772 \right\} 10^{-9} \text{ ohms}$$

shunted capacity reactance:

(a) return conductor at distance l':

$$x_c = \frac{S^2 \log \dfrac{l'}{l_r}}{\pi f l} 10^{-9} = \frac{S^2 \log \dfrac{2\,\pi\, l'}{l_2}}{\pi f l} 10^{-9} \text{ ohms}$$

at extremely high frequencies:

$$x_c = \frac{S^2}{\pi f l} \left\{ \log \frac{S}{2\,\pi f l_r} - 0.5772 - \text{sil} \frac{2\,\pi f l'}{S} \right\} 10^{-9} \text{ ohms.}$$

(b) conductor without return conductor:

$$x_c = \frac{S^2}{\pi f l} \left\{ \log \frac{S}{2\,\pi f l_r} - 0.5772 \right\} 10^{-9} \text{ ohms}$$

inductances:

low frequency internal inductance:

$$L_{10} = \frac{l\,\mu}{2} 10^{-9} \text{ henrys}$$

internal inductance of unequal current distribution:

$$L_1 = \frac{l}{l_1} \sqrt{\frac{0.1\mu}{\gamma f}}\, 10^{-4} \text{ henrys}$$

external inductance:

(a) return conductor at distance l':

$$L_0 = 2\,l \log \frac{l'}{l_r} 10^{-9} = 2\,l \log \frac{2\,\pi\, l'}{l_2} 10^{-9} \text{ henrys.}$$

at extremely high frequencies:

$$L_4 = 2\,l \left\{ \log \frac{S}{2\,\pi f l_r} - 0.5772 - \text{sil} \frac{2\,\pi f' l'}{S} \right\} 10^{-9} \text{ henrys.}$$

(b) conductor without return conductor:

$$L_2 = 2\,l \left\{ \log \frac{S}{2\,\pi f l_r} - 0.5772 \right\} 10^{-9} \text{ henrys.}$$

capacity:

(a) return conductor at distance l':

$$C_0 = \frac{10^9}{2\,S^2 \log \dfrac{l'}{l_r}} \text{ farads}$$

at extremely high frequencies:

$$C_4 = \frac{10^9}{2\,S^2\left\{\log\frac{S}{2\,\pi f l_r} - 0.5772 - \operatorname{sil}\frac{2\,\pi f l'}{S}\right\}} \text{ farads}$$

(b) conductor without return conductor:

$$C_2 = \frac{10^9}{2\,S^2\left\{\log\frac{S}{2\,\pi f l_r} - 0.5772\right\}} \text{ farads}$$

Herefrom then follow the Circuit Constants:

At Low Frequencies: (Machine Frequencies up to 10^3 cycles, approx.)

$$\begin{aligned} r &= r_0 \\ x &= x_{10} + x_0 \\ L &= L_{10} + L_0 \\ C &= C_0 \\ g &= 0 \end{aligned}$$

At Medium Frequencies: (10^3 to 10^5 cycles, approx.)

$$\begin{aligned} r &= r_1 \\ x &= x_1 + x_0 \\ L &= L_1 + L_0 \\ C &= C_0 \\ g &= 0 \end{aligned}$$

At High Frequencies: (10^5 to 10^7 cycles, approx.)

$r = r_1 + r_3$ (with return conductor)

$ = r_1 + r_2$ (without return conductor)

$x = x_1 + x_0$ (with return conductor)

$ = x_1 + x_2$ (without return conductor)

$L = L_1 + L_0$ (with return conductor)

$ = L_1 + L_2$ (without return conductor)

$C = C_0$ (with return conductor)

$ = C_2$ (without return conductor)

(approximately, or represented by x_c and r_c)

g represented by r_c and x_c:

$$g = \frac{r_c}{r^2_c + x^2_c}\,;\; b = \frac{x_c}{r^2_c + x^2_c} = 2\,\pi f C$$

At Extremely High Frequencies: (above 10^7 cycles, approx.)

$r = r_1 + r_4$ (with return conducter)

$ = r_1 + r_2$ (without return conductor)

$x = x_1 + x_4$ (with return conductor)

$ = x_1 + x_2$ (without return conductor)

C and g represented by r_c and x_c, thus:

$$g = \frac{r_c}{r_c^2 + x_c^2}$$

$$C = \frac{x_c}{2\pi f(r_c^2 + x_c^2)}$$

From these follow the Derived Circuit Constants:
Magnetic Attenuation:

$$u_1 = \frac{r}{2L}$$

Dielectric Attenuation:
Usually zero at low and medium frequencies,

$$u_2 = \frac{g}{2C} = \frac{\pi f r_c}{x_c}$$

at high and very high frequencies.
Attenuation Constant:

$$u = u_1 + u_2 = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right)$$

Series Power Factor:

$$\cos\omega = \frac{r}{\sqrt{r^2 + x^2}}$$

Shunt Power Factor:
Zero at low and medium frequencies,

$$\cos\omega_c = \frac{r_c}{\sqrt{r_c^2 + x_c^2}}$$

at high and very high frequencies.
Duration of Oscillation:[1]

$$t_0 = \frac{1}{u} \text{ seconds}$$

$$N_0 = \frac{f}{u} \text{ cycles}$$

---

[1] *Under "Duration" of a transient is understood the time (or the number of cycles), which the transient would last, that is, the time in which it would expend its energy, if continuing at its initial intensity. With a simple exponential transient, this is the time during which it decreases to $\frac{1}{\epsilon}$ or 36.8 per cent of its initial value. It decreases to one-tenth of its initial value in 2.3 times this time.

Presented at the 7th Mid-winter Convention of the American Institute of Electrical Engineers, New York, February 20, 1919.

Copyright 1919. By A. I. E. E.

REVIEW OF WORK OF SUB-COMMITTEE ON WAVE SHAPE STANDARD OF THE STANDARDS COMMITTEE*

BY HAROLD S. OSBORNE

ABSTRACT OF PAPER

The paper gives a review of the work done during the past three years by a sub-committee of the Standards Committee of the Institute appointed to make recommendations for changes in the Institute's rules regarding the wave shape of alternators. After a very careful consideration of the question and a large amount of experimental work the Sub-Committee last spring made recommendations that for the present the ten per cent deviation rule should be retained without change (except in wording), and that trial use should be made of a supplementary wave shape factor. The new factor, based on the relation between voltage wave shape and interfering effect in telephone circuits, when power and telephone lines parallel each other, is called the "telephone interference factor." The committee invites the broadest discussion and trial use of the new factor. No attempt will be made to decide upon limiting values of the factor until after wide experience shall have been obtained in its use, and the committee will welcome assistance on the part of any interested in collecting information to this end.

THIS paper is presented at the request of the sub-committee on Wave Shape Standard to encourage "the trial use of telephone interference factor and the collection of information by any who are interested."

Introduction. Until the past year the only standard of wave-shape included in standardization rules of the American Institute of Electrical Engineers has been the rule that the maximum deviation from the equivalent sine wave should not exceed 10 per cent of the amplitude of the equivalent sine wave. This provision has been in the standardization rules for many years and has done excellent service in preventing the use of machines having grossly distorted wave-shapes.

On a number of occasions papers and discussions have been presented to the Institute criticising the 10 per cent deviation rule. In two papers presented to the Institute in 1913 (*Wave-Form Distortions and Their Effects on Electrical Apparatus,* by P. M. Lincoln, and *Proposed Wave-Shape Standard,* by C. M. Davis) it was pointed out that the deviation standard has two marked disadvantages in that it sets approximately the same limit for harmonics of all frequencies and is relatively difficult of application.

---

*The author acknowledges the assistance of the other members of the Sub-Committee on Wave Shape Standard. The members of this Sub-committee are: C. A. Adams, Frederick Bedell, L. W. Chubb, A. W. Copley, F. M. Farmer, H. S. Osborne, L. T. Robinson and A. E. Silver.

The discussion of these papers has led to the consideration of wave-shape factors other than the deviation and of more convenient means for determining them. The simplest method so far suggested of determining a wave factor is by devising an electrical network such that the current in one branch resulting from a complex voltage wave at the terminals of the network is an indication of the distortion of the voltage wave in question. The harmonics of the wave may be given a desired relative weighting by using a suitable network.

The wave-shape standard proposed by Mr. Davis in the paper cited above employed for its determination the simplest form of such a network. He suggested that the weighting of the harmonics be proportional to their frequency, and that this weighting be obtained by using as a measure of wave shape the ratio of the admittance of a condenser to the complex wave to the admittance of the condenser to a sine wave of the same frequency. The factor so measured is called the differential distortion factor. This and several other means of measuring wave shape were discussed again before the Institute in 1915 (*Distortion Factors,* by Professor Bedell). The attention which had been directed to the subject led in December, 1915, to the appointment by the Standards Committee of a sub-committee to consider this subject and make recommendations.

Outline of the Work of the Sub-committee. The form of wave-shape standard which was suggested by Mr. Davis has a number of characteristics which recommend it. In the form suggested, however, it would be exceedingly difficult of practical application, largely because of errors in measurement caused by the inductance and resistance of the machines under test and of the measuring instrument. The committee first turned its attention to the possibility of developing a satisfactory standard by simple modifications of that proposed by Mr. Davis formed by adding a series resistance or a resistance and inductance to the condenser which is the measuring circuit in that standard. It was hoped that a satisfactory standard of this sort could be developed that would be so simple that it could be set up from standard coils, condensers and meters available in any well equipped laboratory.

The different methods of measuring wave shape, which under these limitations could be given serious consideration, are discussed in a paper presented to the Institute in 1916 by Professor Bedell reporting the progress of the sub-committee's work (TRANSACTIONS, 1916, page 1154). However, it was the conclusion of the sub-committee, based on a large amount of experimental work,—that whatever form of network might be adopted it would need to be set up as a special piece of apparatus with the constants carefully specified. It was also found necessary to use with the apparatus a highly sensitive indicating instrument, such as a vacuum thermocouple with a direct-current milliammeter or a very sensitive alternating-current dynamometer milliammeter. Moreover, a network of any of the forms discussed in the progress report was felt to be not entirely satisfactory because the readings obtained would not vary rapidly with changes in wave shape and also because the relative penalizing of different harmonics was arbitrarily determined by the constants of the network.

These conclusions led the committee to consider the development of a rating of wave shape or wave-shape factor, unrestricted by the requirement of a simple series arrangement of measuring network, which would give as accurately as practicable a measure of the detrimental effect of any wave, taking into account effects on power systems and inductive effects on communication circuits. To such a factor the harmonics of different frequencies would contribute both in proportion to their magnitude and in proportion to the amount of the disturbance which they would individually cause if of the same magnitude.

The committee has found that but very little definite information is available regarding the relative effect of harmonics of different frequencies on the operation of power systems. For the present any modification of the 10 per cent deviation standard for the purpose of weighting the harmonics with reference to their effect on the operation of power systems would have to be based on general judgment and does not seem to be necessary.

On the other hand when telephone systems come within the inductive influence of power systems more trouble has been experienced, and this trouble bears a definite relation to the frequencies of the harmonics in the wave form. Consequently the committee has collected definite experimental data regarding the relative effect of harmonics of different frequencies in producing interference in telephone systems.

In view of these facts, the committee has proposed as a first step in a revision of the rules regarding wave shape and pending the obtaining of further information, that the present deviation factor be retained for all alternators and that an additional factor be adopted to be applied to alternators which will be used on open-wire * transmission and distribution systems. This additional factor is based entirely on the effect of the harmonics in producing interference in telephone circuits when the power and telephone circuits are parallel. In accordance with this conclusion the sub-committee has developed, and the Standards Committee has adopted for trial use, a new wave-shape factor which is called the telephone-interference factor. No attempt has been made to set values of the telephone-interference factor which should be used as limits, it being considered important that a large amount of data and experience be obtained before values are set. The purpose of this paper is to call attention to the new factor, to tell how it was obtained and to describe in detail the apparatus by which it may be measured so that any one interested can assist the committee in obtaining experience with the use of the new factor and in determining what would be the best limiting values of the factor to be adopted as standard.

New Standardization Rules. In the new standardization rules regarding wave-shape the old rule for maximum deviation remains unchanged except in the wording. The new rules are as follows:

Section 17 (new)

The deviation factor of a wave is the ratio of the maximum difference between corresponding ordinates of the wave and of the equivalent sine wave of equal length to the maximum ordinate of the equivalent sine wave when the waves are superposed in such a way as to make this maximum difference as small as possible. (See Section 406)

Section 406 (new)

A deviation factor, as defined in Section 17, of the terminal voltage wave on open-circuit of all synchronous machinery not exceeding 10 per cent is permissible except when otherwise specified.

The new factor, called a telephone-interference factor, is covered by new sections in the Standards' rules, reading as follows:

---

*By open-wire system is meant one which is not in cable.

Section 17 A (new)

The interference factor of a wave is a measure of its departure from the sinusoidal shape based upon its disturbing effects when acting by induction on other circuits. (See Sections 407 and 408).

Section 407 (new)

The telephone interference factor is the ratio of the square root of the sum of the squares of the weighted values of all the sine-wave components (including in alternating waves both fundamental and harmonics) to the r.m.s. value of the wave.

Section 408 (new) (*For trial only*)

The weighting of the sine-wave components of different frequencies is as given in the following curve. (Fig. 1). The telephone interference factor of a voltage wave, corresponding to this weighting, may be measured by the use of the network shown in the following diagram (Fig. 2). With this network the telephone interference factor is the ratio of the current I in micro-amperes in the meter branch of the network to the voltage E applied to the external terminals of the network. The measurement may be made on the low-tension side of a potential transformer. A sensitive vacuum thermo-couple provided with a shunt, and a direct-current milliammeter have been found convenient for measuring the current.

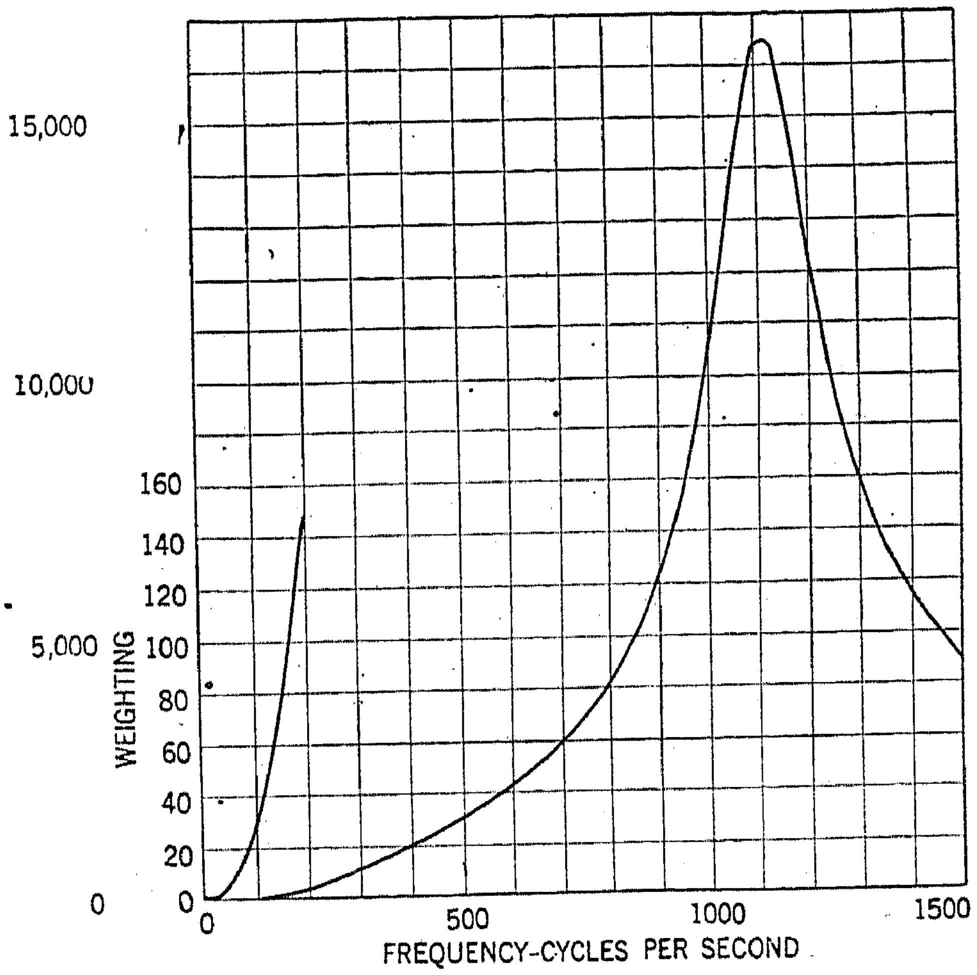


Fig. 1—Frequency-Weighting Curve for Telephone Interference Factor

Section 409 (new)

A telephone interference factor (See Sections 17A and 407) of the open-circuit terminal voltage wave not exceeding * . . . is permissible except when otherwise specified. This rule shall apply in cases where synchronous machinery is required for use in connection with open-wire transmission or distribution circuits and shall be supplementary to 406. The purchaser shall in each case indicate whether or not the conditions are such that this section applies.

---

*The appropriate limiting value of this factor has not yet been determined, but will be recommended after further experience shall have been obtained.

Relation Between Wave Shape and Telephone Interference. The great importance of the harmonics in voltage and current waves of the power system in determining how much interference the power circuit will cause in parallel telephone circuits is well illustrated by the curve given in Rule 408. This curve represents the relative amount of interference caused by voltages of the same magnitude but different frequencies in the power circuit. From the curve it is estimated that at 1100 cycles the interference per volt is approximately 9000 times that at 60 cycles and 60,000 times that at 25 cycles.

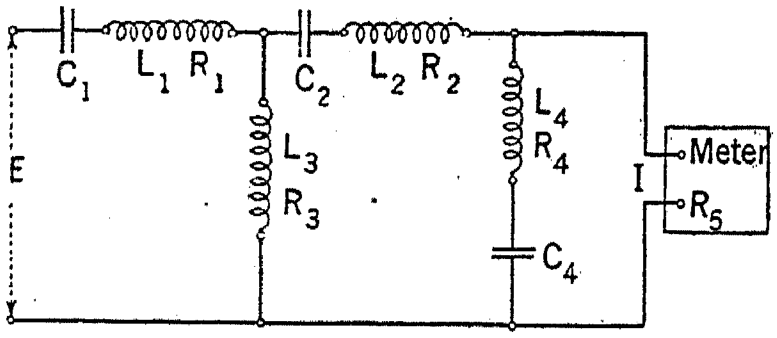


FIG. 2—NETWORK FOR MEASUREMENT OF TELEPHONE INTERFERENCE FACTOR

NETWORK CONSTANTS.

$C_1 = 0.9$ mf.
$C_2 = 0.9$ "
$C_4 = 7.5$ "
± 0.5%

$L_1 = 0.023$ henry
$L_2 = 0.0205$ "
$L_3 = 0.068$ "
$L_4 = 0.019$ "
± 0.5%

$R_1 = 5$ ohms ± 2%
$R_2 = 12$ " ± 2%
$R_3 = 73$ " ± 1%
$R_4 = 22.5$ " ± 2%
$R_5 = 43$ " ± 1%

This means that if the harmonics were entirely absent from a 25- or 60-cycle power system, the problem of inductive interference from the system with telephone service would practically vanish. Although this condition cannot, of course, be obtained, anything which causes an approach to it causes a reduction in the amount of interference under conditions otherwise the same and therefore makes possible a reduction in the expense which is necessary in the plants of the power and telephone systems in order to avoid excessive interference. There is therefore a direct economic advantage from the standpoint of present day conditions in designing and building alternators with as low a telephone interference factor as practicable; provided no material increase in cost is involved.

Determination of the Relative Importance of Frequencies in Telephone Interference. The curve given in new Rule 408 has been developed to represent the variation with frequency of the amount of interference in parallel telephone circuits from a given voltage applied to the power circuit, other conditions being the same. It is based on experimental work carried out under the direction of the chairman of the Sub-committee and which is described in Appendix A. The methods by which these results were obtained may be briefly summarized as follows:

Two methods have been used. In each case known amounts of current of different frequencies have been inserted in the telephone receiver of an ordinary telephone circuit, using for convenience artificial lines. With one method the observers listened to the reading of ordinary news matter over the noisy circuit and then over a quiet circuit of greater length and indicated over which circuit they could understand better. This method, is of course, crude if the results of a single observer be considered and the results obtained by different observers vary widely. However, averaging the results obtained in large numbers of observations the results obtained in two distinct sets of tests check closely. The two sets of tests were made four years apart, with different observers and under somewhat different conditions, and the check of the results obtained indicates strongly that the method is sound.

The second method is based on articulation tests. With this method the goodness of transmission over the telephone circuit is measured by the percentage of syllables spoken over the circuit which are correctly apprehended by the listener. The syllables are arranged in groups which are so selected that each group represents ordinary speech both in the relative number and in the arrangement of the fundamental sounds of which speech is composed. Therefore, the percentage of correctly received syllables is a measure of the degree of intelligibility of the conversations held over the circuit. The percentage of words correctly understood in an actual conversation is, of course, very much higher than that obtained in the articulation tests because the syllables used in the test are without context and, in general, without meaning. The articulation tests made in connection with this experimental work involve 60,000 observations, considering each syllable as an observation.

The articulation tests do not necessarily give a measure of the complete detrimental effect of the noise caused by extraneous currents because in addition to the reduction of intelligibility in the telephone circuit, there is a factor of annoyance to the person using the telephone. It is interesting to note that the relative results obtained by the articulation tests check those obtained by the reading tests for low frequencies, and then with frequencies above 1000 cycles, show progressively less effect. This indicated that the reading tests include in part the factor of annoyance which becomes relatively more important with the high frequencies.

The experimental work which is briefly discussed above gives a curve representing in relative terms the combined effect of varying sensitiveness of the telephone receiver and of the human ear. The shape of the curve depends much, of course, on the constants of the receiver. The receiver used in this work, however, was typical in this respect of all receivers in general use in the country and the curve represents a relation between sensitiveness and frequency of current which will probably be approximately maintained for a number of years.

In developing from this experimental work a curve to be made the basis of a new wave-shape factor, consideration has been given to other elements which help to determine the relative amount of noise caused by generator voltages and currents of different frequencies. The more important of these are outlined below:

a. The current induced in telephone circuits by a given voltage or current in the power circuit is approximately proportional to the frequency. This effect was taken into account in determining the weighting curve.

b. The attenuation of the harmonic components of the current as they are propagated along the power and telephone circuits varies with the frequency. Except in rare instances, this effect is small compared with the others which are taken into account. It was therefore omitted.

c. In some forms of telephone apparatus, there may be appreciable selective action tending to change the relative amount of currents of different frequencies which reach the receiver. Loaded telephone lines generally do not transmit currents above about 2500 cycles, but the effect of these currents is normally inappreciable in the production of noise because of their almost complete absence from the power system. Some forms of telephone apparatus tend to eliminate the currents of low frequencies. These currents have a very small effect on the amount of noise, however, except when present in very large volume. In general, the condition for a good telephone apparatus requires that it transmit with approximately equal efficiency all frequencies important in the reproduction of speech and it is just these frequencies which are important in the production of noise from extraneous sources. The effect of the apparatus is therefore in general small compared with the other factors which have been included and this effect has been neglected in determining the weighting curve.

In accordance with (a) the curve which is presented in Rule 408 was obtained by multiplying the experimental curve of relative interference of currents of different frequencies by a factor proportional to the frequency. As a result of the experimental work which has been done and of the above considerations the committee believes that the curve selected is the best curve for use at this time.

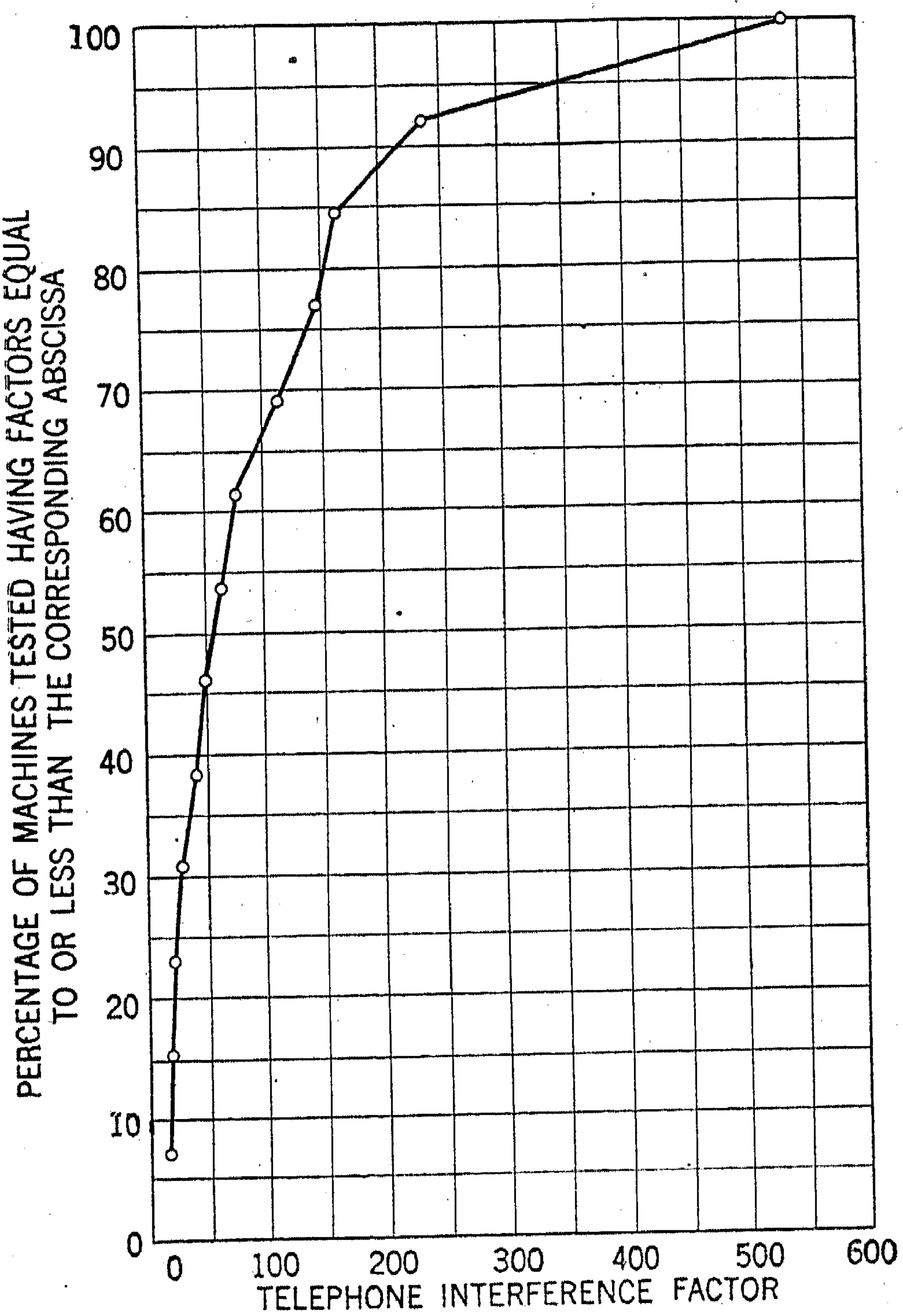


FIG. 3

Description of Apparatus and Method of Use. In order to make the use of the new telephone interference factor a simple matter, the committee has developed apparatus in which the ratio of current through one branch of a network to the voltage applied at its terminals gives a measure of the telephone inter-

TABLE I

RESULTS OF MEASUREMENTS WITH NETWORK

Number	Frequency	Number phase	Volts	Kv-a.	Rev. per min.	Type	Telephone interference factor determined by network	Oscillogram
1	25	3	2,200	6,000	1,500	G	5	
2	60	3	13,200	12,500	1,800	G	11	
3	60	3	600	450	138.5	G	11	
4	60	2	2,300	2,500	3,600	G	11	
5	60	3	13,200	12,500	1,800	G	12	
6	60	3	2,300	3,750	3,600	G	13	
7	25	3	440	800	136.3	G	14	
8	60	3	2,200	925	120	G	14	
9	60	3	13,200	25,000	1,800	G	16	
10	60	2	2,300	35	1,800	G	16	
11	60	3	600	1,250	164	G	16	
12	60	3	6,500	6,250	375	G	16	38
13	60	2	2,600	3,000	600	G	17	
14	60	2	2,400	2,500	1,800	G	18	
15	....	3	2,300	30	1,800	G	18	
16	60	2	2,400	2,500	1,800	G	18	
17	25	3	2,300	50	750	G	18	
18	60	3	13,200	2,350	3,600	G	18	
19	60	2	2,200	180	514	M	18	
20	25	3	2,200	625	500	M	19	
21	30	3	2,200	450.	300	M	19	
22	25	3	6,600	1,440*	500	M	19	9
23	60	2	2,400	625	3,600	G	19	
24	60	3	13,200	25,000	1,800	G	20	
25	60	2	2,300	850	....	G	20	
26	25	3	13,200	9,000	750	G	20	
27	60	2	2,300	850	90	G	21	
28	60	3	2,300	300	1,200	M	22	
29	25	3	2,200	290*	750	M	23	
30	60	3	2,200	625	250	M	23	
31	25	3	13,200	1,800	94	G	24	
32	60	3	13,200	9,000	1,800	G	25	
33	60	3	2,200	6,000	300	M	25	
34	25	3	13,200	1,800	94	G	26	
35	60	3	600	750	1,200	C	26	
36	60	3	13,200	5,000	1,800	G	27	
37	60	1	1,100	187	1,200	G	27	
38	60	3	2,300	935	3,600	G	29	
39	60	2	2,500	500	400	G	30	
40	60	2	2,200	6,000	....	G	30	30
41	60	3	2,300	3,750	300	M	32	
42	60	3	250	1,000	600	C	34	35
43	60	2	2,600	750	90	G	34	
44	60	2	2,400	1,500	1,800	G	35	
45	60	3	7,000	500	100	G	35	
46	60	3	250	1,000	600	C	36	30
47	60	2	2,300	850	90	G	37	
48	60	3	600	450	150	G	38	
49	25	3	390	3,500	300	M	38	
50	50	3	480	400	1,000	C	42	31
51	60	3	6,600	2,500	3,600	G	42	
52	60	3	2,300	100	900	G	44	
53	60	2	2,300	1,000	1,800	G	46	
54	60	2	2,300	1,000	1,800	G	46	
55	60	3	2,200	625	250	M	46	
56	60	3	2,300	150	600	G	48	

TABLE I (*Continued*)

RESULTS OF MEASUREMENTS WITH NETWORK

Number	Frequency	Number phase	Volts	Kv-a.	Rev. per min.	Type	Telephone interference factor determined by network	Oscillogram
57	60	3	240	94	600	G	50	72
58	60	2	2,300	1,000	1,800	G	50	
59	25	3	13,200	5,000	500	G	50	
60	25	3	250	750	750	C	50	
61	60	3	250	300	1,200	C	51	68
62	60	3	250	500	1,200	C	51	23
63	60	3	250	500	1,200	C	51	
64	60	3	250	500	1,200	C	54	54
65	60	3	250	500	1,200	C	54	
66	60	3	250	500	1,200	C	60	
67	60	3	2,300	540	150	G	60	
68	60	3	2,300	50	1,200	G	61	
69	60	3	250	500	1,200	C	61	55
70	60	3	240	500	1,200	C	64	83
71	60	3	250	500	1,200	C	64	
72	60	3	2,300	300	450	G	66	
73	60	3	2,300	500	400	G	66	
74	60	3	3,810	500	720	G	67	54
75	60	3	2,300	150	225	G	67	
76	60	3	240	94	600	G	69	75
77	60	3	1,200	500	1,200	C	70	86
78	60	3	2,400	437	300	G	72	
79	60	1	1,150	30	1,500	G	75	
80	60	3	1,000	50	1,200	G	77	
81	60	....	13,200	20,000	720	G	78	
82	60	3	250	300	1,200	C	80	75
83	60	3	2,200	445*	900	M	83	
84	60	3	250	200	1,200	C	85	106
85	50	3	220	320	187.5	G	92	
86	50	3	2,200	580	750	G	105	
87	60	3	2,300	75	600	G	110	
88	60	3	2,200	225	900	M	111	
89	60	....	13,200	10,000	720	G	113	
90	60	3	600	800	180	G	113	100
91	50	3	5,250	625	....	G	122	139
92	60	3	220	75	77	G	132	140
93	60	3	2,300	1,000	300	G	136	
94	60	3	2,200	4,000	....	G	....	138
95	60	3	13,200	3,000	600	G	140	
96	62.5	3	2,200	2,000	....	G	....	142
97	60	3	250	500	1,200	C	145	57
98	60	3	2,200	225*	900	M	146	146
99	60	3	2,300	325	720	M	147	
100	60	3	2,200	4,000	....	G	....	147
101	60	3	6,600	1,080*	514	M	148	
102	60	3	2,200	225*	900	M	149	177
103	60	3	2,200	225*	900	M	152	
104	60	3	2,200	290*	900	M	153	195
105	60	3	2,200	225*	900	M	155	137
106	60	3	2,200	225*	900	M	156	175
107	60	3	2,200	290*	900	M	160	
108	60	3	2,200	435*	900	M	163	164
109	60	3	2,200	225*	900	M	164	151
110	60	3	2,200	225*	900	M	165	166
111	60	3	2,200	250	300	M	180	

TABLE I (*Continued*)

RESULTS OF MEASUREMENTS WITH NETWORK

Number	Frequency	Number phase	Volts	Kv-a.	Rev. per min.	Type	Telephone interference factor determined by network	Oscillogram
112	60	3	2,200	290*	900	M	181	189
113	60	3	13,200	3,000	720	G	190	
114	60	3	2,200	290*	900	M	191	171
115	60	3	2,200	750	300	M	193	
116	60	3	2,300	1,000	277	G	200	
117	60	3	2,200	225*	900	M	201	139
118	60	3	2,200	225*	900	M	207	221
119	60	3	2,300	300	450	G	230	
120	60	3	240	150	225	G	239	228
121	60	3	480	400	200	G	262	
122	60	3	240	33	1,200	G	299	
123	60	1	1,150	50	1,200	G	305	
124	60	3	600	75	900	G	352	
125	60	3	13,200	1,000	1,200	G	360	
126	60	3	2,200	210*	257	M	375	375
127	60	3	13,200	1,000	1,200	G	430	
128	25	3	11,000	500	1,500	G	450	
129	60	....	13,200	9,000	720	G	490	
130	60	2	1,150	50	900	G	543	

Types of Machine:

G = Generator
M = Synchronous Motor
C = Synchronous Converter
*Rating given in Horse Power.

ference factor. The essential electrical constants of the network are given in the diagram in Rule 406. A detailed description and specification of the apparatus is given in Appendix B to this paper.

The measuring instrument of the apparatus consists of a vacuum thermocouple connected to a direct-current milliammeter. When there are a number of harmonics in the voltage wave applied to the apparatus, this instrument gives a reading proportional to the square root of the sum of the squares of the effects of the individual harmonics. The data which have been obtained regarding the law of combination of the detrimental effects of a number of sources of noise disturbance in a telephone circuit, indicate that this law varies under different extreme conditions. It seems probable that the root-sum-square law best represents the average condition and it is far more practicable for use than any other law because of the characteristics of measuring instruments. It has therefore been adopted.

Summary of Results Obtained with Network. The sub-committee has used this network for measurements of the wave-factor of the open-circuit voltage of a considerable number of machines. Some of the results which have been obtained are given in Table I—Results of Measurements with Network. These results are summarized in the curve of Fig. 3.

In the last column of Table I are given the results obtained in the case of the machines in checking the reading of the network by computing the telephone interference factor from the analysis of oscillograms. In order to obtain this check it is necessary to carry the analysis out to the 25th harmonic, and in some cases higher, in order to include the tooth harmonics. These analyses have shown that the use of the network involves not only much less work but also gives greater accuracy than the computation of the factors from the analysis of the oscillograms.

Approximate analyses which were made in a number of cases show very clearly that in general the machines with high telephone interference factors are machines with large slot harmonics. It is evident that this should be so from the shape of the admittance curve of the network. This is illustrated by the following table which gives the relative amounts of harmonics of different orders which would give the same value of telephone interference factor assuming that one harmonic only in addition to the fundamental is present in the wave.

Harmonic	Approximate relative amount
3	100
5	26.7
7	10.4
9	6.6
11	4.4
13	3.0
15	1.86
17	.96
19	.73
21	1.25
23	1.90
25	2.57

Determination of Limiting Values. The question of what limiting values of telephone-interference factor should be adopted for the construction of new machinery will, of course, be an economic one. There will be a point beyond which further reduction in the telephone-interference factor of machines, would involve an increase in cost greater than would be warranted by the damage caused or greater than the cost of outside-plant methods of obtaining equally effective results. It is proposed by the sub-committee not to recommend any limiting values for use until adequate data shall have been obtained. In the meanwhile, the sub-committee wishes to enlist the cooperation of any who are interested in the work of gathering as much information as possible which can contribute to the result Practicability in application seems to require that the measurements of telephone-interference factor be made with the machines operating on open-circuit. Load on a generator modifies the wave-form and results obtained indicate that in general, the telephone-interference factor of the voltage wave of the power system is less than the no-load factor of the worst machine connected to it.

The sub-committee is particularly anxious to get from the operating engineers data regarding the relation between the telephone-interference factor of the system voltage under the usual load conditions and the factors of the individual generators and motors connected to it, as measured on open circuit. Valuable information can thus be obtained by measuring the wave factors of the generators and other important pieces of rotating machinery on open-circuit and at rated speed, and recording also the measurements of resultant interference factor on the system at regular intervals, and as far as it can be determined, the load conditions of all generators at the time the measurements are made and other conditions on the system which would affect the results. The data submitted should, when possible, include the following information regarding individual machines:

1. Telephone interference factor.
2. Complete name plate data and name of manufacturer.
3. Terminal voltage at time of measurement.
4. Speed at time of measurement.
5. Load condition at time of measurement.

Attention is particularly called to the fact that in many cases the telephone-interference factor varies rapidly with the speed. The speed should therefore be carefully controlled and should be as accurately measured as possible.

The members of the sub-committee will be pleased to hear from anyone who is willing to cooperate in this work, and to supply any desired information regarding apparatus and methods.

Future Work. Future work will involve the collection of the information necessary for recommendations regarding the limiting values of telephone-interference factor to be applied to new alternating-current rotating apparatus. Such values should not be recommended until a large amount of information shall have been obtained and full opportunity given all interested to acquaint themselves with the new factor and to offer comments. Furthermore, the application of the new rules should be so timed that manufacturers will not be inconvenienced in adjusting themselves to them. Future work should include also the collection of any available information regarding the effect of different harmonics on the operation of power systems in order that the desirability of modifying the present deviation factor may be reviewed.

In the present rules, the deviation factor and the telephone interference factor are applied only to synchronous machinery. A complete consideration of the subject would require a study of the effect of other classes of electrical machinery upon the wave forms of the currents and voltages of the systems to which they are connected.

APPENDIX A

RELATIVE INTERFERING EFFECTS OF SINGLE-FREQUENCY CURRENTS IN THE TELEPHONE RECEIVER

The frequency-weighting curve for the telephone-interference factor (Fig. 1) is based upon several experimental investigations to determine the relative interfering effect of single-frequency currents in a telephone receiver. These investigations were conducted at different times, and by different methods, by the Engineering Department of the American Telephone and Telegraph Company. The general principles of the different methods are given in the body of the paper. Details of procedure and data are given in this appendix.

"Reading" Tests of 1914. In the first series of tests* a typical telephone circuit was set up, using artificial cable, and arranged so that different single-frequency currents could be separately introduced in the telephone receiver at the listening end of the circuit, simultaneously with the normal telephone current. By throwing a switch the listener could remove the disturbing current from the circuit and at the same time reduce the volume of sound output from the receiver, the amount of this reduction and the amount of the disturbing current being set by the reader or caller and being unknown to the listener.

---

*Made in 1914 at the request of the (California) Joint Committee on Inductive Interference.

The procedure was for the caller to read over the circuit while the listener switched from the noisy condition to the quiet condition and the reverse, until the latter made his judgment as to which of the two circuits would be preferable for carrying on a conversation. This judgment was signaled to the caller, who then changed the amount of volume-reduction or the noise and the procedure was repeated. By this means, and after an average of about five different settings of reduction in volume, the listener's opinion as to what reduction equaled in effect the given amount of single-frequency disturbing current was indicated. In these tests frequencies of 60, 180, 400, 800, and 1500 cycles were used. Four strengths of disturbing current of each frequency were employed, with an average of eight observers. This involved a total of 160 determinations, each of which was based on an average of five separate judgments, making a total of 800 judgments. The results obtained in these tests indicated a linear relationship between the amount of disturbing current and the volume reduction expressed in terms of miles of cable added to the standard testing circuit. The relative interfering effects of equal currents of the different frequencies employed in these tests are shown by the points marked "X" in Fig. 4.

"Reading" Tests of 1918. The tests made in 1914 were not considered final and in 1918 they were supplemented by further studies. In these latter tests it was felt that particular regard should be paid to 1100 cycles because the resonance point of telephone receivers is in the neighborhood of this frequency. The circuit was arranged, by using an amplifier, so that receivers for a number of observers were connected to the same circuit simultaneously while someone at the calling end switched from the noisy condition to the quiet condition with reduced volume, or the reverse, every 15 seconds. After some experimenting, it was found satisfactory to have the caller read for a total of 2 minutes, thus giving four periods of 15 seconds each for each condition. At the end of this time each observer recorded his judgment as to the relative desirability of the two conditions. For each frequency of interfering current, similar comparisons were made with two callers and for five reductions in volume on the quiet circuit. In this series of tests the comparisons were made for frequencies of 180, 400, 800, 1100, and 1500 cycles with from 8 to 11 observers, involving in all over 1000 judgments. For three of these

frequencies more than one value of current was used but for the other two only one value was used. As the previous tests established a linear relationship between current strength and reduction in volume expressed in miles of standard cable, the data could all be reduced to loss per unit current. The results obtained in this series of tests, together with those obtained in 1914, are represented by curve *A* of Fig. 4. It should be noted that for the frequencies common to the two series of tests, namely 180, 400, 800, and 1500 cyles, the results check very closely.*

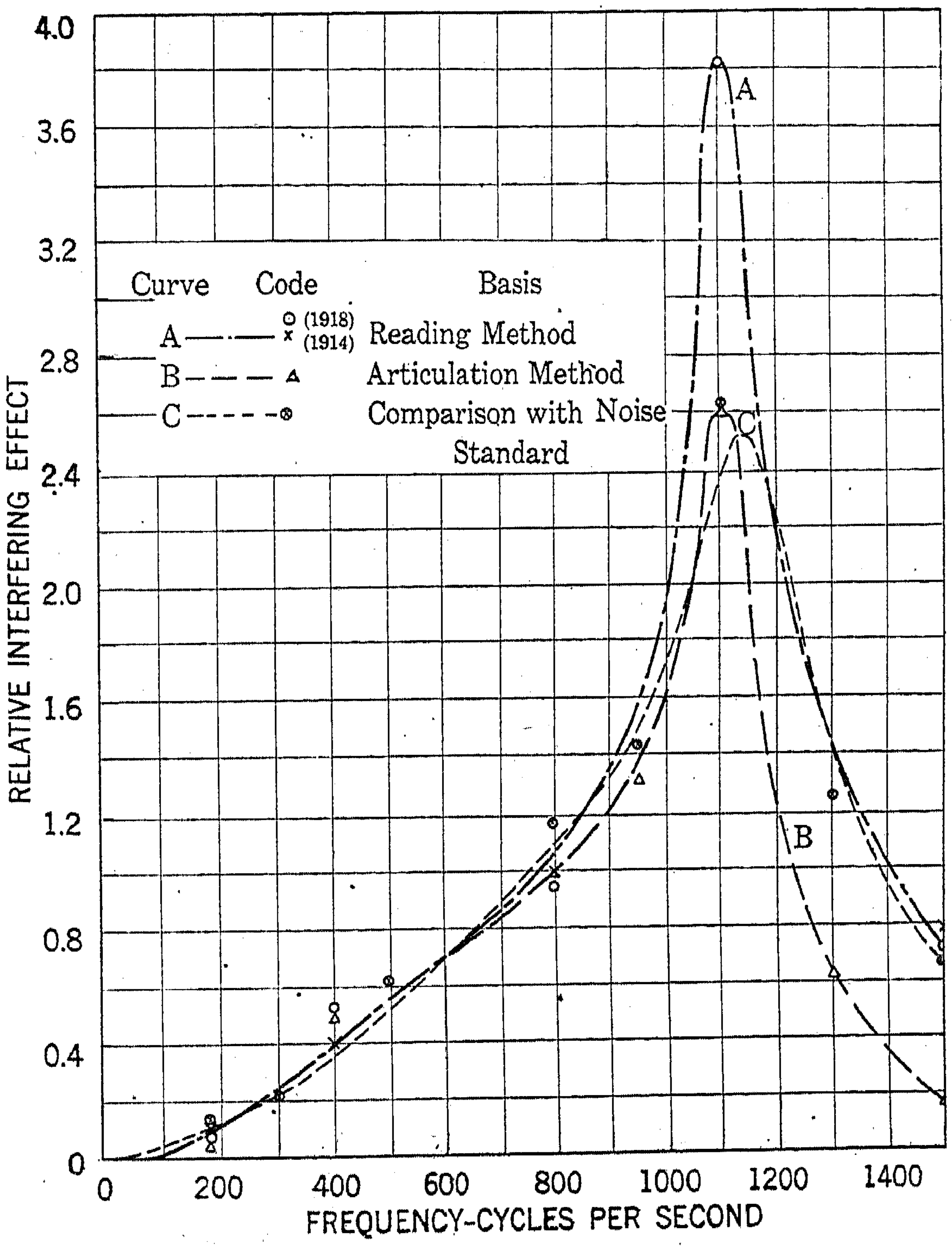


FIG. 4—RELATIVE INTERFERING EFFECT OF SINGLE FREQUENCY CURRENTS IN A TELEPHONE RECEIVER

---

*The preliminary results obtained in 1914 have been presented to the Institute in the form of a curve (TRANSACTIONS, Vol. 34, page 1182, Fig. 4, Curve *C*). Within the precision of the data, this tentative curve agrees with the curve given here below 800 cycles, but is quite different in the range, between 800 and 1500 cycles, in which no observations had been made in the 1914 tests.

"Articulation" Tests. These tests were made by having a caller at the transmitting end of a typical telephone circuit speak into the transmitter lists of syllables which were phonetically balanced in groups of 50, while a number of observers, with receivers connected to the receiving end of the circuit through an amplifier, recorded the sounds which they heard. A comparison of these records with the lists of sounds called shows the percentage of syllables correctly received, and this is taken as the measure of the articulation for the condition of the circuit at the time. Articulation measurements of this sort were made with different single-frequency disturbing currents in the receivers at the listening end of the circuit and also without these disturbing currents but with conditions otherwise the same. Tests of this nature were made for values of disturbing currents of the following frequencies: 180, 400, 800, 950, 1100, 1300, 1500 and 1700 cycles. Two callers and five observers were employed. Using the percentage articulation over the quiet circuit as the basis of reference, the effects of the disturbing currents are expressed as the differences in the percentage articulation for the noisy and quiet circuits. The ratio of these differences for equal values of disturbing currents then gives a measure of the relative effects of currents of different frequencies. Curve *B* of Fig. 4 is a plot of these results.

With the articulation method, tests were made for two different lengths of circuit which gave a variation in the telephone energy delivered to the receivers at the listening end in the ratio of approximately 9:1. The results obtained with the two lengths of circuit varied in their absolute values, but the relative effect of different frequencies was practically the same for both circuits. In the whole series of articulation tests about 270 lists of 50 syllables each were called over the circuits, and about 60,000 syllables were recorded.

"Noise" Comparisons. Since the adoption of a frequency-weighting curve by the sub-committee, some tests have been made to compare the relative noise-producing powers of equal single-frequency currents in the telephone receiver with their relative interfering effects as determined by the methods just described.

In brief, these tests comprised measurements of the "noise" caused by the different single-frequency currents in a telephone receiver using the apparatus ordinarily employed by the telephone companies for measuring the magnitude of the noise on circuits subject to induction. In measuring the noise on a given circuit the observer listens with the receiver connected alternately to the noisy circuit and to a source of standard noise and adjusts the amount of the standard noise until in his opinion the noise from the actual circuit and from the standard would produce equal interference with a conversation if carried on over the circuit. By this method much is left to the judgment of the observer and the results obtained by any one observation can be only approximate. For convenience in this series of tests the method was slightly modified in that the setting of the noise standard shunt was fixed and the observer allowed to vary the magnitude of the single-frequency current until he judged the two of equal interfering effect. Nine observers were used, each making the comparison for

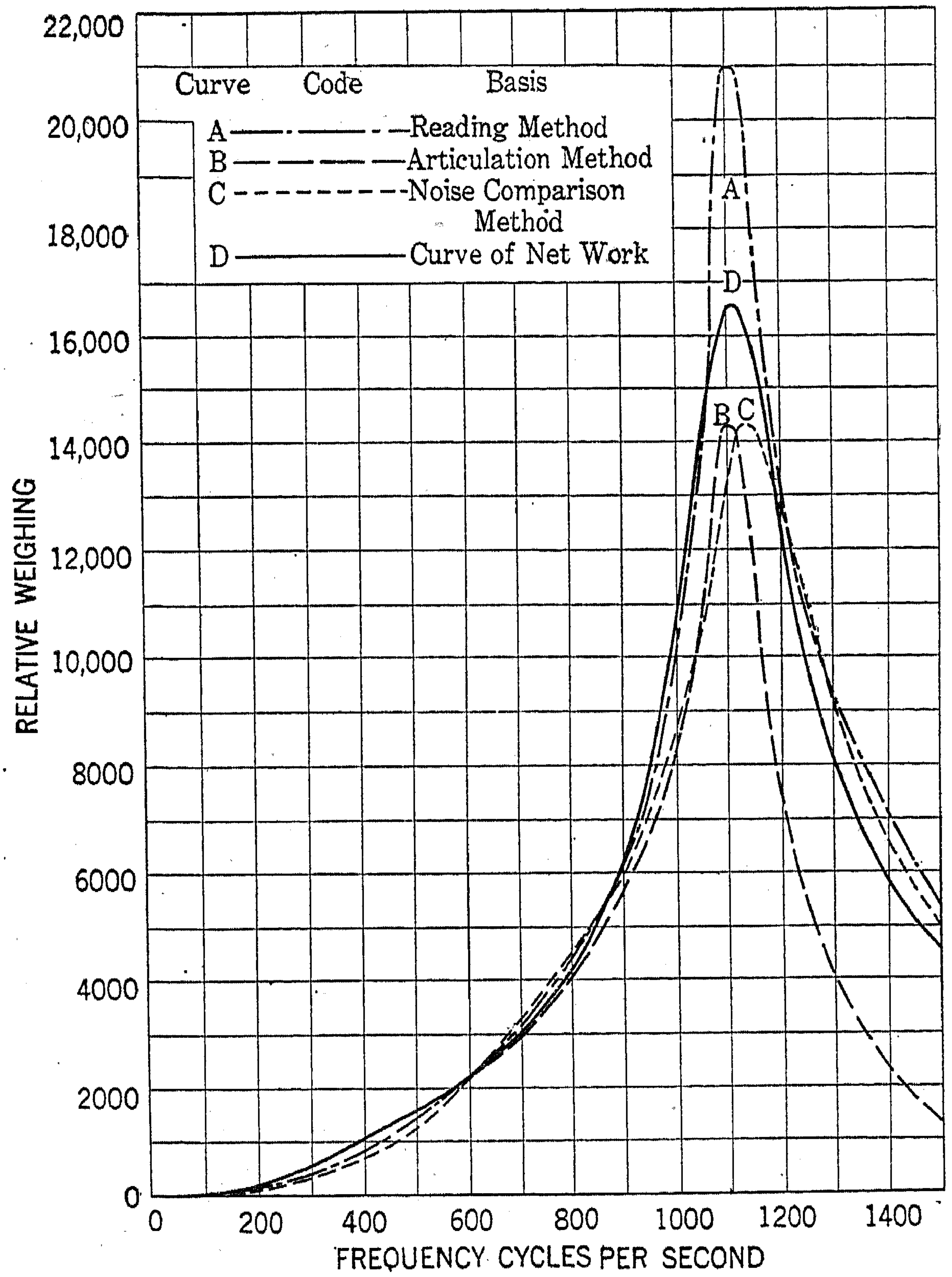


Fig. 5—Comparison of Frequency Weighting Curves

three different values of noise and for single-frequency currents of 180, 300, 500, 800, 950, 1100, 1300 and 1500 cycles per second. The results are shown by curve *C* of Fig. 4.

Comparison of Results. The different methods above described afford relative measures of the effects of different frequencies. In plotting the results thus obtained to a common scale, as shown by Fig. 4, the reduction factors were chosen so as to make the different curves coincide as nearly as practicable through the range of frequencies below 800 cycles. The results obtained in the several tests are in surprisingly good agreement.

To facilitate a comparison of all these results with the frequency-weighting curve given by the network, which was chosen with curves *A* and *B* as a basis, the ordinates of all the curves of Fig. 4 multiplied by a factor proportional to the frequency are used as the ordinates of the corresponding curves of Fig. 5, together with the frequency-weighting curve adopted.

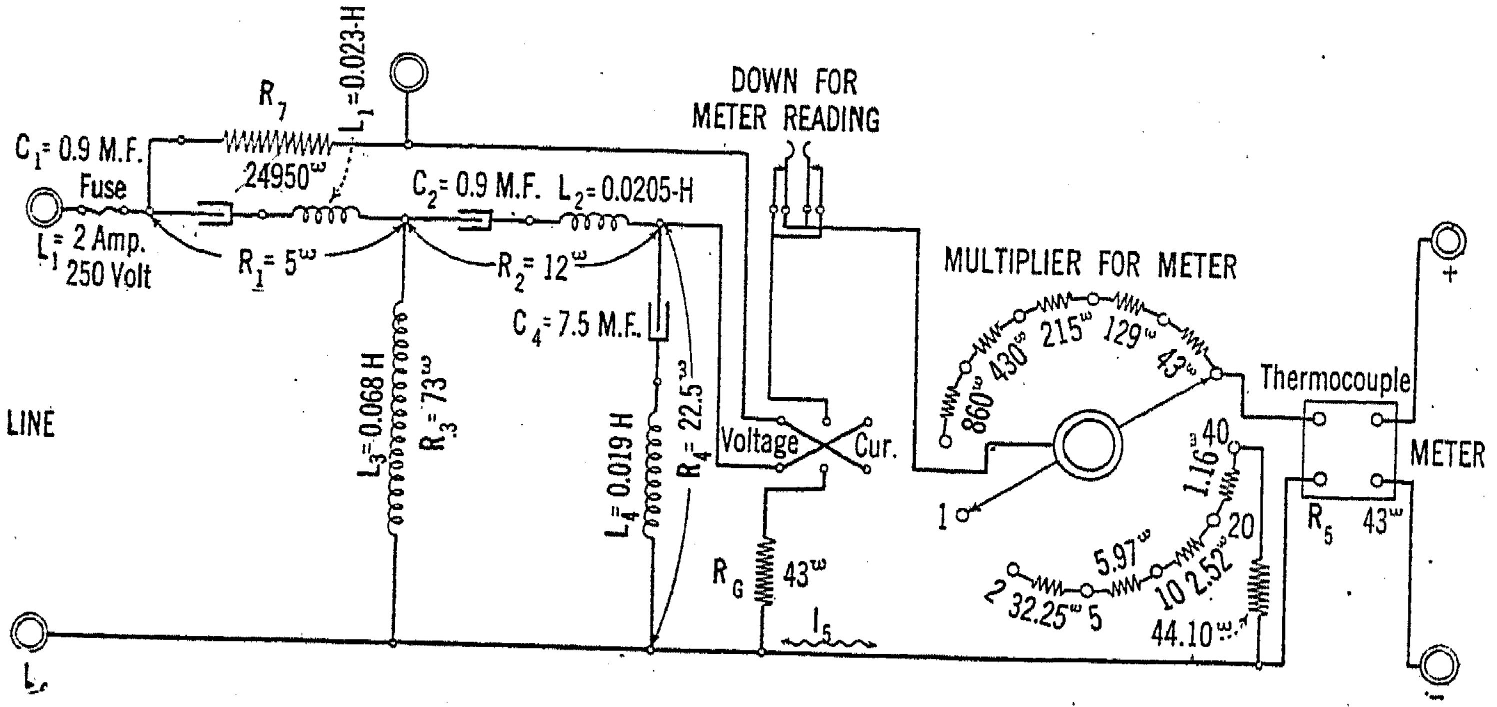


FIG. 6—APPARATUS FOR MEASUREMENT OF TELEPHONE INTERFERENCE FACTOR-DIAGRAM OF CONNECTIONS

APPENDIX B

SPECIFICATIONS OF APPARATUS FOR MEASURING TELEPHONE INTERFERENCE FACTOR

These specifications describe in greater detail the electrical requirements for the network designed to measure the telephone-interference factor which is described in Rule 408 and illustrated in Fig. 2. In addition to the electrical requirements of the essential parts of the network, certain auxiliaries are also described which have been found to be very useful. The network including auxiliaries consists of the following parts: three condensers, four inductance coils, two resistances, one multiplier composed of a six-position dial switch and ten resistances, one vacuum thermocouple, one double-pole double throw switch, one voltmeter contact key and one fuse. These parts are connected electrically as shown in the wiring diagram of Fig. 6.

In a series of fine print notes are given data regarding a network which has been constructed and which satisfactorily meets all of the requirements specified. This network has been designed to give the most satisfactory results at 110 volts as most of the measurements will be made on the low-voltage side of the potential transformers. It may be used however for any voltage not exceeding 700.

Condensers. C_1 should be a mica type condenser having a capacitance of 0.9 microfarad, which should be correct to within 0.5 per cent at 900 cycles. At this frequency the deviation of its phase angle from 90 degrees should not exceed two minutes. The condenser should be rigidly mounted so that its constants will not change, and should be dipped in some suitable moisture-proof compound. The voltage rating of this condenser should equal that of the maximum voltage with which the network is to be used and should include a reasonable factor of safety.

> This condenser consists of alternate sheets of tin-foil and mica securely fastened between brass mounting plates. It is rated to withstand 1000 volts direct. Its outside dimensions are 3¼ by 3⅝ by 3⅞ in. Its capacitance measures 0.901 microfarads at 900 cycles, and its effective resistance 0.09 ohms corresponding to a phase angle deviation of 1.7 minutes.

C_2 is a mica type condenser having the same requirements as C_1 except that the voltage rating need not exceed 500.

> This condenser is constructed similarly to C_1. It is rated to withstand 500 volts direct. Its outside dimensions are 1⅛ by 3⅝ by 3⅞ in. Its capacitance measures 0.898 microfarads at 900 cycles and its effective resistance 0.09 ohms corresponding to a phase angle deviation of 1.7 minutes.

C_4 may be a paper type condenser having a capacity of 7.5 microfarads, and should be correct to within 0.5 per cent at 900 cycles and at a temperature of 25 deg. cent. The deviation of its phase angle from 90 degrees should not exceed 30 minutes at 900 cycles. The condenser should be suitably mounted in a container which will protect it from mechanical injury and which should be sealed with a suitable moisture-proof compound.

This condenser is made up of selected standard rolled paper units of nominal 2 microfarads and 0.5 microfarads sizes. Each unit is mounted in standard sheet metal containers of the following over-all dimensions: 2 microfarads: 1 25/32 by 1 21/32 by 5 in.

0.5 microfarad: 1¾ by 1 7/32 by 4⅞ in. and sealed with suitable moisture proof compound. The total capacitance measures 7.49 microfarads at 900 cycles and the effective resistance 0.12 ohms at 900 cycles and a temperature of 25 deg. cent. This effective resistance corresponds to a phase angle deviation of 18 minutes.

Inductance Coils. All of the four inductance coils should have non-metallic cores. It is quite desirable that one at least should be of the toroidal type, preferably L_3. In specifying the resistance of the coils allowance is made for the effective resistance of the associated condensers.

L_1 should have an inductance of 0.023 henry which should be correct to within 0.5 per cent at 1100 cycles, and an effective resistance of 4.9±0.1 ohms at 1100 cycles. If this coil is constructed of stranded wire it may be adjusted at 900 cycles.

This coil as constructed is of the solenoid type wound with approximately 550 turns of single silk-covered stranded conductor, composed of 25 strands of No. 32 B. & S. gage black enameled copper wire, and has the following approximate dimensions:

Inside diameter ....2 7/12 in.
Outside " ....5 in.
Winding length ....1 ½ in.

The measured value of the inductance is 0.0230 henry and the effective resistance 4.9 ohms at 900 cycles.

L_2 should have an inductance of 0.0205 henry which should be correct to within 0.5 per cent and an effective resistance of 11.9±0.2 ohms at 900 cycles.

This coil as constructed is of the solenoid type wound with approximately 700 turns of No. 24 B. & S. gage double cotton-covered copper wire and has the following approximate dimensions:

Inside diameter ....1 7/16 in.
Outside " ....3 in.
Winding length ....1 in.

The measured value of the inductance is 0.0204 henry and the effective resistance 11.8 ohms at 900 cycles.

L_3 should have an inductance of 0.068 henry which should be correct to within 0.5 per cent and an effective resistance of 73.0±0.5 ohms at 900 cycles.

This coil as constructed is of the toroidal type. It is wound with approximately 4400 turns of No. 27 B. & S. gage double cotton-covered copper wire on a core of rectangular cross-section, having external and internal diameters of 2¾ in. and 1¾ in. respectively and a height of 1 in. The over-all dimensions of the coil are approximately as follows:

Outside diameter	3 5/16 in.
Axial height	1 11/16 in.

The measured value of the inductance is 0.0680 henry and the effective resistance 73.5 ohms at 900 cycles.

L_4 should have an inductance of 0.019 henry which should be correct to within 0.5 per cent and an effective resistance of 22.0±0.5 ohms at 900 cycles.

This coil as constructed is of the solenoid type wound with approximately 700 turns of No. 27 B. & S. gage double cotton-covered copper wire and has the following approximate dimensions.

Inside diameter	1 7/16 in.
Outside "	3 in.
Winding length	1 in.

The measured value of the inductance is 0.0190 henry and the effective resistance 21.9 ohms at 900 cycles.

Resistances. R_6 is provided so that for either position of the double-pole double-throw switch, the impedance of all branches of the network will remain the same. It should be non-inductively wound and have a resistance of 43 ohms which should be correct to within 0.5 ohm. It should have a continuous current carrying capacity of 0.1 ampere.

R_7 is provided as a multiplier for measuring the terminal voltage. It should have a d-c. resistance of 24,950 ohms which should be correct to within 0.5 per cent. Its inductance should not exceed 0.5 henry at 60 cycles. The unit should be designed to withstand the maximum voltage with which the network is to be used, and to dissipate 20 watts intermittently.

This resistance as constructed consists of approximately 2360 feet of No. 37 B. & S. gage double silk covered manganin wire wound in two sections on a wooden spool. The over-all dimensions of the completed coil are as follows:

Outside diameter	1½ in.
Length	4⅛ in.
Winding depth	½ in.

The resistance and inductance of this unit as measured are 24,960 ohms and 0.5 henry respectively.

Multiplier for Meter. This multiplier should consist of suitable shunt and series resistances so arranged that the input resistance will be 43.0±0.2 ohms for all settings and the ratio of its input to output current variable by definite steps so as to give satisfactory meter readings for any current from 0.001 to 0.1 ampere. The actual ratios of input to output currents should not differ by more than 1 per cent from the nominal values marked on the multiplier.

The multiplier as constructed consists of a six-position dial switch, together with 10 non-inductive units having the following resistances; 860, 430, 215, 129, 43, 32.25, 5.97, 2.52, 1.16, and 44.1 ohms. These should be connected as shown in the wiring diagram of Fig. 6.

Meter. The measuring instrument used with this network should consist of a non-inductive meter having a resistance of 43.0±0.2 ohms at all frequencies and all current values. It should measure the effective value of currents having any wave-shape and should, together with the multiplier give satisfactory readings for all currents from 0.001 to 0.1 ampere.

The measuring instrument used consists of a vacuum thermocouple whose heater is composed of material having a very low temperature coefficient, and which has a resistance of 43.0 ohms. The thermocouple is quite substantial in construction and should be able to withstand without a material change in calibration a heater current three times as large as that which gives a full scale deflection on the meter. The meter used with the thermocouple is a d-c. milliammeter having a sensitiveness which will give, with the thermocouple the current range specified.

Switches. The double-pole double-throw switch shown in Fig. 6 should be of such a design that there will be very little possibility of appreciable contact resistance developing as a result of extensive use. The voltmeter contact key should be inserted as shown in Fig. 6. This key will add materially to the protection of the thermocouple in reducing the possibility of overheating due to excessive currents.

Fuse. The fuse should be designed to operate on a voltage equal to the maximum for which the meter is to be used and with a sustained current of not over 2 amperes.

The fuse as used with the instrument has the following rating:

250 volts
2 amperes.

It is of the cartridge type and is mounted inside the box. When the network is used for a voltage higher than that of the

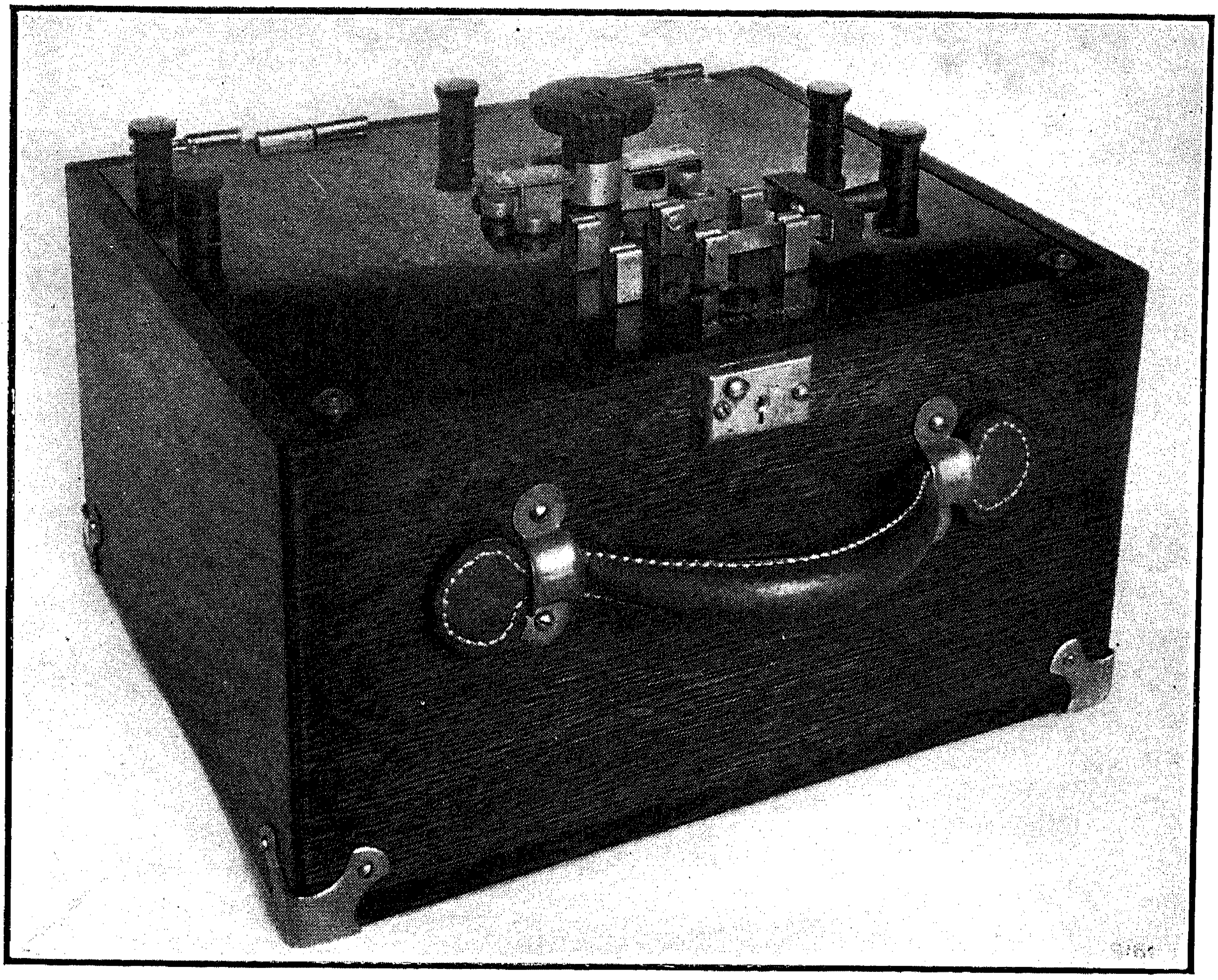


Fig. 8

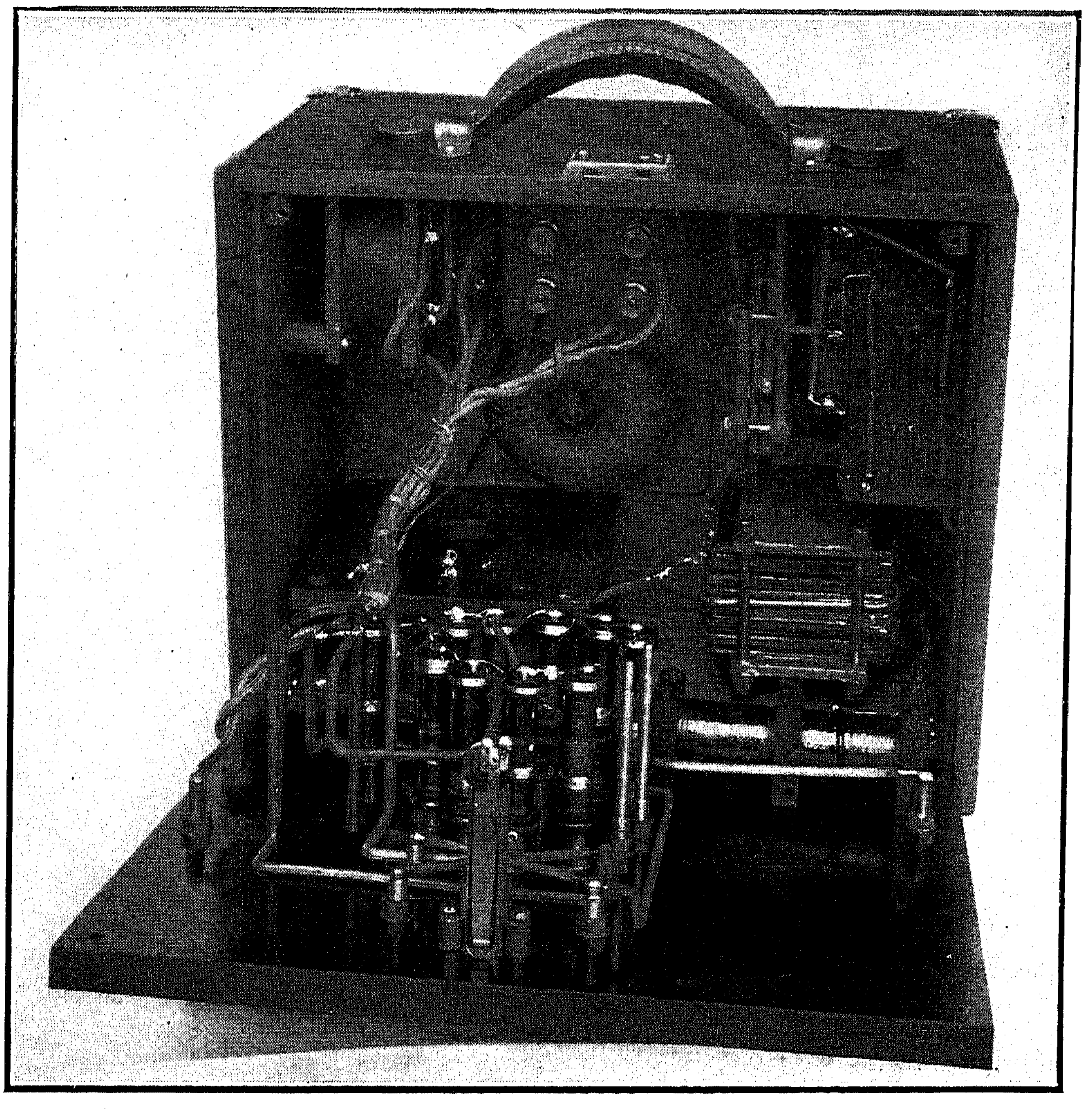


Fig. 9

[OSBORNE]

fuse rating, this fuse should be short-circuited and an external fuse of the proper size used.

Binding Post R. This terminal is provided so that R_7 may be shunted by a resistance connected between R and L_1. This permits the measurement of a lower terminal voltage than would otherwise be possible, which is particularly desirable in calibrating the network. In addition, with the switch in the voltage position, the posts R and L_G provide terminals directly connected to the multiplier. This arrangement is very useful in calibrating the multiplier and the thermocouple.

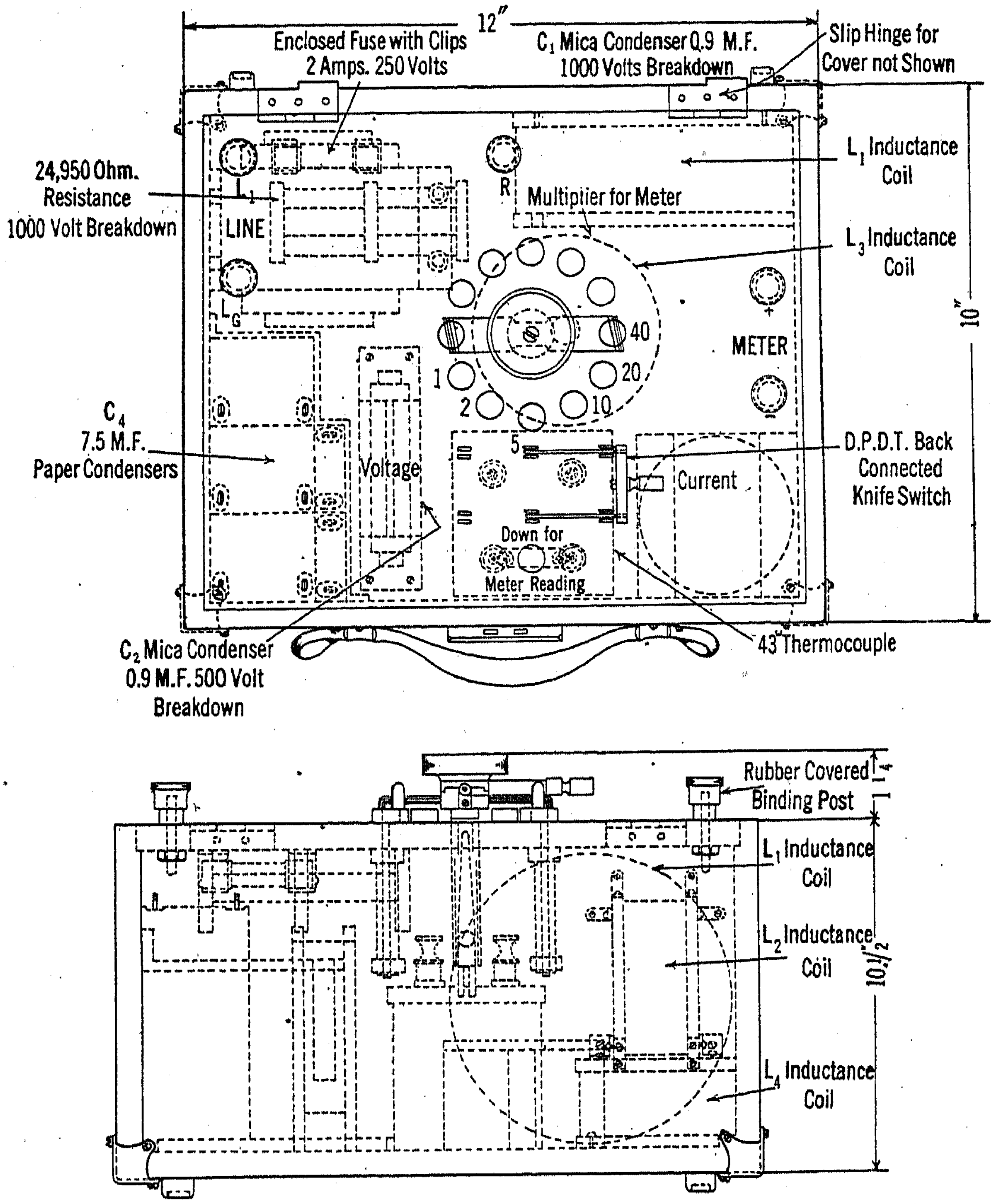

12″
Enclosed Fuse with Clips 2 Amps. 250 Volts
C1 Mica Condenser 0.9 M.F. 1000 Volts Breakdown
Slip Hinge for Cover not Shown
L1 Inductance Coil
24,950 Ohm. Resistance 1000 Volt Breakdown
R
Multiplier for Meter
L1
LINE
L3 Inductance Coil
LG
10″
40
METER
1
20
2
10
C4 7.5 M.F. Paper Condensers
5
Voltage
Current
D.P.D.T. Back Connected Knife Switch
Down for Meter Reading
43 Thermocouple
C2 Mica Condenser 0.9 M.F. 500 Volt Breakdown
Rubber Covered Binding Post
L1 Inductance Coil
L2 Inductance Coil
10 1/2
L4 Inductance Coil

Fig. 7—Portable Apparatus for Measurement of Telephone Interference Factor

Assembly. The parts should be securely mounted in a suitable box which should be as small as possible to make the instrument convenient for field use. All parts should be as accessible as possible. The coils should be arranged so that there is practically no mutual inductance between any two of them and also so that their inductance and effective resistance will not be appreciably increased due to the presence of iron or other metal in their magnetic fields. The wiring should be well insulated and arranged as simply as possible.

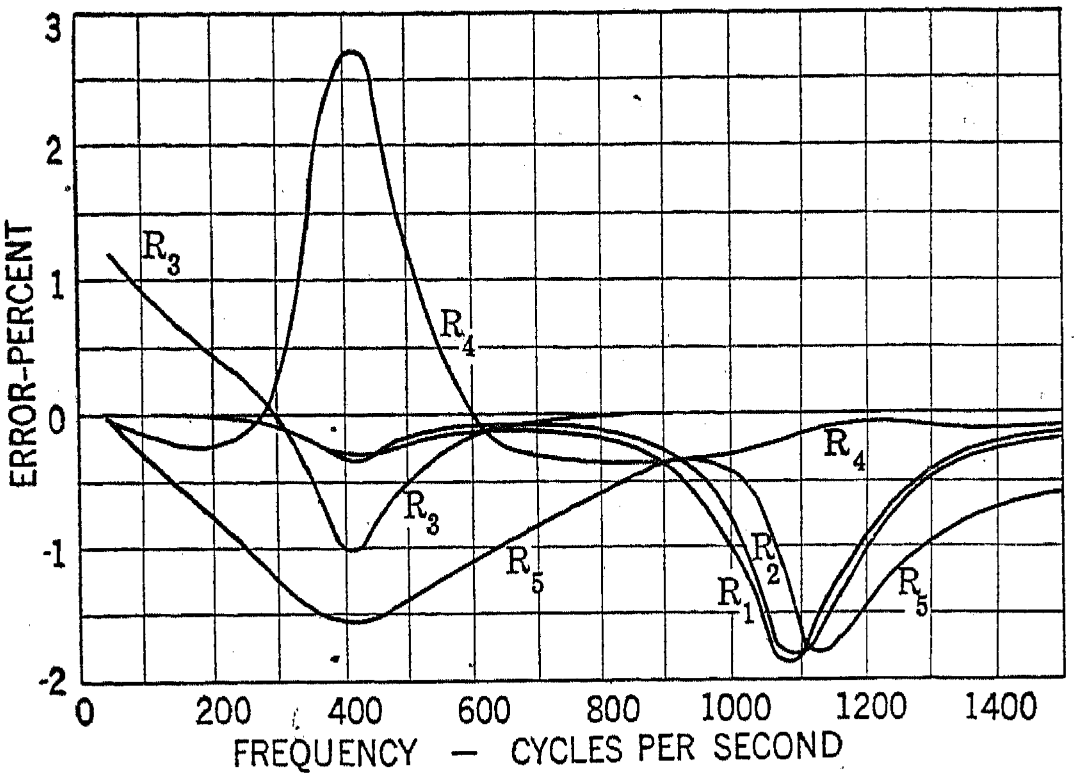


Fig. 10—Effect of a One-ohm Increase in Resistance of Branches of Network

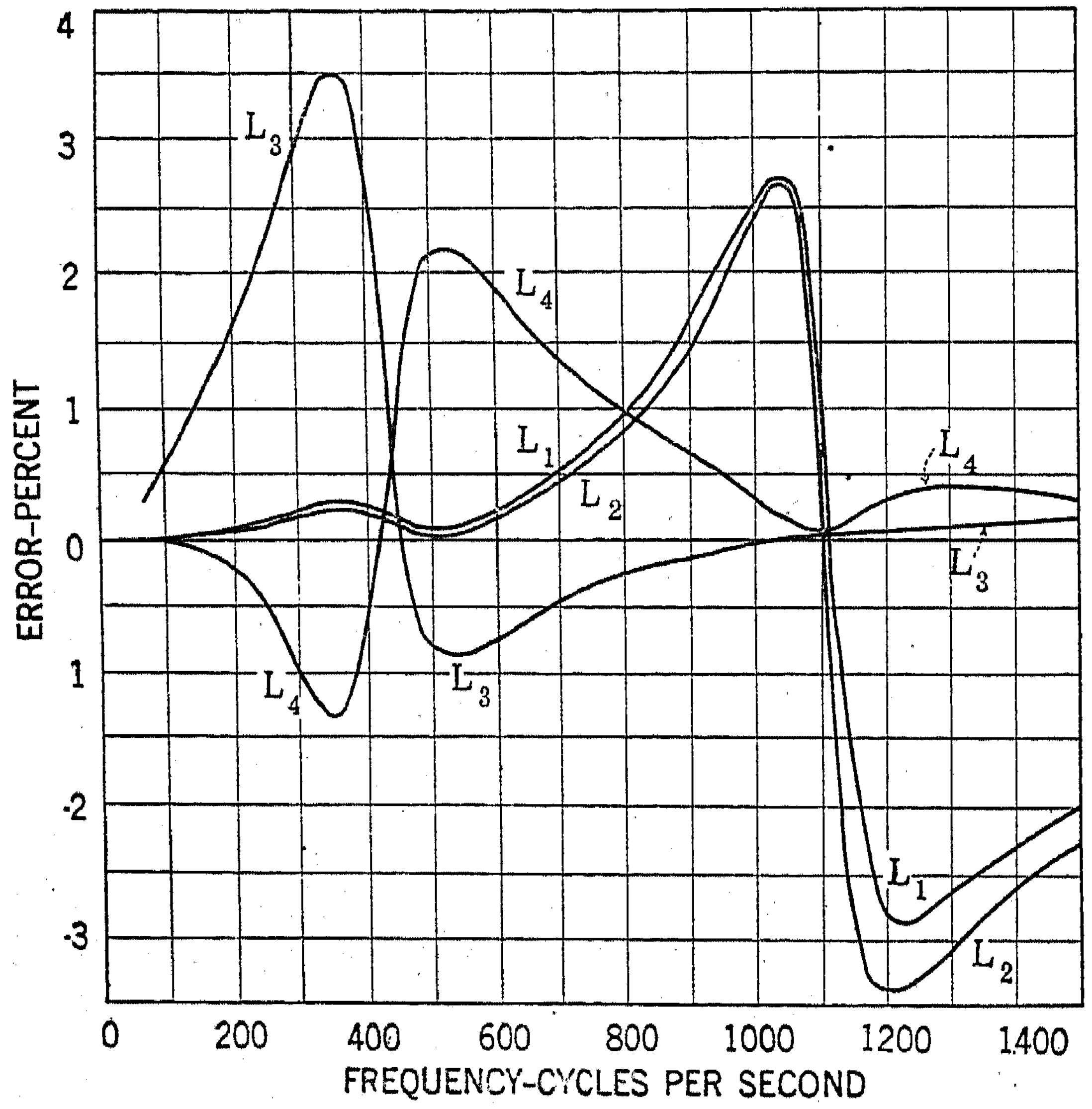


Fig. 11—Effect of a Two-Percent Increase in Inductance of Coils

In Fig. 7 are given working drawings of the apparatus as constructed showing the general arrangement of the parts. All switches are mounted on a hinged top which may be easily lifted to facilitate the inspection of the various parts. The solenoid-type coils are mounted with their axes mutually perpendicular so as to reduce mutual inductance between them. The fourth coil is of the toroidal type which has a negligible stray field. All coils are kept as far as possible from the condensers to reduce any effect which might be produced by their mounting plates or containers. Figs. 8 and 9 are from photographs of the completed apparatus. Its weight is 27 pounds.

Calibration. The network should be calibrated by connecting it to a source of single-frequency alternating current of known and variable frequency. The calibration of the network should include a curve showing the ratio of the current input to the multiplier to the terminal voltage for all frequencies from 0 to 1500 cycles which should agree with the theoretical curve shown in Fig. 1 and described in Section 408 of the Standardization Rules to within 5 per cent at all frequencies. In addition to this frequency-weighting curve the calibration curve of the thermocouple and meter should be given in case the combination is not direct reading. Care should be taken that all higher harmonics are practically eliminated from the testing voltage by the use of filters.

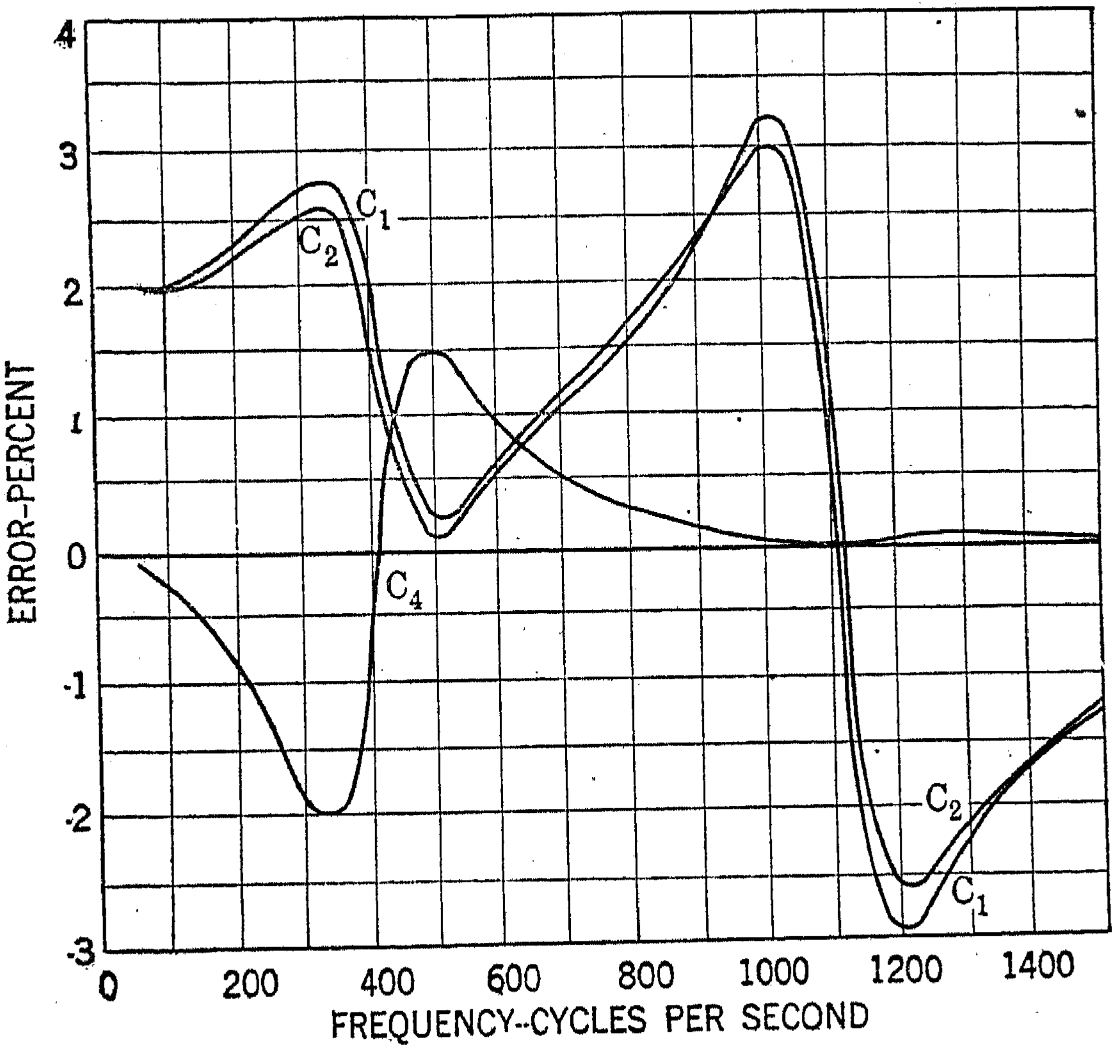


FIG. 12—EFFECT OF A TWO-PER CENT INCREASE IN CAPACITANCE OF CONDENSERS

The following table shows the calibration of the network constructed as described above, together with a comparison with thoretical values.

RATIO OF CURRENT TO TERMINAL VOLTAGE.
MICROAMPERES PER VOLT

Frequency	Theoretical	Actual*	Error per cent.
200	147.8	142.2	3.6
400	998.1	1,010	0.3
600	2,109	2,117	0.4
800	4,143	4,160	0.4
900	6,264	6,340	1.2
1,000	10,530	10,630	0.9
1,100	16,490	16,670	1.1
1,200	12,400	12,350	0.4
1,300	7,983	7,960	0.2
1,500	4,526	4,440	1.8

*At temperature of 21.0 deg. cent.

Effects of Errors in Network Constants. In Fig. 10, 11 and 12 are shown the effects on the measured value of the telephone-interference factor of small changes in the constants of the different elements of the network taken separately. In every case the ordinates represent the percentage increase in the indicated interference factor. In Fig. 10 is shown the effect of an increase of 1 ohm in the resistance of each of the five branches of the network. Fig. 11 shows the effect of a 2 per cent increase in each of the four inductances, and Fig. 12 the effect of a 2 per cent increase in each of the three capacitances. As may be seen from the figures the percentage effect on the interference factor is, in general, less than the percentage error in the element involved.

DISCUSSION ON "THE GENERAL EQUATION OF THE ELECTRIC CIRCUIT-III" (STEINMETZ) AND "REVIEW OF WORK OF SUB-COMMITTEE ON WAVE SHAPE STANDARD OF THE STANDARDS COMMITTEE" (OSBORNE), NEW YORK, N. Y., FEBRUARY 20, 1919.

C. F. Scott: There are certain things in circuits that we want, and as long as we get them and are satisfied with them, well and good, but we often get things we do not want. At first, they are very troublesome. They interfere with operation. Presently, theoretical study as it advances takes up these indefinite and unknown things, and finally it utilizes them and in many cases, a new order of electrical application arises. From the direct current we passed to the alternating current, and from ordinary frequencies to the higher frequencies, each opening up new fields. And the man who gives the advanced theory deals with things which are away beyond present practise and which explains the difficulties and the things not understood, announces what presently becomes the ground work of future text books and the foundation for further advance. It is a paper of this type which Dr. Steinmetz has given us.

Charles R. Underhill: I wish to supplement Prof. Scott's remarks. A few years ago I heard Dr. Milliken state that there was a gap between physics and electrical engineering which needed to be bridged. The physicists give us the so-called fundamental laws, or the theoretical information upon which electrical engineering is based, but this information often has to be interpreted or simplified so that it can be expressed in terms which the electrical engineer can apply to practical design.

For instance, Maxwell gave us an equation for the mechanical force of an electromagnet, but it could not be used by engineers until Dr. S. P. Thompson interpreted it in a form which engineers could use, but even then the results only were approximate.

About eight years ago Dr. Steinmetz gave us a general equation for the electromagnet which is more comprehensive than Maxwell's and which holds in every possible case.

Here, at least, is a case where the engineer has explained a physical phenomenon in a more comprehensive manner than the physicist, and the oftener that physical phenomena are studied by engineers and brought down to engineering standards, the greater will be the benefit to engineers.

A. E. Kennelly: Dr. Steinmetz's paper largely employs logarithmic scales in the illustrations, this is often a great advantage. The method should be aided by standardizing its nomenclature. If, in dealing with any physical magnitude expressed in numbers, we call the common logarithms of those numbers the orders of their magnitude, it would be very helpful in description. Thus a frequency of one million cycles per second would be a frequency of the sixth order. A frequency of 1000 would be of the third order. A frequency of 60 would be of the first order, or more precisely, of the 1.78th order. Again, a precision of the fourth order in measurements of any kind would be to one part in ten thousand.

The paper is of great value in clearly indicating the physical nature and essential relations of radiation resistance. It is perhaps the first paper to set the matter clearly from an engineering point of view. In particular, the paper brings the subject into the fields of electric lighting, power distribution and communication; whereas hitherto, radiation resistance has been regarded as the exclusive property of radio engineering. The paper shows that every pair of wires, such as the going and returning conductor of a single-phase alternating-current circuit inherently possesses radiation resistance; but that in the steady state, this radiation resistance is negligible at ordinary separating distances between the wires, until the frequency is very high. From this point of view, the paper should be widely read and studied.

There is a discrepancy on page 207 which, if its meaning is not misunderstood, should be corrected. Reference is there made to the skin-effect extra resistance of copper conductors of different shapes of cross section. It is intimated that the high-frequency linear resistance of all copper conductors having the same periphery will be the same. This we know is far from being the case. The linear resistance of a copper strap, say 2 mm. thick and 30 mm. wide, is very much greater than that of a copper tube or hollow cylinder of 1 mm. wall thickness and 60 mm. in periphery. This great difference is due to edge effect. The edge effect in many forms of copper conductors gives them much more linear resistance at high frequencies, owing to distorted current-density distribution, than does the simple skin effect.

The paper throws great light upon the rapidly increasing attenuation of very high-frequency or steep-front waves. This difficult subject badly needed elucidation, and we are all indebted to the paper for a relatively easy explanation of the shearing backward of a steep slope, as a vertically fronted wave runs along a conductor.

Dr. C. P. Steinmetz: I am very much gratified to have Dr. Kennelly come out so strongly in favor of the geometric scale, the scale of powers of 10, or logarithmic scale. I have urged this scale for a number of years as the most rational.

It is the scale of one of the most important and oldest human activities,—music. The musical scale is the geometric scale, for the reason that the perception of our senses follows this scale, that is, equal differences in the intensity of perception of any of our senses correspond to equal intervals on the geometric scale.

For this reason the most successful industrial scales have been geometric, as, the Brown & Sharp wire gauge, in which each successive number differs from the preceding one by a constant proportion. This is difficult to do in a merely numerical scale and herein lies the weakness which has always handicapped the merely numerical scales in gages.

As to the question of Dr. Kennelly regarding the skin effect of flat ribbon and round conductor, I refer to this in the beginning of Appendix A, where I say that there is an intermediate range between no skin effect and complete skin effect in which it is difficult to calculate it, but compared with the total range of frequencies of importance in our investigation, this range where there is still complete penetration but a decrease of current density towards the interior, is so small (less than a unit in the logarithmic scale), as to be negligible in its effect on the result, and we can therefore change abruptly from the equation of constant current density, at low frequency, to that of complete skin effect, at high frequency.

J. B. Whitehead: The first question on the Osborne paper which I would like to ask relates to the matter of noise. Mr. Osborne suggests that by the use of telephone receivers noise may be measured quantitatively. Can noise be measured quantitatively, and independently of the frequency, by means of a telephone receiver?

The other question that arises in my mind is with reference to the lower frequencies. For example, the third harmonic of a 25-cycle circuit (75 cycles), is it possible by means of the telephone receiver to measure a frequency as low as that, so that it may be applied in the form of factor that is suggested? And if not, does it not mean that the method will not take account of wave shape variations in the range mentioned?

W. J. Foster: The part of Mr. Osborne's paper that is of special interest to me is Table I. Would the author be willing to tell me whether Nos. 9, 24, 31 and 34 belong to the same company,—a large public utility company. I suspect they do, from the ratings of the machines. If so, it may be interesting to you to know that Nos. 31 and 34 which have a very low telephone interference factor, as determined by this instrument, are machines of very poor potential waves. They are machines which would have an interference factor higher than any given in the table, providing that the weighting was in the neighborhood of about 300 instead of 1140. Those machines would not come within a ten per cent deviation from a sine wave. On the other hand, at the lower end, of the table where the interference factor is 400 and higher I notice a large machine or two that to the eye have good potential waves as they appear on the oscillogram. The trouble, is, that these particular machines were built before any agitation had come up regarding telephone interference, before we knew what periodicities to avoid. The next to the last machine in the table, 9000 kv-a., 720 rev. per min., is a generator that has three slots per phase per pole, which, is a number we must steer clear of in the future in sixty-cycle generators unless we can guard against the slot frequency and neutralize its effect by introducing corrective features into the design.

I am curious to know, and I should like Mr. Osborne to tell us in closing, whether he has come to any conclusion in his own mind, as to what will be satisfactory as far as interference with the telephone is concerned, in the matter of the interference factor, whether it should be placed at 100 or 200. What would he suggest?

In looking at the matter from the standpoint of the designer, I see no reason why in any particular case in the design of a new generator it is not possible to attain a very low factor for the telephone interference factor. At the same time there have been many machines, such as those that appear here at the beginning of the table, with the low telephone interference factors, that have very poor potential waves, and have been in service for many years, and so far as I know they have never caused trouble of any kind. Consequently we want to be a little bit careful, when we establish our telephone interference factors, that some other interests will not come forward and prove a case of interference from some other periodicities than those which interest the telephone people.

John B. Taylor: Mr. Osborne's paper is a fair and open attempt to solve the problem in which we have all been interested for some time, and it is one of the few occasions on which a practical telephone man has stated these technical matters in a way to give information and to invite further discussion.

More specifically, as relates to the Standardization Rules themselves, as they appear in the new edition, I feel that there are points here which are not as they should be. In the first place we have an Interference Factor defined in Sec. 17a, as quoted in the first part of the paper, which states virtually that a sine wave will have an interference factor of zero, and then under that definition of interference factor, we have a Telephone Interference Factor, Sec. 407, which is something altogether different. It is not properly "Wave Shape" though under that heading. The essence of the T. I. F. is not one of wave shape, it is a matter of frequency, pure and simple, and if you read the paper carefully you will find where the matter is explained, not on the basis of shape, but on the basis of frequency.

To make this clear, under the rule referred to, a sine wave of 500 cycles would have a zero interference factor. This wave according to the table would have a telephone interference factor of, in round figures, about 1500. A 1000-cycle perfectly good sine wave would be listed as having no interference factor at all, because it is a sine wave, and yet its telephone interference factor would be 10,000.

Furthermore, in the presentation of the paper there is, I think, confusion all the way through between harmonics and what should be properly frequency. I will not attempt to point out all these spots, but merely cite an instance which occurs at the middle of page 273. The data are tabulated with reference to harmonics, but with nothing to tie them to any frequency. I presume we will find it is based on 60-cycles. The rules should be either given different names or worked differently, because sooner or later there will come up confusion over what is not wave shape but frequency.

From my talks with power transmission engineers, I know they are glad to do what they can to reduce interference, but most of them are not trained telephone men and they feel that the situation is advancing over their heads, so to speak. A better feeling can be brought about and a better working program arranged if in presenting a paper like this or similar papers the telephone data are presented a little more completely and clearly.

For example, the curves of Fig. 4 present test results on the sensibility of telephone receiver to currents of different frequencies and these curves are the basis of the proposed T. I. F. But though the strength of the current was doubtless recorded and the observations made in terms of miles of standard cable, the curves show relative values only and the strength of current employed is not given. If these results are offered as foundation for formal Standardization Rules, numerical values should be given so that the observations may be verified or compared with results of similar investigations.

Attention may be called to the fact that the proposed telephone interference factor rests not on the observed relation between disturbing current of a given value in the telephone circuit and corresponding noise, but on an assumption that induced currents in the telephone apparatus will be greater in direct proportion as the frequency of the harmonics in the the pressure wave at the terminals of the generator are higher.

On this assumption there is left out of consideration the effect of reactance, as the high-frequency ripple must pass through generator, protective reactance, step-up transformer and line; it will be soaked up to some extent by the capacity of the cables and lines; then it must be transferred by induction to the telephone line; the highest frequencies will again be soaked up by capacity of telephone line, and damped by reactance of telephone line and apparatus. Apparently the telephone interference factor curve is not based directly on the observations, but frequency values are all increased in proportion to their frequency, on top of what the tests indicated. Is there justification for increasing the observed values in direct proportion to the frequency employed?

Jesse J. Linebaugh: Mr. Osborne's paper is the most concise expression thus far in the attempt to obtain the quantities known as interference factor and telephone interference factor, but I believe if Mr. Osborne would elaborate on one or two points it will lead to a clearer appreciation of just what the telephone interference factor is supposed to cover. In Appendix A under *Noise Comparisons* Mr. Osborne refers to the "standard noise." Standard noise must have some definition of value as concerns the telephone receiver, and I think if Mr. Osborne would kindly define the standard noise, or give more information as to what makes this a standard noise, that it would help the ordinary engineer to gain a clearer appreciation of what the telephone interference factor is supposed to cover.

In Fig. 5 certain weightings also have been given. I would like to ask Mr. Osborne if he cannot in a few words show how these weightings can be transferred back or referred to noises in the telephone

E. E. F. Creighton: Since this is a report from a subcommittee of the Standards Committee, I wish to refer to the use of the term "open wire" or "open wiring," meaning the wires placed apart. I have always found it difficult to express that condition by the use of a single word. I attempted the use at one time of "proximate conductors" and "non-proximate conductors," but it seems to me if we are obliged to use a compound word in describing this condition, that if we say "near-spaced" and "widely spaced" wires we will avoid the confusion and misunderstanding that will naturally come from calling it "open wire" or "open wiring."

Looking at this problem from the standpoint of electrical protection, I think there are certain parts of the problem that must be solved by each side. In the matter of continuous interference between the power circuit and the telephone circuit it would seem that sooner or later some sort of a shunting condition could be created on the power circuit which would short-circuit the high frequencies near the station and absorb them to a sufficient degree to bring the interference to a workable value in the telephone circuits which parallel the power circuits.

On the other hand there is the problem that seems to have no direct solution at the present time, the question of interference during transient conditions in a lightning storm. It is one thing to have a few little harmless harmonics in the circuit, so far as the power circuit is concerned, a very harmless quantity in the circuit, and it is quite a different thing to have hundreds of thousands of kilovolt-amperes shunted to ground and returned by the earth. That must necessarily throw out the electro-magnetic waves to a very great distance and the telephone circuit must be affected during that period.

Now, on the power side I think it is going to be possible to limit the duration of these disturbances to about one-tenth of a second.

I should like to ask Mr. Osborne what has been done on the telephone side to develop some sort of a protective device which will absorb this relatively great amount of energy during the period of transient troubles on the line? It seems to me that something can be done to produce a protective device for the telephone circuits which for the brief period of one tenth of a second, would neutralize this condition. It seems to me that would be a fairly definite straightforward problem.

Dr. C. P. Steinmetz: This paper gives a very interesting and valuable investigation of the effect of the wave shape, or rather its constituent harmonics, in telephone interferences, but, as pointed out by the author, it is really not a wave-shape standard, and has practically nothing to do with wave-shape standards.

To begin with, it refers only to one feature, and one which, however important it may be, is really an extraneous feature. It has nothing to do with the purposes for which the alternating wave was created, or with its usefulness, or the troubles which it may create,—in short, it is not a wave-shape standard at all, to be considered by the electrical power engineer who desires to accomplish certain results with the greatest efficiency and the least trouble in his circuits.

It was stated that the old standard of less than 10 per cent deviation from sine shape has proven satisfactory. I beg to disagree with that, and I believe that the reason why no more trouble occurs is rather due to the development of the industry, which required very much larger machines, where designing conditions have led to a more complete sine shape by the use of distributed and fractional pitch windings, and so have eliminated inherently, without intention and without specification, any very great deviation from sine shape.

This I believe has saved the situation and in power engineering there seems no immediate need in most cases of wave-shape standards. It almost begins to look as if power engineering is not yet in the position to consider a wave-shape standard. Such a standard must deal, not with one particular feature, but with all features which come into consideration.

We might very quickly choose a standard by considering one feature only, specifying for example, that the maximum value of the actual wave must not exceed that of the equivalent sine wave by more than x per cent. This would mean that the insulation stresses produced by such wave would not be greater than those of the sine wave by more than an increase of voltage of x per cent.

Or, we may specify that the mean value of the actual wave must not exceed that of the equivalent sine wave by more than x per cent. This means that the energy losses in the magnetic circuit, excited by waves of this standard, would not exceed those of the sine wave by more than 1.6 x per cent.

Or, we may specify that the amplitude, in per cent, of the third harmonic and all its multiples must not be more than $3x$

times the percentage of internal reactance of the circuit, which would mean that the triple frequency circulating current in the system would not be more than $1/x$ of the full-load current.

Or, we may make specifications that the generator circuit must not be capable of resonating with frequencies above 10 kilo-cycles, and various other specifications which would only be of importance with regard to one particular application or use of the alternating circuit, but none of these is a complete wave-shape standard.

It may be that instead of considering a general wave-shape standard, for the next few years anyhow, the sub-committee of the Standards Committee which is dealing with these matters might be of more utility to the industry by studying, individually, the various conditions imposed by insulation, by energy losses in alternating circuits, by resonance, by triple frequency, and clear up each of these features in the same way and with the same completeness as they have cleared up that one feature of the telephone interference in the paper presented at this time, and so, instead of dealing further with the general wave-shape specification, give us information on the liability to troubles owing to various conditions, and the method of avoiding them, and the limitations which might be imposed in the individual conditions to overcome them.

Now, after all, when you come back to the problem, we must not forget that while we consider the sine wave as the standard, this standardization of the sine wave is, to a very much larger extent than we usually appreciate, based on sentiment and not on actual experience. There is almost no feature in which the sine wave is the preferred wave. It seems, and at least sentiment tells us so, that all around, it is the best wave, but there is more sentiment in that than we usually concede to ourselves.

A. E. Kennelly: This paper seems to proclaim two things. First it calls for non-interference with the frequency of 1100 cycles per second. A glance at Figs. 4 and 5 shows that the observed and experimental telephonic disturbance curve appears to have a sharp maximum near to this frequency. For this reason the telephone engineer requests complete relinquishment of that frequency in his favor, and that if any harmonics of this frequency are accidentally generated in neighboring power systems, that they shall be ruthlessly suppressed. Although there is a sharp peak in the interference curve near that frequency; yet the curve is not a mere vertical belt at this region. It descends in each direction to frequencies of the first or second order on the low side, and to frequencies of the 3.6th order on the high side; so that, in asking for free right of way on that peak frequency, the telephone engineers make a plea for operating as far away from the peak in both directions as may be practicable.

Secondly, and in accordance with the same suggestion, the paper seems to indicate that the prevalent notion of telephony being evenly carried on over a wide spectrum or range of electric frequencies is inaccurate. On the contrary, Figs. 4 and 5 suggest that electric telephony is mainly carried on at one frequency—the 1100-cycle frequency here so prominently displayed,—which is near to the resonant frequency of the diaphragm in the standard telephone receiver. I should like to inquire whether the experimental curve or mean curve of Fig. 4, is not approximately a simple resonance curve of the telephone receiver diaphragm to an alternating current of constant strength, but of adjustably varied frequency. It would seem as if telephony were mainly carried on at the resonant frequency of the diaphragms. Of course, all of the frequencies in vocal tones must be reproduced, within recognizable limits, by a receiver diaphragm; but these may be due to mere overtones or changes in nodal lines. The electric frequencies giving rise to these modulations of diaphragm vibration, may be very feeble relative to the dominant frequency of mechanical resonance, so that they may be swamped by the amplitude of resonant frequency, in the amplitude frequency curve.

If these ideas are even only roughly correct, they mean that the telephone engineers are justified in coming to the general body of electrical engineers and in bespeaking reservation for their special third-order frequency with as wide a margin of reservation as may be practicable. On the other hand, their brother electrical engineers are justified in saying that they will try to keep off this reservation, but that the telephone engineers in return should try to keep within the prescribed limits, lest after some years, when the alternating-current power generators may have been carefully designed to avoid reservation, the telephone receivers may be found to have preempted some new frequency.

L. W. Chubb: I will add a word to Mr. Foster's remarks. The designing engineer today in working up larger machines has practically a new design for each large machine. Now, if he knows what frequencies give trouble, he can hold the voltage wave shapes, with very little extra trouble, to quite reasonable limits of telephone interference factor.

There is one thing to be remembered, however, that is, the manufacturing company in order to give the customer quick and good service and to use tools, dies, etc. economically, must combine standard rotor dies and pole dies in different combinations for different ratings. This necessarily puts the small and intermediate machines in a class where the combination of existing parts may cause rather high telephone interference factors. If each new rating had to have a new pole, a new armature die, corresponding windings and dampers, it would be quite difficult to manufacture the various ratings economically.

I was interested in what Dr. Steinmetz just said about the other considerations and limitations for wave-shape standard.

Mr. Taylor's criticism of the general rule of interference factor is well founded, as frequency and not shape is considered in the specific definition of Telephone Interference Factor. The other considerations and limitations of shape mentioned by Dr. Steinmetz have a definite bearing on the work of the subcommittee. He spoke of the effects of wave shape on insulation stresses and iron losses. Such things have been considered by the committee and the only practicable thing seems to be to base tests on the pure sine wave and later if necessary place limits on crest factor and form factor. A distorted wave in a grounded system especially hard on the insulation or causing low core losses sent through a star-delta transformer changes from a peaked to a flat wave, giving the opposite conditions. Similar but more pronounced effects are produced by transforming by the Scott connection and it therefore seems impracticable to specify limits of crest factor and form factor unless all attending circumstances are given.

C. A. Adams: I understand that this is merely a discussion of telephone interference factor and not broadly a discussion of wave shape. The basis of the fears expressed or implied by Mr. Foster is not such as to give any real cause for uneasiness. Open-circuit voltages only are being discussed. Fortunately, under most operating conditions, most of these higher harmonics are largely absorbed by the reactance of the machines and transformers.

The committee intends later to extend its work into the field of load conditions; but that is a task of some magnitude and will take a long time to accomplish.

L. T. Robinson: When you have a certain condition to meet, the only thing to do, in a practical way, is to do what this sub-committee did, to find out what the trouble is, what you can do to find a cure for it, and leave the more complicated questions to be taken care of later on.

Apparently there is only one earth for us both to use and power must be transmitted and communication must be successfully undertaken. Therefore, we have to find some way to get along, and I think what has been started is a very definite step which will lead to the right end.

As President Adams said, there must necessarily be a good deal more to do, there is interference with telephone systems from direct current and other things in connection with power systems, aside from the machines, so that ultimately we must come to some basis where we can determine in advance what the disturbing factors of the system, as a whole, will be. That is, we can specify the machinery, and perhaps we can specify other things to avoid difficulties, and we finally, I hope can get into a position where we can predetermine what the disturbing effect of the whole system will be and overcome it more or less completely.

During the progress of this work, and as it nears completion, it will be undoubtedly possible to expand the general subject until we have more complete standards; that is, I feel that before we get through we should reach out and take in the telephone side more, that is, take into account the thing that is disturbed as well as the thing that disturbs. Without knowing what form it will finally take, I can only outline at this time what seems to be an ideal condition, and that is that we have some fairly definite idea of what the disturbing effect of a power line is, expressed in terms of this telephone interference factor, as applying to the power system as a whole, rather than to the machines alone, and to correspond with that, some form of standard for the telephone system, as well.

We are becoming more familiar with the difficulties involved in the general question of inductive interference and when we have progressed further with the work and thought enough about the general solution in the light of results obtained I feel that there should be no difficulty in agreeing on reasonable requests that should be imposed on both the power and communication circuits.

With regard to the wave shape standard for alternating-current generators I think the speakers have justified the position that the sub-committee took, that we should not do anything with this standard just now, and that consideration of the subject should be deferred to a later date. There is little to indicate that any change is needed.

C. F. Harding: (communicated after adjournment.) Although the measurement proposed in Mr. Osborne's paper may leave much to be desired as a general standard of wave shape, because of the fact that it confines the standardization to but one of the several effects of distorted wave form operating under no-load conditions only, the data which might be derived from the interference tests made throughout the country with apparatus similar to that proposed might prove very enlightening.

It occurred to the writer that the various University Engineering Experiment Stations, or even the A. I. E. E. Branches might be interested in making the desired tests in their respective localities, thereby accumulating a vast amount of valuable data upon the interference problem. In the case of Institute Branches, the student members, under the direction of their instructors for thesis work or as a laboratory test, might investigate conditions in several neighboring central stations. If this were done however, it would be advisable to accumulate a larger amount of data than that indicated in the paper in order that interference factors under various conditions of load might be determined. Little advance can be made if standards are confined to no-load conditions, when the disturbing wave is ordinarily that resulting from a loaded generator.

Referring now to the design of the equivalent branched circuit proposed in the paper, it is difficult to understand why the accuracy of the apparatus, Fig. 6, is limited by the use of a paper condenser at C_4. Some recent tests carried on at Purdue University upon commercial paper condensers indicate not only a variation in capacity in different condensers of the same make and rating, but a variation of from 10 to 15 per cent with different methods of measurement. The capacity was also found to vary with change of frequency and temperature. Reference to Circular No. 36 of the Bureau of Standards indicates paper condenser variation in capacity of 5 per cent, phase angle of one degree and power factor over a ratio of six to one with frequencies between 100 and 2900 cycles.

The paper further points out that the error in the final factors varies approximately with the error in the capacity C_4, at critical frequencies. To substitute a paper condenser for one of mica under these conditions for variable frequency work would seem to be unduly sacrificing accuracy for a slight gain in the economy and compactness of the instrument.

Harold S. Osborne: Professor Whitehead suggests that the proposed method is at fault because currents of low frequency, such as 75 cycles, cannot well be measured with a telephone receiver. However, with this method the current is measured by a vacuum thermocouple, not by a receiver. With the proposed network very little current of 75-cycle frequency will be caused to flow through the measuring instrument, unless the impressed voltage is relatively large. This is because 75-cycle voltage has very little effect in making telephone circuits noisy, compared with higher frequencies. The deflection of the meter is, for each frequency, proportional to the noise effect.

Professor Whitehead asks whether noise can be measured quantitatively by means of a telephone receiver. If he means measurement of the energy content and wave shape of acoustic waves, rather than their psychological or physiological effect, the answer is, "Yes." The procedure consists in using the receiver as a transmitter and measuring the electrical energy delivered by it. In an ordinary telephone receiver, the response varies greatly with frequency. The response at each frequency can be accurately determined, however, and the instrument is, therefore, satisfactory for measuring single-frequency sustained tones. With the complex wave shapes which correspond to what are commonly called "noises," it is much more satisfactory to use telephone apparatus having an approximately uniform response for the range of frequency under investigation. One instrument which is very valuable for work of this sort is described in a paper published in the "*Physical Review*," July, 1917, by E. C. Wente, entitled, A" Condenser Transmitter as a Uniformly Sensitive Instrument for the Absolute Measurement of Sound Intensity."

Mr. Foster asks whether in Table I machines numbered 9, 24, 31 and 34 do not belong to the same company. I can check the fact that machines Nos. 9, 31 and 34 do belong to the same company. Machine No. 24, however, was tested in the factory, and I have no record of its destination.

Mr. Foster asks whether the author has come to any conclusion regarding satisfactory values of telephone interference factor. He has not. There does not seem any probability that it will prove economical to establish limiting figures low enough to eliminate all interference with telephone circuits, even in severe cases of parallelism. In so far as the telephone interference factor can be reduced, it will reduce the number of cases of interference and the amount of outside plant expense necessary to avoid interference in cases where telephone and power lines must parallel each other. The limiting values should be set at the point giving an economical balance between cost of generators and expense in outside plant of power and telephone companies, insofar as it is practicable to determine the balance point. It is the effort of the sub-committee now to gather information with this in view.

Mr. Taylor is correct in assuming that the table on page 273 is based on a 60-cycle frequency. He is mistaken, however, in stating that the data of Fig. 4 are expressed in miles of cable. As marked on the figure, these data are expressed in relative amount of interfering effect of the same amount of current of different frequencies. The method by which the interfering effect was measured is described for each curve. Mr. Taylor suggests that in determining the shape of the weighted curve factors should have been used other than those selected by the Committee. This matter was carefully considered by the Committee, and a discussion of various factors included and omitted is given on pages 268 and 269 of the paper. It is, of course, erroneous to think of 1100-cycle current as being soaked up by the capacity of the telephone line and damped by the reactance of the telephone apparatus. Eleven hundred cycles is in the most important part of the band of frequencies which must be transmitted by telephone circuits and apparatus, and they are designed to be as efficient as possible for this frequency and for the entire band between 300 and 2000 or more.

Mr. Linebaugh asks for a definition of standard noise. It is pointed out in the paper that this term is used to refer simply to an arbitrary unit of noise which is in use by the telephone companies. This unit is the noise caused in a telephone receiver by a given amount of current of a given wave shape. The wave shape is made very complex so that the noise may as nearly as possible resemble that generally heard over telephone circuits. Its complete description would require details of wave shape in which I suppose Mr. Linebaugh is not here interested. Mr. Linebaugh's question about the relation between weightings and noise is answered above in connection with Mr. Taylor's discussion.

It would seem that Mr. Creighton has misinterpreted the meaning intended to be given to the term, "open wire" in this paper. It does not refer to proximity or lack of proximity between the wires, but is a telephone term for non-insulated wires strung on insulators, as contrasted with insulated wires in a cable.

Mr. Creighton asks what has been done to develop a protective device for the telephone lines, which will absorb the relatively great amount of energy transferred to the telephone lines during the period of transient trouble from the power service. A device of this sort must, of course, differentiate between the induced currents and the telephone voice currents, which are themselves transient and very rapidly varying and which must not be absorbed by the device. The basis for differentiating is voltage; in other words, the device must absorb high voltages and not affect low voltages. Very careful study has been given to this problem, and there have long been in use on telephone lines various forms of "open space cutout" which perform this function very satisfactorily. When high voltages are produced these devices ground the line. The difficulty remains that in so doing, they necessarily throw the line out of service for the time being. Furthermore, at the instant of grounding the line there is often produced a sharp rush of current through the telephone receiver, which produces very severe acoustic shocks to the operator or subscriber using the telephone. There are numerous cases on record of very serious injuries to people by these acoustic shocks. The problem of avoiding them under such circumstances is still unsolved.

Dr. Steinmetz gives an interesting list of a number of important effects dependent upon wave shape. The fact that the telephone interference factor is concerned with a single effect of wave shape rather than all effects of wave shapes does not in the opinion of the Committee invalidate its use as a wave shape standard. Of course, it is not a complete standard. Even in the present rules one other standard, namely the deviation factor, is provided and I think that a complete consideration of the matter would require the establishment of a series of standards, as suggested by Dr. Steinmetz. The telephone interference factor does, of course, depend upon both wave shape and frequency, but this would be true of many wave shape standards set up on the basis of particular effects of wave shapes.

I believe there is something much stronger than sentiment which leads to the adoption of the sine wave as the standard of wave shape. In a circuit containing appreciable inductance and capacity, the sine wave is the only alternating current wave which does not change in wave shape. A periodic wave of complex form could be analyzed into a series of triangular waves, of flat topped waves, or waves of practically any form desired. In studying the propagation of a complex wave over an electric circuit, however, it is practically necessary to resolve it into sine waves because of their property of distortionless transmission. The propagation of each of these sine wave components can be obtained by relatively very simple mathematical treatment, and their recombination at the end of the circuit gives the form, amplitude, and phase of the transmitted wave.

Professor Kennelly asks whether the curve of Fig. 4 is not approximately a simple resonance curve of the telephone diaphragm. It is not. The form of this curve is very considerably different from a curve showing the response of the telephone receiver to currents of different frequencies. This is naturally the case, because the interfering effect on telephone conversations of a disturbing alternating current is not a simple linear function of the intensity of the sound which it produces, but varies in a complicated way with the intensity and frequency of the disturbing noise. Professor Kennelly is mistaken in the impression that the form of the curves of Fig. 4 indicates that telephone conversation is carried on mainly at one frequency. If only a narrow band on each side of the 1100 cycle frequency were transmitted, telephone conversations would be absolutely unintelligible. There is no close relation between the form of this curve and the curve showing the relative importance for the clearness of speech of the transmission of currents of different frequencies.

Professor Harding's discussion of paper condensers indicates that it is important, if paper condensers are to be used in the network designed for measuring telephone interference factor, to be very careful in the selection of these condensers. The condensers used in the models which have been made up have no such variations in capacity as were noted by Professor Harding. The variation of capacity with frequency between 300 and 3000 cycles is in these condensers less than 1 per cent. The variation in capacity with temperature is less than 0.2 of 1 per cent below 75 deg. fahr. At temperatures of 100 degrees or higher, the variation of capacity becomes relatively large, so that it is important in using these condensers to avoid excessive temperatures. The purpose of using a paper condenser is, of course, simply economy and increased portability, and condensers of the type discussed above would seem to give satisfactory performance for most conditions.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 21, 1919.

Copyright 1919. By A. I. E. E.

RADIO TELEPHONY

BY E. B. CRAFT AND E. H. COLPITTS

ABSTRACT OF PAPER

This paper is divided into two parts. The first part describes the development of the art of radio telephony by the American Telephone and Telegraph Company and the Western Electric Company to the accomplishment of Trans-Atlantic telephony, followed by demonstrations of the use of radio telephony between ships and of methods of connecting radio and wire telephone systems. This first part covers work principally of an engineering research nature, the most important problems being the development of systems of generation, modulation, transmission and reception of radio telephone signals.

The second part is concerned almost entirely with the work of producing radio telephone and allied apparatus for the Army and Navy in the late war. The major problem of the second part was the design of light and compact sets which could be produced rapidly and in large numbers, for the main features of the generating, modulating and receiving systems were already well understood.

Historical

RADIO telegraphy has aroused so much popular interest that the main facts of its development to a commercial stage are known to every engineer and to many laymen. A detailed history of the growth of the art is therefore unnecessary in this paper and it is only for the sake of securing the proper point of view for a description of some recent developments in radio telephony that a short sketch of the fundamental principles involved will be presented. The work of Lodge, Marconi, Braun, Fessenden, Fleming, De Forest and many others is described in books written with this professed purpose; in fact a book of considerable dimensions is required to do justice to the historical side of the subject. These remarks are made because the present paper is concerned almost entirely with the work of the Bell Telephone System in radio telephony and little attempt is made to trace to their ultimate origins all the ideas which have entered into the building of the structure as it is at present.

Electromagnetic ether waves of "radio" frequency were first intentionally produced and studied by Hertz, who was guided by the electromagnetic equations of Maxwell, constructed twenty years earlier, which predicted electromagnetic wave propagation and the identity of light and electromagnetic radiation. Hertz succeeded in reflecting, refracting and polarizing his waves and in producing interference. The waves were generated by successive disruptive discharges of a condenser formed by two halves of an extended system, or oscillator, and were therefore damped waves occurring in trains. At each discharge a damped oscillatory current flowed to and fro along the oscillator and consequently set up outside the oscillator variable electric and magnetic states which, according to Maxwell's theory, were propagated as waves to remote regions. They were detected by means of a secondary circuit consisting of a nearly complete conducting ring broken by a minute spark gap.

This was the first radio telegraph system, and although Hertz was not concerned with any immediately practical consequences of his method of producing electrical waves, others were quick to see the commercial possibilities. Passing over the very extensive body of scientific work which was started by the researches of Hertz, the next notable achievements were the discovery by Braun of the coherer for making the waves evident, and the invention by Marconi of the present form of antenna which allows the radiation of large power by long waves. These two steps undoubtedly made radio telegraphy a commercial possibility.

Because of the light which is thereby thrown upon subsequent efforts, it is worth while to examine the situation as it existed at that time. Trains of damped waves were produced by the charging of the oscillator system to a high voltage and allowing an oscillatory discharge to take place after each charge. The charges occurred with a frequency which produced audible sounds while the radiated waves were damped out in a rather small number of oscillations. As a result a graph of the wave amplitude would show a succession of violent and rapidly damped disturbances separated by many periods of no disturbance. This method of generation had two effects: first the average power radiated could be made large only by greatly increasing the amplitude of each separate train. This was limited by the voltage which could be allowed on the antenna and associated apparatus. Secondly, a damped wave differs so much from a simple harmonic wave that sharp tuning was impossible and the advantages known to attend its use could not be realized. These difficulties could both be overcome if a sustained wave generator of sufficient power were available; consequently this was the most important technical problem at this time and resulted finally in the development of the high-frequency alternator and the arc generator.

In the meantime it had been found that bodies, such as crystals, having partial unilateral conductivity, could be used in connection with a telephone receiver to make the signals audible. In this case what was heard was the note produced by the succession of wave trains, the effect of the crystals being to rectify the received current. The rectified high-frequency current varied sufficiently slowly to produce in the telephone an audible sound which continued as long as the key at the sending station was depressed. It is not necessary that the detector be completely unilateral, but only that an applied voltage shall produce a greater current when acting in one sense than when acting in the other. It was soon found that this method of reception employing a telephone receiver and rectifier or detector was far better than that using a coherer, partly because of the greater sensitiveness, but more fundamentally because reception of an audible note by ear allows greater opportunities for discriminating against disturbances.

But with the advent of " continuous " or sustained waves, such a method could not be used without modification, because the rectified current in the telephone receiver was either zero (when the sending key was open) or constant and equal to the average value of the rectified current when the key was closed. Such a flat-topped wave of telegraph frequency produces no audible sound in a telephone receiver except at the make and break. Two solutions of the problem thus presented were given. The first consists in breaking up the received wave into groups at such a rate that the group frequency is in the audible range. This was done by means of a commutator. The other solution, which is extremely elegant and useful, consists in superposing upon the received signal at the receiving station a locally generated wave of frequency slightly different from that of the received signal. Beats are thereby produced which are of audible frequency, and by rectifying this composite wave the current in the telephone is made to vary at any frequency desired. Since the local oscillator is not required to furnish an appreciable amount of power it may be small and may, in fact, be combined with the tuning circuit of the receiver. A more important advantage is that the signals are amplified by this "heterodyne" method of receiving.

This brief sketch brings us to the early attempts at radio telephony. It was realized at an early date that a crystal rectifier, or any similar detector, in combination with a telephone, would produce in the telephone currents whose amplitudes varied approximately as the amplitude of the received high-frequency wave; consequently if the amplitude of this wave could be varied at the transmitting station in accordance with speech, radio telephony would be possible. The receiving apparatus was therefore in existence and all that was needed was a means of varying the amplitude of the transmitted wave, that is, the amplitude of the current in the transmitting antenna. Now, in wire telephony, the amplitude of a direct or alternating current is so varied by means of a microphone, and this device was tried for radio telephony, the method being to insert a microphone, or a combination of them, in some part of the antenna circuit traversed by high-frequency currents. This attempt was handicapped because of the fact that microphones, even when specially designed, cannot easily be made to carry large currents and still function properly. Their use, however, suggested that it might be possible to devise somewhat similar apparatus, and as a result there appeared numerous voice-operated resistance-varying devices. Using methods of this kind considerable success was attained; thus in 1912 Vanni succeeded in telephoning 1000 kilometers.

FUNDAMENTAL PRINCIPLES

In order, however, to understand the nature of the telephone problem, a short discussion is necessary to make clear what are some of the limiting factors in the transmission and reception of speech. Consider first the simplest case of the familiar wire telephone system, which will be taken as a speech-actuated microphone, a battery and an electromagnet in series. A certain current, I, flows in this series circuit and produces a pull, proportional to the square of the magnetic field strength, and consequently proportional to I^2, upon the armature. Now let the microphone be actuated and suppose its design is so good that a sinusoidal variation in air pressure produces a sinusoidal variation in the *carrier current* I. Then the result of singing a pure note of frequency $n/2\pi$ into the microphone will be to produce a current *variation* of amplitude kI, say, so that the current in the circuit is now

$$(1 + k \sin nt)\, I$$

The force on the armature will be proportional to

$$I^2 + 2\,k\,I^2 \sin n\,t + 1/2\,k^2 I^2 + 1/2\,k^2 I^2 \cos 2\,n\,t$$

This is made up of a constant part, which does not sound the telephone, a part, $2k\,I^2 \sin n\,t$, which is a perfect copy of the voice, and a harmonic, $1/2\,k^2 I^2 \cos 2\,n\,t$. The distortion is relatively less the smaller the quantity k, which is called the fraction of complete modulation because if $k = 1$, the amplitude of the carrier current is periodically reduced to zero. It is clear that to completely modulate the carrier current the resistance of the microphone would need to vary from zero to infinity. This is done in telegraph sending, since the key is opened, but it can only be approximated with a microphone. There are two bad results of this incomplete modulation: The full power of the system is not used since it is only the variable part which transmits signals, and any irregularities or ripples in the carrier current have a relatively greater effect the smaller the modulation, since they are themselves modulations. There was therefore an imperative need of a resistance or current-varying device which would produce variations comparable with those produced by a telegraph key.

In our first circuit it is not necessary that the carrier current be constant; thus suppose it is represented by $I \cos r\,t$, in which $r/2\pi$ is a high (radio) frequency and hence inaudible. Upon modulation the current becomes

$$(1 + \sin nt)\, I \cos rt = I \cos rt + 1/2\,k\,I \sin (r + n)\,t - 1/2\,kI \sin (r - n)\,t$$

which defines a frequency band from $r - n$ to $r + n$. Thus if speech is to be transmitted (which requires all frequencies to about 3000) the high-frequency system must be capable of transmitting, not only the frequency $r/2\pi$ but a band extending 3000 cycles on each side of this frequency. This fact imposes a limit upon the sharpness of tuning allowable in carrier-wave telephony.

The above modulated current produces in the receiver (or an e.m.f. of this type produces in a rectifying detector) an effect roughly proportional to its square. Thus if there is impressed at the receiver another current, $A \cos r\, t$, in phase with $I \cos r\, t$, or equivalently, if the polarization of the receiver in the telephone case is made stronger, the audible frequency terms of the desired frequency will be reinforced while those of undesirable frequency will not play so prominent a part. Consequently, the received signal will be amplified with a minimum of distortion.

It is not intended to give here the theory of the modulation and detection of carrier waves, which should properly be treated in a separate paper, but this very brief sketch has shown the importance of complete modulation, the fact that radio telephony is concerned with the transmission of a high-frequency band instead of a single high frequency, and has indicated the possibility of new methods of transmission and reception.

INTRODUCTION OF THE AUDION

In 1906 Dr. Lee De Forest brought out a vacuum tube device which he called the "audion" and which has been described in the A. I. E. E. TRANSACTIONS. This was a very sensitive detector of electric oscillations and consisted of three electrodes in an exhausted envelope. One of the electrodes could be heated to incandescence with the result that electrons were emitted by it (the Edison effect). A battery connected between this electrode, as cathode, and another as anode resulted in a convection current of electrons from one to the other. Since negative electricity only was present, current could flow in but one direction. This is so far the action of the Fleming valve which also makes use of the Edison effect, but in the audion an epoch making advance was made in that the third electrode allows us to completely *control* the strength of the electron current *without consuming appreciable energy at that electrode or in its circuit.* In other words an inappreciable amount of power applied to the third electrode, or grid, will result in large changes in power in the anode circuit. Moreover, since the electrons have no appreciable inertia, the response in the anode circuit to stimuli in the grid circuit is practically instantaneous.

Now this kind of device is exactly what is needed as a repeater in telephony. It was therefore considered by the Bell

System and it was found that, provided the device could be so improved as to be able to handle the power required for wire telephony, the audion would make an ideal amplifier. In 1912 our audion development work was started with this in view, and the new device soon became a competitor of the so-called "mechanical" repeater, which it is displacing. This development of the telephone amplifier, fortunately, required a considerable amount of research in the field of high vacua and electron emission and also a rather extensive study of the electrical characteristics of thermoionic tubes.

This had two results immediately applicable to radio telephony. First, an application of the early studies of amplifying systems and circuits and particularly a study of "singing" or freely oscillating repeater systems by Dr. G. A. Campbell, resulted in a successful form of thermoionic oscillator, suitable for all frequencies, and second, through the work of Mr. E. H. Colpitts early in 1914, there was produced a thermionic modulation system giving approximately undistorted speech. Several related forms of modulating systems were soon after developed by van der Bijl and others.

Before describing the modulator it should be remarked that when one has a perfect amplifier and a perfect modulator, the problem of radio telephony is solved, at least theoretically, for all that is necessary is to modulate the output of an oscillator circuit and then, if necessary, amplify the modulated current. Or one may first generate oscillations of large power and modulate them by means of the amplified output of the telephone transmitter. It should also be remarked that a "perfect" modulator, *i.e.* one which may produce completely modulated undistorted carrier waves, is also a perfect detector of these waves.

Modulator. In order to understand the modulator, consider Fig. 1 in which is plotted a characteristic curve, C, of the audion. This curve shows the variation of anode, plate or output current with grid-filament or input voltage. Now let the tube be connected as shown in the circuit diagram in which two voltages are impressed simultaneously upon the grid. The first of these is due to a telephone transmitter, T, and is sufficient to vary the grid voltage from zero to the value $-E$. The second is of radio frequency and smaller than that due to the transmitter. The total voltage on the grid will then be shown, as a function of time, by the corrugated sine curve plotted vertically. The characteristic curve shows us that the resulting current in the output circuit will be represented by the variably corrugated curve extending to the right. The rapidly variable part of this curve represents a completely modulated high-frequency current which may be led to an antenna or amplifier by the transformer at the right. It will be seen that the modulator acts essentially as an amplifier whose amplification is not constant but varies from zero (at the point $-E$) to a maximum at the extreme right. The object of introducing the battery e in the grid circuit is to insure that the grid will not become positive with respect to the filament, for as long as it is negative no electrons can flow to it and there will be practically no consumption of power in the grid circuit. It is, however, sometimes desirable to allow current in the grid circuit, in which case the battery may be adjusted.

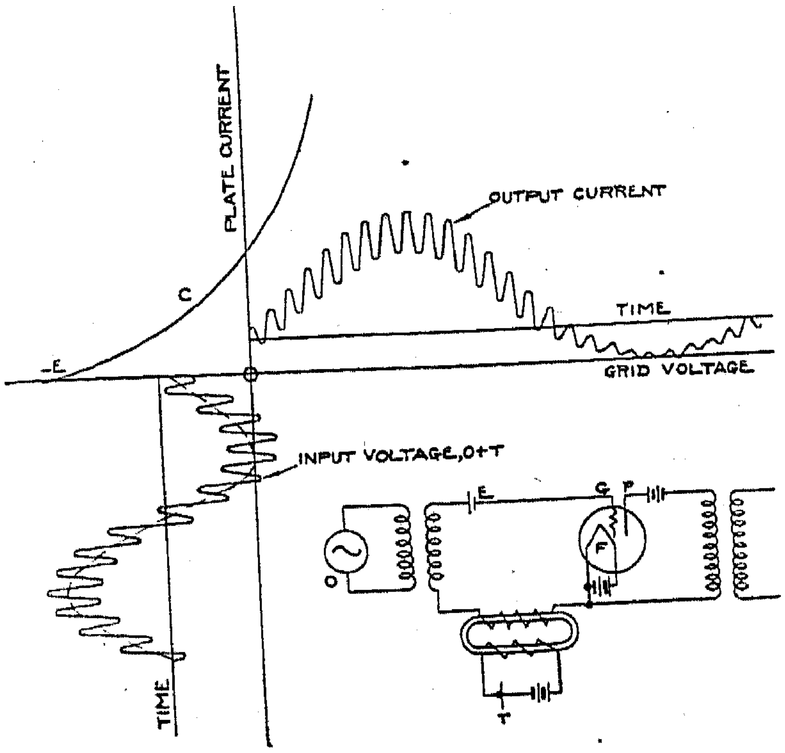


Fig. 1—Modulator Schematic

During the years 1912, 1913, 1914 a great deal of research work was carried on in connection with the development of the audion for telephony. There were many different problems involved in the manufacture of practical tubes which could handle the larger amounts of power necessary for amplifiers of telephone and radio currents. In the way of applications, laboratory apparatus was set up for amplifying telephone currents under long line conditions, a study of the detecting and rectifying properties of the tubes was made, circuits were designed for inter-connecting radio and wire telephone systems, oscillator circuits for both high and low frequencies were built, the audion was applied as a power-limiting device to protect circuits, and a number of other systems of modulation were devised.

FIRST FIELD TRIALS OF RADIO TELEPHONY

In 1914 it was decided that the apparatus and methods developed in the laboratory were sufficiently promising to warrant an attempt at Trans-Atlantic telephony.

In order that a practical demonstration of the method might be obtained an experimental station was constructed at Montauk, L. I., and a receiving station was constructed on the roof of the Dupont Building, in Wilmington, Del. A trial was made on April 4, 1915 in the presence of Professor Millikan and Colonel Reber, as well as Messrs. Carty, Gherardi, Jewett and others of the executives of the Engineering Departments of the American Telephone and Telegraph and the Western Electric Companies. The transmission was, of course, one way only. These gentlemen, therefore, witnessed the operation of the transmitting equipment at Montauk on one day and then on the following day visited the receiving station at Wilmington and listened to the incoming transmission. Wire connections between Montauk and Wilmington permitted the listeners at Wilmington to report immediately what words they had heard, or, as on the second day, it permitted the speaker at Montauk to report what he had said. The wire connection thus admitted of the verification of the experiment.

Following this demonstration the range was extended from Montauk to St. Simons Island off the coast of Georgia where a receiving antenna had been erected. For the purpose of this experiment the radio receiving circuit at St. Simons was connected to a telephone circuit leading to New York. Similarly, the radio transmitting equipment at Montauk was connected to a telephone line leading from New York. The speaker at New York talked over a wire circuit to Montauk, by wireless to St. Simons and then by wire to a listener in New York.

The method which was used in transmission was essentially that which was followed in later experiments when the Navy station at Arlington was used as a transmitting station. Prior to this time the inability of inventors to modulate large amounts of power in such a manner that the modulated current, when detected at the receiving station, could be obtained essentially free from distortion, had proven a serious limitation on the increase of range of transmission. It had been shown, however, that this limitation could be overcome by the use of the audion amplifier and modulator. The general method proposed, therefore, and the method finally developed was as follows. A small current of high-frequency was to be generated by means of a vacuum tube oscillator. This high-frequency current was to be modulated, essentially completely, by the voice current from the telephone transmitter. The resulting modulated current was to be amplified, or successively amplified, by distortionless amplifiers, of the vacuum tube type, until the energy which it represented was sufficient for transmission from an antenna over the desired distance.

For the production of a sustained high-frequency current a vacuum tube was used. With such a tube, provided its input and output circuits are coupled together and one of them contains a tuned circuit, the operation is one of successively amplifying its own output. This successive amplification will result after a moment in the development of a steady state of oscillation in which there is a definite maximum of output current. This maximum amplitude of the generated alternating current depends for its value upon the characteristics of the tube itself. An oscillation generator for producing alternating current is shown in Fig. 2. Power may be obtained from it by coupling loosely to the inductance of its circuit another inductance which in turn is connected to the point where this power is to be utilized.

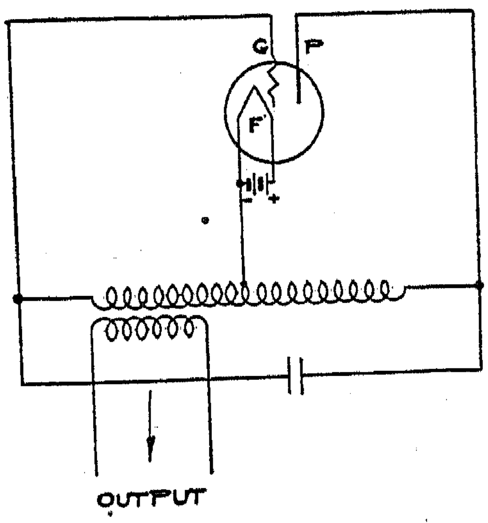


FIG. 2—OSCILLATION GENERATOR SCHEMATIC

As noted above, the study of the vacuum tube had also indicated that although it was possible to use the tube as a distortionless amplifier, it was also possible to adjust its voltages and the impedances to which it was connected so as to produce a distortion of the input current in a manner suitable for the purposes of radio telephony. Under such conditions the output of the tube contains a component which is proportional to the product of such voltages as may be simultaneously impressed upon its input or grid-filament circuit. If one of these inputs is obtained from the oscillation generator just described and the other from a telephone transmitter, it is then possible to obtain an output which varies harmonically with the frequency of the oscillation generator, and also varies harmonically in amplitude with the frequency of the voice current impressed by the transmitter.

A vacuum tube, properly designed for the impedances between which it was to be connected and properly adjusted so as to emphasize the modulator characteristics, was used as a modulator in the system of transmission which was formally demonstrated at Montauk. To its input was coupled inductively the tuned circuit of the oscillation generator and also telephone transmitter circuit containing a local battery.

In order to obtain a sufficient amount of energy for transmission over appreciable distances, it is necessary to amplify the energy output of the modulator before impressing it upon the antenna. In case considerable amplification is desired it may be obtained by impressing the voltage from the modulator upon an amplifying system formed by two amplifiers in tandem. It was also realized that amplifiers might be worked in parallel, that is, with their input circuits connected in parallel and with their output circuits connected in parallel, without introducing disturbing interactions, provided that certain precautions are taken. The method of amplification adopted for the Montauk tests and later used for the Arlington experiment was therefore a system of two stages of amplification. At Montauk the first stage was obtained by a single tube and the second stage by a number of tubes in parallel. In Fig. 3 is shown a view of the experimental apparatus used at Montauk. On the left in the foreground is a rack containing the amplifiers of the second stage. On the right, immediately above the battery racks, are four boxes containing the other radio apparatus. The upper right-hand box contains the oscillation generator. The box immediately below this holds the modulator and its associated apparatus; the upper box on the left contains the vacuum tube of the first stage of amplification; and the box below it has a tuned circuit and certain controlling resistances which enter into the transmission system. Another view of this apparatus is shown in Fig. 4. On the left of this is seen again the rack of amplifiers, but now in such a view as to show the vacuum tubes themselves. These tubes, which were similar to those later used in the Arlington demonstration, were especially designed for this purpose and are much larger than the vacuum tube amplifiers of telephone practise.

THE ARLINGTON-DARIEN EXPERIMENT

The tests from Montauk to Wilmington and to St. Simons Island were so successful that it appeared practicable to carry the method to its logical conclusion and to extend the range of radio telephony to much greater distances. For this purpose experiments were undertaken in conjunction with the Navy Department. For the initial steps in making such arrangements and for assistance throughout the experiments, we are indebted and distinctly grateful to Admiral R. S. Griffin, Engineer in Chief, U. S. Navy, Captain now Admiral W. H. G. Bullard, Superintendent of the Radio Service, and to Commander now Captain Hepburn and Lieutenant now Commander S. C. Hooper of the Bureau of Steam Engineering, U. S. N.

For the purpose of these experiments a small operating house was constructed beside the main operating building of the Navy Wireless Station at Arlington, Va. An antenna switch was provided so that the antenna might be connected to the Western Electric equipment in this experimental station. Captain Bullard also arranged for observers from the Western Electric Company to be present at the Navy stations at Darien, San Diego and Mare Island. The necessary apparatus was installed in the early summer of 1915 and preliminary experiments were started at once. On August 27th successful transmission was obtained to Darien on the Isthmus and was received by Mr. R. H. Wilson of the Western Electric Company and by Lieutenant R. S. Crenshaw of the Navy.

On this day Colonel Reber, Lieutenant Bryant of Captain Bullard's office and Mr. G. H. Clark, Radio Expert of the Bureau of Steam Engineering, had been asked to speak from Arlington in the hope that communication would be established with Darien. Immediately prior to the time these gentlemen spoke, two selections were played on a phonograph placed in front of the telephone transmitter. These were correctly recognized by Wilson and Lieutenant Crenshaw. Each of the visitors then spoke for a minute or two. The voices of the speakers were not familiar to the observers at Darien. Wilson, however, recognized the change in voice caused by a change in speaker and received correctly several phrases and some scattered words. These words were also verified by Lieutenant Crenshaw. A report of these results was sent immediately from Darien by Navy code to Arlington and was compared by the speakers with their records of their spoken words.

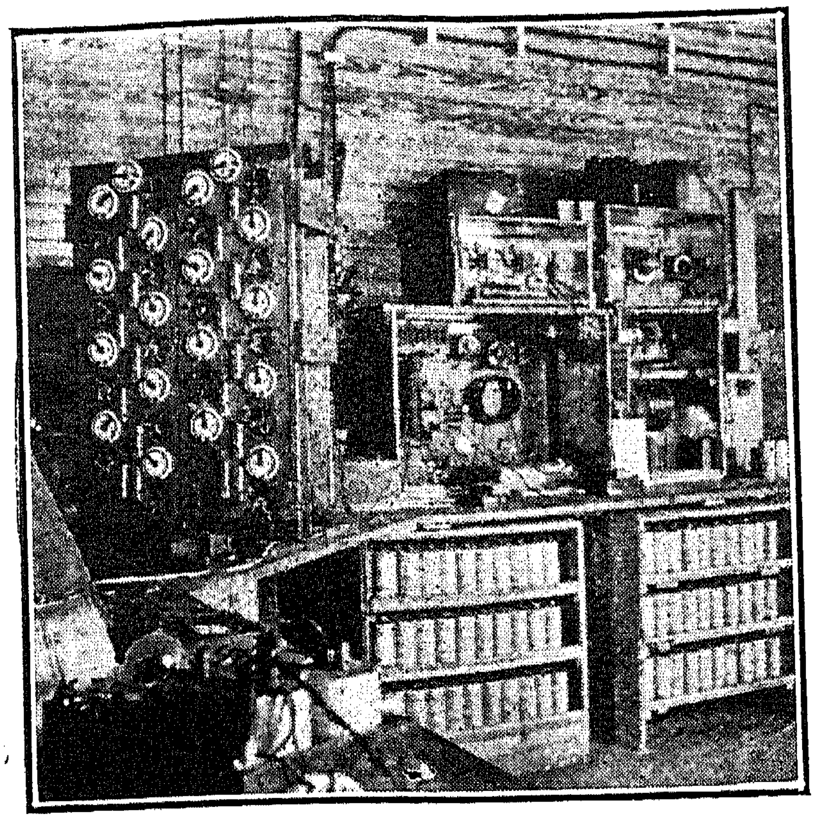


FIG. 3—MONTAUK EXPERIMENTAL APPARATUS

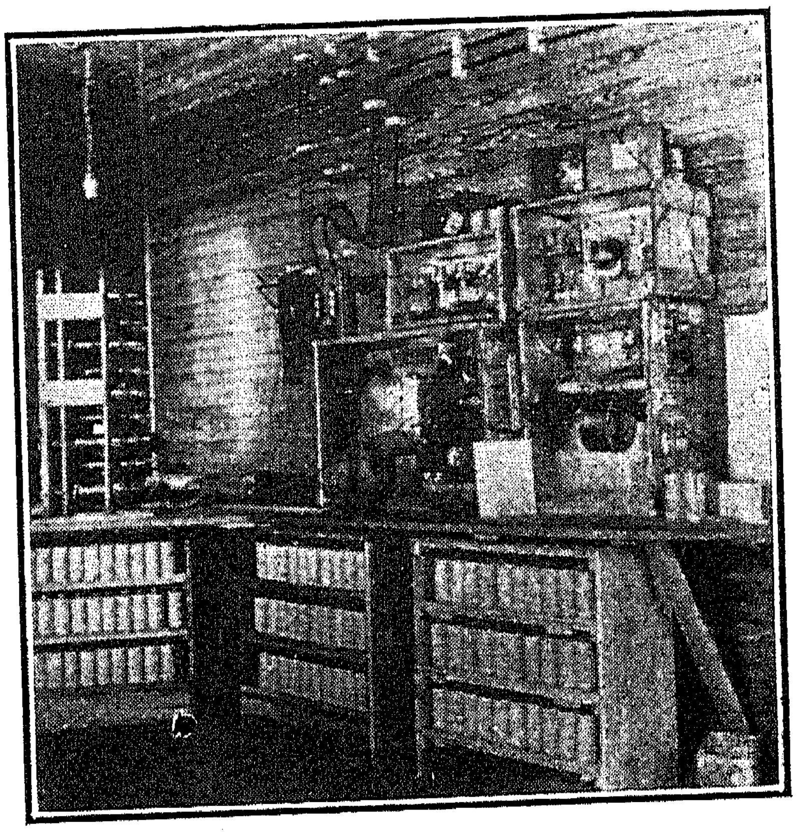
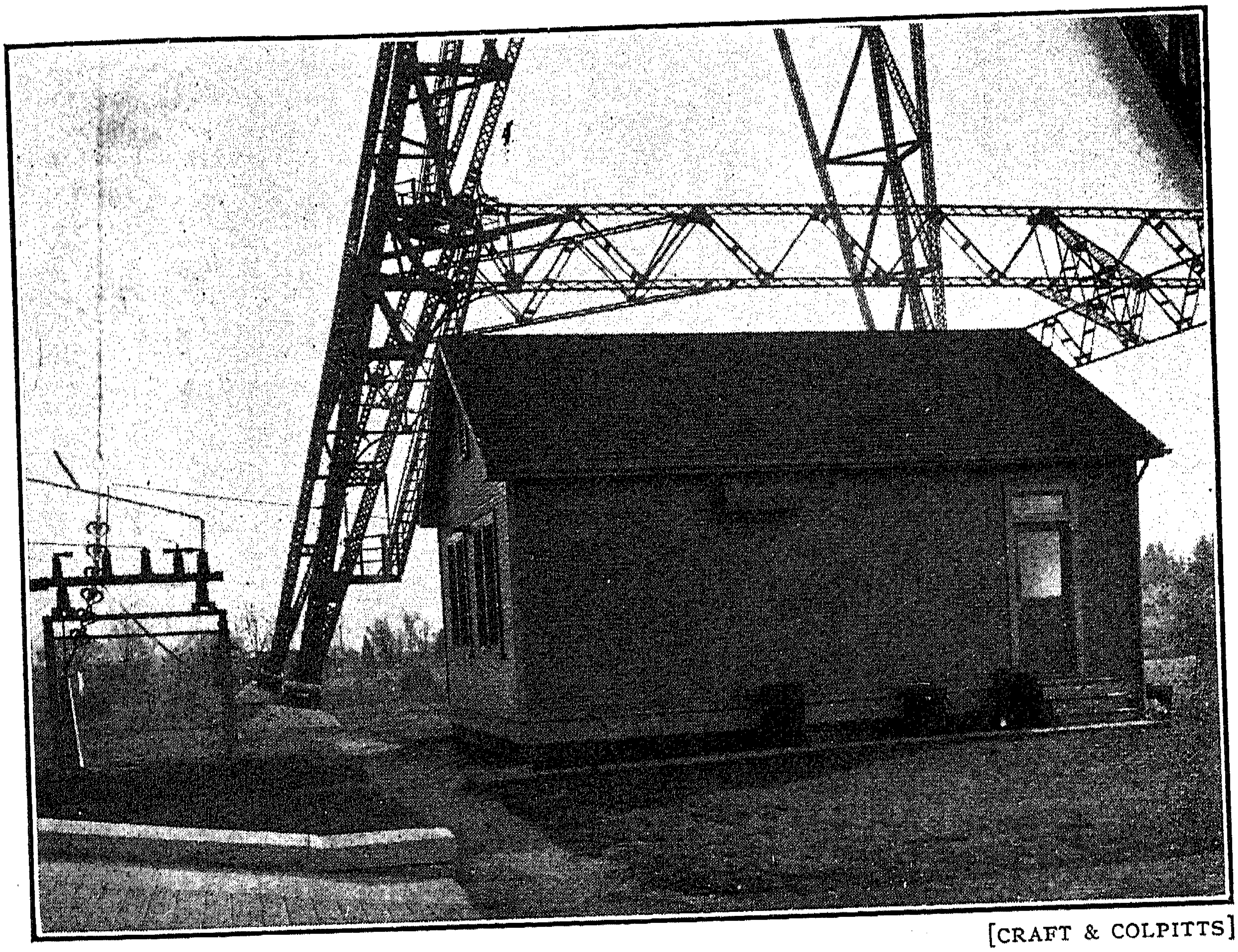


FIG. 4—MONTAUK EXPERIMENTAL APPARATUS



[CRAFT & COLPITTS]

FIG. 5—RADIO EXPERIMENTAL STATION, ARLINGTON, VA.

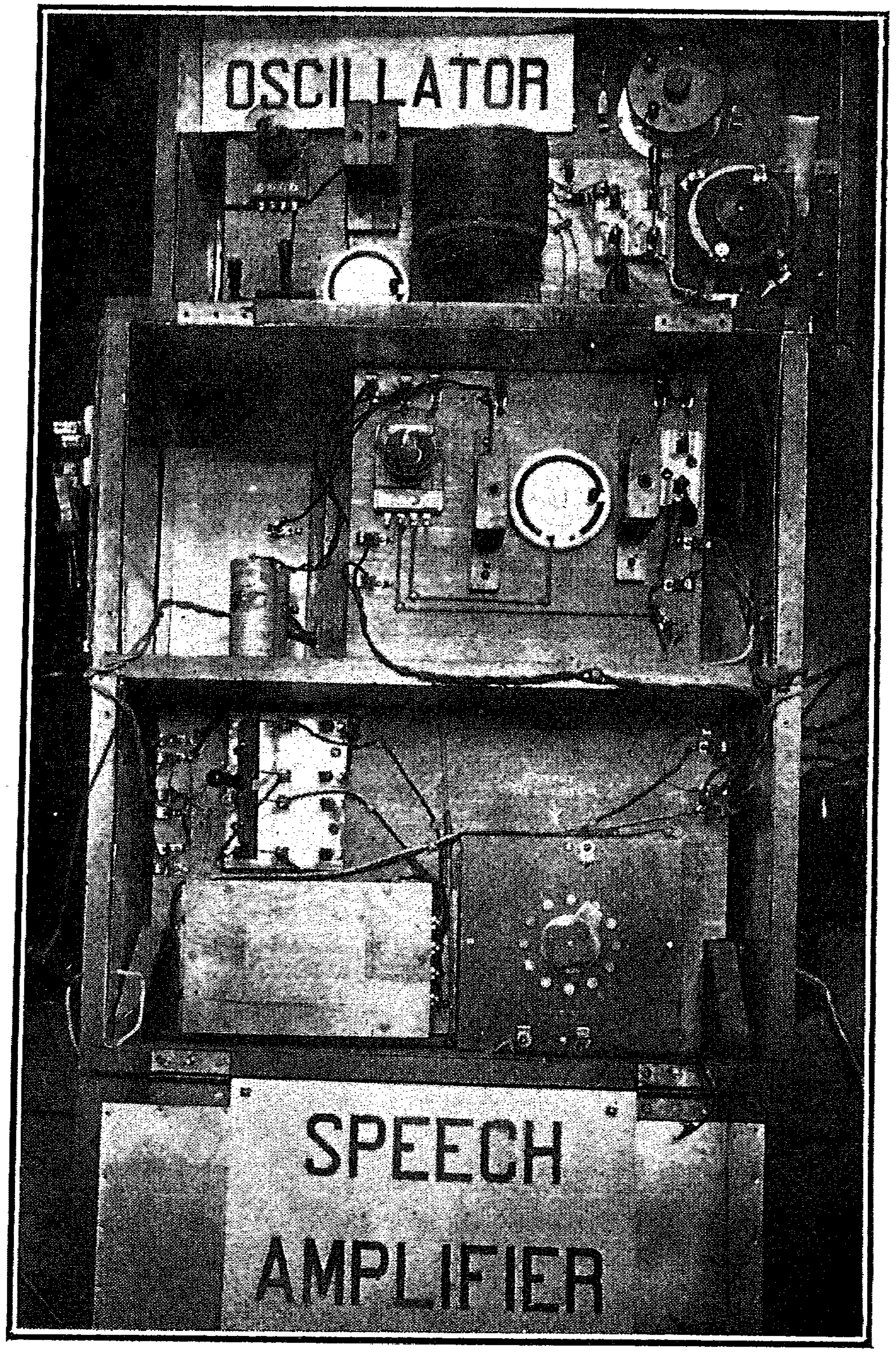
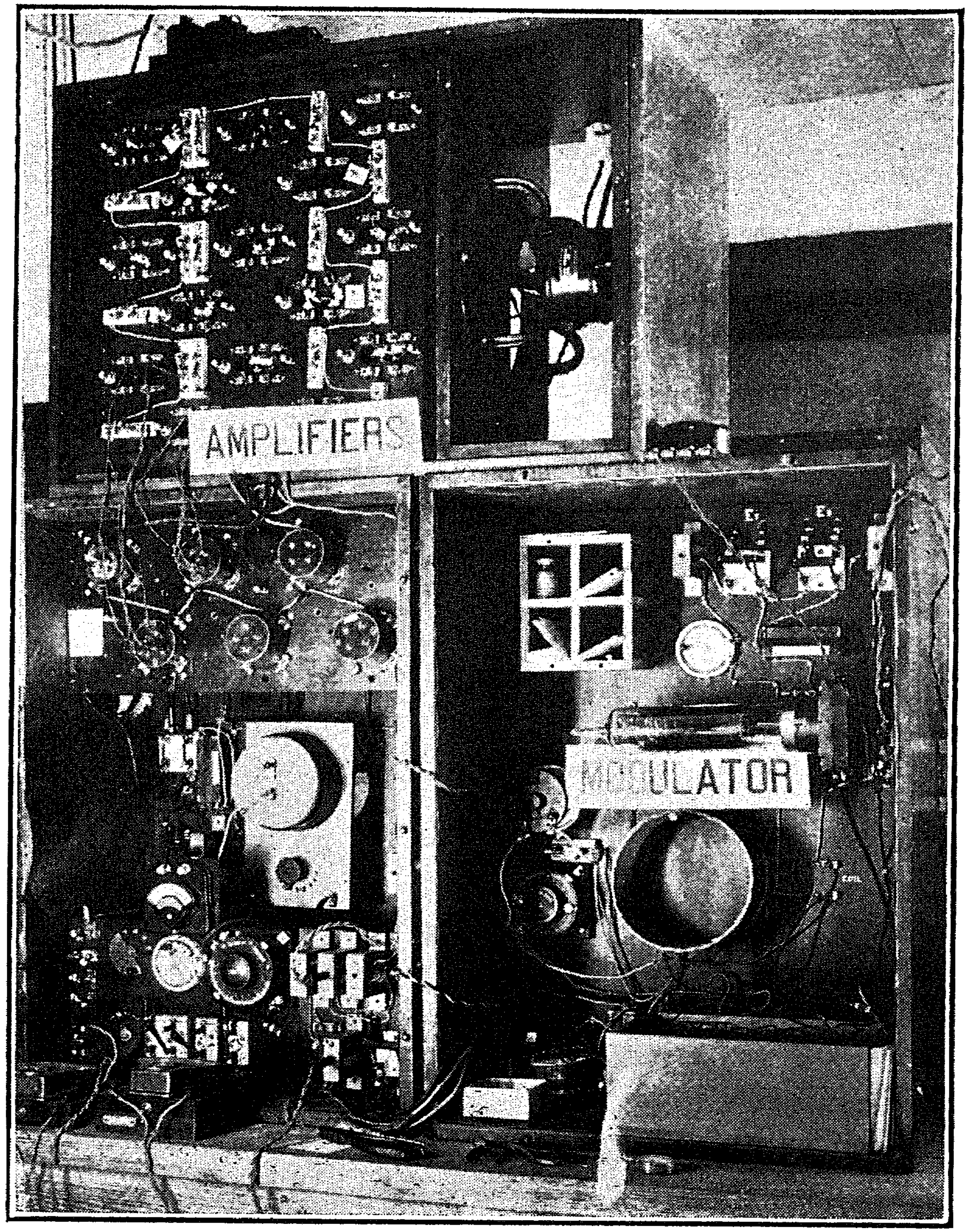


FIG. 6—SPEECH AMPLIFIER—RADIO EXPERIMENTAL STATION, ARLINGTON, VA.



[CRAFT & COLPITTS]

FIG. 7—MODULATOR—RADIO EXPERIMENTAL STATION, ARLINGTON, VA.

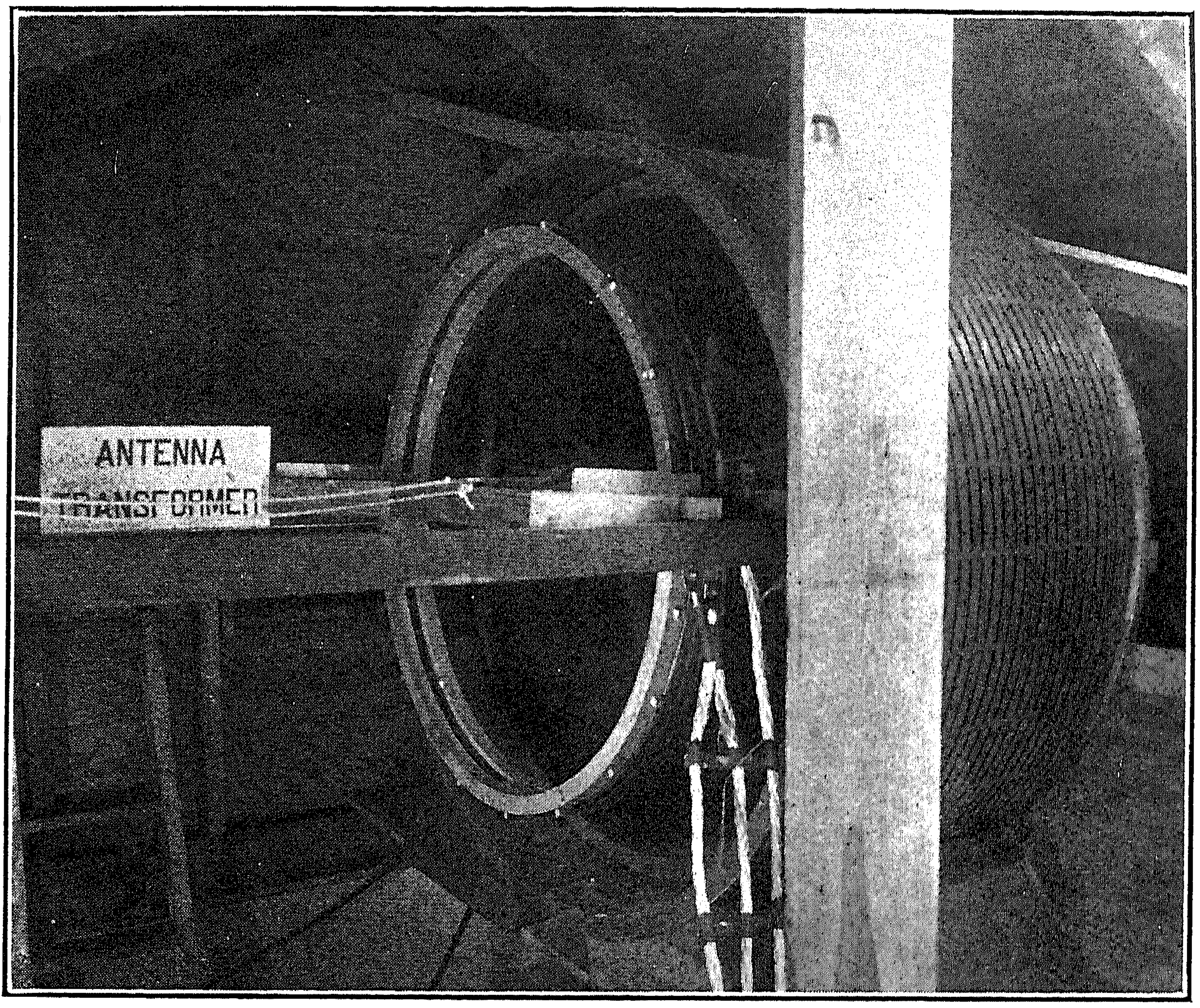
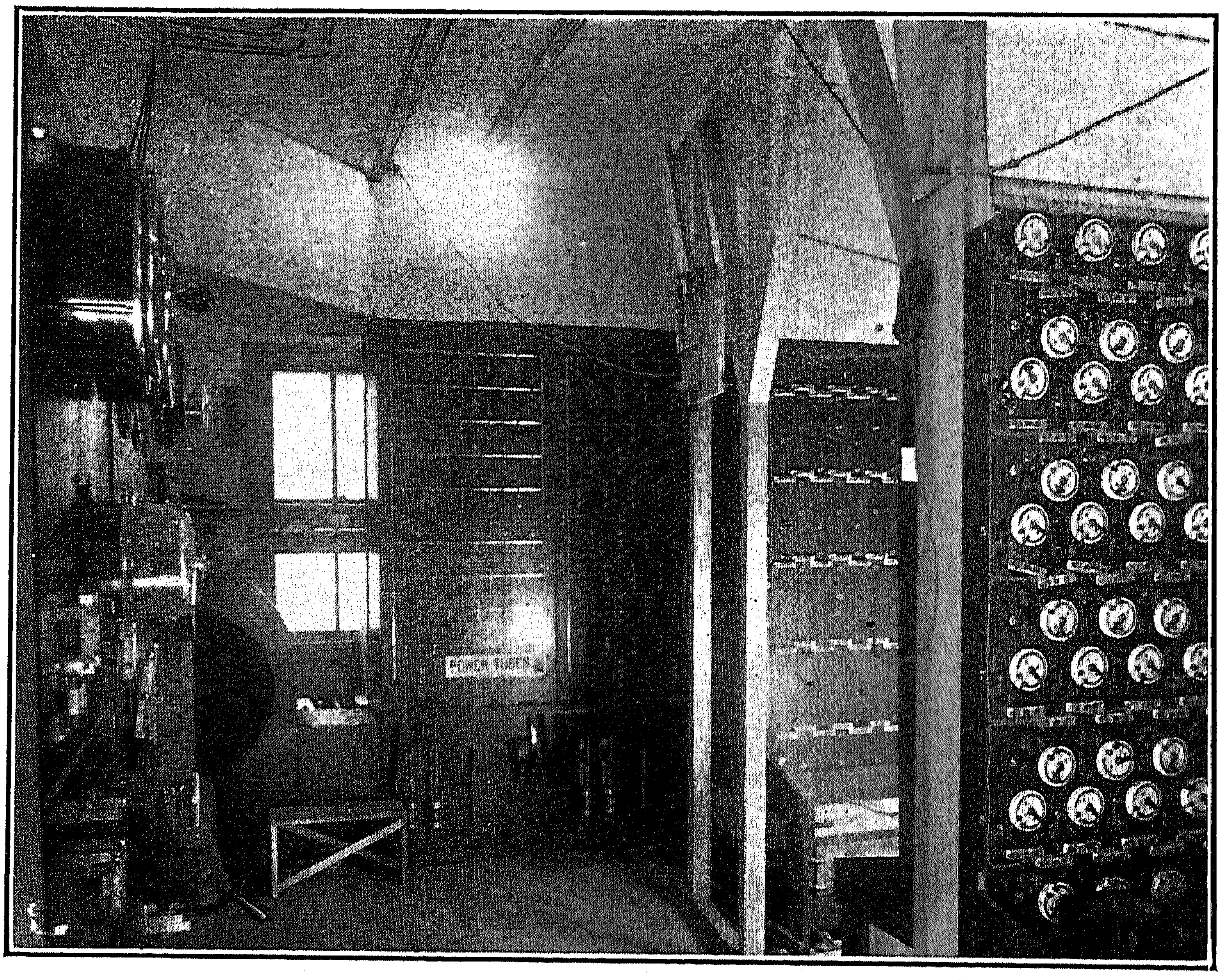
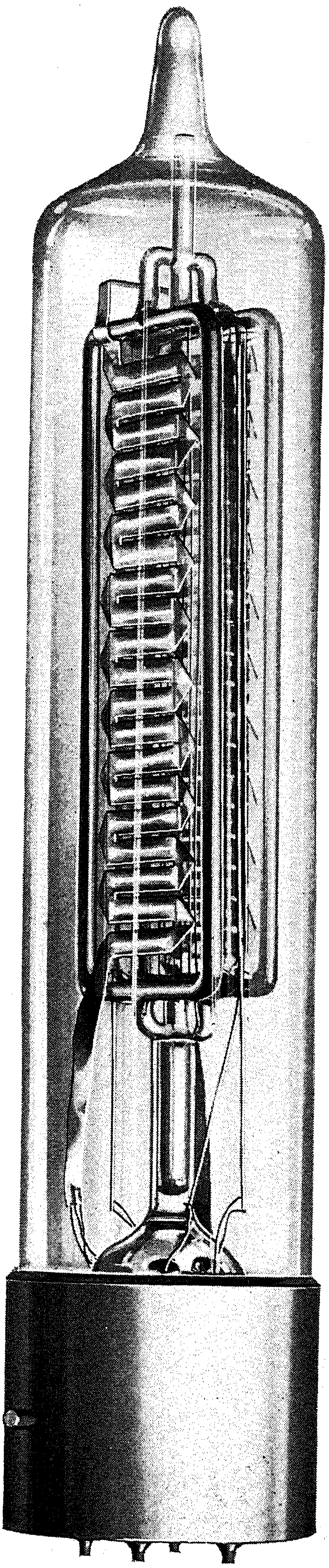


FIG. 8—ANTENNA TRANSFORMER—RADIO EXPERIMENTAL STATION, ARLINGTON, VA.



[CRAFT & COLPITTS]

FIG. 10—POWER TUBE RACK—RADIO EXPERIMENTAL STATION, ARLINGTON, VA.



[CRAFT & COLPITTS]

FIG. 9—TYPE OF VACUUM TUBE USED IN LONG DISTANCE RADIO TELEPHONE EXPERIMENTS

The method of transmission was essentially that described above in connection with the Montauk experiment. The experimental station at Arlington is shown in Fig. 5. The transmitters used in these tests were of the ordinary commercial type, or of a type which was then in a stage of development and which it was thought might reproduce more exactly some of the higher harmonics of the voice. A phonograph was also used for the transmission of music. The output of the transmitter was amplified in a speech amplifier of the audion type, which is shown in Fig. 6. The high-frequency oscillator is shown in this figure above the amplifier where the tuning condensers and the coil for coupling to the inductance are plainly visible. The output of this speech amplifier and of the high-frequency oscillator were both impressed upon a modulator of the vacuum tube type, shown in Fig. 7. The voltage of this amplifier was not, however, high enough for the main amplyfying tubes, and it was stepped up by an intermediate group of amplifiers, of which six as a rule were used, which are shown at the left of the figure. The modulator output thus amplified was impressed upon a number of power tubes in parallel and then upon the antenna transformer shown in Fig. 8. The type of power tube is illustrated in Fig. 9. The amplifier tubes preceding the power tube and the modulator were also of the same form, but were of different constants, properly adapted to their purposes.

The current by which the filaments were heated was supplied by a local power company and brought in by underground cable to the operating station. Alternating current was used for heating the filament and any possibility of the superposition of a 60- or 120-cycle note, due to the frequency of this current, was eliminated by the use of a special scheme of connections. The transformers which were inserted in this circuit are shown at the left of the door in Fig. 5. Between the plate and the filament of the power tube a constant voltage of about 500 volts was impressed. It was obtained from one of the motor generators used by the Navy in operating the large Poulsen arc, with which they were then transmitting to Darien and other remote points. A large amount of energy, of course had to be dissipated in such an equipment because of the energy supplied to the filaments, and a blower was installed for this purpose as shown in Fig. 10 where the racks containing the power tubes are also visible. At the left of this figure is the switchboard for operating remotely the motor generator set in the main station of the Navy, for controlling the alternating current supplied to the filament, for operating the blower, and for metering the currents used for lighting the plant and for similar purposes.

The method of reception was varied from time to time as it was still in process of development. In some cases an amplifier for the high-frequency currents was introduced between the detector and the tuned circuit which was coupled to the antenna. In some cases a low-frequency amplifier was inserted between the detector and the receivers. In some of the later experiments a feed back circuit was used for the detector so that the detector tube itself acted as an oscillation generator as well as a detector. In that case the frequency of the oscillations was adjusted to be the same as those of the oscillation generator at the transmitting station.

THE NEW YORK-ARLINGTON-MARE ISLAND DEMONSTRATION

After obtaining successful communication with Darien, a distance of 2100 miles, attention was directed towards reaching the Pacific Coast stations of Mare Island and San Diego. As a test of the transmission efficiency, records were made at the W. E. Co's. laboratory in New York of the quality and intensity of the speech from Arlington, each time transmission was attempted. Records were also made occasionally at Darien, but no attempt was made to fit the time of transmission into the schedule of operation of that station. An order wire was set up between Arlington and the two Pacific Coast Stations, so that instructions as to the schedule of tests and reports as to its results might be conveniently interchanged.

Radio telephone transmission between Arlington and Mare Island was formally demonstrated on September 29th. On this date, Mr. J. J. Carty was inspecting the station at Mare Island. A demonstration for Mr. T. N. Vail was therefore arranged for this date.

The station at Mare Island consisted of a small house near enough to the operating house of the Navy so that the experimental apparatus might be connected to the antenna. The observers at the Mare Island station were Commander G. C. Sweet and Lieutenant Kenshit of the Navy, Mr. A. H. Babcock, Electrical Engineer, Southern Pacific Railroad, Mr. J. J.

Carty and others. Messrs. Arnold and Hartley of the Western Electric Company were the experimenters.

Arrangements had been made whereby Mr. Vail's apartment in New York could be connected with the radio apparatus at Washington. A telephone line extended from the transmitter assigned to Mr. Vail's use to the radio telephone apparatus at Arlington. This telephone circuit was connected to the radio apparatus in place of the telephone transmitter which had formerly been used at Arlington. From Arlington transmission was through the ether to Mare Island. The telephone set which Mr. Carty used at Mare Island consisted of a transmitter connected to a telephone line leading, via the New York test board, to a receiver in Mr. Vail's apartment. The receiver of Mr. Carty's set at Mare Island was one of several which were supplied in parallel by the detector of the radio receiving set. Arrangements were thus made whereby Mr. Vail and Mr. Carty might carry on two-way conversation, but what Mr. Carty and the other observers heard at Mare Island was transmitted from Arlington by wireless.

At the New York toll test board a telephone line was bridged onto the circuit leading from Mr. Carty to Mr. Vail. This line extended to Arlington and contained one-way repeaters so that transmission could take place only from New York to Arlington. At the latter place it supplied a group of telephone receivers which were also connected through a one-way telephone repeater with the line which bore Mr. Vail's speech to the radio apparatus. In this way some dozen observers at Arlington could hear both sides of this unique conversation. The use of one-way repeaters prevented any interaction between the oppositely directed speeches.

At the Arlington station the conversation was listened to by Captain Bullard, Colonel Reber, Commander Hooper, Lt. R. B. Coffman, Lt. Bastedo, Commander Bryant, and members of the telephone system. Messrs. T. N. Vail, U. N. Bethel, J. I. Waterbury, Bancroft Gherardi, and H. P. Charlesworth all conversed with Mr. Carty through the ether.

THE ARLINGTON-HONOLULU EXPERIMENT

Having successfully transmitted over water to Darien, a distance of 2100 miles and over land to Mare Island, a distance of 2500 miles, attention was now directed towards reaching the remoter points of Honolulu and Paris. Schedules were therefore arranged for experiments with these stations. With Paris, however, the arrangements as to schedules were necessarily delayed because of the difficulties in cable transmission occasioned by the war.

With Honolulu, on the other hand, arrangements for a transmitting schedule had been made shortly before the demonstration of the New York-Arlington-Mare Island transmission. The first deliberate attempt to communicate with this station occurred on the night following the demonstration to Mare Island. Mr. Lloyd Espenschied of the American Telephone and Telegraph Company, who was the experimenter at this point, reported by cable the following morning, giving a record of the words which he had heard spoken, and also the name of the speaker, whose voice he had recognized.

THE ARLINGTON-PARIS EXPERIMENT

At Paris were Mr. Shreeve and Mr. Curtis of the Western Electric Company, in which connection it may be mentioned that they are again in France, the former as Lieutenant Colonel and the latter as Lieutenant in the Signal Corps. These experimenters had installed their apparatus in the Eiffel Tower station of the French Army. This courtesy had been accorded our representatives by Lieutenant Colonel Ferrié, Director of the Military system of Radio Telegraphy, to whom they had been introduced by Commander W. R. Sayles, Naval Attaché of the American Embassy. The magnificent spirit which France has shown throughout this bitter struggle is well illustrated by the ability of that nation to consider scientific developments, which apparently were not of immediate military value, and to assist in them despite the demands of the war. The amount of time during which our experimenters could use the antenna was, of course, very limited, and was small as compared to their necessities. The permission to use the antenna at a time when France was wholly engaged in this bitter struggle represents a contribution to science altogether disproportionate to the time which the antenna was available to our men. Unfortunately, however, much of the time when Mr. Shreeve and Mr. Curtis had access to the Eiffel Tower antenna there was interference from other high-power stations on the Continent, such as Nauen, Eilvese, Clifden, Norddeich, Lyons and Vienna which were transmitting upon the same range of wave lengths as it was necessary for us to employ. Instructions from Paris to Arlington, to alter the wave length of the transmission, could be forwarded only by cable, and were, of course, subject to days of delay, so that a satisfactory and flexible working schedule was not to be hoped for. Further, it was possible to transmit in any one day only for a brief period. In spite of these difficulties, promising results in the way of clearly received speech at Paris were secured during short periods that it was possible to arrange for working, between October 12th and October 21st. On October 23d as a final demonstration a number of observers, including Commander Sayles and representatives of the French Army, listened to the telephone transmission from the Arlington station 3600 miles away. The wave length used was about 6000 meters and the antenna current at Arlington about 50 amperes.

On certain of the occasions when we had worked to Paris, Mr. Espenschied at Honolulu had been informed as to the schedule of tests. He had therefore listened in and copied the words spoken at Arlington. The conditions at his stations were, of course, much better that at Paris because of the absence of interference from high-power stations in the neighborhood. His observations were upon transmission over a distance some 25 per cent greater than that from Arlington to Paris, and in part over land as well as sea water.

PRACTICAL DEVELOPMENTS

The impending danger of war put an end to the experiments as far as they were concerned with increase in range of transmission. The possibility of applying the principles which had been successfully developed in connection with these experiments in trans-Atlantic and trans-Pacific radio telephony to problems of immediate and patriotic value was of course evident. Subsequent developments and experimental tests were therefore directed solely with an aim toward their military value to the Army and the Navy. Some of these experimental developments which we made in this connection and the success with which they were accompanied may be of interest.

The first opportunity to demonstrate the use of the wireless telephone in war was presented early in 1916. Secretary Daniels desired a demonstration of what could be accomplished in the way of mobilizing the telegraph and telephone facilities of the country in case of war-time need. This was carried out by the American Telephone and Telegraph Company in May, 1916, and included as part of it wireless telephone communication with a battleship at sea.

The part played by the wireless telephone was to show the possibilities of quick communication by word of mouth between vessels at sea and Headquarters on land. A complete set was built and installed on the battleship New Hampshire. The transmitting to the battleship was done from our station at Arlington. The signals from the battleship were received at the Navy Yard at Norfolk with a set built for that purpose, and were automatically transferred to land telephone lines to Washington. The connections were brought directly to the mobilization switchboard at the Navy Department Annex. When it was desired to talk to the battleship, the regular telephone circuit from the Navy Department was connected up with the lines to Arlington and Norfolk, thus allowing communication by telephone to be held directly with the Captain of the vessel. It was found entirely practicable to hold two-way radio communication as effectively as over wire lines and to secure information or to give orders. Captain Chandler received his orders for manoeuvering in this experiment by wireless telephone from Secretary Daniels and Admiral Benson, and he reported to them each hour his position by the same means. The fact that the lines to the transmitting and receiving land stations came to Washington where these gentlemen were located did not mean that such an arrangement only was possible. This radio system was attached to the long distance telephone line and Capt. Chandler talked with Lt. Snyder of the Great Lakes Naval Station. It was then connected with the Transcontinental Telephone Line and he talked directly with Capt. F. M. Bennett, Commandant of the Mare Island Navy Yard. The ship during these conversations was 50 miles from Norfolk and outside the Capes.

The set installed on the New Hampshire differed from the set at Arlington only in size. The general circuit arrangement and modulation system was the same. The set was installed on the lower bridge deck and the telephone transmitter and receiver were installed on the bridge itself. This allowed the Captain, while on the bridge and directing the movement of the ship, to converse without leaving his post.

BATTLESHIP EQUIPMENT

The possibilities of the wireless telephone in naval maneuvering were quickly recognized by the Navy Department.

Commander Hooper of the Bureau of Steam Engineering asked us to cooperate in making the wireless telephone a useful piece of equipment in naval service. It was decided to build two experimental sets, install them on battleships and give the operating staff an opportunity to investigate and criticize them from the operating point of view. The construction of two sets was begun in December 1915 and they were completed about the middle of January. A short description of these sets will not be out of place at this time. Each set consisted of complete transmitting and receiving sets with a motor-generator. The receiving set was built to be set on top of the transmitting set. A photograph of the two thus placed is shown in Fig. 11. An extension circuit was provided so as to allow placing the hand set either on the bridge or in the Admiral's or Captain's cabin so that it would not be necessary for them to go to the radio room to talk. A duplicate hand set was provided for the operator in the radio room as well as a head telephone for listening-in. The normal condition of the circuit was with the antenna connected to the receiving set. Any call thus coming in would be received. When a call was received and it was desired to talk instead of to listen, a button on the hand set was pressed which caused the relay operated antenna switch to disconnect the antenna from the receiving set and connect it to the transmitting set and at the same time start the transmitting set operating. This operation required only a fraction of a second. It was thus possible to transmit and receive suc-

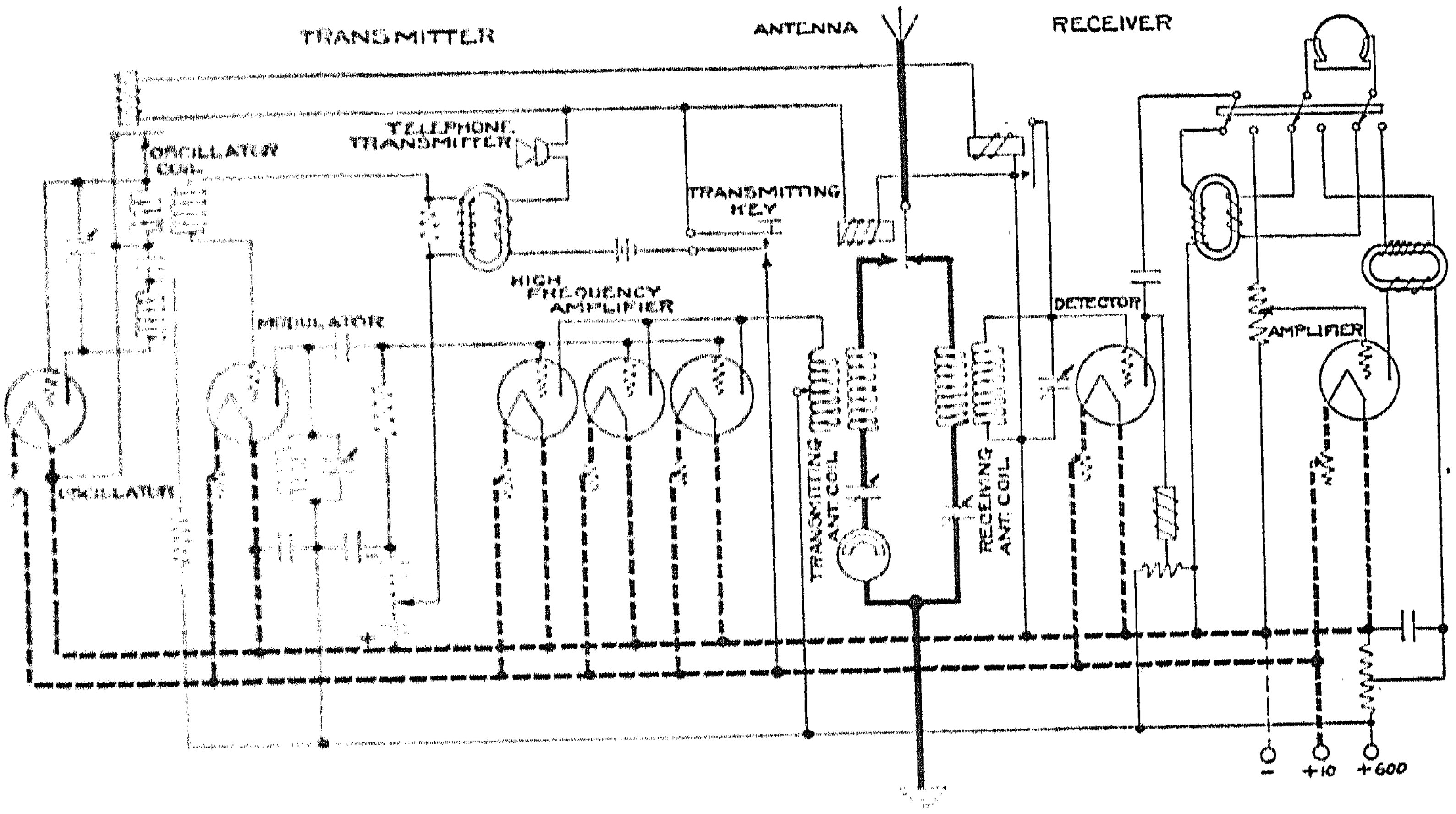


Fig. 12—Circuit Schematic—Battleship Set

cessively on the same antenna by pressing the button whenever it was desired to talk. The wave length range of the set was 600 to 1200 meters. The circuit is shown in Fig. 12. This system was likewise similar to that at Arlington, but was, of course, quite small and produced only one and one-quarter amperes in the antenna. These sets were installed on the battleships Arkansas and Florida at Guantanamo, Cuba, the first part of February, 1916. It was found entirely practicable to hold a two-way conversation between vessels over 30 miles apart. There is no reason to think that was the limit of communication as some of these conversations were overheard in Jamaica, a distance of 175 miles. No attempt was made for range as that was not the object in building these sets.

MULTIPLEX WORKING

This experiment pointed out many things in the use of the wireless telephone which it was highly desirable to know. As the result, the Bureau of Steam Engineering approached us later in the same year and requested that a second step be made in the development of a satisfactory set. The fact that this set worked on the same wave lengths as their regular radio telegraph sets, caused considerable interference. It was desired in the telephone field to provide separate possibilities in communication for the Navy and not encroach upon the telegraph. It was, therefore, thought desirable to go to much shorter wave lengths and provide a set which would not interfere at all with the telegraphic operation of neighboring vessels and interfere very little with the telegraphic operations on the same vessel on which the telephone was being used. A few field experiments were made with short-wave antennas on two ships in July 1916 to provide data necessary for building these sets and their design and construction was started in October. The new set differed from the old one in that it operated on three wave lengths, 150, 189, and 238 metres and also in that it was multiplex and allowed nine conversations to be carried on three wave lengths. The basis of the multiplex system which was originated by Mr. R. A. Heising is as follows:

Suppose a wave of 25,000 cycles is modulated according to a speech signal. This wave when received in the receiving set and rectified reproduces the desired speech. If, however, instead of radiating this 25,000-cycle wave directly, we modulate a 150-meter wave with it and radiate the 150-meter wave, on

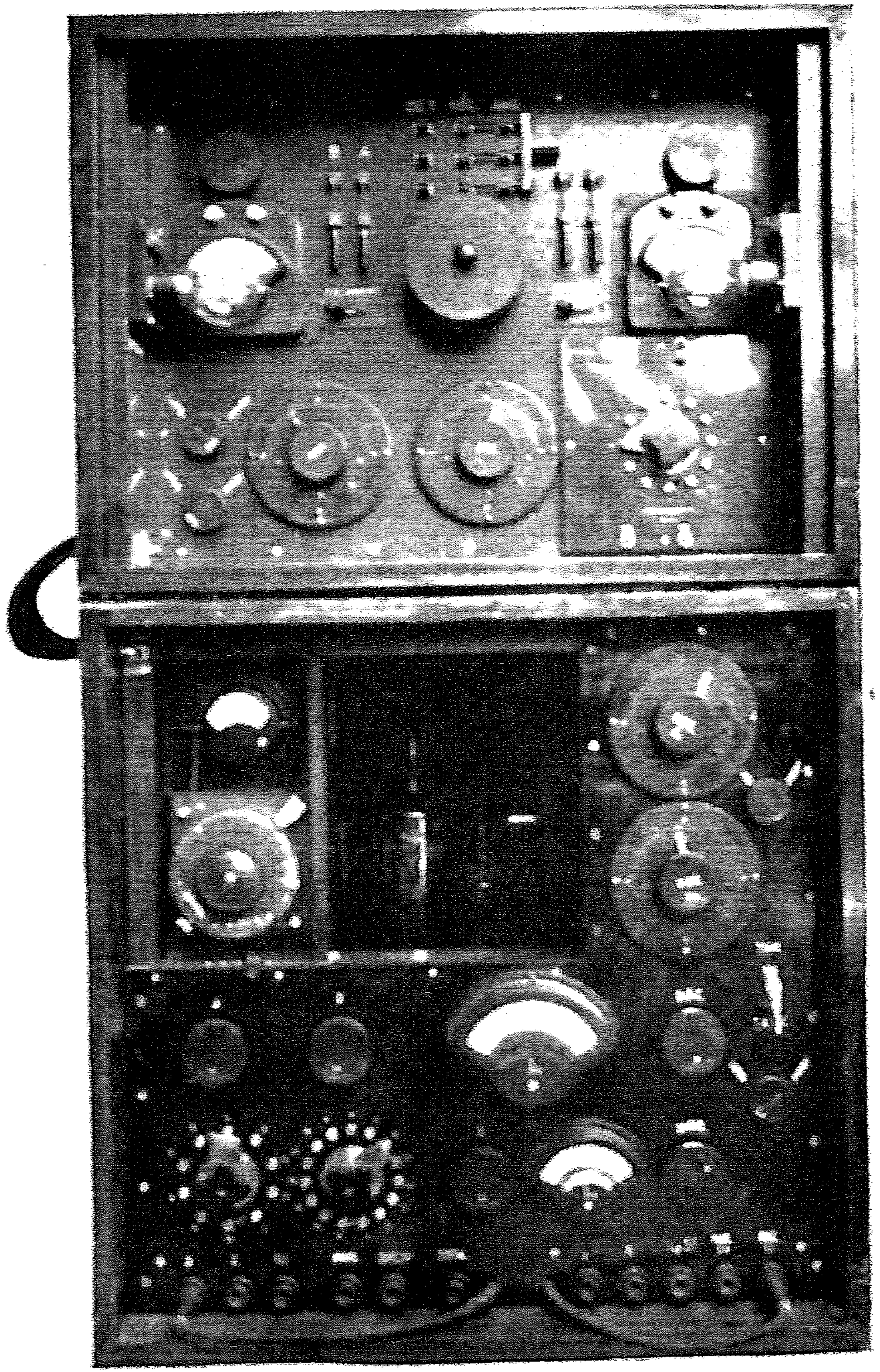
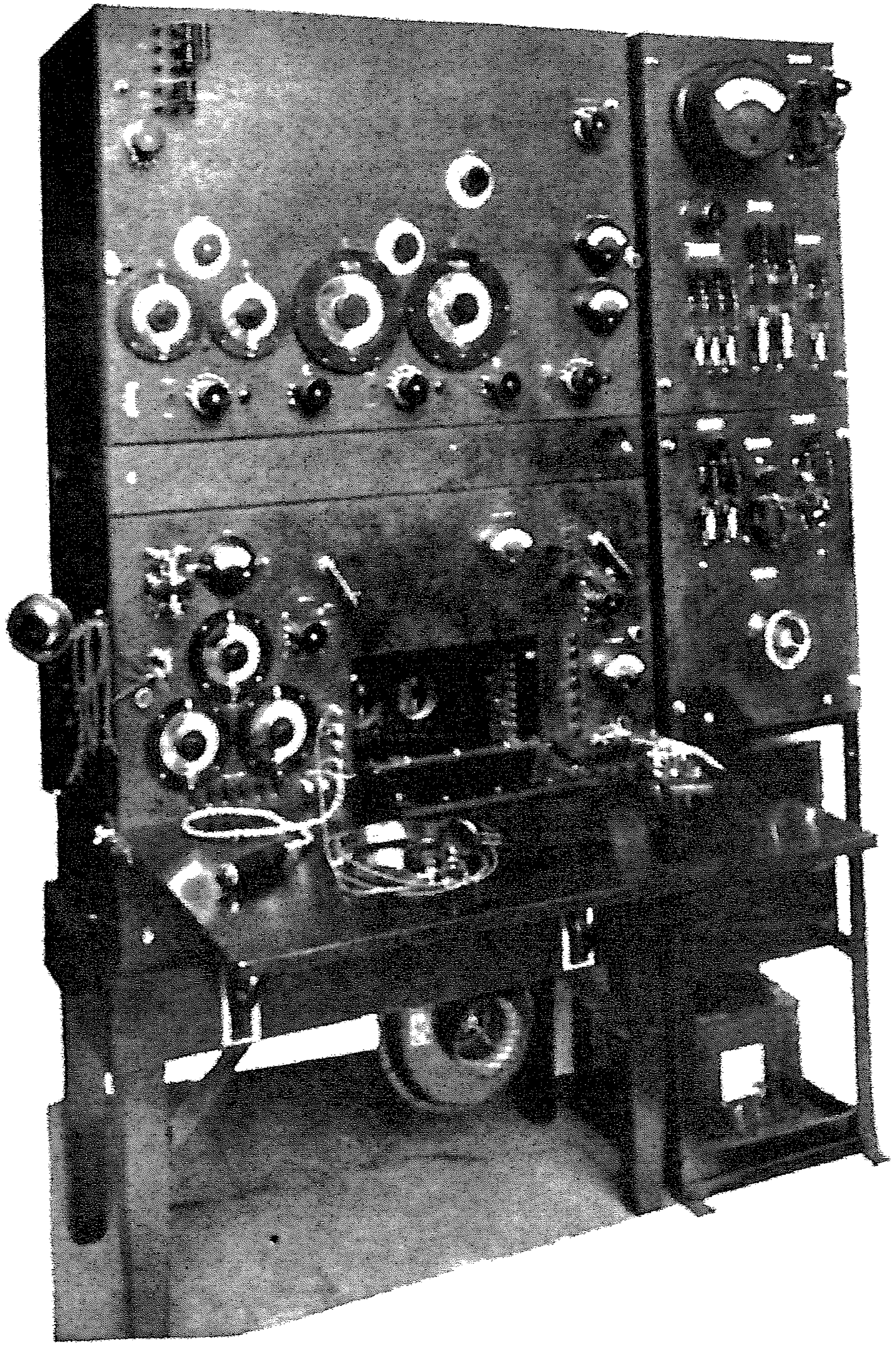


FIG. 11—BATTLESHIP RADIO TELEPHONE TRANSMITTING AND RECEIVING SET



[CRAFT & COLPITTS]

FIG. 15—MULTIPLEX TRANSMITTING AND RECEIVING SET

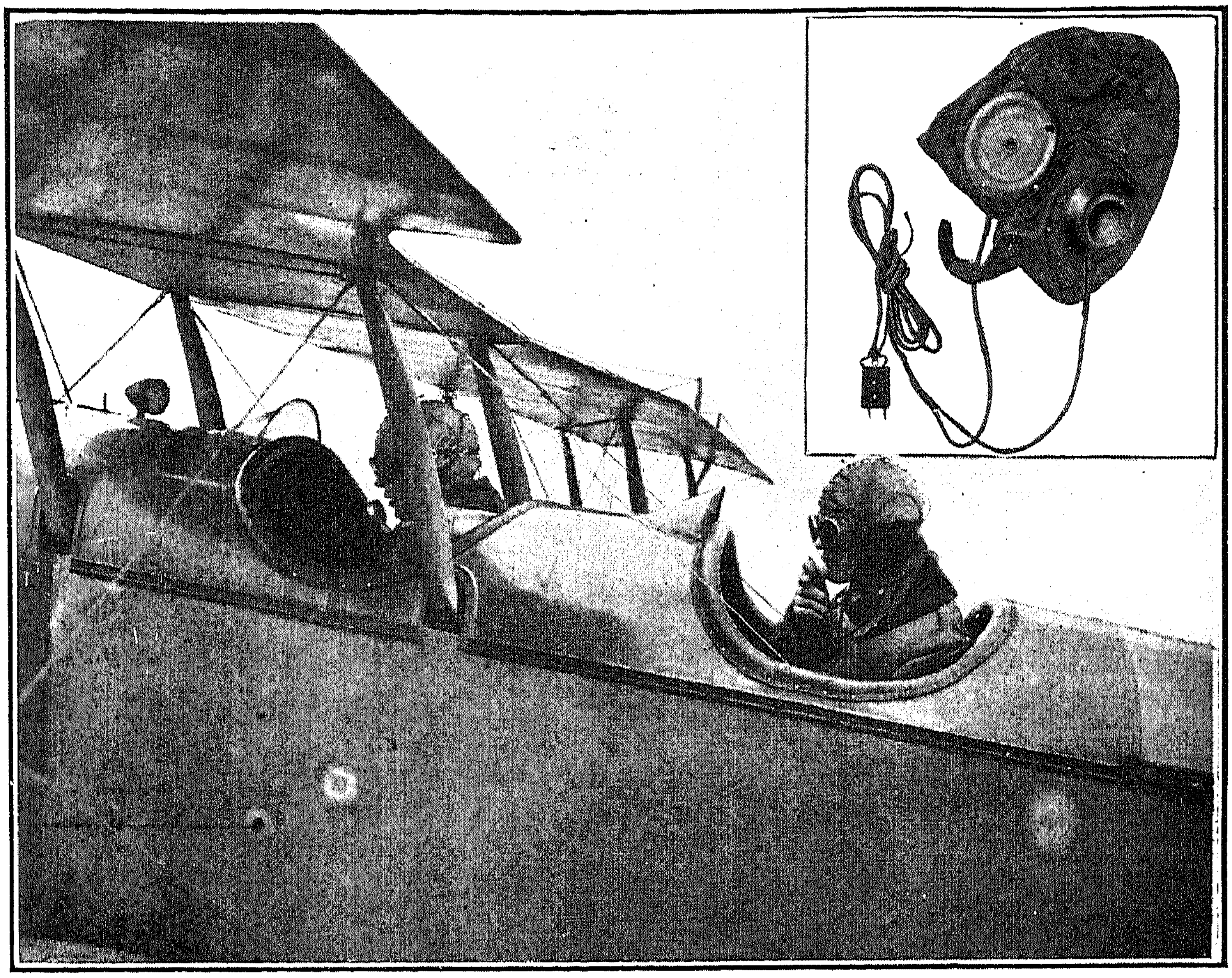


FIG. 17—TELEPHONE RECEIVING HELMET FOR AIRPLANE RADIO

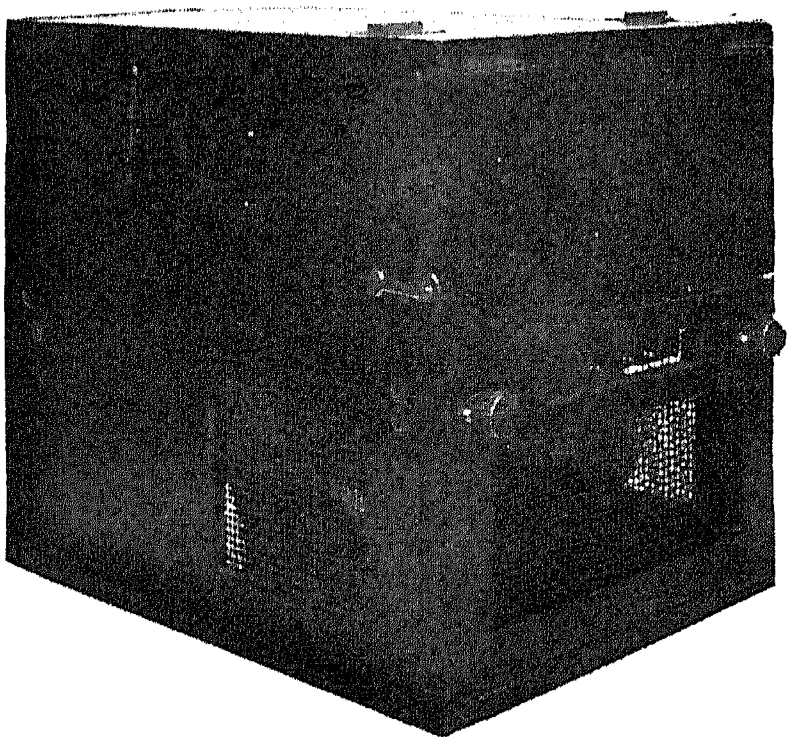
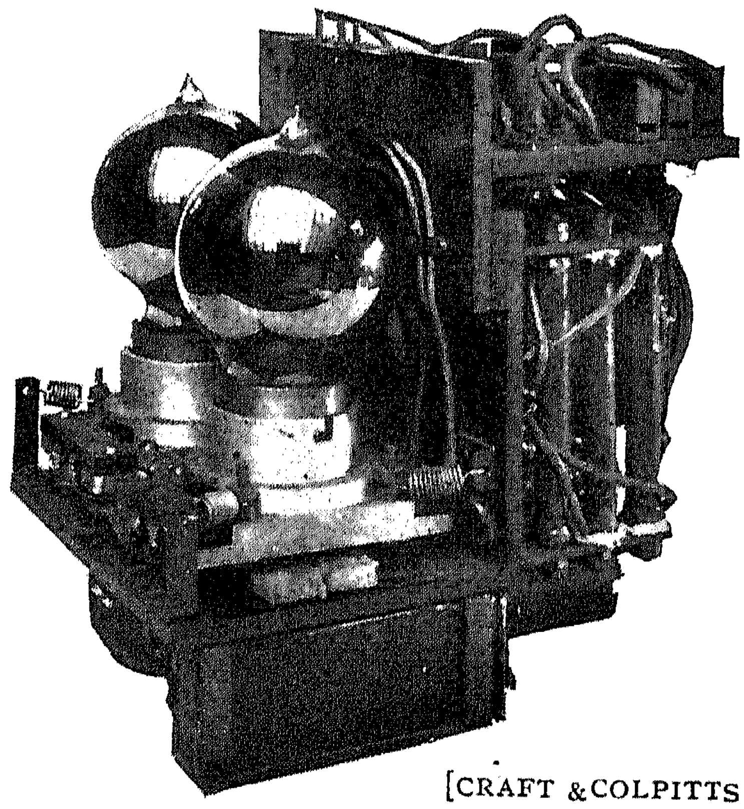


FIG. 19—EARLY FORM OF AIRPLANE TRANSMITTING SET



[CRAFT & COLPITTS]

FIG. 20—TRANSMITTING SET—OPEN VIEW

receiving and detecting at the receiving station the 150-meter wave there will be produced the wave of modulation, or the 25,000 cycle wave. This second wave, is, therefore, produced in the receiving station not by its direct radiation and reception, but is produced there by being carried by the shorter carrier wave. If it is then selected by tuned circuits and detected, the speech signal becomes audible. Therefore, with these sets provided, it was possible to have three ships transmit simultaneously on the same wave length, 150 meters for instance, one ship using an intermediate frequency of 25,000 cycles, the second an intermediate frequency of 35,000 cycles and the fourth, an intermediate frequency of 45,000 cycles. A receiving station tuned to 150 meters would receive all three double modulated waves. By using a second tuned circuit, one of the stations could pick out any one of these three conversations by tuning the second tuned circuit to 25,000, 35,000 or 45,000 cycles and detecting and would not be interfered with by the other two stations on the same wave length. A schematic diagram of the system is given in Fig. 13 and a circuit diagram of the receiving system is given in Fig. 14.

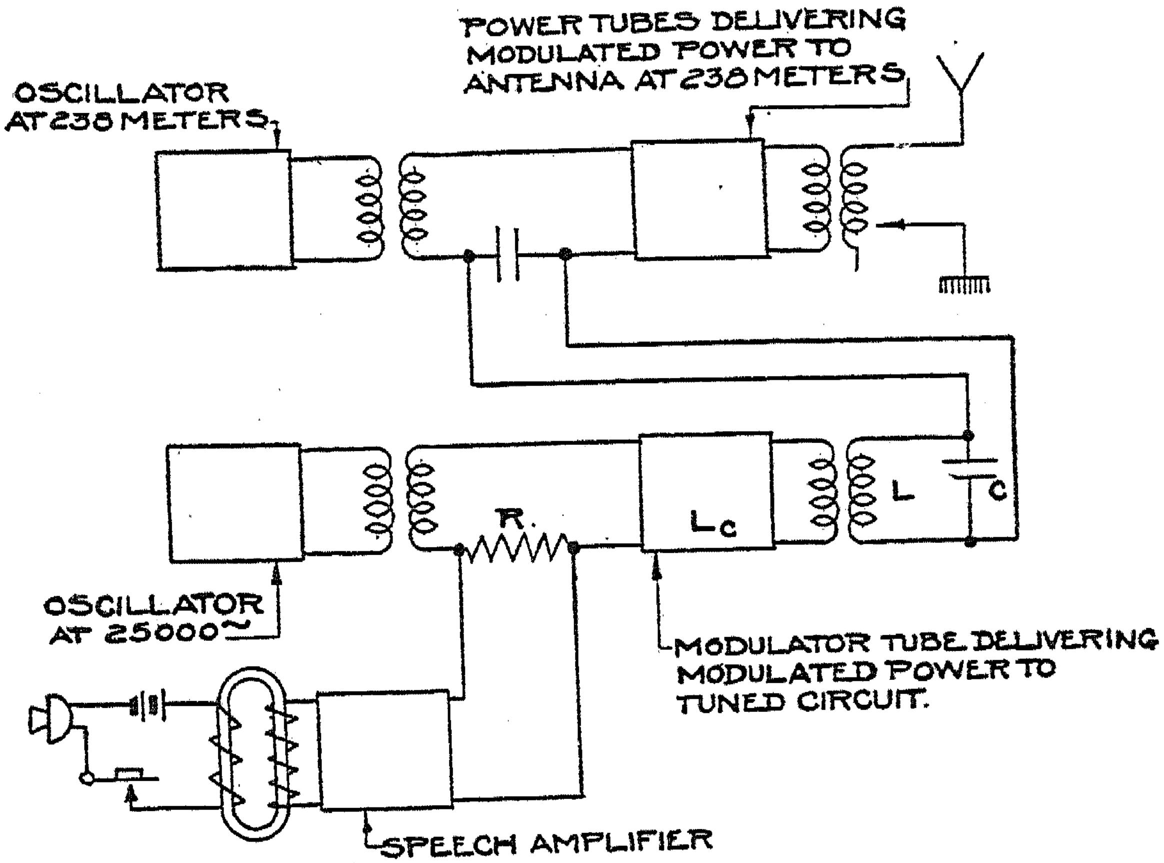

POWER TUBES DELIVERING MODULATED POWER TO ANTENNA AT 238 METERS
OSCILLATOR AT 238 METERS
R
Lc
L
C
OSCILLATOR AT 25000~
MODULATOR TUBE DELIVERING MODULATED POWER TO TUNED CIRCUIT.
SPEECH AMPLIFIER

FIG. 13—SCHEMATIC OF MULTIPLEX ARRANGEMENT

These sets were built upon the same plan as the previous ones in that transmitting and receiving sets were built to be placed one upon the other. An illustration is shown in Fig. 15. These sets were installed on the U. S. S. Pennsylvania, Wyoming and Seattle in January 1917, but scarcely had they been installed than diplomatic relations with Germany were severed and the assistance in the development of apparatus by the Operating Staff of the Navy was necessarily curtailed.

RADIO TELEPHONE APPARATUS FOR MILITARY USE

Beginning with the entry of this country into the war, the demands for radio apparatus for military purposes have resulted in the vary rapid commercial development of this type of equipment. While nothing fundamentally new has resulted from this work, there has been a very practical and valuable result in that the commercial development and manufacture of radio telephone apparatus in general has been put on a basis which otherwise could not have been reached for many years to come.

The first apparatus for war pruposes was built at the request of the Navy Department in March, 1917, and consisted of fifteen experimental sets which were proposed for use on submarine chasers. These sets were of the continuous wave type and were intended primarily for telegraph communication, but were also equipped with a telephone modulating attachment. The transmitting element employed four tubes of the type used in the trans-Atlantic experiments, and delivered approximately 1.5 amperes into the small antenna of the submarine chaser. In view of the fact that these sets were immediately replaced by an improved type which will be described later, the detailed construction of these sets need not be discussed, and they are mentioned merely to indicate the Navy Department's practical interest in this type of apparatus at a comparatively early date.

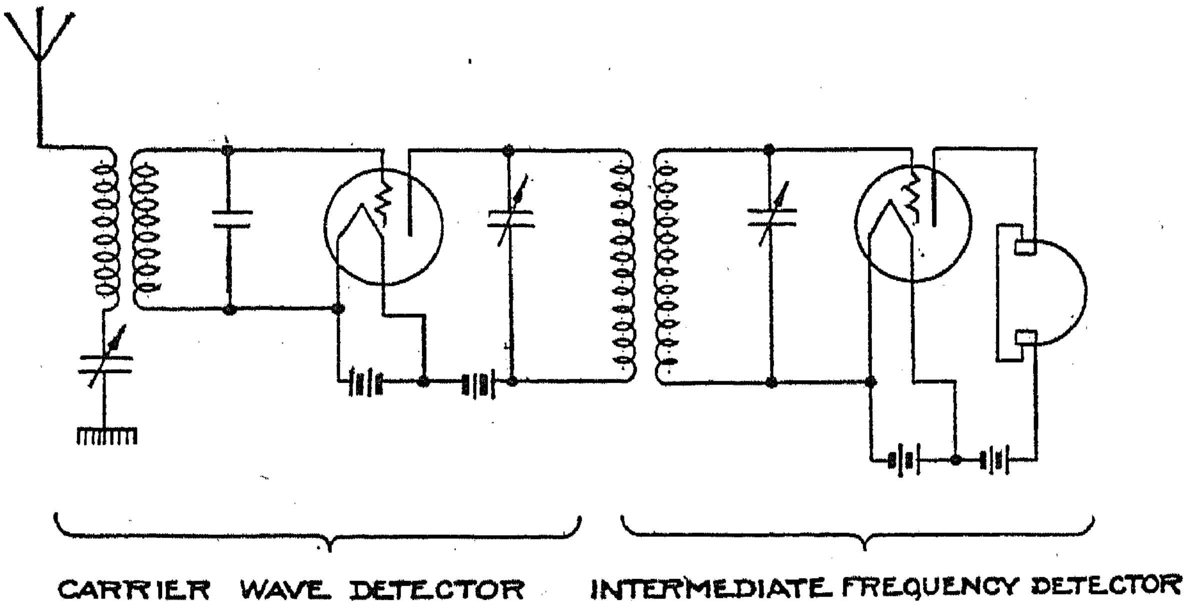


FIG. 14—SCHEMATIC OF MULTIPLEX RECEIVING CIRCUIT

AIRCRAFT TELEPHONY

The most prominent and probably most spectacular development of radio telephony during the war was in connection with its use on military aircraft. While radio telegraphy had been used before for fire control and scouting purposes, its field had not gone beyond that of one-way communication, and was confined practically to the use of simple spark gap types of telegraph apparatus.

Anticipating possible employment of radio telephony in this field, the Western Electric Company Engineering organization had done a considerable amount of experimental work in the way of applying the principles demonstrated in connection with earlier experiments in long distance radio telephony to short range work. This work had been carried on for some time under the direction of Mr. H. W. Nichols, and had resulted in the development in experimental form of satisfactory short range apparatus.

On May 22, 1917, Major General Squier, Chief Signal Officer of the Army, called a conference at Washington to consider the feasibility of intercommunication between airplanes while in flight by means of radio telephony. There were present at this conference besides General Squier, Colonel Rees of the Royal Flying Corps of Great Britain, Captain, now Colonel C. C. Culver of the Signal Corps and Major later Lieut. Colonel F. B. Jewett and Captain later Major E. B. Craft who had been assigned from the Western Electric Company Engineering Organization to engage in this work. At this time plans were in the making for the tremendous aircraft program which was later undertaken by the Army, and it was clear to all that a successful means of communication between battle planes when flying in squadrons would be of inestimable value and would greatly increase the efficiency with which these squadrons could be maneuvered. The early work had shown such promise that there was justification for assuring the Signal Corps that this means of communication could be worked out successfully and applied to aircraft of various types. As a result of this conference, orders were issued by the Signal Corps for the Western Electric Company to undertake the development of a wireless telephone system for the purpose. The experimental equipment available as a laboratory proposition at this time, served as a basis for the first experimental trials in the air. This transmitting set did not use the Colpitts system of modulation but the "constant current" system of R. A. Heising which was found to be more compact and easily operated. This system comprises a modulator and an oscillator tube with their plate circuits essentially in parallel for audio frequency currents. They are supplied from a generator through a choke coil whose function is to maintain a practically constant current to the two tubes no matter how the plate currents may vary individually. A circuit diagram is shown in Fig. 16 and the operation is as follows.

When the transmitter is not actuated there is a certain normal value of voltage impressed upon the grid of the modulator tube, this value being adjusted until the plate current of the modulator is about the same as that of the oscillator. Now

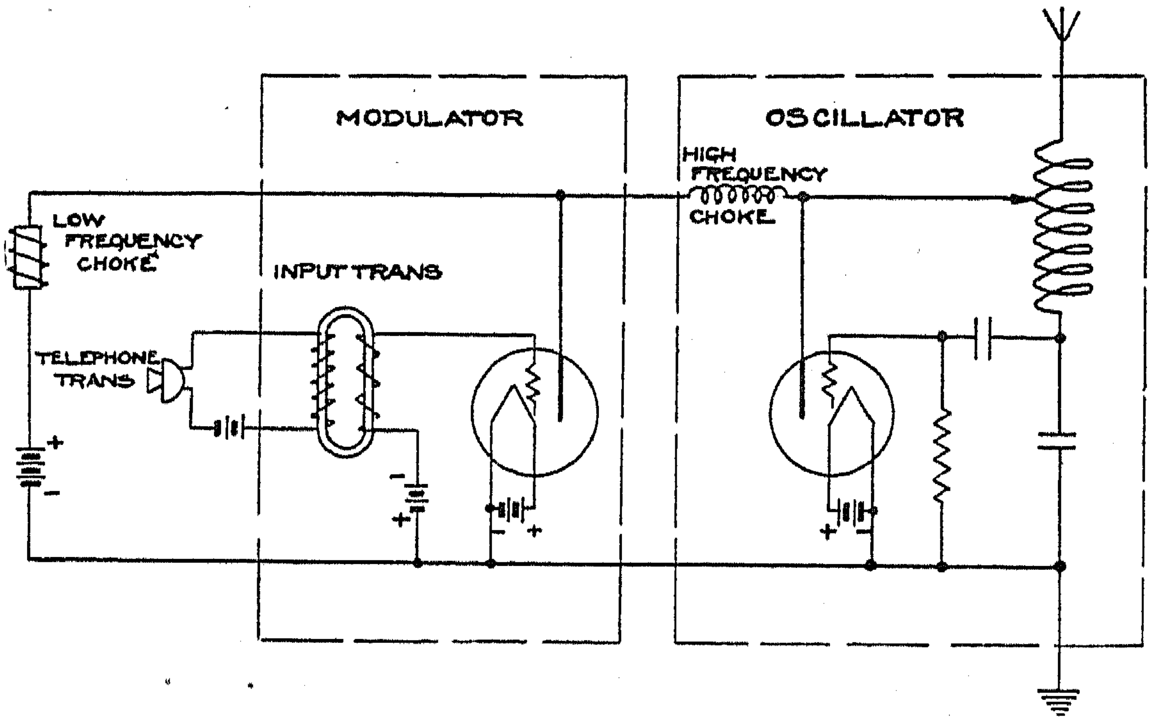


FIG. 16—SCHEMATIC OF HEISING MODULATION SYSTEM

the characteristic curve of Fig. 1 shows that as the grid becomes more negative the plate current decreases at constant plate voltage, and with positive grid voltage it increases. This may be described by saying that the resistance of the plate circuit may be varied by varying the grid voltage, and inspection of the curve shows that this variation may be from a very high value (point $-E$) to a small value for positive grid voltages. In the modulating system of Heising the transmitter voltage acts upon the grid of the modulator and causes the resistance of the plate circuit to vary through a wide range in accordance with the speech voltage. Since this circuit shunts the plate circuit of the oscillator tube (at audio but not at radio frequencies) the oscillator will be robbed of current or have additional plate current forced through it in accordance with the speech voltage. Its output will thus be varied and it has been found that speech is modulated in this way with very good quality. The efficiency of this system is high and greater than that of an "absorption" system in which power is variably diverted from an oscillating circuit and wasted in a modulating device. In the constant-current system the voltage and current in the oscillator rise periodically to nearly twice their values when the transmitter is not acting, which fact accounts for the increased efficiency.

EXPERIMENTAL WORK

This fundamental plan seemed to be the most feasible one to pursue and on June 5, 1917, another conference was held at Washington to agree upon the various technical features and make plans for carrying out field tests.

It was early realized that the principal difficulty in airplane telephony would be the noises due to the motor and the wind, so that attention was concentrated on the problem of providing against this difficulty both in the transmitting and receiving ends of the combination. Laboratory work was directed particularly toward producing a form of telephone transmitter or microphone which would be as insensitive as possible to these extraneous noises and at the same time be responsive to the voice frequencies. In order that this work might be carried on at high speed, a sound proof room was constructed and a device provided which reproduced very accurately the noises of the engine exhaust. By the first of July experiments looked sufficiently promising to warrant tests in the field. On July 2nd a complete radio telephone transmitting equipment was taken into the air and speech of good volume and quality was received on the ground, with the transmitting plane two miles away. The development of a suitable receiving head set was carried on simultaneously and it was found possible to devise a leather helmet with the receiving elements so disposed and screened from external noises that the weak radio signals could be readily observed. On July 4th experimental receiving equipment was taken into the air and Mr. L. M. Clement of the Western Electric Company successfully received speech from the ground at a distance of several miles.

As stated before, the apparatus used in these tests was of an experimental form built in the laboratory. The transmitting set had an output of 0.7 amperes at a wave length range of

200 to 400 meters, the antenna being a trailing wire about 100 meters along.

DEVELOPMENT OF APPARATUS

With the information resulting from these tests, the development of a practical airplane set began at once. During the previous month, comparative tests had been made on a number of proposed modulating systems, with the result that the constant-current system previously described seemed best for the purpose. The problem now was to produce sets of minimum size and weight, physical structures which would withstand the extreme vibrations and jars encountered in flying, especially in landing, the most convenient disposition of control elements, suitable sources of power for both high and low voltage, and a form of antenna which would not interfere with the evolutions of a plane in squadron formation.

It was realized that the solution of the last mentioned problem would consume more time than was at our disposal, and while work was immediately started in the field on antenna measurements and study, the design of the sets proceeded on the basis of using a trailing wire antenna.

The working out of a practical helmet design proved to be more difficult than the success of the earlier experiments indicated. It was found that the degree of interference experienced changed very rapidly with slight imperfections in fit. The problem was also complicated by the necessity of providing means for using oxygen at high altitudes, and of providing for the comfort of the wearer over a period of several hours. It was found that a very slight amount of pressure on certain portions of the ear caused excessive pains and headaches after a very short period, and the final design was a compromise between comfort and efficiency as to sound insulation. Fig. 17 illustrates the form of helmet that was eventually developed and adopted.

The problem of power supply was an interesting one. It was required that the weight should be reduced to a minimum, which precluded the possibility of employing storage batteries. There are obvious objections, also, to attaching any form of generating device to the propelling engine. Consideration of all the factors led to the adoption of a wind-driven generator for the purpose. This generator of about 100 watts direct-current output was required to produce a potential of 300 volts for the plate circuits of the vacuum tubes, and a potential of 25 volts for the filament circuits. It is necessary, to insure most efficient operation, that the filament current be kept constant, and the fact that the specifications called for operation with airplane speeds varying from 40 to 160 miles per hour made the problem of voltage regulation loom large. Ordinary forms of electro-mechanical regulating devices did not prove to be successful and while it was probable that something of this type would eventually have been developed, the problem was solved by a very ingenious arrangement proposed by Mr. H. M. Stoller, in which the vacuum tube is the essential element. Fig. 18 shows, schematically, the generator and regulator circuit. The voltage of the generator is held approximately constant by means of a vacuum tube regulator, which controls the field flux. Two field windings are provided, the main field, which is in series with the filament of the regulator tube, and a differential field which is in series with the plate circuit. At minimum speed the differential field is inactive and the generator behaves like an ordinary shunt machine, except that the main field has a small resistance in series due to the regulator tube filament. This filament is so designed

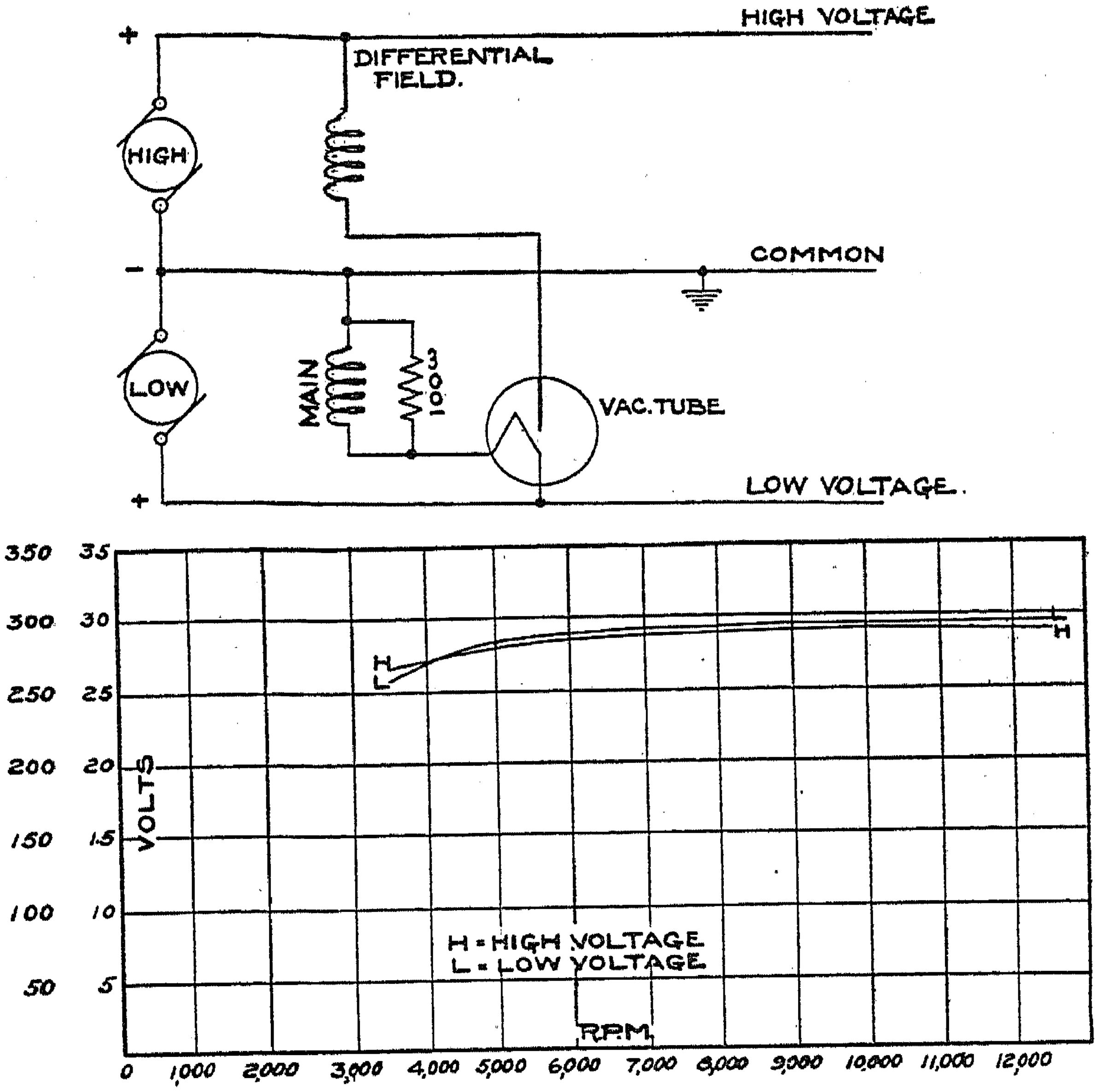


FIG. 18—VOLTAGE REGULATION OF WIND-DRIVEN GENERATOR

that the main field current heats it to a temperature which gives a small electron current. This current flows through the differential field and reduces the resultant flux. At minimum speed, the differential current is small, but as the speed increases, the main field current tends to rise (as in any shunt generator) and this increases the temperature of the regulator tube filament. The electron current is, therefore, considerably increased, which current, flowing through the differential field, reduces the generator flux and thus restricts the rise in voltage. Due to the fact that the electron current of the regulator tube increases very rapidly with increase in filament current, the voltage is held practically constant between 4000 and 12,000 rev. per min. The 1.5-ohm resistance units are provided so that the regulator may be set to give different voltages by cutting them in or out of the main field. The 100 ohms shunt resistance is used to prevent hunting.

Upon the completion of the next set of models, field tests were resumed and on August 20th the first two-way telephone conversation between two planes in the air was successfully accomplished. After this first trial, Major Bartholf and Lt. Stevens of the Signal Corps held two-way conversation between planes with very satisfactory results.

In Figs. 19 to 24 are shown the apparatus that was employed in the August tests at Langley Field. At this time two general schemes of control were considered,—manual means for transferring from the transmitting to the receiving position, and automatic means for accomplishing the same result through the operation of a remote control relay. It was concluded, because of its simple construction, to employ the manual means of control, the idea being to locate the set in the observers position in the plane.

On August 22nd an informal demonstration of talk from airplane to ground was given for Secretary of War Baker, General Scott, and Colonel Baker. These experiments conclusively demonstrated the practicability of the system and apparatus, and we felt justified in proceeding with its commercial development.

Commercial Development

VACUUM TUBES

While the vacuum tubes that had been developed in the 1915 and 1916 experiments were satisfactory in operation under normal conditions, it was found that the mechanical vibration

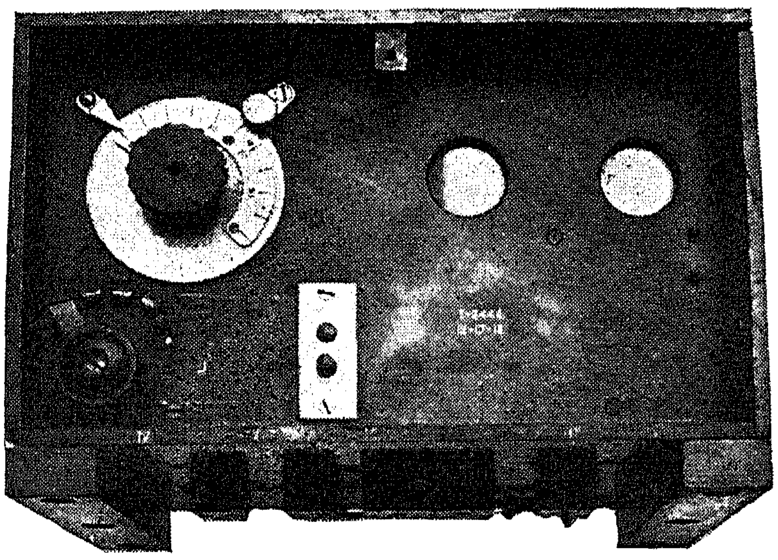


FIG. 21—EARLY FORM OF AIRPLANE RECEIVING SET

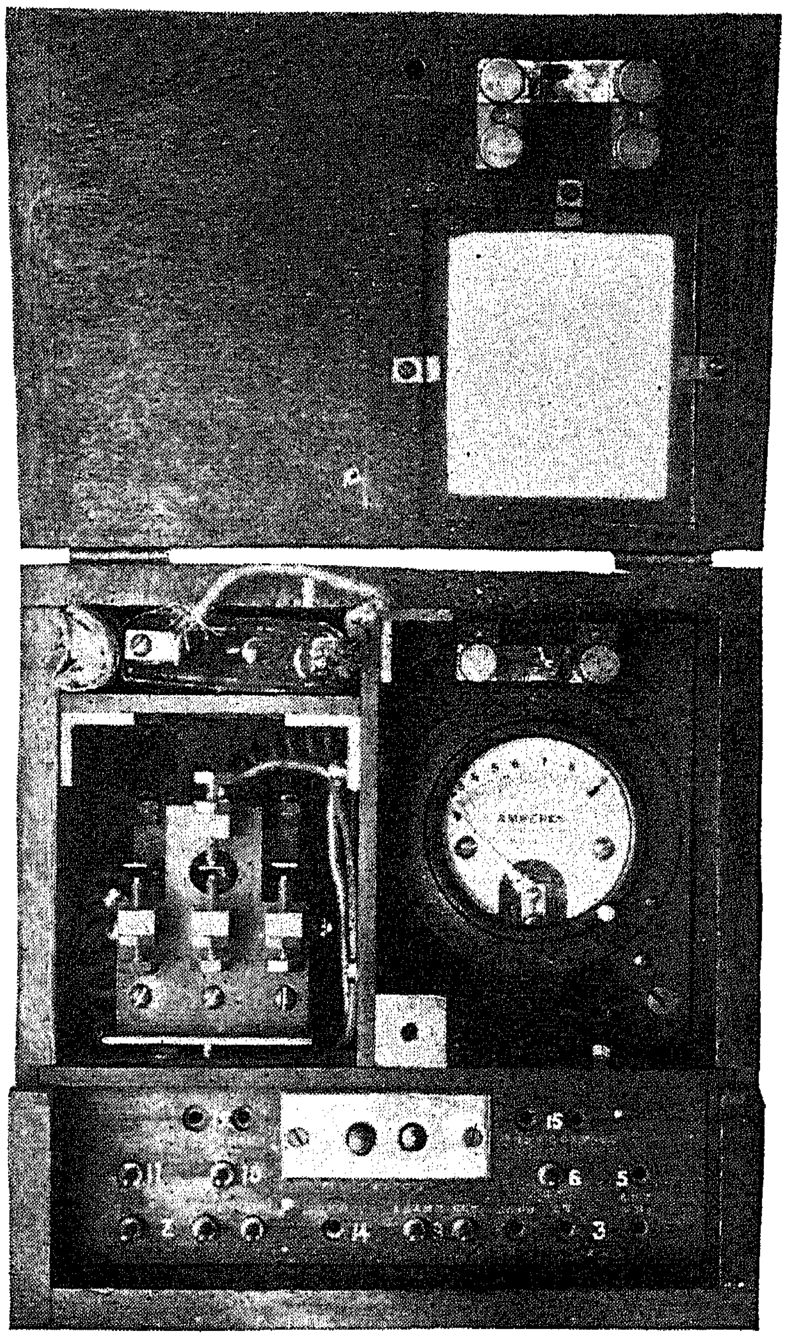


FIG. 24—REMOTE CONTROL BOX—AIRPLANE RADIO

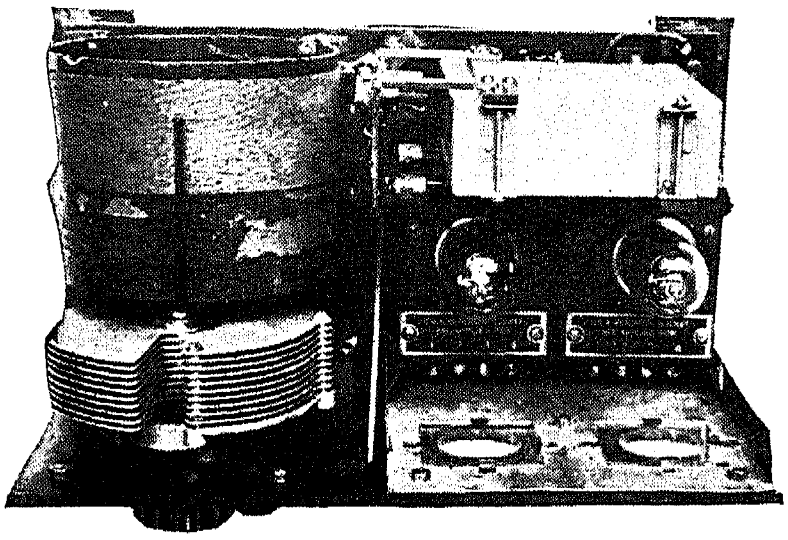


FIG. 22—AIRPLANE RECEIVING SET—OPEN VIEW

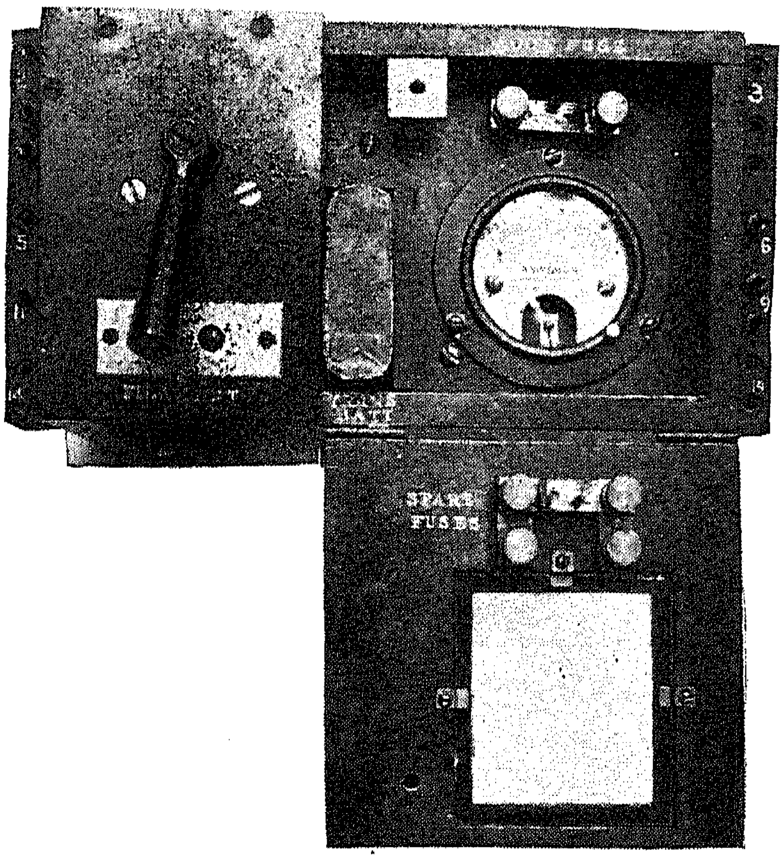
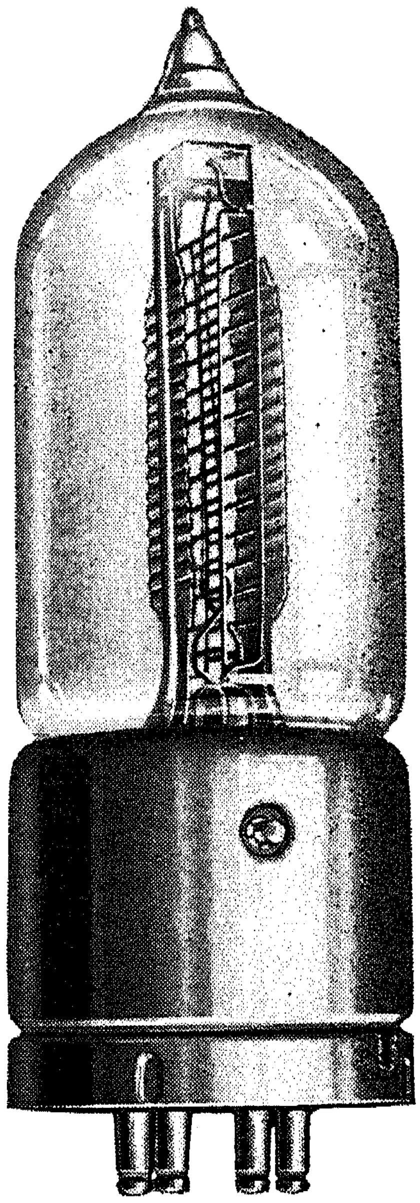


FIG. 23—MANUAL CONTROL BOX—AIRPLANE RADIO



[CRAFT & COLPITTS]

FIG. 25—WESTERN ELECTRIC RECEIVING TYPE VACUUM TUBE—ARMY AND NAVY STANDARD

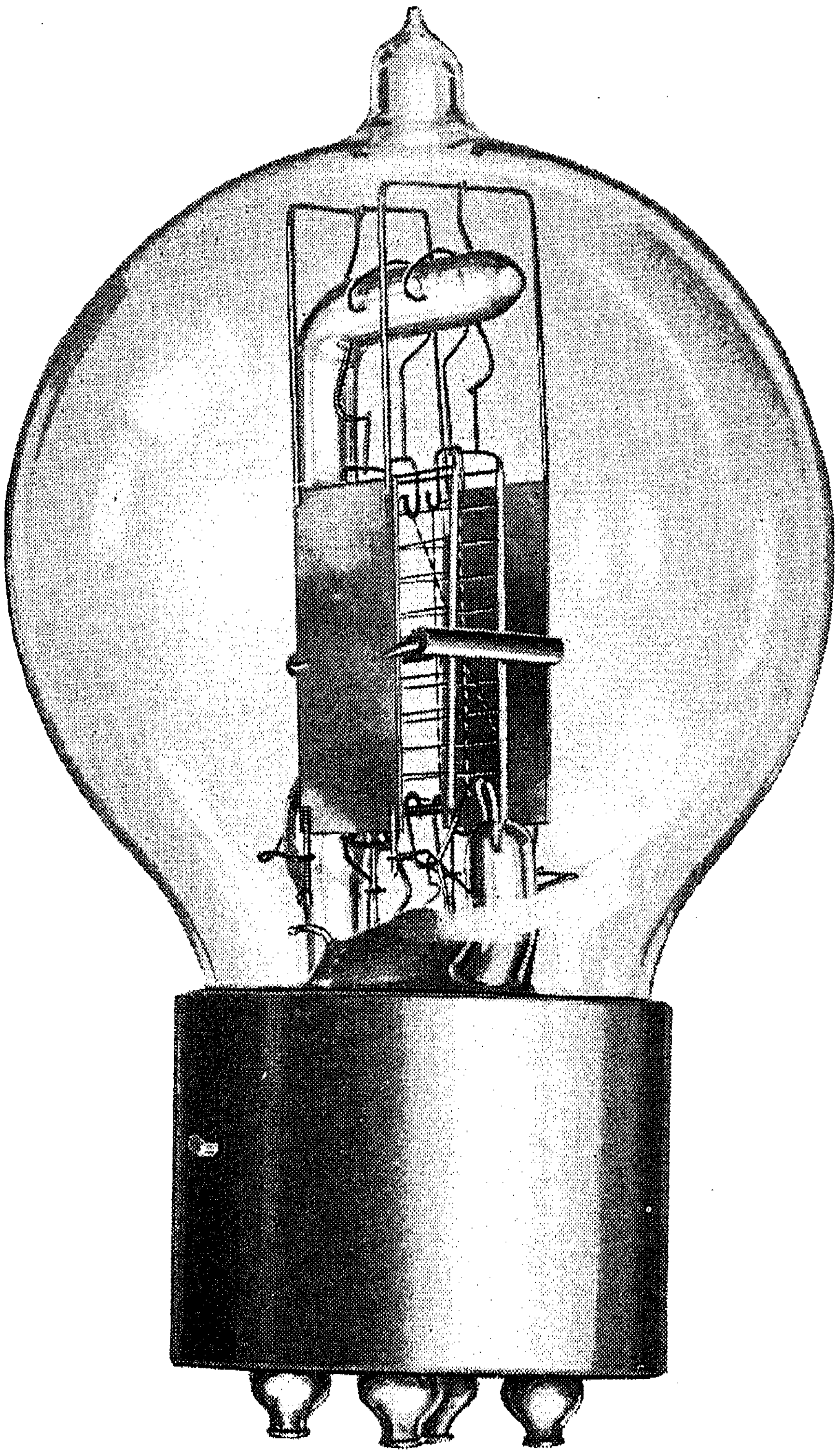


FIG. 26—EARLY FORM OF WESTERN ELECTRIC TRANSMITTING TYPE VACUUM TUBE

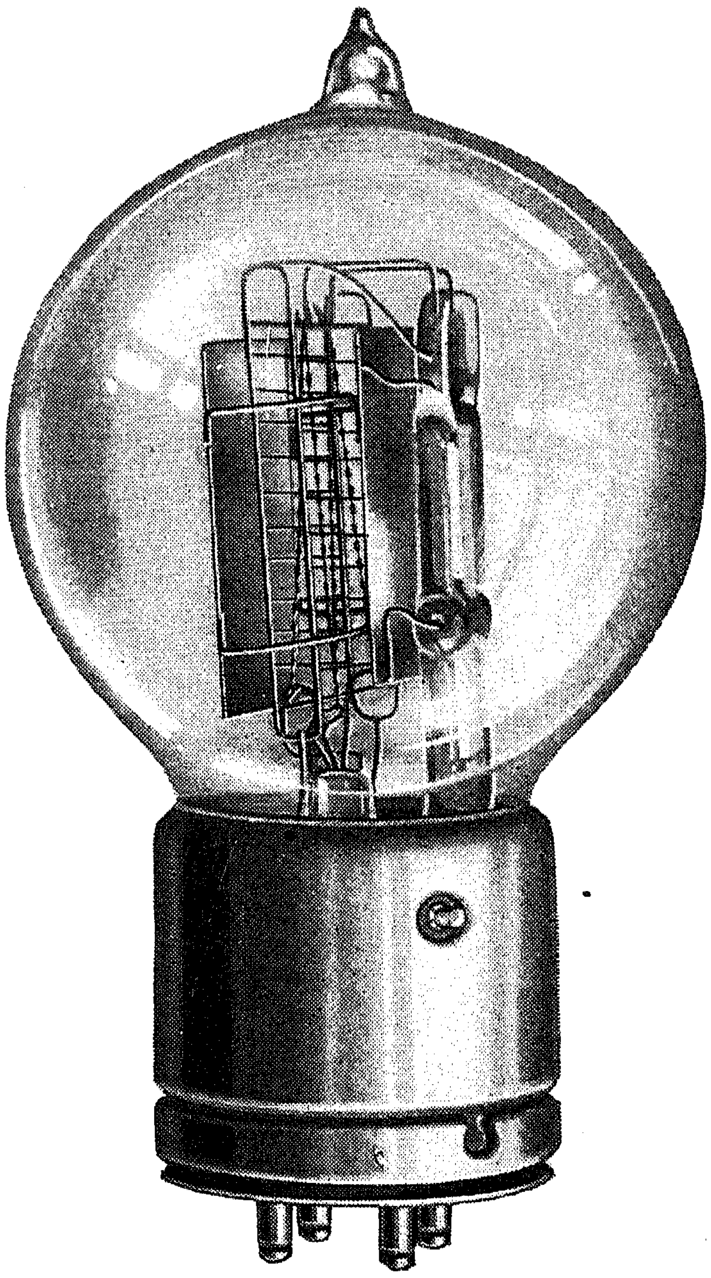
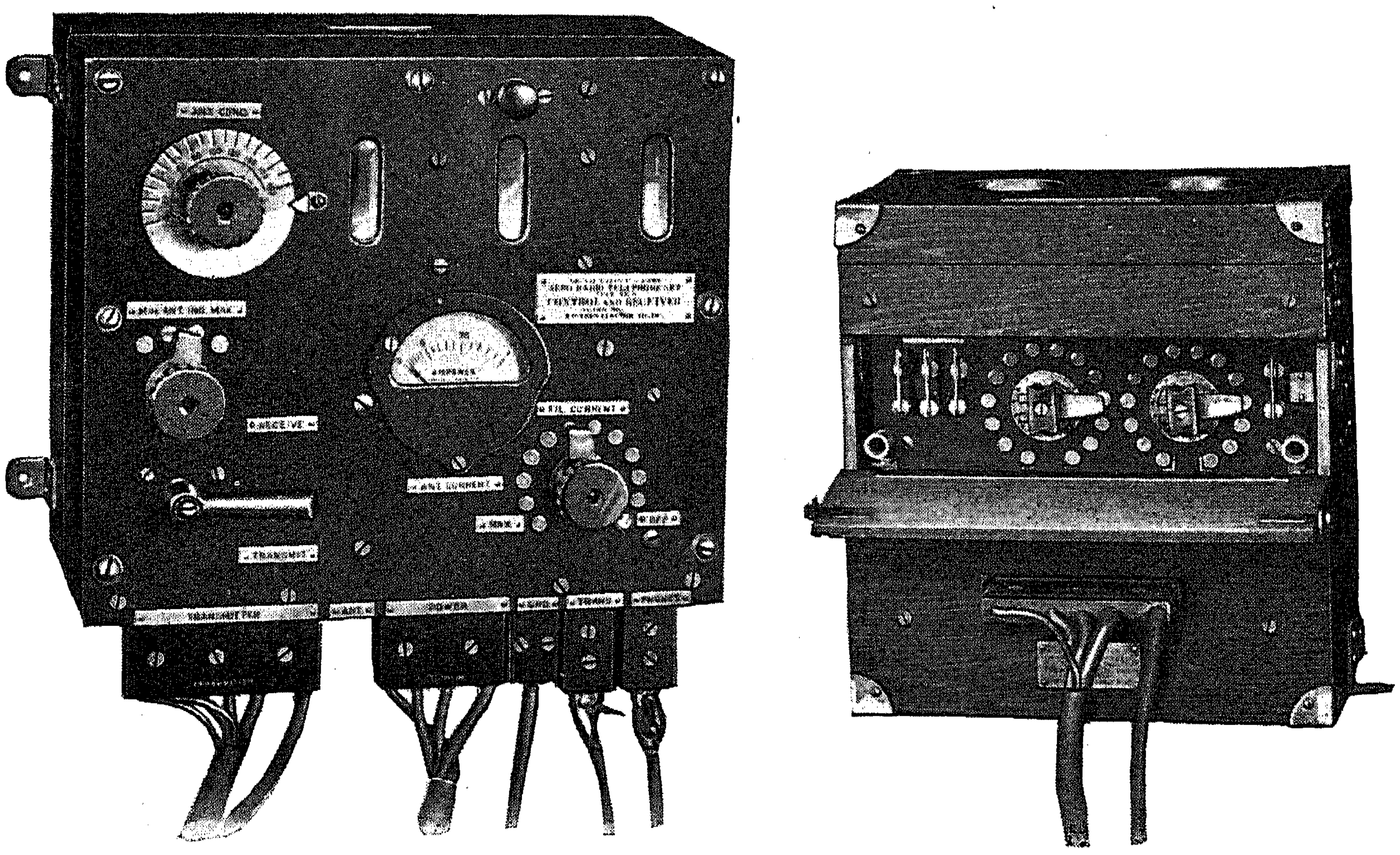


FIG. 27—WESTERN ELECTRIC TRANSMITTING TYPE VACUUM TUBE —ARMY AND NAVY STANDARD



[CRAFT & COLPITTS]

FIG. 30—WESTERN ELECTRIC AIRPLANE TRANSMITTING AND RECEIVING SET—FIRST STANDARD TYPE ADOPTED BY THE SIGNAL CORPS

encountered in the air was such as to necessitate special structures to withstand them. New forms of vacuum tubes, which successfully met these requirements were developed. Fig. 25 shows the receiving tube in which the elements are supported in a metallic framework, thus eliminating a great deal of breakage which occurred in the earlier structures. Fig. 26 shows the first form of power tube which was developed, later followed by the type shown in Fig. 27.

The vacuum tubes used in all this work were of the so-called "Wehnelt Cathode" type; that is, the electron-emitting cathode consisting of a metal filament—usually platinum—coated with a mixture of oxides which, when heated to a moderate temperature, gives off electrons in great numbers. When properly constructed and treated, such a type has several great advantages:

1. For a given electron emission, the temperature is not so near the point of destruction due to evaporation, etc., as in the case of a pure metal—for instance, tungsten. The result is longer life in operation.
2. For a given change in filament current at working ranges the change in electron emission is smaller in the case of the Wehnelt cathode, permitting of closer regulation in power output.
3. Evaporation of the filament does not go on so rapidly, consequently its electrical behavior is more uniform throughout its life.
4. The electron emission for a given amount of energy supplied to the cathode is larger, resulting in more economical operation of the sets.

It was found that for transmitting and receiving purposes, two types of tube were sufficient. The tube for receiving purposes was designated as VT1 by the Signal Corps and as the CW933 by the Navy. A transmitting tube capable of delivering a moderate amount of power—say from 3 to 5 watts high-frequency output—was known as the VT2 by the Signal Corps and as the CW931 by the Navy. Vacuum tubes are defined, as far as electrical characteristics in normal operation are concerned, by plate and filament voltages, plate current with normal plate voltage and zero grid voltage, normal filament current, filament life at this current, and by what is known as the amplification constant, which is approximately the ratio, μ, of that change in plate voltage which produces a given change in plate current to the change in grid voltage to produce the same change in plate current.

Some of these relations are conveniently shown in characteristic curves. The plate current-grid voltage characteristics of the *VT*1 and *VT*2 tubes are shown in Figs. 28 and 29. On these illustrations are given also the other data mentioned above.

As indicating some of the problems involved in the commercial production of this entirely new type of equipment, it is interesting to note that prior to August, 1917, the total output of commercial vacuum tubes of this general type was approximately 200 per week, their use being practically confined to long distance wire telephony and to radio detection purposes. On November 11th, 1918, deliveries were being made at the rate of 25,000 per week. This involved the organization of equipment and personnel to do a class of work for which there was practically no experienced talent available and the situation in many features was analogous to that in connection with airplane production.

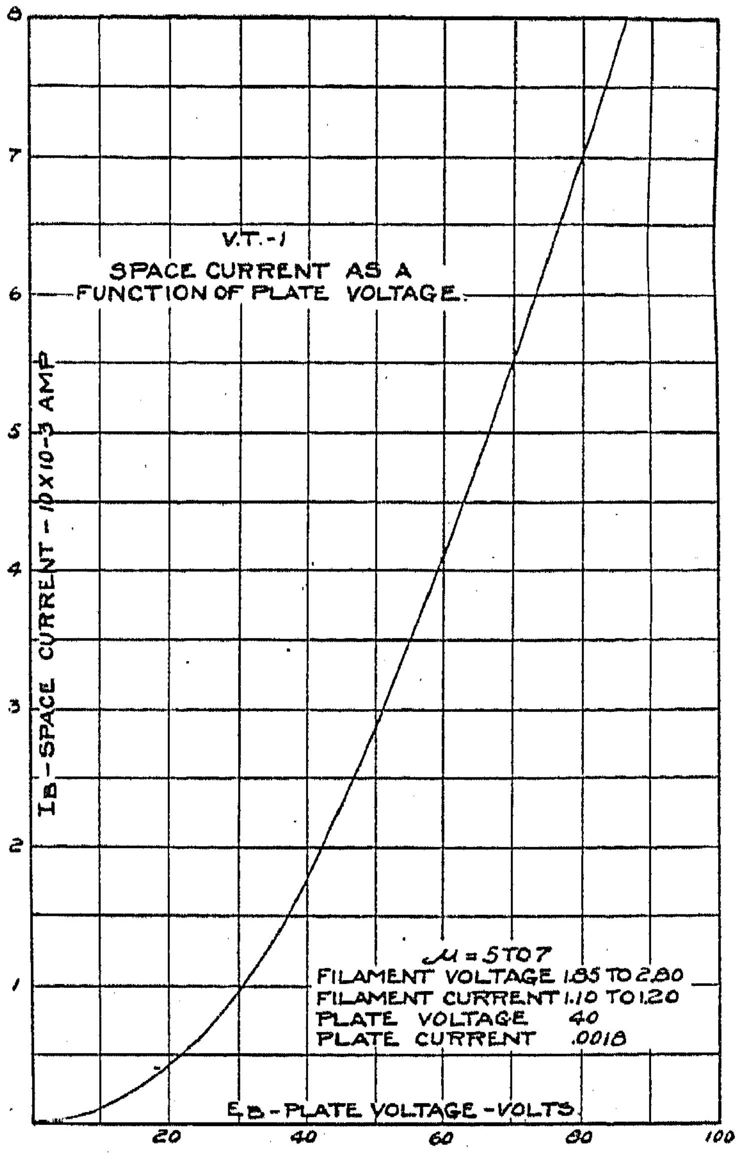


FIG. 28—CHARACTERISTICS OF RECEIVING TUBE *V T*-1

APPARATUS

Tests of the first standardized sets for Signal Corps use were made at Langley Field on October 6th. These sets operated very successfully and official demonstrations of two-way communication were made. This set was made up of a combined control panel and receiver with two stages of amplification and a separate transmitting set, the electrical connections being made by flexible cords. These units are illustrated in Fig. 30. By means of the multi-contact manual switch located on the control panel, the operator can receive or transmit as desired. The receiver consists of a single tuned circuit of the very simplest type and the amplifier is made adjustable for the convenience of the observer.

While the original requirement was that communication should be maintained at a distance of 2000 yards, all of the tests indicated that successful communication could be obtained at much greater distances. On October 16th an official distance test was made, and communication between planes was maintained at a distance of 23 miles, and from plane to ground, 45 miles. The conditions under which these tests were made were extremely favorable, and are noted merely to show the possibilities of this particular equipment. The figure that was finally established as being representative of what would be required in service, was three miles.

A number of sets were immediately constructed, and complete equipment sent overseas with Signal Corps Officers,

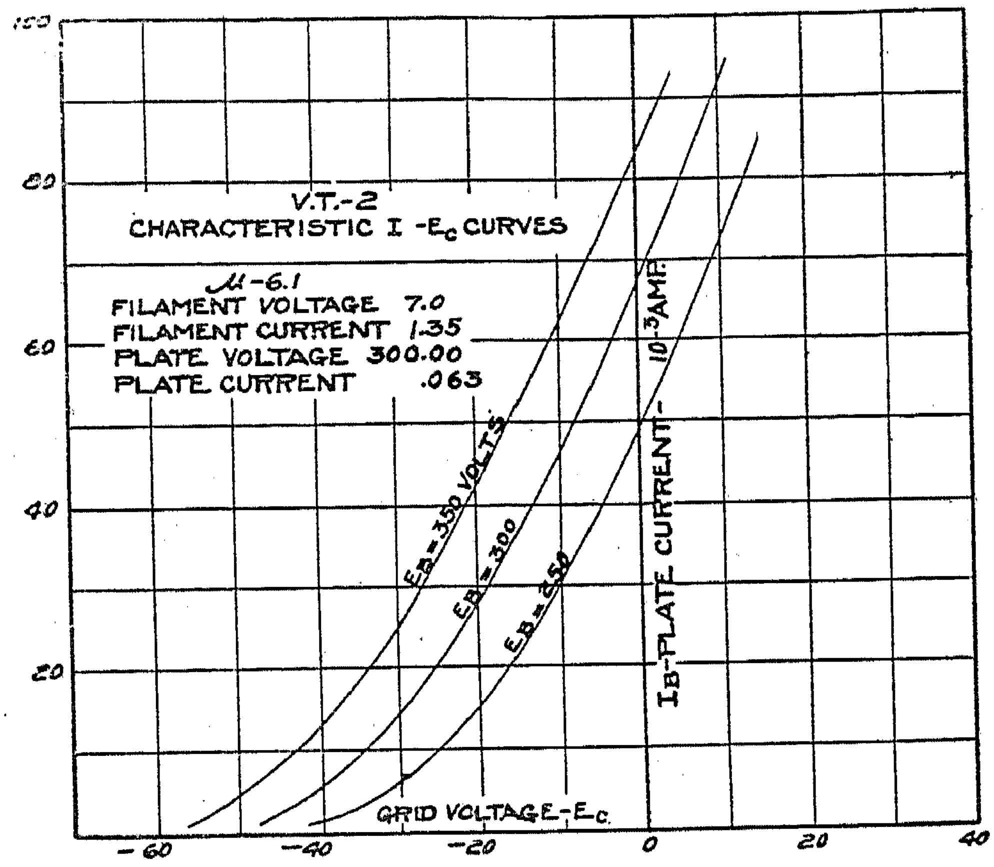


FIG. 29—CHARACTERISTICS OF TRANSMITTING TUBE *V T*-2

where they were submitted for the criticism of our military forces.

Several more demonstrations of this apparatus were made, culminating in the official trials at Dayton, Ohio, on December 2nd, 1917. There were present members of the Aircraft Production Board and the joint Army and Navy Technical Board, and various Signal Corps Officers. The demonstration consisted of a three-cornered conversation between two planes in the air and a ground station. At the ground station a loud speaking receiver was connected to the radio set in such a way that the entire party of about thirty could overhear the conversation between the fliers and also the speech between the planes and the ground. Under orders transmitted from the ground station, the fliers performed various evolutions in the air, and the observers could see these orders carried out and hear the acknowledgments. In these tests the pilot and observer in each plane were also connected, so that there were five people in constant communication. The maximum distance of the planes from the ground station was about eight miles, at which distance they were invisible. These tests so conclusively demonstrated the possibilities of the successful use of this apparatus, that quantity orders were immediately placed by the Signal Corps.

The time elements were very short and the problem of adapting the designs to commercial quantity manufacture were many and intricate. This probably is the first instance on record in which the production of radio apparatus, either telephone or telegraph, has been put on a manufacturing basis, in quantities comparable with that obtaining in ordinary lines of electrical manufacture.

The apparatus represents not what would be designed and built under normal conditions with time available for proper study of all the technical features involved, but the best compromise, bringing in such factors as the use of standard parts, available manufacturing facilities and finally the imperative need of haste.

Information from abroad was at times conflicting and inadequate, so that the Signal Corps Officers in Washington were often required to render decisions on important points under exceedingly difficult circumstances. We are greatly indebted to Lt. Col. N. H. Slaughter and other officers of the Signal Corps for their effective cooperation in the solution of difficult problems arising from all these causes.

The final standardized form of two-way airplane set is known by the Signal Corps as the SCR-68 set. Fig. 31 shows the complete equipment for a two-place airplane. The elements are so arranged that they can be mounted in the space available in various parts of the machine. The main set, however, necessitates being accessible to the operator.

The switch box, shown in the center, is to provide for communication between pilot and observer by means of the same helmets and microphones as are used for radio communication. This is one of the interesting by-products resulting from the radio development. Heretofore the only practicable means of communication between pilot and observer was by means

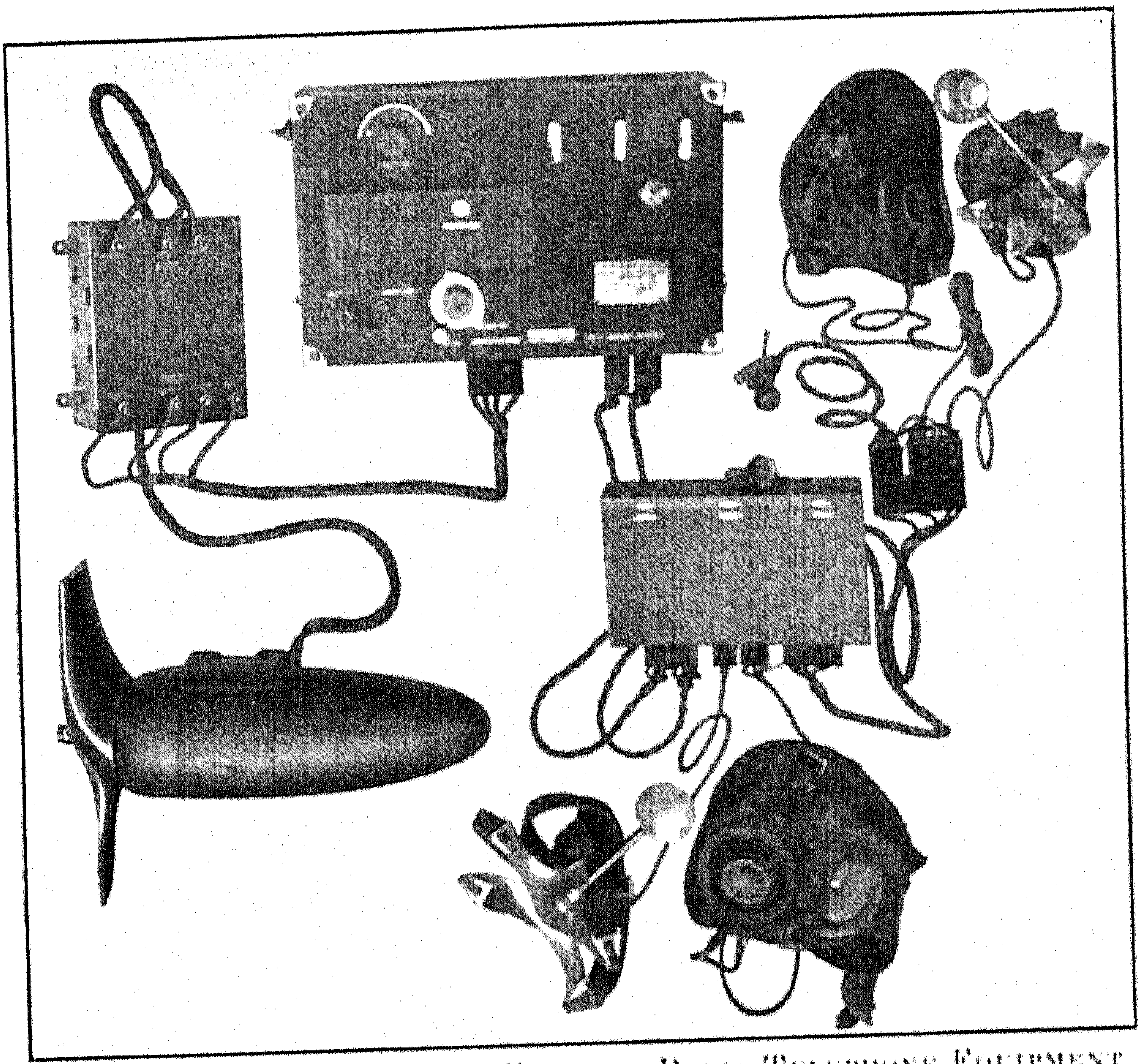


Fig. 31—Complete Western Electric Radio Telephone Equipment for Two-Place Airplane—Signal Corps Standard SCR-68

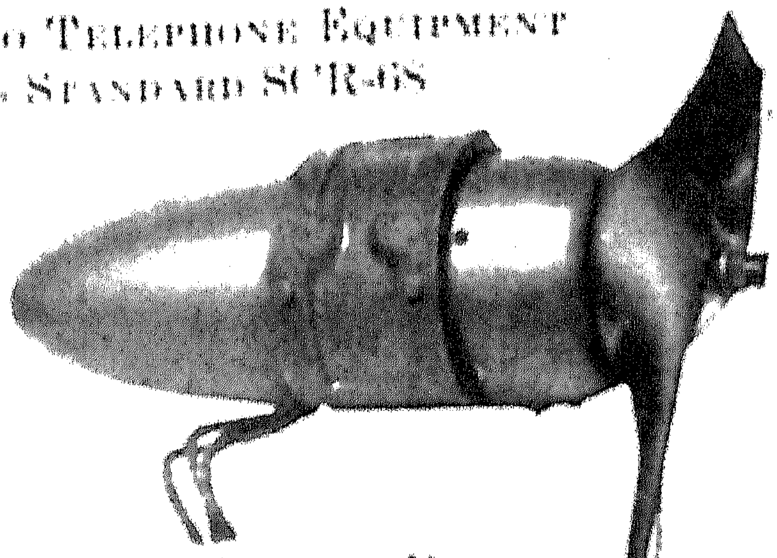


Fig. 33—Double Voltage Wind-Driven Generator

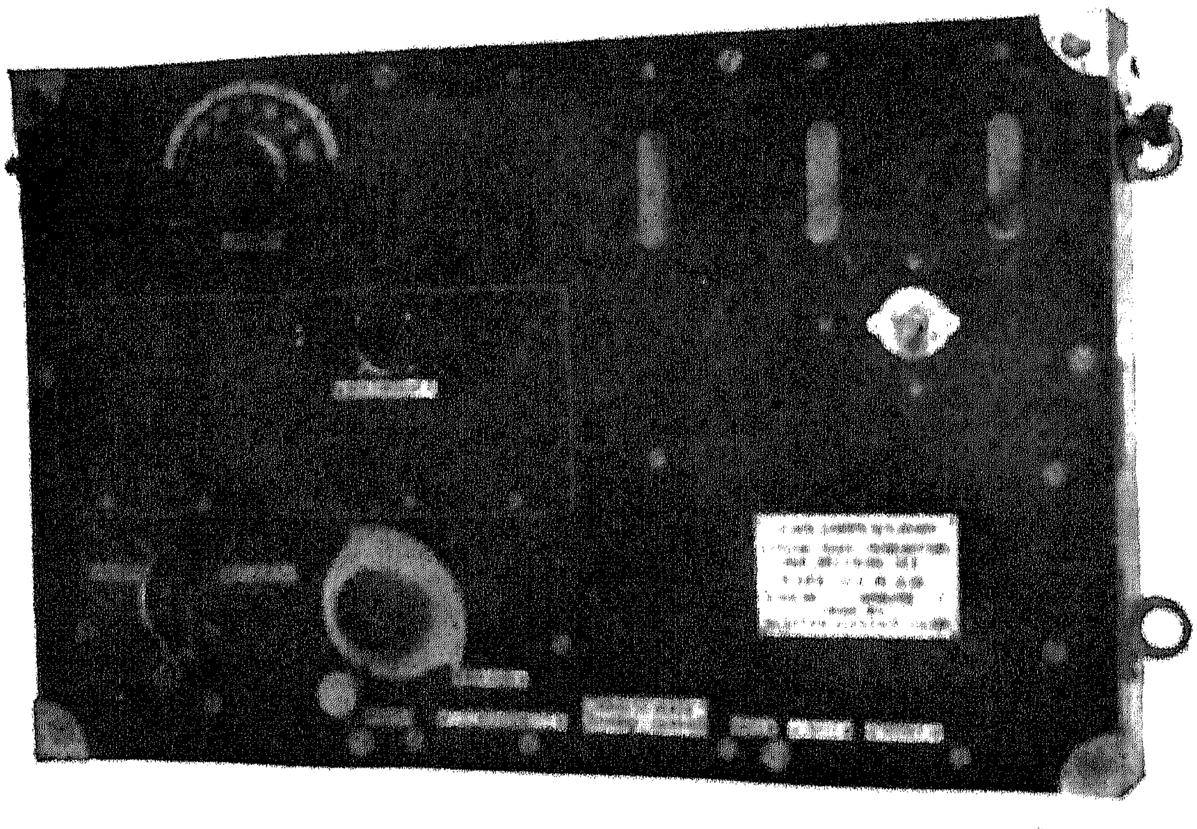


Fig. 32—Transmitting and Receiving Set Box—Signal Corps SCR-68

[CRAFT & COLPITTS]

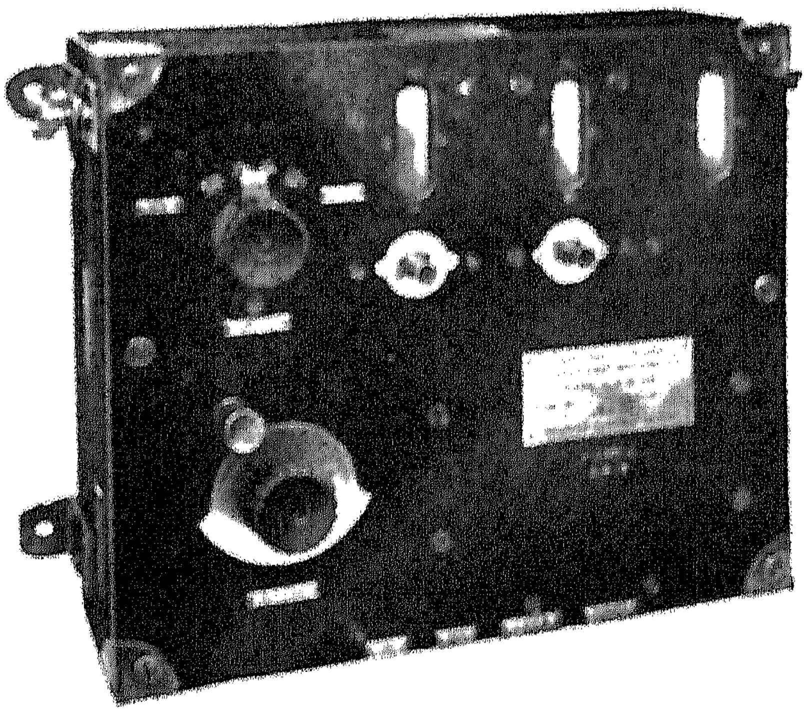


Fig. 37—Western Electric Radio Telephone Receiving Set—Signal Corps S C R-59

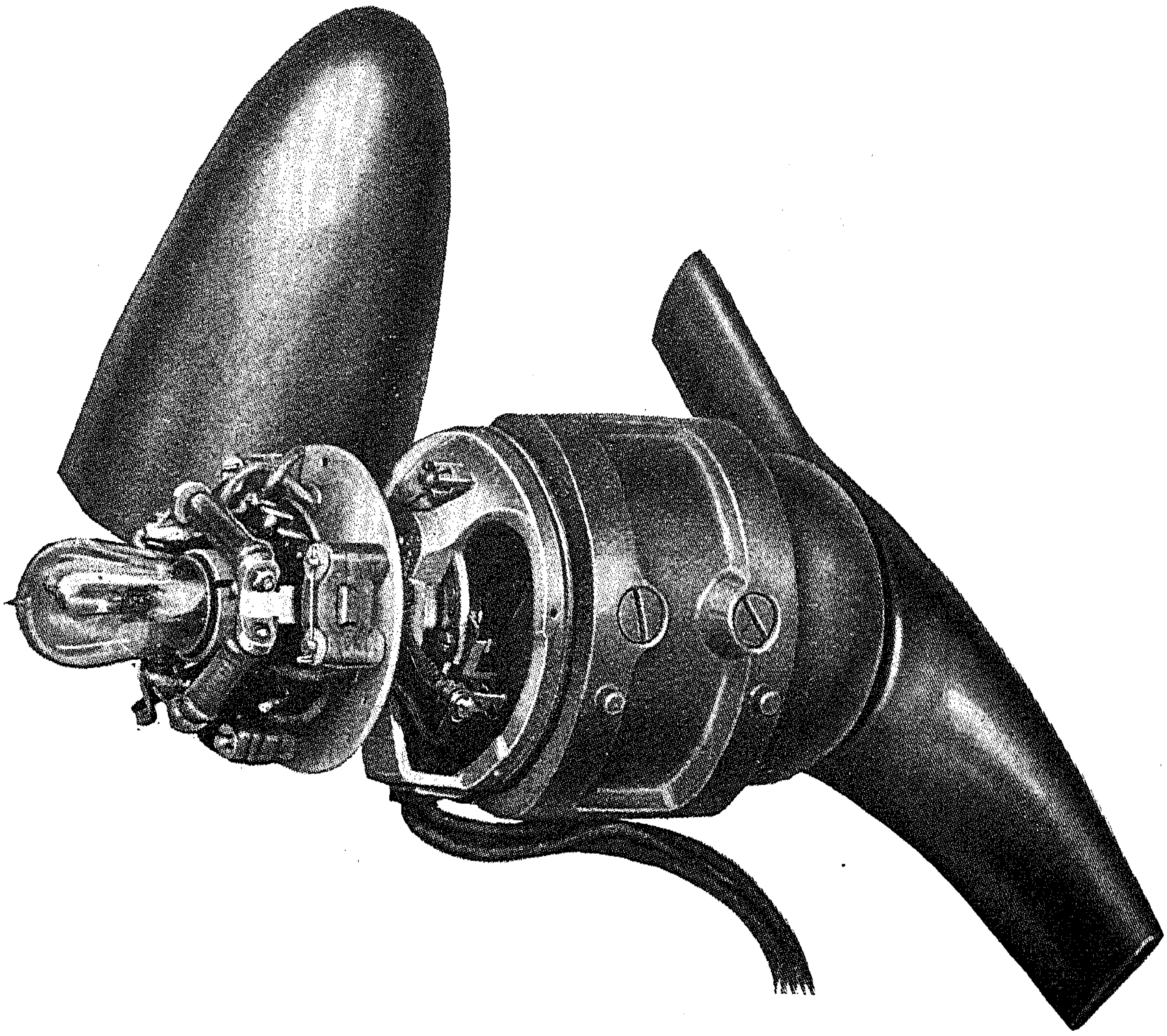
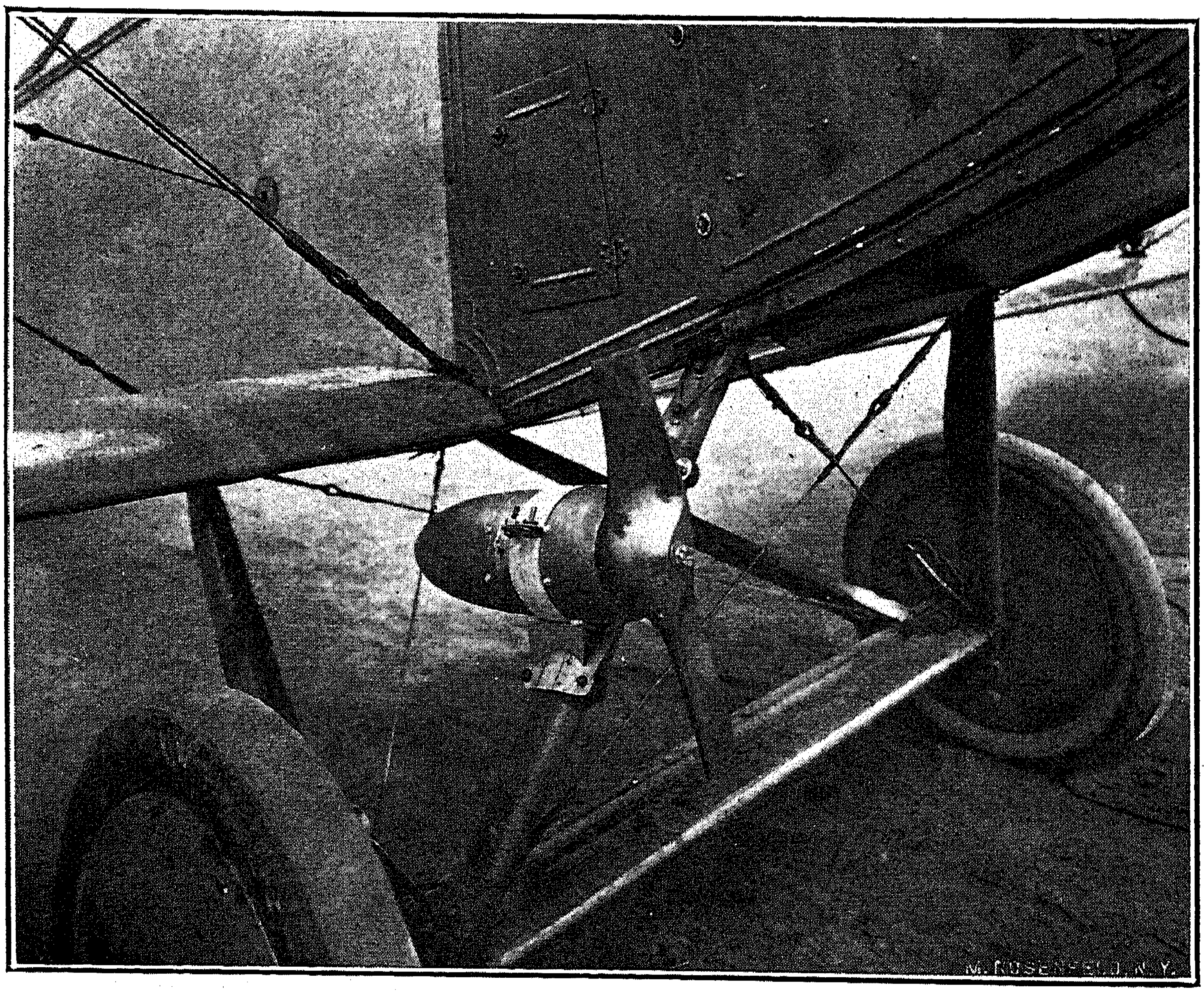


FIG. 34—WIND DRIVEN GENERATOR SHOWING VOLTAGE REGULATING TUBE



[CRAFT & COLPITTS]

FIG. 35—METHOD OF MOUNTING WIND DRIVEN GENERATOR

of manual signals. With the telephone equipment, satisfactory intercommunication could be provided at all times. This has been found very useful in connection with the training of fliers as well as in military operations.

The transmitting and receiving set proper is shown in Fig. 32. This set is approximately 17 by 10 by 7 in. in dimensions, and weighs 21 lb.

The wind-driven generator is shown in Fig. 33, and in Fig. 34 the tail cover has been removed showing the vacuum tube regulating device.

The generator is usually mounted in the slip stream of the propeller on one of the struts of the landing gear, as shown in Fig. 35.

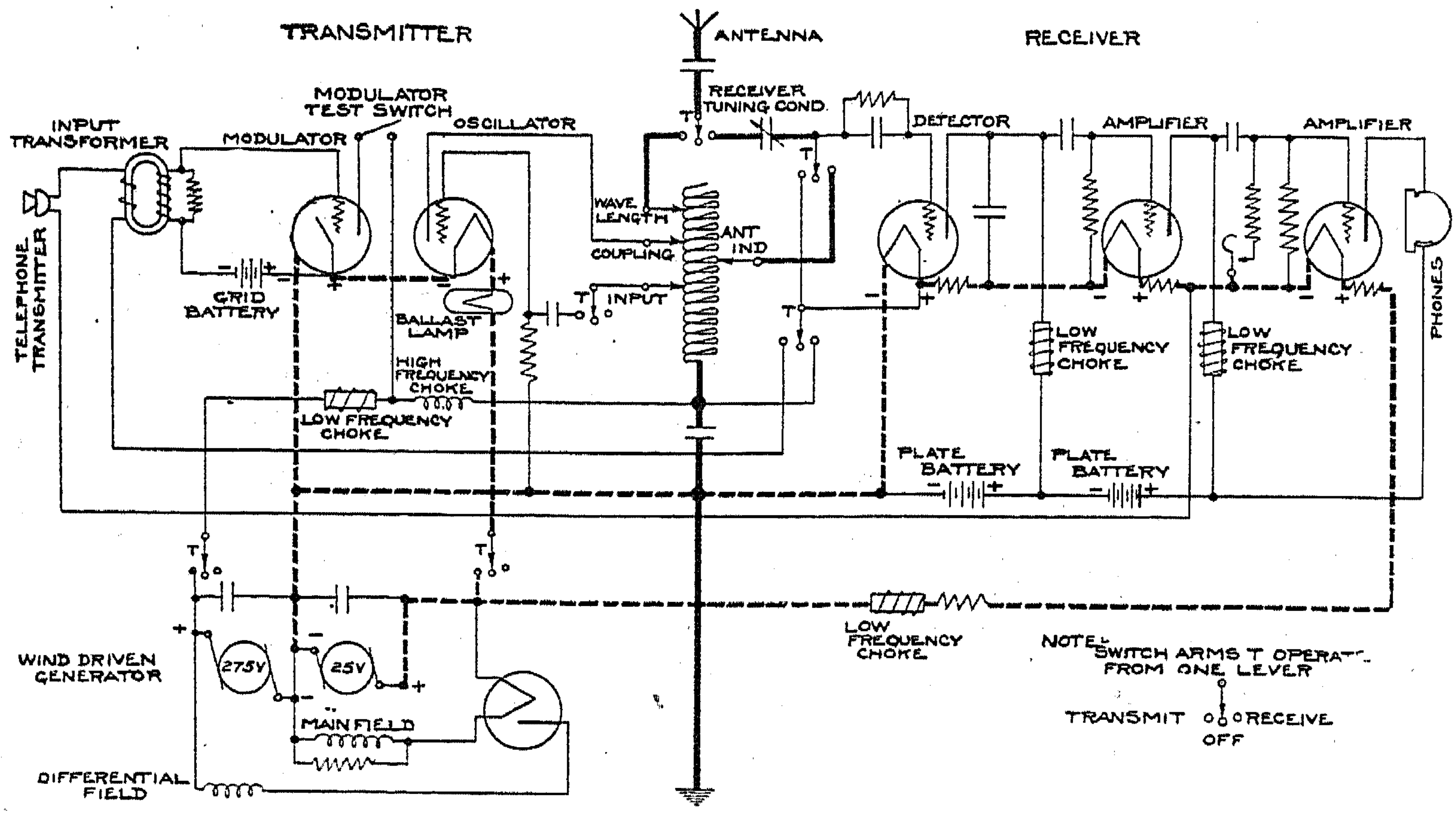


Fig. 36—Schematic Circuit Diagram—Radio Telephone Transmitting and Receiving Set—Signal Corps SCR-68

The weight of the complete equipment including generator and two operators sets is approximately 58 pounds.

Fig. 36 shows, in simplified form, the electrical circuits.

When receiving only is required, the transmitting portion is omitted, and power is supplied to the filaments by a small storage battery. The receiving set is shown in Fig. 37.

Modifications of the SCR-68 sets adapted to different classes of service were developed and manufactured, one of the principal adaptations being a combination transmitting and receiving set for use on the ground. This is illustrated in Fig. 38. This set was employed principally for training purposes. With this means of communication at hand, the performance of the student could be observed in the air and his faults could be corrected by the instructor on the ground, thus greatly accelerating this training work. Energy for this set was provided by storage batteries, a dynamotor being operated by the storage battery to supply the plate potential.

Up to the time of the signing of the Armistice, many thousands of these sets had been manufactured and delivered.

SHORT WAVE TYPE

Since the first experiments the trailing wire antenna had been employed and wave lengths of from 200 to 500 meters were used. It was realized that a long trailing wire antenna was not well adapted for use on military airplanes for tactical reasons, and it would be very desirable to limit the physical dimensions of the radiating system. This, of course, implies a corresponding reduction in the length of wave radiated if an efficient system is to be used, and steps were taken to design such a set. There is no difficulty in securing extremely short waves with the vacuum tube oscillator but there is some difficulty in designing a very compact set, with wave length adjustable over a considerable range and containing amplifiers and modulators, and at the same time avoiding excessive losses. In October, 1917, some laboratory sets were made for wave lengths of the order of 70 to 150 meters, and early in 1918 the Signal Corps requested the development of a short wave set. This was built in April and was electrically essentially the same as the longer wave sets except for minor changes due to the use of higher frequencies of the order of 4,000,000 cycles.

Trials of this system were made at Camp Alfred Vail at wave lengths of 60 meters and above. The antenna first used was a very short structure on the top of the plane with two wires extending to the tail. This antenna had a natural wave length of 32 meters and a resistance of only one ohm. To increase the radiating qualities the rear portion of the antenna was raised and the resistance thereby brought up to nearly 3 ohms at 75 meters. This structure did not materially increase the head resistance of the plane and did not interfere with its operation. As described later, this form of antenna was soon replaced by another.

The use of short wave lengths brings froward prominently the problem of location of component parts of the set in the plane, for a few unnecessary feet of connecting wire carrying these high-frequency currents may cause the failure of the set. For this reason considerable thought was given to the question of location of transmitter and control box. Fig. 39 shows the final form of these short wave sets and it will be observed that the multiple unit plan of assembly has been carried further than heretofore, and this was due to the fact that difficulty was encountered in mounting the SCR-68 type set in the single place combat planes.

Fig. 40 shows the interior construction of the short wave transmitting and receiving units.

RADIATING SYSTEMS

The whole subject of proper radiating systems for use on planes was one to which more time should have been applied than was available at the time of development and manufacture of the first apparatus. A considerable amount of work, however, was carried on and valuable data obtained. This work was started in the summer of 1917 at Langley Field, and was later transferred to Camp Alfred Vail, N. J. The work was in charge of Mr. A. A. Oswald of the Western Electric Co. under the direction of the Signal Corps, and resulted in the accumulation of a considerable amount of useful data on many types of antenna. There are three important conditions to be met in designing an antenna for use on airplanes.

1. It must be an efficient radiator.
2. It must not be directive.
3. It must not interfere with complicated evolutions of the plane.

The third condition practically prohibits the use of a long trailing wire with a weight, because of the danger of fouling the propellor. This type of antenna is also quite directive.

The details of this antenna investigation should properly be covered in a separate paper. It may be well to state, however, that the best system, all things considered, was found to consist of two short unweighted wires, one from each wing tip. The two wires were joined in parallel above the fuselage, and worked against the conducting portions of the plane as a counterpoise. It was found that the use of one such wire as antenna and the other as counterpoise was not good.

In Fig. 41 the distribution of intensity of radiation is shown with the single long trailing wire and the two-wire type of antenna.

Ignition Interference

Another problem arose when sets began to be installed. This was that of noise in the receiving sets due to the engine ignition, and was very serious in some cases. Its solution was complicated by the fact that it was not allowable to alter the plane equipment in any way, for instance by changing the position of one magneto to secure greater shielding, but it was found that by covering the ignition wires with a flexible conducting tube grounded at intervals this trouble could be practically eliminated.

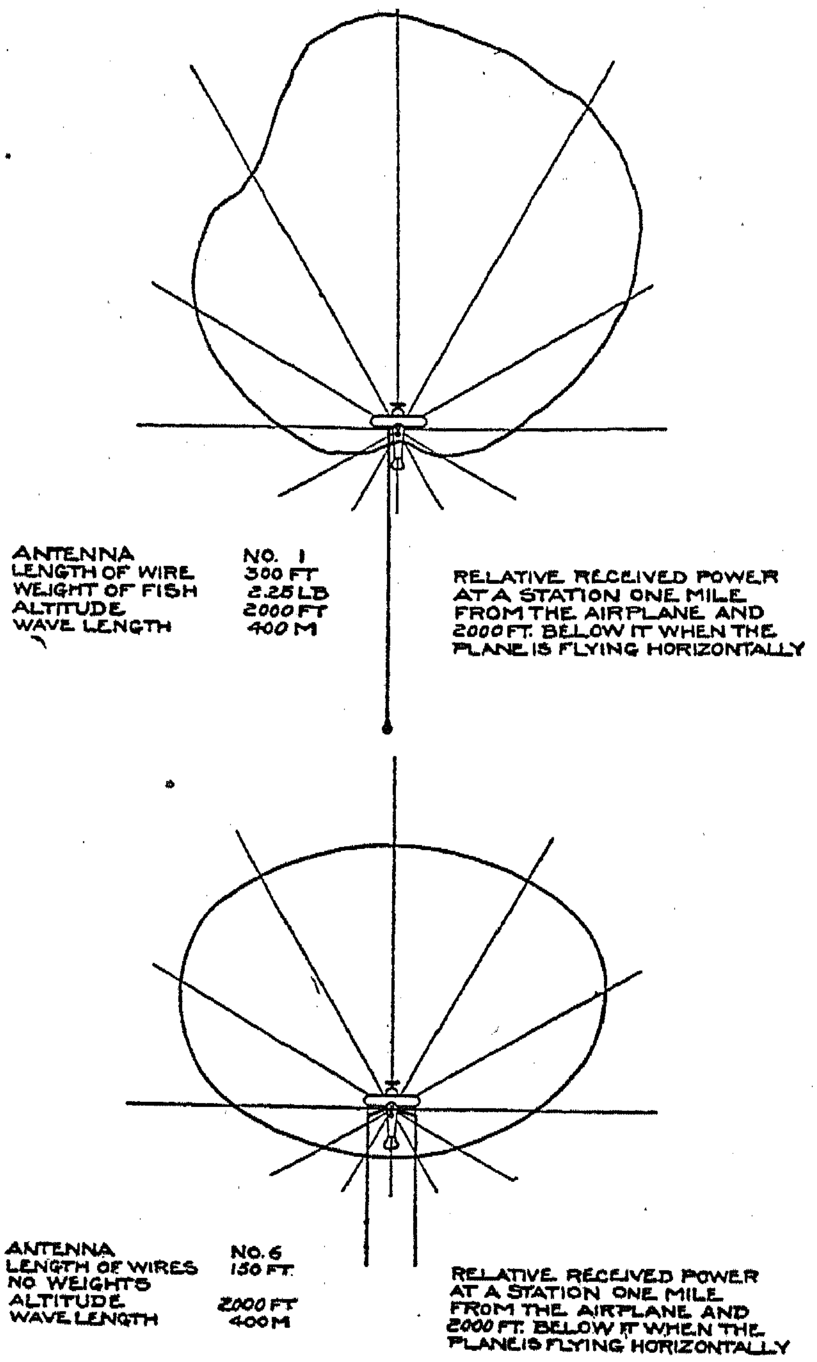


Fig. 41—Directional Characteristics of Airplane Antenna

It was found that in some cases the ignition spark started in the ignition system high-frequency oscillations which were of proper frequency to affect the receiver. This happened on the submarine chasers, which are driven by gasoline engines, and the trouble was remedied by inserting small iron core choke coils in the ignition leads to change the frequency of the oscillations. It is obvious that the "radio signals" sent out by the ignition spark are many millions of times more intense than those to be received and it is remarkable that they do not

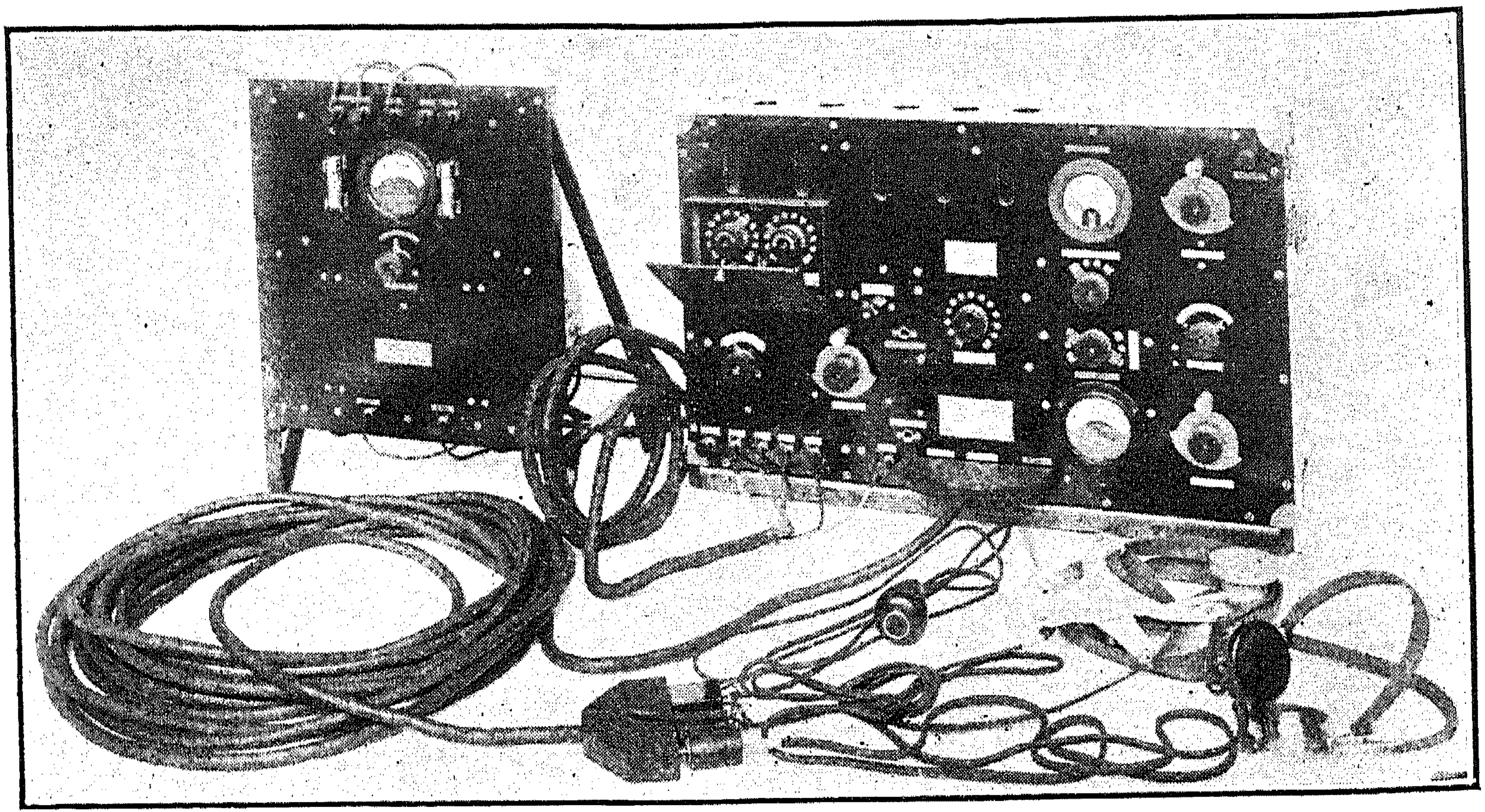


FIG. 38—RADIO TELEPHONE TRANSMITTING AND RECEIVING SET FOR GROUND USE—SIGNAL CORPS S C R-67

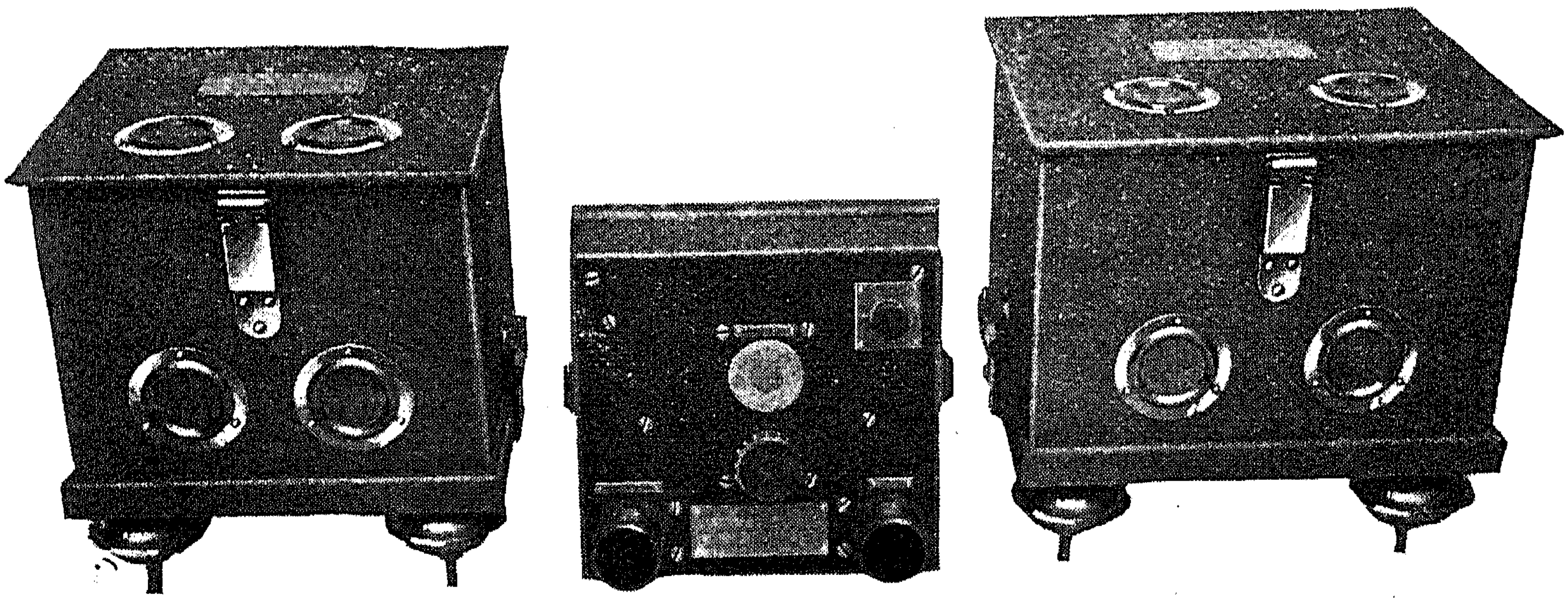
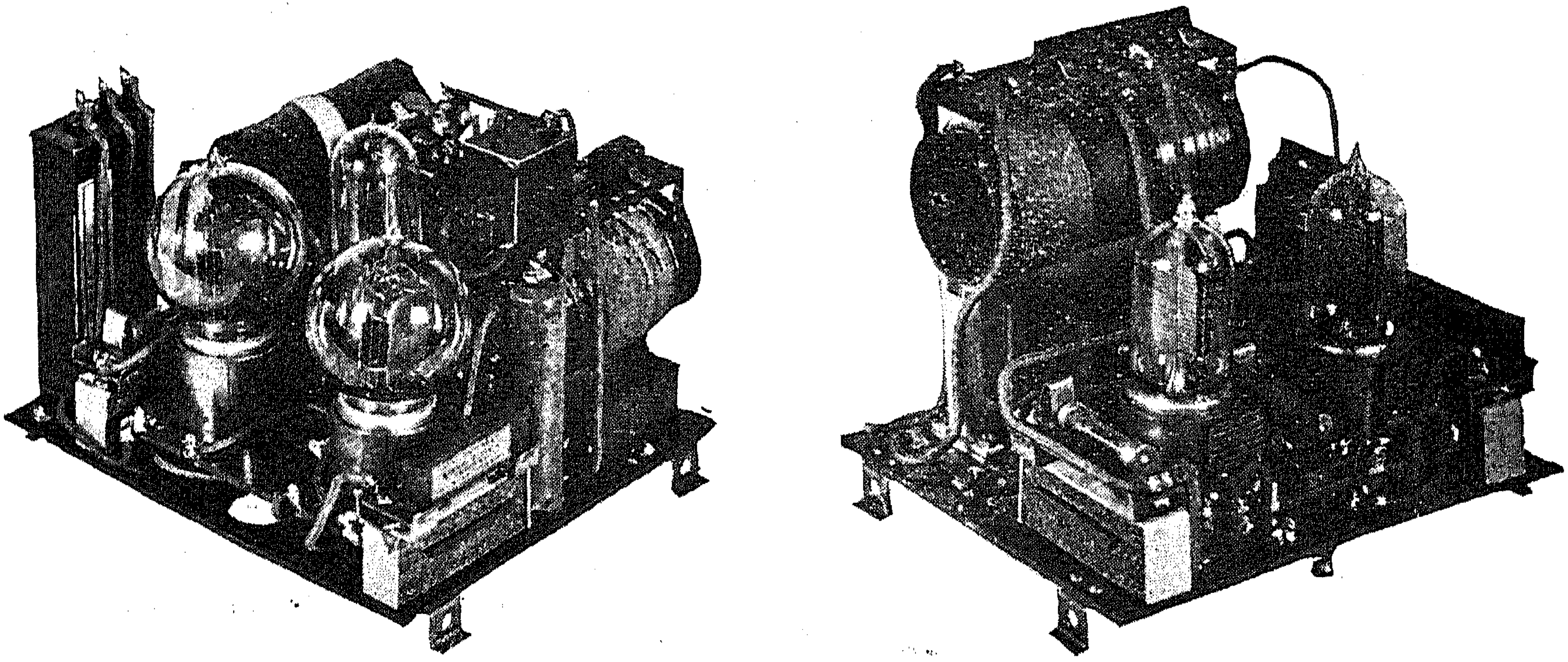


FIG. 39—SHORT WAVE TYPE AIRPLANE RADIO TELEPHONE SET



[CRAFTS & COLPITTS]

FIG. 40—INTERIOR CONSTRUCTION OF SHORT WAVE TRANSMITTING AND RECEIVING UNITS

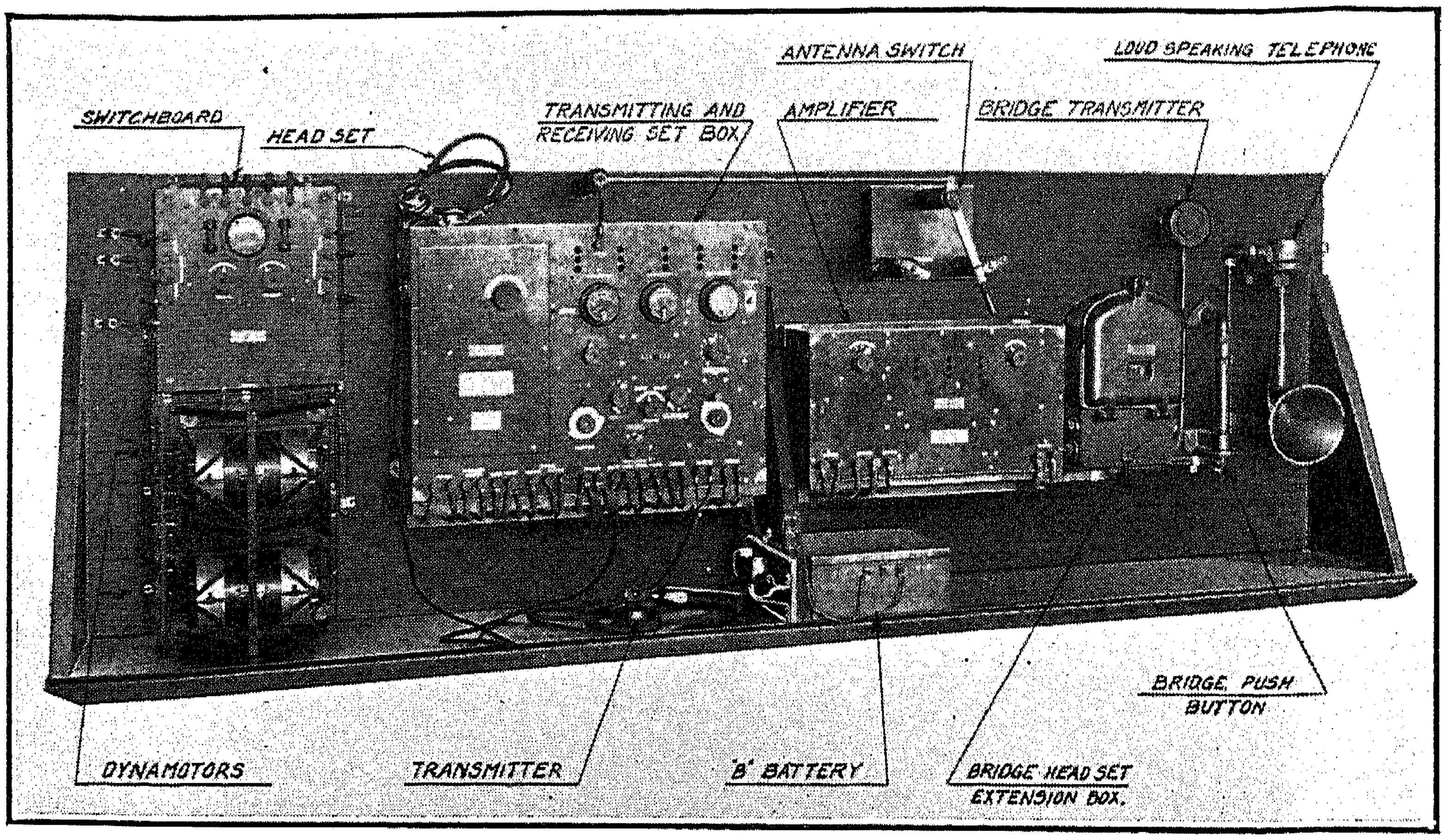
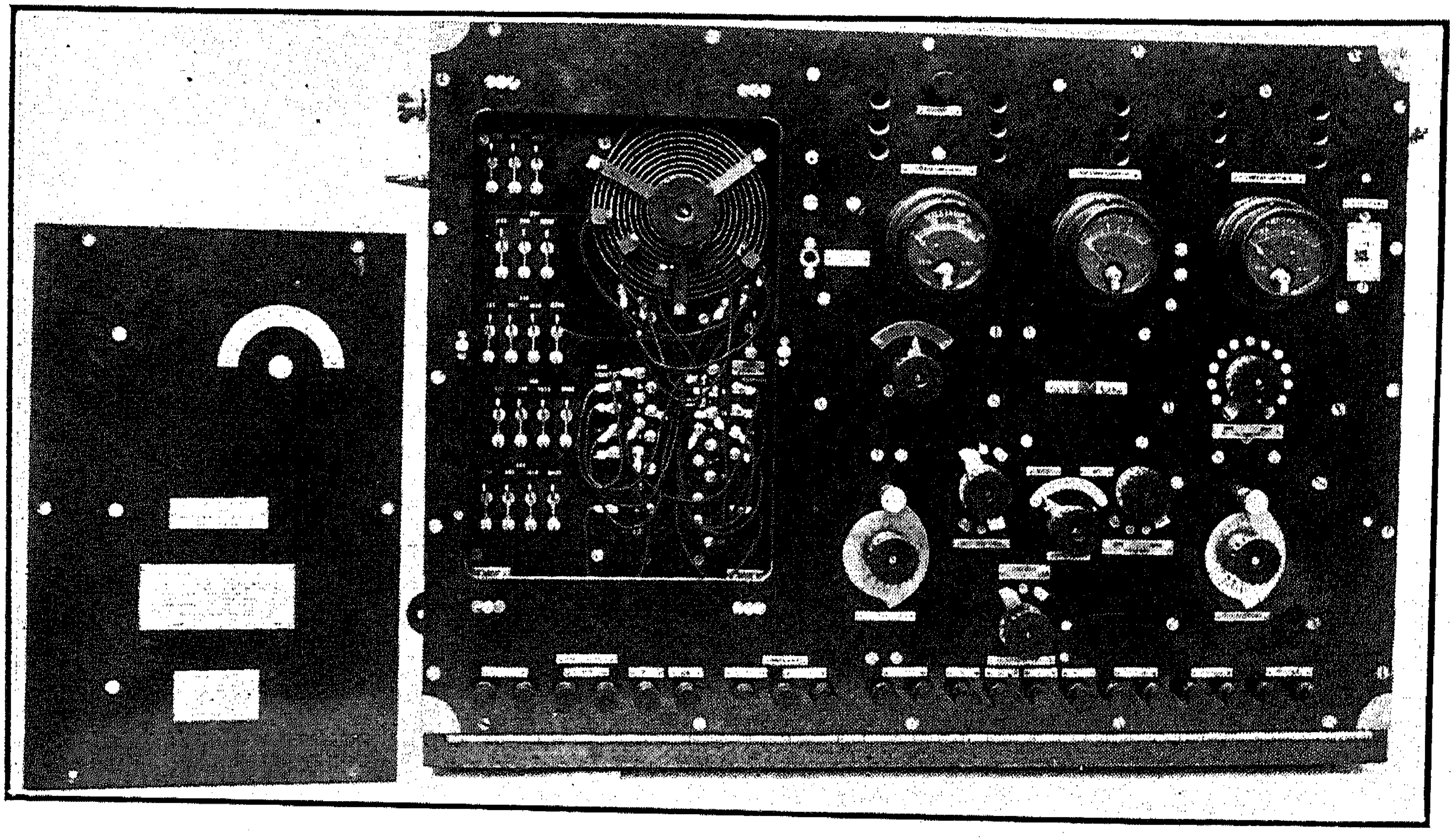


FIG. 42—COMPLETE WESTERN ELECTRIC TWO-STATION RADIO TELEPHONE EQUIPMENT FOR SUBMARINE CHASERS—BUREAU OF STEAM ENGINEERING STANDARD C W-936



[CRAFT & COLPITTS]

FIG. 43—SUBMARINE CHASER TYPE TRANSMITTING AND RECEIVING SET BOX—C W-936

entirely prevent reception. As a matter of fact, successful airplane telegraphy has been carried on in an experimental way by using the ignition magneto as a radio transmitter.

SUBMARINE CHASER EQUIPMENT

During the war important use has been made of the wireless telephone in connection with naval work. Small 110-ft. craft had been largely employed in connection with anti-submarine operations. In order to make the most effective use of the various listening devices, and to co-ordinate the operations of the various units, it is very necessary that instantaneous and direct communication be established at all times. Under the direction of the Special Submarine Board of the Navy Department, modified forms of radio telephone equipment were developed for this purpose. Early in November 1917, the first practical trials were made, and satisfactory operation between chasers approximately five miles apart was obtained. These trials demonstrated the extreme value of this means of communication and sample equipments were immediately dispatched overseas for further trials under actual war conditions.

The circuits of this set are practically the same as the airplane equipment, except that power is obtained from small dynamotors operated from the 30-volt storage batteries with which the chasers are equipped. The set itself is located in the radio room, where it can be attended to by the regular radio operator. A telephone transmitter and receiver is, however, located in the pilot house so that the commanding officer can hold direct conversation with other vessels. The radio operator monitors the conversation and performs all tuning operations, thus leaving the commanding officer free to use the radio telephone as an ordinary wire line, except that he must press a button located at the side of the telephone when talking. The ordinary telephone head set is supplemented by a loud-speaking telephone receiver, which makes it possible for incoming signals to be heard without the use of head gear. Fig. 42 shows an assembled view of the various elements that go to make up a standard submarine chaser equipment. To provide sufficient energy for operating the loud speaking receiver a three-stage amplifier connected to the output side of the radio set is required. It is possible to connect the receiving portion of the radio set to either the ordinary head receiver or to the amplifier with its loud speaker.

Fig. 43 shows a close-up view of the radio transmitting and receiving set proper.

This equipment has an effective operating range of about ten miles when used on the 110-ft. submarine chasers. One feature not incorporated in the airplane sets consists of a wave length control gear by means of which the set can be operated on any one of five different wave lengths, ranging from 250 to 600 meters. This permits of a certain amount of selective operation within a chaser squadron.

A number of problems were met with in connection with this particular application not the least of which was the suppression of electrical disturbances in the antenna system due to the ignition system of the three propelling engines. Several thousand of these sets have been produced, and practically all submarine chasers sent overseas have been equipped with this apparatus. This apparatus is also being used for short range work on other types of naval vessels, with eminently satisfactory results. Modifications of the airplane type of equipment have also been made for use on naval seaplanes. For some classes of service the demand for longer range work has made necessary the employment of more powerful types of vacuum tubes.

The foregoing covers the most extensive application of vacuum tube radio telephony, the same general type of circuit and apparatus being used for short range telephony and telegraphy on land and water and in the air. For reasons already mentioned, quantity production has been limited to the earlier models, in spite of the fact that investigations carried on during production have indicated many changes which would improve the electrical and mechanical efficiency of the apparatus.

The Future of Radio-Telephony

The possibility of communication by speech between any two individuals in the civilized world is one of the most desirable ends for which engineering can strive. For this reason it is particularly desirable to form some opinion of the part which radio telephony may play in securing this universal service. It is clear that the elimination of the Morse operator, which is accomplished by the use of radio telephony rather than radio telegraphy, is necessary for universal and direct communication.

Radio telephony and wire telephony offer several sharp contrasts. The latter requires fixed channels of communication whose construction and maintenance necessitates an accessible path between stations, but the results obtained include secrecy, power efficiency, selection of a desired station and freedom from interference. A large item of expense is the line. On the other hand radio telephony requires neither fixed nor accessible channels and no cost at all for line construction and maintenance, but it is non-secret in the practical sense of the word, its power efficiency is low, selection is at present not practicable except in a limited way by wave length, and freedom from interference is not at present an attained fact. Thus while the two systems may be contrasted they are not comparable but each is useful in its own field. It is easy to see that radio telephony can never compete with wire telephony in densely populated districts, while wire telephony is a physical impossibility at sea and in the air. Fortunately, however, the connection of a wire system to a radio system is no more complicated than connecting two wire lines by means of a repeater and therefore these two fields, although distinct, are adjacent. Leaving aside for the moment the particular methods by which radio telephone communication is to be carried on, it is clear that the establishment of communication between two given individuals will be most efficiently realized through the use of a combination of wire transmission on a network extending over perhaps 99 per cent of the stations and radio transmission to those relatively few stations to which it is either impossible or impracticable to build lines. These stations will be of two kinds:

1. *Moving*, such as ships, airplanes, trains, trucks.
2. *Fixed but inaccessible*, such as on islands, in deserts and in very sparsely settled regions.

A third class of service is that which is concerned, not with single individuals, but with groups; such service as the broadcasting of news, time and weather signals, and warnings. In some cases one objection to radio telephony would be an advantage in this class of service.

The choice of the particular kind of transmitting apparatus which will ultimately be used in thus extending the range of communication will involve careful consideration of costs and other engineering factors; its feasibility has, however, been conclusively demonstrated.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 21, 1919.

Copyright 1919. By A. I. E. E.

THEORY OF THE TRANSIENT OSCILLATIONS OF ELECTRICAL NETWORKS AND TRANSMISSION SYSTEMS

BY JOHN R. CARSON

ABSTRACT OF PAPER

The purpose of this research was to make a broad theoretical study of transient phenomena with a view to developing methods of calculation directly applicable to engineering problems. The investigation starts with the problem of formulating the current in an electrical network or transmission system in response to a suddenly applied e.m.f. of arbitrary form. A simple formula is derived which expresses this current in terms of two independent functions: one, the applied e.m.f. expressed as a time function, and the other a characteristic function of the constants and connections of the system, this latter being termed the "indicial admittance" of the system. A knowledge of the indicial admittance, therefore, completely determines the behavior of the system to all types of applied voltages including both transient and steady states. A systematic investigation of methods for solving and computing the indicial admittance follows, in the course of which original solutions for transmission and artificial lines are derived and a new method involving integral equations is developed. This latter is particularly well adapted to handle the difficult problem of taking into account the effects of terminal impedances in transmission systems.

Introduction and Statement of Problem

WHEN an electromotive force is suddenly applied to an electrical network or system the current flowing in each and every part of the system is, at every instant of time, a determinate function of the form of the applied e. m. f. and of the connections and constants of the system. If the applied voltage is periodic (including, as a limiting case, the uniform steady voltage of zero periodicity) the system ultimately executes periodic oscillations of the same frequency as that of the applied voltage. These "steady-state" or periodic oscillations are the subject of the theory of alternating currents which takes up such a large part of the literature of electrical engineering. A more or less appreciable time after the application of the voltage elapses, however, before the proximate steady state is arrived at, and the oscillations which the system executes in this interval have been roughly termed "transient oscillations," this term implying, of course, that this "unsteady" state dies away or rather merges into the final steady state.

From a practical standpoint a knowledge of the steady state phenomena is sufficient in the great majority of problems. Theoretically, however, the so-called "transients" are of fundamental importance, and in certain problems, notably those of long distance telephony and telegraphy, the steady state theory is quite inadequate and the investigation of transient conditions becomes a matter of practical importance as well. The present paper is the outgrowth of an investigation, undertaken in connection with the writer's work in the Engineering Department of the American Telephone and Telegraph Company, of speech-excited transients in telephone systems, with particular reference to long, loaded cable circuits.

Stated in its broadest terms, our problem is to formulate, as a function of time, the current which flows in any part of an electrical system in response to a voltage of any specified form or type which is suddenly impressed on the system. After the development of the general theory and formulas, a considerable and by no means the least important part of the paper is devoted to deriving fundamental solutions, which admit of easy interpretation and computation, for the transmission line having continuously distributed inductance, capacity, resistance and leakage and the corresponding artificial line in which these constants are lumped. Formulas (**32**) to (**36**) and (**52**) to (**64**), which give the solutions for the current and voltage waves in all the important types of transmission system, are all the original results of the present research with the exception of formulas (**57**) and (**62**) which are the solutions for the current and voltage waves in the non-inductive, leakageless cable, first derived by Kelvin and Heaviside and formula (**53**) for the current wave in the leakageless line, which was independently derived by Heaviside and Poincare.

The investigation naturally falls into five parts:

1. The derivation of a general formula expressing the current as a function of the form of the applied voltage and the connections of the particular system under consideration. It will be shown that the current is simply expressed in terms of two functions, one the applied voltage, considered as a time

function, and the other a characteristic function of the system which is independent of the form of the applied voltage. This characteristic function will be termed the "indicial admittance" of the system, a terminology whose oppositeness will be justified by the physical and mathematical significance of the function. The word "indicial" indicates that the function is really an index of the system, and completely determines the behavior of the system to all types of applied voltages. On the other hand, the word "admittance" indicates that the function is of the nature of an admittance in a generalized sense, and that it has very real physical significance.

2. In practical problems, the applied voltage of the greatest importance is the mono-periodic or sinusoidal. For this particular type of e. m. f. special formulas are derived and a mechanical method of numerical solution is developed and described.

3. In (1) and (2) a knowledge of the "indicial admittance" is implied. A systematic investigation of the possible methods of solving for this function is taken up, and appropriate methods for particular types of systems, notably transmission systems, are developed.

4. The indicial admittance functions for all the important types of transmission systems, including the periodically loaded line are derived, and a large number of graphs of these functions is shown.

5. The appropriate treatment to take care of the effects of terminal apparatus in transmission systems is developed and applied to a number of specific problems.

I. General Theory and Formulas. The mathematical basis of the following investigation is furnished in a paper published by the writer in the *Physical Review* (Sept., 1917, pp. 217). A brief resume of the theory and formulas there developed will now be given as a necessary preliminary to the subsequent investigation; for a more detailed treatment the reader is referred to the original paper.

The current in any part of an electrical system or network is related to the impressed voltage (or voltages) by a set of linear differential equations in the variable t (time). If the applied voltage is taken as $E\,\epsilon^{pt}$, and if the differential operator $\frac{d^n}{d\,t^n}$ is replaced by p^n, the differential equations reduce to a set of simultaneous algebraic equations, and by purely algebraic processes we get the symbolic formula or solution

$$I = \frac{E\ \epsilon^{pt}}{Z\ (p)} \tag{1}$$

In the symbolic formula (**1**), $Z\ (p)$ is the impedance function; it is the ratio of two determinants (or the limit of such a ratio), and the equation $Z\ (p) = 0$, which is of frequent occurrence, will be termed the determinantal equation. If p is taken as a pure imaginary, formula (**1**) is simply the usual complex expression for the steady state current, and the real part of (**1**) the periodic current, corresponding to an impressed voltage of frequency $\frac{p}{2\pi}$. At present, however, no restriction is imposed on the operator p, and (**1**) is to be regarded as purely symbolic.

Starting with formula (**1**), the writer showed, in the paper referred to above, that, if the voltage is impressed on the system at time $t = 0$, the resultant current is expressed by the formula:

$$I = E\left[\frac{\epsilon^{pt}}{Z\ (p)} + \Sigma_m \frac{\epsilon^{p_m t}}{(p_m - p)\ Z'\ (p_m)}\right] \tag{2}$$

where p_m is the mth root of $Z\ (p)$, the summation is extended over all the roots of $Z\ (p)$ and

$$Z'\ (p_m) = \left[\frac{d}{d\,p} Z\ (p)\right]_{p = p_m}$$

In formula (**2**), p is no longer a mere operator and is assigned the value, real, imaginary or complex, which characterizes the impressed voltage. Thus if the impressed voltage is a unit d-c. voltage (zero before, unity after $t = 0$), $E = 1$, $p = 0$, (**2**) and becomes:

$$I = A\ (t) = \frac{1}{Z\ (0)} + \Sigma_m \frac{\epsilon^{p_m t}}{p_m\ Z'\ (p_m)} \tag{3}$$

which is the Heaviside expansion formula.

The function formulated by (**3**) is fundamental to our whole problem and will be defined as follows:

The *indicial admittance* of an electrical network or system is equal to the current, expressed as a time function, which flows in the system in response to a unit d-c. voltage (zero before, unity after $t = 0$) applied to the system at time $t = 0$. It is mathematically defined and formulated by (**3**) and will be denoted by $A(t)$ with appropriate subscripts. Thus if the applied voltage is inserted in the j^{th} mesh or branch of the network, the current in the k^{th} mesh or branch (assuming a unit d-c. voltage) will be denoted by $A_{jk}(t)$. The indicial admittance $A(t)$ is of the dimensions and character of an admittance in a generalized sense and is subject to the reciprocal relation $A_{jk}(t) = A_{kj}(t)$. When the two subscripts are the same the function may conveniently be referred to as the *driving point indicial admittance*, while when the subscripts are different, it will be termed the *transfer indicial admittance*, thus correlating our conceptions and terminology with those commonly employed in the theory of steady state or periodic phenomena. An important distinction, however, exists between the indicial admittance, as defined above, and the steady state admittance in that the former is a function of time while the latter is a function of frequency.

We shall now show that the "indicial admittance" completely determines the behavior of the system to all types of impressed voltages; in other words, that a knowledge of the indicial admittance alone is all that is required to completely specify the system, and predict its performance under all circumstances. The proof is based on a well known partial expansion which is employed and fully discussed in my paper referred to above. By a slight extension of the theory there developed it may be shown that all the impedance functions with which we are concerned admit of the partial fraction expansion:

$$\frac{1}{Z(p)} = \frac{1}{Z(\infty)} + \sum^{m} \frac{1}{(p - p_m)\, Z'(p_m)} \tag{4}$$

whence:

$$\frac{1}{Z(0)} = \frac{1}{Z(\infty)} - \sum^{m} \frac{1}{p_m\, Z'(p_m)} \tag{5}$$

Comparing (**3**) and (**5**) it is at once evident that

$$\frac{1}{Z(\infty)} = A(0)$$

$\frac{1}{Z(\infty)}$, and hence $A(0)$, is usually zero, except when, due to the absence of finite inductance, the current corresponding to $A\ (t)$ has an initial jump to a finite or even infinite value.

Returning now to equation (2) and replacing $\frac{1}{Z\ (p)}$ by its value as given by (4) we get:

$$I = E\left[\frac{\epsilon^{pt}}{Z\ (\infty)} + \epsilon^{pt}\ \Sigma_m\ \frac{\epsilon^{(p_m - p)t} - 1}{(p_m - p)\ Z'\ (p_m)}\right]$$

which is clearly equal to:

$$I = E\left[\frac{\epsilon^{pt}}{Z\ (\infty)} + \epsilon^{pt}\int_0^t \epsilon^{-p\lambda}\ \Sigma_m\ \frac{\epsilon^{p_m\lambda}}{Z'\ (p_m)}\ d\ \lambda\right] \tag{6}$$

But by (3)

$$\Sigma_m\ \frac{\epsilon^{p_m t}}{Z'\ (p_m)} = \frac{d}{d\ t}\ A\ (t) = A'\ (t)$$

$$\frac{1}{Z\ (\infty)} = A\ (0)$$

Substitution of these identities in (6) gives

$$I = A\ (0)\ E\ \epsilon^{pt} + \int_0^t E\ \epsilon^{p\ (t-\lambda)}\ A'\ (\lambda)\ d\ \lambda \tag{7}$$

By obvious transformations (7) may also be written in either of the equivalent forms,

$$I = \frac{d}{dt}\int_0^t E\ \epsilon^{p\ (t-\lambda)}\ A\ (\lambda)\ d\ \lambda$$

$$= \frac{d}{dt}\int_0^t E\ \epsilon^{p\lambda}\ A\ (t - \lambda)\ d\ \lambda \tag{8}$$

which are sometimes more convenient to handle than is (7).

If the impressed voltage is the mono-periodic or sinusoidal function

$$E \sin\ (p\ t + \theta)$$

it is easy to show (by writing the sine function is its complex exponential form) that (**7**) becomes,

$$I = A\,(0)\,E \sin\,(p\,t + \theta) + \int_0^t E \sin\,(\,p\,(t - \lambda) + \theta\,) \cdot A'\,(\lambda)\,d\,\lambda \qquad \textbf{(9)}$$

More generally if the impressed voltage $f\,(t)$ is not a pure sinusoid but is expressible (for $t > 0$) as a Fourier series:

$$f\,(t) \qquad = \Sigma^k\ a_k\ \sin\,(p_k\,t + \theta_k)$$

$$f\,(t - \lambda) = \Sigma^k\ a_k\ \sin\,(\,p_k\,(t - \lambda) + \theta_k\,)$$

equation (**9**) becomes, since the relations are linear,

$$I = A\,(0)\,f\,(t) + \int_0^t f\,(t - \lambda) \cdot A'\,(\lambda) \cdot d\,\lambda \qquad \textbf{(10)}$$

while (**8**) becomes

$$I = \frac{d}{dt} \int_0^t f\,(t - \lambda) \cdot A\,(\lambda)\,d\lambda$$

$$= \frac{d}{dt} \int_0^t f\,(\lambda)\,A\,(t - \lambda)\,d\lambda \qquad \textbf{(11)}$$

Equations (**11**) are the fundamental formulas which mathematically relate the current to the type of applied voltage and the connections of the system, and constitute the solution of the first part of our problem. The most important deductions from these formulas are expressed in the following theorems:

1. *The indicial admittance of an electrical network or system completely determines, within a single quadrature, the behavior of the system to all types of applied voltages.* As a corollary, a knowledge of the indicial admittance is the sole information necessary to predict completely the performance and characteristics of the system.

2. *The applied voltage and the indicial admittance are similarly and coequally related to the instantaneous current flowing in the system.* As a corollary, the form of the current may be modified either by changing the connections and constants of the system (and thus modifying $A\,(t)$) or by modifying the form of the applied voltage.

Since the applied voltage may be discontinuous formulas (**11**) not only determine the building up of the current in response to an applied voltage, but also its subsidence to equilibrium when an applied voltage is removed and the system is left to itself. In brief, *formulas* (**11**) *reduce the whole problem to a determination of the indicial admittances of the system,* a problem which forms the subject matter of succeeding sections of this paper.

The type of applied voltage of the greatest practical importance is the mono-periodic, or sinusoidal. In this case denoting the voltage applied at time $t = 0$, by $E \sin (p\,t + \theta)$, (**9**) is the appropriate formula. This may be written:

$$\begin{aligned} I = {} & A\,(0)\,E \sin (p\,t + \theta) \\ & + E \sin (p\,t + \theta) \int_0^t \cos p\,\lambda \cdot A'\,(\lambda)\,d\,\lambda \\ & - E \cos (p\,t + \theta) \int_0^t \sin p\,\lambda\, A'\,(\lambda)\,d\,\lambda \qquad (\mathbf{12}) \end{aligned}$$

which is the formula most convenient in practise. Another formulation which is sometimes preferable is derivable from the first of (**11**)

$$\begin{aligned} I = {} & E \sin \theta\, A\,(t) \\ & + E\,p \cdot \cos (p\,t + \theta) \int_0^t \cos p\,\lambda \cdot A\,(\lambda)\,d\,\lambda \\ & + E\,p \sin (p\,t + \theta) \int_0^t \sin p\,\lambda \cdot A\,(\lambda)\,d\,\lambda \qquad (\mathbf{12a}) \end{aligned}$$

Except in rare cases, however, formula (**12**) is preferable and further consideration will be limited to it. From formula (**12**) the final *steady state* current is given by

$$\begin{aligned} & A\,(0)\,E \sin (p\,t + \theta) \\ & + E \sin (p\,t + \theta) \int_0^\infty \cos p\,\lambda\, A'\,(\lambda)\,d\,\lambda \\ & - E \cos (p\,t + \theta) \int_0^\infty \sin p\,\lambda\, A'\,(\lambda)\,d\,\lambda \qquad (\mathbf{13}) \end{aligned}$$

While the transient distortion (*i. e.*, the difference between the actual and the steady state current) is given by

$$-E\sin(pt+\theta)\int_t^\infty \cos p\lambda\, A'(\lambda)\,d\lambda$$

$$+E\cos(pt+\theta)\int_t^\infty \sin p\lambda\, A'(\lambda)\,d\lambda \qquad \textbf{(14)}$$

The resolution of the total current into two terms, first the steady state current as given by (**13**), and second, the transient distortion, as formulated by (**14**), is often of considerable practical advantage. In fact in many practical problems we are primarily concerned with the character and duration of the transient distortion. It should be observed that the "transient distortion" is formally defined by (**14**); it is the difference between the actual total current and the final steady state current.

Formula (**13**) gives the steady state current; this, of course, is derivable by other and far simpler methods so that (**13**) would, in practise, never be used to evaluate the steady state current. However, when equated with the usual steady state formulas, it furnishes an excellent check on the correctness of the theory and formulas, and also a means of evaluating some very complicated infinite integrals, the latter property being of very considerable mathematical interest.

Formula (**13**) also enables us to express the indicial admittance of the system in terms of its steady state admittance as follows. If the applied voltage is $E\sin(pt+\theta)$ and if

$$\frac{1}{Z(ip)} = \alpha(p) + i\,\beta(p)$$

the expression for the steady state current is by the usual methods

$$I = \alpha(p)\,.\sin(pt+\theta) + \beta(p)\cdot\cos(pt+\theta)$$

But this is equivalent to (**13**); hence equating the sine and cosine terms,

$$\alpha(p) = A(0) + \int_0^\infty \cos p\lambda\, A'(\lambda)\,d\lambda$$

$$\beta(p) = -\int_0^\infty \sin p\lambda\, A'(\lambda)\,d\lambda$$

$\alpha(p)$ and $\beta(p)$ are, of course, the real and imaginary components of the steady state admittances. The first of these equations can be solved giving

$$A(t) = A(0) + \frac{2}{\pi} \int_0^\infty (\alpha(p) - A(0)) \frac{\sin(tp)}{p} \, dp$$

A very simple example must suffice for the present to illustrate the foregoing theory. Let the system consist of an inductance L in series with a resistance R. Then, by the expansion theorem,

$$A(t) = \frac{1}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right)$$

$$A'(t) = \frac{1}{L}\epsilon^{-\frac{R}{L}t}$$

Substituting in (**12**), the current is given by

$$I = \frac{E}{L}\sin(pt+\theta)\int_0^t \epsilon^{-\frac{R}{L}\lambda}\cos p\lambda \, d\lambda$$

$$- \frac{E}{L}\cos(pt+\theta)\cdot\int_0^t \epsilon^{-\frac{R}{L}\lambda}\sin p\lambda \, d\lambda$$

which integrates to

$$I = \frac{E}{(pL)^2+R^2}\left\{R\sin(pt+\theta) - pL\cos(pt+\theta)\right\}$$

$$+ \frac{E\epsilon^{-\frac{R}{L}t}}{(pL)^2+R^2}\left\{pL\cos\theta - R\sin\theta\right\}$$

Here the first term is the steady state and the second term the transient distortion which is exponentially damped, so that the two components—steady state and transient distortion—are automatically separated by the process of integration.

Still limiting our consideration to sinusoidal voltages, the reader may have remarked that we can get the formal solution for the current directly from (**2**) by making p a pure imaginary and thus dispense with the definite integrals of formula (**12**) altogether. When the number of degrees of freedom of the system is small, and when the roots of the determinantal equation $Z(p) = 0$ are easily located, the direct evaluation of formula (2) is indicated; in the great majority of problems, however, we shall find that the employment of the definite integral formula (**12**) effects a great saving of labor, and that, in fact, the solution obtained directly from (**2**) does not admit, practically speaking, of either interpretation or computation. Without going into details, the chief reasons for this are briefly as follows:

1. The direct evaluation of the current from formula (**2**) requires that the indicial admittance be expressed as a sum of characteristic oscillations or vibrations, a form of solution which is not only usually to be avoided for practical reasons but which in a great many problems cannot be obtained.
2. *The definite integrals of formula* (**12**) *admit of mechanical integration when the indicial admittance is known.* On this fact rests the practical, as distinguished from the purely theoretical, value of the formula.

In order to explain the mechanical evaluation or integration of the integrals appearing in (**12**), we remark that they are of precisely the same form as the harmonic components of a Fourier series, differing therefrom only in that no periodicity on the part of $A(t)$ is either assumed or implied and in that the upper limit of integration is variable. The suggestion to employ a harmonic analyser is immediate and obvious, and upon investigation it was found that a Coradi harmonic analyser of the rolling sphere type was adapted to evaluate directly the integral terms of (**12**) when $A(t)$ is known.

The analyser is applied to the problem of mechanical integration and operates as follows: Having plotted $A(t)$ to a suitable time scale (depending on the frequency $p/2\pi$ which is specified, and the geometrical dimensions of the machine), the curve is traced with the pointer of the analyser, precisely as we trace a curve with an integraph or planimeter. As we trace this curve the recording mechanism of the analyser gives directly the progressive values, corresponding to any particular value of t, of the integrals

$$\int_0^t \cos p\,\lambda\, A'(\lambda)\, d\,\lambda$$

$$\int_0^t \sin p\,\lambda\, A'(\lambda)\, d\,\lambda$$

the value of p depending on the time scale to which $A\ (t)$ is plotted. But these are precisely the integrals we require to evaluate formula (**12**), and the analyser therefore furnishes the means for the mechanical solution of our problem in a simple direct manner with a minimum of labor. The analyser employed by the writer has three separate harmonic rollers so that formula (**12**) can be evaluated for three different frequencies simultaneously.

I do not wish to give the impression that the labor of computation is reduced to a negligible quantity, for it is still considerable by reason of the very nature of the problem. This follows from the fact that we require, not a single numerical value of I for a particular value of t, as is the case for steady state conditions, but a complete history of the current, as a function of time, from its start up to its steady state value. What I do claim, however, is that the labor is by no means prohibitive and that mechanical integration as described above, furnishes a thoroughly practicable, if not the only practicable means of numerical solution.*

II. The Determination of the Indicial Admittance $A\ (t)$

By virtue of formulas (**11**) and (**12**), derived and discussed in the preceding section, our problem is reduced to a determination of the indicial admittance $A\ (t)$. In other words, our problem is now restricted to the formulation of the current flowing in any part of the electrical system or network in response to a unit d-c. voltage (zero before, unity $t = 0$) applied to the system. To the solution of this problem Heaviside devoted the greater part of the first two volumes of his "Electromagnetic Theory" and no student of this subject can fail to express the greatest admiration for his researches, an acquaintance with which is indeed essential to any investigation of the problem. While new problems have presented themselves, the study of which has resulted in new methods and original solutions, Heaviside was the first to treat the problem comprehensively and to insist on solutions which admitted of physical and numerical interpretation, as distinguished from merely formal mathematical expressions which conceal the real nature of the solution as effectually as do the original differential equations of the problem. Furthermore, his expansion theorem alone is a powerful instrument and forms the basis, not only for the special methods of solution developed in this section but also for the transmission line formulas derived in the succeeding sections:

---

*See Note 1 at end of paper.

The formal solution of our problem, the determination of the indicial admittance $A(t)$ is furnished by the expansion theorem (**3**):

$$A(t) = \frac{1}{Z(0)} + \Sigma^m \frac{\epsilon^{p_m t}}{p_m Z'(p_m)}$$

For systems or networks which have a relatively small number of degrees of freedom, and the roots of whose determinantal equation $Z(p) = 0$ admit of evaluation, formula (**3**) furnishes the working or practical as well as the formal solution. In the great majority of problems, however, the expansion theorem, as it stands, is of restricted utility as a working solution for several reasons, the chief of which are that it requires the location of the roots of $Z(p)$ and that it involves a prohibitive amount of labor for computation when the number of degrees of freedom is large. On the basis of the expansion theorem we therefore proceed to develop derived methods of solution.

1. *The Power Series Solution.* The solution for $A(t)$ can always be written down in the form of a power series in the variable t (time) without evaluating the roots of the determinantal equation $Z(p) = 0$, provided the function $\frac{1}{Z(p)}$ admits of asymptotic expansion* in the variable p. To prove this theorem and derive the power series solution we proceed as follows:

Return to formula (**3**) and expand the exponential functions in the usual power series. Since this expansion is absolutely convergent, the resultant expansion is absolutely convergent. Making use of the relation (**5**), formula (**3**) then becomes,

$$A(t) = \frac{1}{Z(\infty)} + \frac{t}{1!} \Sigma \frac{1}{Z'(p_m)} + \frac{t^2}{2!} \Sigma \frac{p_m}{Z'(p_m)}$$
$$+ \ldots . + \frac{t^n}{n!} \Sigma \frac{{p_m}^{n-1}}{Z'(p_m)} + \ldots$$

Now take equation (**4**) and write it as

$$\frac{1}{Z(p)} = \frac{1}{Z(\infty)} + \frac{1}{p} \Sigma \frac{1}{Z'(p_m)} \left(1 - \frac{p_m}{p}\right)^{-1}$$

---

*See Note 2.

Expanding the second factor of the summation by the binomial theorem, and arranging in inverse powers of p, we get,

$$\frac{1}{Z\,(p)} = \frac{1}{Z\,(\infty)} + \frac{1}{p} \sum \frac{1}{Z'\,(p_m)} + \frac{1}{p^2} \sum \frac{p_m}{Z'\,(p_m)}$$

$$+ \,.\,.\,.\,.\, + \frac{1}{p'} \sum \frac{{p_m}^{n-1}}{Z'\,(p_m)} + \,.\,.\,.\,.\,.$$

Comparing these two expansions we see at once that the coefficient of $\frac{t^n}{n\,!}$ in the former is identical with the coefficient of $\frac{1}{p^n}$ in the latter. Furthermore, the latter is nothing more than the asymptotic expansion of $\frac{1}{Z\,(p)}$. We therefore arrive at the following rule:

Expand the function $\frac{1}{Z\,(p)}$ *asymptotically in the form*

$$\frac{1}{Z\,(p)} = \sum_{0}^{\infty}{}^{k} \frac{a_k}{p^k}$$

Then the indicial admittance is given by

$$A\,(t) = \sum_{0}^{\infty}{}^{k}\, a_k \frac{t^k}{k\,!}$$

which is obtained from the asymptotic expansion by replacing $\frac{1}{p^n}$ *by* $\frac{t^n}{n\,!}$.

Those familiar with Heaviside's Electromagnetic Theory Vol. II, will recognize in the foregoing expansion the equivalent of his operational solution. Heaviside himself refrained from giving any justification of the solution beyond the pragmatic one that "it works," and so far as I am aware, the above furnishes the first formal proof of its correctness.

As regards the practical value of the power series expansion as a working solution, its utility is rather restricted (except when the expansion can be identified with known functions) by reason of the slow convergence of the series. A second disadvantage lies in the fact that the asymptotic expansions are usually difficult to obtain and of complicated form. Finally, it should be remarked that while $\frac{1}{Z(p)}$ always admits of asymptotic expansion for networks having a finite number of degrees of freedom, the asymptotic expansion of $\frac{1}{Z(p)}$ for transmission lines, where the number of degrees of freedom is infinite and trancendental functions are involved, is impossible. The method is, however, of very great mathematical interest, and is useful in computing $A(t)$ for small values of t.

We cannot at present take up the more developed series solutions of Heaviside where fractional powers of t appear. These solutions are of great mathematical interest and will be referred to later in connection with the treatment of transmission line problems. It may be remarked, however, in passing, that their chief application is in connection with the non-inductive cable which is of minor practical importance and whose solution is otherwise derived in Section IV.

2. *The Solution from Fourier's Integral.** It is always possible to express the current as a Fourier integral of the following form:

$$I = \frac{1}{2\pi i}\int_{-\infty}^{\infty} f(\lambda)\,d\lambda \int_{-\infty i}^{\infty i} \frac{\epsilon^{p(t-\lambda)}}{Z(p)}\,dp$$

where $f(t)$ is the impressed voltage and $Z(p)$ is the impedance function. It can be shown that this double integral is capable of reduction to the expansion theorem solution or the power series solution (for either integral or fractional powers of t), and it thus furnishes a check on these fundamental formulas.

3. *The Synthetic Method.* In many problems which do not admit of a working solution by direct methods of attack, the solution can be built up from the solutions for the component parts of the system under consideration by a method recently developed and employed by the writer. A notable example of the type of problem to which the method is applicable is furnished by the transmission line. Here, as we shall show later, the solution can be obtained for a line of any length if the voltage is applied directly to one end of the line and the other

---

*See Note 3.

end is short-circuited. When, however, the voltage is applied through terminal apparatus, and the other end is closed through terminal apparatus, or when apparatus is inserted at any point along the line, the solution by direct frontal attack is impossible. The effects of such terminal or intermediate apparatus can be solved for, both formally and numerically, by the synthetic method which promises to be an instrument of value in the solution of practically important problems. Again the system may be a "compound" system or network which is made up of two separate networks, coupled or connected in any specified manner. This method furnishes us a means for determining the indicial admittances of the compound system in terms of the indicial admittances (assumed known) of the component networks, when direct methods are incapable of yielding a solution. A particular problem of this character is the determination of the modifying effects of inserting additional impedance elements in a network.

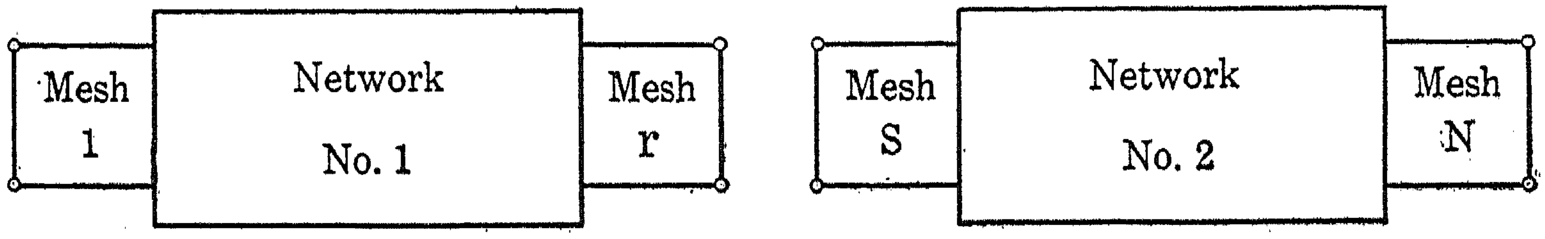
The method, as the name implies, is synthetic and physical in character, and the formulas are indeed based on physical reasoning rather than on mathematical analysis. From the mathematical standpoint, however, the method is of interest in that it applies the theory of integral equations* to the transient problem.

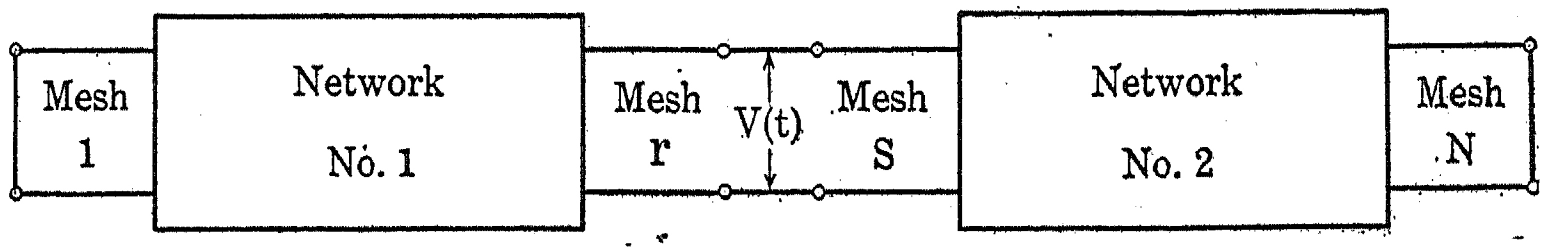
Briefly stated, the principle underlying the method consists in building up the solution for a network or system from the known solutions for its component parts. We now proceed to a discussion of the theory and a derivation of the formulas.

To make the application as general as possible we shall assume that we have two networks or systems, and we shall further assume that the indicial admittances of each are known. Our problem is to determine the indicial admittances (and hence the complete behavior and characteristics) of the compound network or system which is formed by connecting the two given systems in any specified manner. A sufficiently general manner of connecting the two networks is to open any mesh or branch of one network and any mesh or branch of the other network, and to connect the corresponding open terminals of the two networks, thus forming a compound network. This manner of coupling is illustrated schematically in the accompanying sketches.

---

*See Note 4.





To proceed with the theory, let network No. 1 consist of m meshes or branches, numbered from 1 to m, and let network No. 2 consist of n meshes or branches, numbered from I to N. Let the indicial admittances of No. 1 be denoted by $a_{jk}(t)$ (the subscripts depending, of course, on the meshes with which we are concerned) and those of No. 2 by $b_{JK}(t)$. Now to couple these networks open any mesh, say the r^{th} of No. 1, and any mesh, say the S^{th} of No. 2, and connect the open terminals of mesh r to open terminals of mesh S. The two networks are thus connected together through their r^{th} and S^{th} meshes respectively and form a compound system. We require the indicial admittances of the compound system in terms of the known indicial admittances of the two component networks.

To formulate the required admittances, assume, as usual, a unit e.m.f. inserted at time $t = 0$ in any mesh of the compound system, say the 1st mesh of network No. 1; then by definition the current flowing in mesh j of network No. 1 is $A_{1j}(t)$ and the corresponding current flowing in mesh J of network No. 2 is $A_{1J}(t)$.

Now an unknown voltage $V(t)$ exists across the common terminals which connect the two networks; hence the current flowing in any mesh of network No. 1 is the same as that which would flow if the network were isolated and actuated by a unit e.m.f. inserted in mesh 1 and an e.m.f. $-V(t)$ in mesh r. Hence by reference to formula (**11**)

$$A_{1r}(t) = a_{1r}(t) - \frac{d}{dt}\int_0^t V(\lambda)\, a_{rr}(t-\lambda)\, d\lambda \qquad \textbf{(a)}$$

By precisely the same physical reasoning we get

$$A_{1S}(t) = \frac{d}{dt}\int_0^t V(\lambda)\cdot b_{SS}(t-\lambda)\, d\lambda \qquad \textbf{(b)}$$

since network No. 2, regarded as isolated, is subject only to the impressed voltage $V(t)$ in mesh S.

But owing to the connections of the two networks, the currents corresponding to equations (**a**) and (**b**) are necessarily equal, whence:

$$A_{1r}(t) = A_{1S}(t) \qquad \text{, and}$$

$$\frac{d}{dt}\int_0^t V(\lambda)\left[a_{rr}(t-\lambda) + b_{SS}(t-\lambda)\right] d\lambda = a_{1r}(t) \qquad (\mathbf{c})$$

Now this is an integral equation in the unknown voltage $V(t)$ and is reducible to the canonical form

$$V(t) = f(t) + \int_0^t V(\lambda)\, u(t-\lambda)\, d\lambda \qquad (\mathbf{d})$$

where $f(t)$ and $u(t)$ are known functions.

The formal mathematical solution of this integral equation can be written down, by the method of successive substitutions, in the form of an absolutely convergent series. Furthermore, without solving the integral equations, we can immediately derive the power series solution (in case a power series solution in integral powers of t exists) by repeated differentiations of equation (**d**). When the power series solution involves fractional powers of t, this fact is at once evident from the form of the known functions appearing in (**d**), and the correct series solution can be derived from the integral equation as it stands. Having solved in some such manner for $V(t)$ the required indicial admittances are obtained from equations (**a**) and (**b**) by a single quadrature.

The practical value of this method of solution, as distinguished from its purely theoretical and mathematical interest, lies in the possibility of the numerical integration of the integral equations in cases where the formal and power series solutions are impracticable from the standpoint of interpretation or computation. The numerical integration works out well in practise, it may be made as accurate as we please, and gives the required numerical solution directly and with a minimum of labor. In practise, furthermore, the method is not so complicated as might appear from the sketch of the theory and often various short cuts are suggested by physical considerations. It is also applicable to problems other than that explicitly considered above; for example it yields a relatively easy numerical solution of an important problem in telegraphy, namely, the required form of impressed voltage to produce a prescribed form of arrival curve.

Space forbids further consideration of this method which the writer hopes to treat in greater detail in a future paper, and I must content myself with giving the simplest possible example of the method. Let network No. 1 consist of a resistance R and No. 2 of an inductance L so that the "compound" system consists of a resistance R in series with an inductance L. The indicial admittance of this system is, of course, known to be

$$\frac{1}{R}\left(1-\epsilon^{-\frac{R}{L}t}\right).$$

This solution will now be verified by the method just discussed. We have

$$a\,(t)=\frac{1}{R}$$

$$b\,(t)=\frac{1}{L}\,t$$

Substitution in (**c**) gives

$$\frac{d}{dt}\int_0^t V\,(\lambda)\left[\frac{1}{R}+\frac{1}{L}\,(t-\lambda)\right]d\,\lambda=\frac{1}{R}$$

Differentiating and rearranging

$$V\,(t)=1-\frac{R}{L}\cdot\int_0^t V\,(\lambda)\,d\,\lambda$$

By the method of successive substitutions the solution of this integral equation is:

$$V\,(t)=1-\frac{R}{L}\cdot\int_0^t d\,\lambda+\left(\frac{R}{L}\right)^2\cdot\int_0^t\int_0^\lambda d\,\lambda\,d\,\lambda_1$$

$$-\left(\frac{R}{L}\right)^3\cdot\int_0^t\int_0^\lambda\int_0^\lambda d\,\lambda\;d\,\lambda_1\;d\,\lambda_2+\ldots$$

$$=1-\left(\frac{R}{L}\right)\frac{t}{1\,!}+\left(\frac{R}{L}\right)^2\frac{t^2}{2\,!}$$

$$-\left(\frac{R}{L}\right)^3\frac{t^3}{3\,!}\;+$$

$$=\epsilon^{-\frac{R}{L}t}$$

Hence

$$A(t) = \frac{1}{R}(1 - V(t))$$

$$= \frac{1}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right)$$

which is the correct solution.

4. *Special Methods for the Solution of Transmission Systems.* In treating the problem presented by the transmission system, embracing, on the one hand, the transmission line having continuously distributed constants, and on the other, the artificial line whose constants are lumped or localized, and including as intermediate between these two the periodically loaded cable, we shall derive the basic solutions for the simplest possible configuration, namely when the voltage is applied directly to the line and the distant end is short-circuited. It will then be shown that the effect of terminal and intermediate apparatus can be formulated and numerically computed from the basic solution by aid of the synthetic method of treatment just discussed. The solution for the line itself is therefore fundamental.

Now limiting for the present our consideration to the line which is short-circuited at one end, and excited by the direct application of a voltage at the other end, the symbolic formula is readily written down; its general form is well known to be

$$\frac{1}{K}\,\frac{\cosh \gamma\,(s - x)}{\sinh\,(\gamma\, s)} \cdot \epsilon^{pt}$$

This is the symbolic formula for the current at point x of a line which is short-circuited at $x = s$ and is excited by a voltage applied at $x = 0$. K and γ are functions of the operator p, and the line constants. (If we are dealing with an artificial line s and x are to be interpreted as number of sections instead of distance; the form of function is, however, the same in both cases.)

Now for the smooth line (having continuously distributed constants) and for the artificial line, the expansion theorem solution can be written down without any trouble. As a working or practical solution this is, however, of no utility as it stands, owing to the fact that the number of terms in the summation is infinite for the smooth line (and very large for long artificial lines) and the convergence very slow. Physically this difficulty arises from the fact that the expansion theorem solution formulates the current or indicial admittance, as a sum of characteristic oscillations or vibrations, a formulation which is mathematically correct but physically artificial. In reality the current travels out in the form of a wave which is reflected at the distant end, and which is independent of the distant termination for a definite time. Each one of the modes of characteristic oscillation involves, however, the character of the distant termination. Investigation shows that if we deal with the *infinitely* long line the sum of characteristic oscillations is converted into the direct wave which alone exists in the infinitely long line, and that this wave is expressible in terms of known functions.

To handle the case of the infinitely long line we set up the expansion theorem solution for a line of length s and then let s approach the limit infinity. As s approaches infinity, the summation of the characteristic oscillations is converted into a definite integral, which, for the practically important types of lines considered in succeeding sections, admits of evaluation in terms of known functions.

To now apply the solution for the infinitely long line to the case of the line of finite length we remark that the current, say $\phi_x(t)$, which exists at point x and time t of a line of infinite length with unit d-c. voltage (zero before, unity after $t = 0$) applied at $x = 0$, is the solution of the problem whose symbolic formula or solution is

$$\frac{1}{K} \cdot \epsilon^{-x\gamma}$$

Now, take the formula given above, for the line of finite length s, express the hyperbolic functions in their equivalent exponential form, and expand; we get

$$\frac{1}{K}\,\epsilon^{-\gamma x} + \sum \frac{1}{K}\,\epsilon^{-\gamma(2ms+x)} + \sum \frac{1}{K}\,\epsilon^{-\gamma(2ms-x)}$$

Now, by definition, $\phi_x(t)$ is the solution for the first term; hence the solution for the term, $1/K\ \epsilon^{-(2ms+x)}$, is

$$\phi_{2ms+x}(t)$$

which is gotten from $\phi_x(t)$ (a function of x and t) by replacing x with $2\,m\,s + x$. Hence the solution for the short-circuited line of length s is simply

$$A_x(t) = \phi(t) + \Sigma\,\phi_{2ms+x}(t) + \Sigma\,\phi_{2ms-x}(t) \qquad \textbf{(15)}$$

The first term in the above is the direct wave (the only wave which would exist if the line were infinitely long) the first summation represents the series of waves reflected from the sending end, and the second summation the series of waves reflected from the other end of the line. The summations are rapidly convergent, and the above furnishes the working solution which we shall employ.

If, now a voltage $V_1(t)$ is applied at $x = 0$, and a voltage $V_2(t)$ at $x = s$, the current $I_x(t)$ at point x is the aid of formula (**11**):

$$
\begin{aligned}
I_x(t) = {} & \frac{d}{dt}\int_0^t V_1(t-\lambda)\left\{\begin{array}{c}\phi_x(\lambda) + \Sigma_m\ \phi_{2ms+x}(\lambda) \\ + \Sigma_m\ \phi_{2ms-x}(\lambda)\end{array}\right\} d\lambda \\
& - \frac{d}{dt}\int_0^t V_2(t-\lambda)\left\{\begin{array}{c}\phi_{s-x}(\lambda) + \Sigma_m\ \phi_{(2m+1)s-x}(\lambda) \\ + \phi_{(2m-1)s+x}(\lambda)\end{array}\right\} d\lambda
\end{aligned}
\qquad (\mathbf{16})
$$

This is a formula of fundamental importance to the whole theory of transmission systems. If the line is of great length (large attenuation) and the applied voltages are alternating, the reflected waves are negligible and we have the approximate formula,

$$
\begin{aligned}
I_x(t) = {} & \frac{d}{dt}\int_0^t V_1(t-\lambda)\,\phi_x(\lambda)\,d\lambda \\
& - \frac{d}{dt}\int_0^t V_2(t-\lambda)\,\phi_{s-x}(\lambda)\,d\lambda
\end{aligned}
$$

The *driving point* indicial admittance of the line is obtained from (**15**) by setting $s = 0$; denoting it by $A_1(t)$ it is

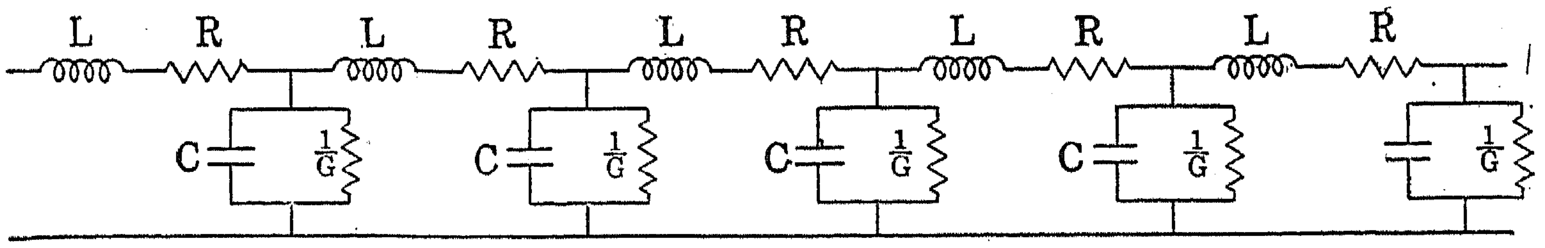
$$A_1(t) = \phi_0(t) + 2\sum_1^{\infty}{}_m\ \phi_{2ms}(t) \qquad (\mathbf{16})$$

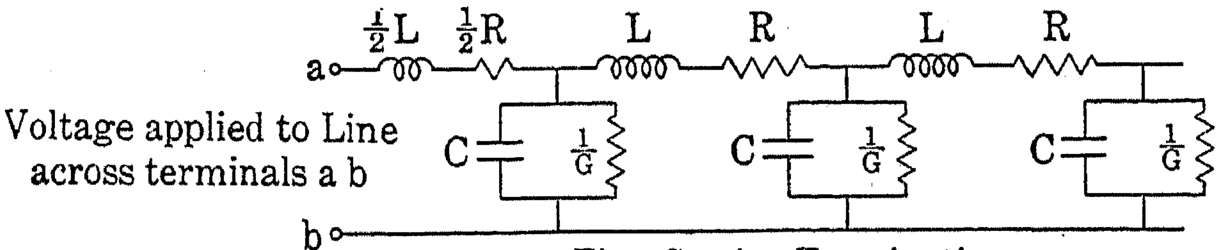
The *transfer* indicial admittance; *i. e.* the current at $x = s$ with line short-circuited at $x = s$, is obtained from (**15**) by setting $x = s$. Denoting this function by $A_2(t)$ it is:

$$A_2(t) = 2\sum_1^{\infty}{}_m\ \phi_{(2m-1)s}(t) \qquad (\mathbf{16a})$$

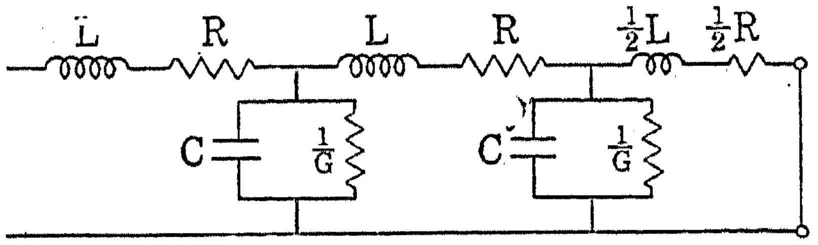
III. Solution for the Artificial Line

The artificial line is a periodic structure of the character shown in the sketch herewith.





First Section Termination



n^{th} Section Termination

Each section of the structure consists of an inductance L and a resistance R in series with the line, and the parallel combination of a capacity C and a leakage conductance G in shunt across the line. The structure which we shall consider is of n sections in length, and the voltage is applied at "mid-series" position, and the nth section is short-circuited at mid-series as shown in the sketches herewith. These terminations are chosen not only because they simplify the analysis somewhat but because they are of practical importance.

A study of the transient properties of the artificial line is important for several reasons:

1. The artificial line is often used as a model of an actual transmission line and it is therefore of importance to determine theoretically the degree of correspondence between the two.
2. The solution for the corresponding transmission line with continuously distributed constants is derivable from the solution for the artificial line by keeping the total inductance, resistance, capacity and leakage constant or finite, and letting the number of sections approach infinity.
3. The artificial line is very closely related, in its properties and performance, to the periodically loaded cable, and its solution is, to a first approximation, a working solution for the loaded cable.

4. The structure is of great importance in its own right and is one form of "wave filter" employed in telephone systems.*

The starting point of the solution is the symbolic formula for the current in the qth section of a line of n sections in length, terminated as shown above; it is

$$-(C_p+G)\cdot\frac{\cos(n-q)\,\Gamma}{\sin(\Gamma)\cdot\sin(n\,\Gamma)}\,\epsilon^{pt} \tag{17}$$

where

$$\begin{aligned} v &= \frac{1}{\sqrt{L\,C}} \\ \rho &= \frac{R}{2\,L}+\frac{G}{2\,C}\,. \\ \sigma &= \frac{R}{2\,L}-\frac{G}{2\,C} \end{aligned} \tag{17a}$$

and

$$\cos\Gamma = 1+\frac{1}{2}\,\frac{(p+\rho)^2-\sigma^2}{v^2} \tag{17b}$$

This formula may be deduced in a number of ways; perhaps the simplest is a follows: Considering the qth section, write down the circuital equation for the voltage drop around the closed section; after replacing $\frac{d}{dt}$ by p we get (denoting the circuital currents in the $(q-1)st$, qth and $(q+1)st$ sections by I_{q-1}, I_q and I_{q+1} respectively);

$$I_q\left(L\,p+R+\frac{2}{C\,p+G}\right)-\frac{1}{C\,p+G}\,I_{q-1} - \frac{1}{C\,p+G}\cdot I_{q+1}=0$$

or

$$I_q\left(\frac{(p+\rho)^2-\sigma^2}{v^2}+2\right)-I_{q-1}-I_{q+1}=0 \tag{18}$$

Equation (**18**) is a difference equation relating the currents in the three adjacent sections; in accordance, therefore, with usual methods we assume a solution of the form

$$I_q = A\,\epsilon^{iq\Gamma}+B\,\epsilon^{-iq\Gamma} \tag{19}$$

---

*See Note 5.

where A and B are arbitrary constants and $i\,\Gamma$ is the exponential propagation factor which is to be determined. Substitution of (**21**) in (**20**) gives after simplification:

$$\left(A\,\epsilon^{iq\Gamma} + B\,\epsilon^{-iq\Gamma}\right)\cdot$$
$$\cdot\left(1+\frac{1}{2}\left(\frac{(p+\rho)^2-\sigma^2}{v^2}\right)-\frac{\epsilon^{i\Gamma}+\epsilon^{-i\Gamma}}{2}\right)=0$$

Clearly this equation is satisfied and furthermore leaves two arbitrary constants A and B at our disposal to satisfy any specified terminal conditions if we set

$$\frac{\epsilon^{i\Gamma}+\epsilon^{-i\Gamma}}{2}=\cos\Gamma=1+\frac{1}{2}\,\frac{(p+\rho)^2-\sigma^2}{v^2}$$

which agrees with (**17b**)

If A and B are now determined in accordance with the terminal conditions illustrated above we get, without any trouble, the symbolic formula (**17**) for the current in the qth section of the structure.

Assuming now a unit d-c. voltage applied to the line at time $t = 0$, the current in the qth section is by definition the indicial admittance of the line and will be denoted by $A_q\,(t)$, the double subscript being omitted for convenience; $A_q\,(t)$ is formulated by applying the expansion theorem to the symbolic formula (**17**) with the result that we get without any trouble

$$\begin{aligned} A_q\,(t) = &-\sum \frac{C\,p_m\cos\,(q\,\Gamma_m)}{n\cdot p_m\cdot\left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)\sin\,\Gamma_m}\,\epsilon^{p_m t} \\ &-\frac{G}{C}\sum\frac{C\cdot\cos\,(q\,\Gamma_m)}{n\cdot p_m\cdot\left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)\sin\,\Gamma_m}\,\epsilon^{p_m t} \qquad (\mathbf{20}) \\ &+\frac{G}{C}\sum\frac{C\cdot\cos\,(g\,\Gamma_m)}{n\cdot p_m\left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)\sin\,\Gamma_m} \end{aligned}$$

In equation (**20**) p_m and Γ_m are determined by the determinantal equation

$$\sin\,(n\,\Gamma) = 0$$

whence

$$\Gamma_m = m\;\pi/n \qquad\qquad m = 0, 1, 2, \ldots\ldots n$$

and

$$\frac{(p_m + \rho)^2 - \sigma^2}{2\,v^2} = -\,(1 - \cos\Gamma_m)$$

It will be observed that (**20**) may be written

$$A_q\,(t) = -\sum \frac{C \cdot \cos\,(q\,\Gamma_m)}{n \cdot \sin\Gamma_m \cdot \left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)}\,\epsilon^{p_m t}$$

$$- \frac{G}{C}\int_0^t d\,t_1 \cdot \sum \frac{C \cdot \cos\,(q\,\Gamma_m)}{n \cdot \sin\,\Gamma_m \cdot \left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)} \cdot \epsilon^{p_m t_1} \qquad \mathbf{(21)}$$

since the last two terms are, within a constant, the integral of the first term. This effects a great simplification and reduces our problem to a solution of the first summation of (**20**) or (**21**).

After some straightforward simplification and rearrangement we get

$$F\,(t) = -\,C\sum \frac{\cos\,(q\,\Gamma_m)}{n \cdot \sin\,\Gamma_m \cdot \left(\dfrac{d\,\Gamma_m}{d\,p_m}\right)}\,\epsilon^{p_m t}$$

$$= \frac{\sqrt{2}}{\pi}\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\sum \frac{\pi}{n} \cdot$$

$$\cdot \frac{\cos\left(q\,m\,\dfrac{\pi}{n}\right)}{\sqrt{1 - \cos\left(m\,\dfrac{\pi}{n}\right) - \dfrac{\sigma^2}{2\,v^2}}} \cdot$$

$$\cdot \sin\left(\sqrt{2}\,v\,t\sqrt{1 - \cos\left(m\,\frac{\pi}{n}\right) - \frac{\sigma^2}{2\,v^2}}\right)$$

Now in accordance with our previously discussed mode of treatment let n approach infinity and set

$$\frac{\pi}{n} = d\,\lambda$$

$$m\,\frac{\pi}{n} = \lambda$$

In the limit the summation passes into the definite integral and we get:

$$F(t) = \frac{\sqrt{2}}{\pi}\sqrt{\frac{C}{L}}\,\epsilon^{-pt}\int_0^{\pi}\frac{\cos(q\,\lambda)}{\sqrt{1-\cos\lambda-\dfrac{\sigma^2}{2\,v^2}}}\,\cdot$$

$$\cdot\,\sin\left(\sqrt{2}\,v\,t\sqrt{1-\cos\lambda-\frac{\sigma^2}{2\,v^2}}\right)\cdot d\,\lambda$$

If we now write for convenience

$$\begin{aligned}\rho/2\,v &= \eta\\ \sigma/2\,v &= \zeta\\ 2\,v\,t = p_c\,t &= z\end{aligned} \qquad \textbf{(22)}$$

and observe that

$$1-\cos\lambda = 2\sin^2\left(\frac{\lambda}{2}\right)$$

we get finally, after a suitable change in the variable and limits of integration,

$$F(z) = \sqrt{\frac{C}{L}}\,\epsilon^{-\eta z}\cdot\frac{2}{\pi}\cdot\int_0^{\pi/2}\frac{\cos(2\,q\,\lambda)}{\sqrt{\sin^2\lambda-\zeta^2}}$$

$$\sin(z\cdot\sqrt{\sin^2\lambda-\zeta^2})\cdot d\,\lambda \qquad \textbf{(23)}$$

and

$$A_q(t) = F(z) + (\eta-\zeta)\int_0^z F(z_1)\cdot d\,z_1 \qquad \textbf{(24)}$$

where now $A_q(t)$ is equal to the current in the qth section of an *infinitely* long line instead of the line of finite length with which we started. It will be observed that the integral term of (**24**) vanishes if the leakage conductance G is put equal to zero and that within a constant it is merely the progressive integral of the first term. For both the artificial line and the smooth line the effect of leakage is to introduce an integral term of this character. This integral term is readily evaluated by mechanical integration as soon as the first term of the formula is derived.

Returning to equation (**23**) we observe that

$$\int_0^{\pi/2}\frac{\cos(2\,q\,\lambda)}{\sqrt{\sin^2\lambda-\zeta^2}}\,\sin(z\cdot\sqrt{\sin^2\lambda-\zeta^2})\cdot d\,\lambda =$$

$$\int_0^z d\,z_1\int_0^{\pi/2}\cos(2\,q\,\lambda)\cdot\cos(z\cdot\sqrt{\sin^2\lambda-\zeta^2})\cdot d\,\lambda$$

whence $$F(z) = \sqrt{\frac{C}{L}}\,\epsilon^{-\eta z} \int_0^z \Phi(z_1) \cdot d z_1 \tag{25}$$

where $$\Phi(z_1) = \frac{2}{\pi} \int_0^{\pi\,2} \cos(2 q \lambda) \cdot \cos(z_1 \sqrt{\sin^2 \lambda - \zeta^2}) \cdot d \lambda \tag{26}$$

Our problem is thus reduced to the solution of the definite integral of (**26**)

The key to the solution of this integral is furnished by the known value of the definite integral:

$$\frac{2}{\pi} \int_0^{\pi/2} (\cos(2 q \lambda) \cdot \cos(z \cdot \sin \lambda) \cdot d \lambda = J_{2q}(z) \tag{27}$$

where $J_{2q}(z)$ is the Bessel function* of the first kind of order $2q$ and the argument z. To make use of this known function in solving (**26**) we proceed as follows: Expand the second cosine function appearing in (**26**) in the usual power series. We get

$$\Phi(z_1) = \frac{2}{\pi} \int_0^{\pi\diagdown 2} d \lambda \cdot \cos(2 q \lambda) \cdot$$

$$\Sigma_m \; (-1)^m \frac{z_1^{2m}}{(2m)\,!} (\sin^2 \lambda - \zeta^2)^m$$

expanding $(\sin^2 \lambda - \zeta^2)^m$ by the binomial theorem and arranging in powers of ζ^2 this becomes:

$$\Phi(z_1) = \frac{2}{\pi} \int_0^{\pi/2} d \lambda \cdot \cos(2 q \lambda)$$

$$\left\{ \begin{array}{l} \Sigma (-1)^m \dfrac{z_1^{2m}}{(2m)\,!} \sin^{2m} \lambda \\ + \dfrac{\zeta^2}{1\,!} \Sigma (-1)^m (m+1) \dfrac{z_1^{2m+2}}{(2m+2)\,!} \sin^{2m} \lambda \\ + \dfrac{\zeta^4}{2\,!} \Sigma (-1)^m (m+1)(m+2) \dfrac{z_1^{2m+4}}{(2m+4)\,!} \sin^{2m} \lambda \\ + \ldots\ldots \end{array} \right. \tag{28}$$

---

*See Note 6.

$$= \varphi_0(z_1) + \frac{\zeta^2}{1\,!}\,\varphi_1(z_1) + \frac{\zeta^4}{2\,!}\,\varphi_2(z_1) + \ldots$$

$$+ \frac{\zeta^{2n}}{n\,!}\,\varphi_n(z_1) \qquad (29)$$

where

$$\varphi_n(z_1) = \frac{2}{\pi}\int_0^{\pi/2} d\,\lambda \cdot \cos(2\,q\,\lambda)\,\cdot$$

$$\Sigma^m\,(-1)_m\,(m+1)\ldots(m+n)\,\frac{{z_1}^{2m+2n}}{(2\,m+2\,n)\,!}\,\sin^{2m}\lambda$$

We observe that

$$\varphi_1(z_1) = \frac{z_1}{2}\int_0^{z_1} \varphi_0(z_2)\cdot d\,z_2 \,.$$

$$\cdots\cdots\cdots$$

$$\varphi_n(z_1) = \frac{z_1}{2}\int_0^{z^1} \varphi_{n-1}(z_2)\cdot d\,z_2$$

and

$$\varphi_0(z_1) = \frac{2}{\pi}\int_0^{\pi/2} \cos(2\,q\,\lambda)\,.\,\cos(z_1\,.\,\sin\lambda)\,.\,d\,\lambda$$
$$= J_{2q}(z_1)$$

Hence (**28**) may be written:

$$\Phi(z_1) = \varphi_0(z_1)$$
$$+ \frac{\zeta^2}{1\,!}\cdot\frac{z_1}{2}\int_0^{z_1}\varphi_0(z_2)\,.\,d\,z_2$$
$$+ \frac{\zeta^4}{2\,!}\cdot\frac{z_1}{2}\int_0^{z_1}\frac{z_2}{2}\,d\,z_2\int_0^{z_2}\varphi_0(z_3)\,.\,d\,z_3$$
$$+ \frac{\zeta^6}{3\,!}\cdot\frac{z_1}{2}\int_0^{z_1}\frac{z_2}{2}\,d\,z_2\,.\int_0^{z_2}\frac{z_3}{2}\,d\,z_3\,\cdot$$
$$\int_0^{z_3}\varphi_0(z_4)\cdot d\,z_4 + \ldots\ldots \qquad (30)$$

Reversing the order of the multiple integrals appearing in (**30**) we get, after repeated integration and simplification, as the expression for $F(z)$ of equation (**25**)

$$F(z) = \sqrt{\frac{C}{L}}\,\epsilon^{-\eta z} \int_0^z d\,z_1 \cdot J_{2q}(z_1) \cdot \left\{ \begin{aligned} & 1 + \frac{\zeta^2 (z^2 - z_1^2)}{2^2} \\ & \quad + \frac{\zeta^4 (z^2 - z_1^2)^2}{2^2 \cdot 4^2} \\ & \quad + \frac{\zeta^6 (z^2 - z_1^2)}{2^2 \cdot 4^2 \cdot 6^2} \\ & \quad + \ldots\ldots \end{aligned} \right. \qquad \textbf{(31)}$$

The series appearing in the above is the expansion of the known function $I_0 (\zeta \sqrt{z^2 - z_1^2})$ which is the Bessel function of order zero and pure imaginary argument $i \zeta \sqrt{z^2 - z_1^2}$ (the function itself is entirely real since even powers alone are present). Hence,

$$F(z) = \sqrt{\frac{C}{L}} \cdot \epsilon^{-\eta z} \int_0^z J_{2q}(z_1) \cdot I_0 (\zeta \sqrt{z^2 - z_1^2}) \cdot d\,z_1$$

and finally, by reference to (**24**), the final solution for $A_q(t)$ is

$$\begin{aligned} A_q(t) = {} & \sqrt{\frac{C}{L}}\,\epsilon^{-\eta z} \int_0^z J_{2q}(z_1) \cdot I_0 (\zeta \sqrt{z^2 - z_1^2}) \cdot d\,z_1 \\ & + (\eta - \zeta) \cdot \int_0^z d\,z_1 \cdot \sqrt{\frac{C}{L}} \cdot \epsilon^{-\eta z_1} \cdot \\ & \qquad \int_0^{z_1} J_{2q}(z_2) \cdot I_0 (\zeta \sqrt{z_1^2 - z_2^2}) \cdot d\,z_2 \end{aligned} \qquad \textbf{(32)}$$

This is the complete solution for the current in the qth section of an artificial line of infinite length when a unit voltage is applied to the line at time $t = 0$. The second term of (**32**) is merely the integral of the first term multiplied by $(\eta - \zeta)$, and for that reason offers no difficulties in the way of computation. The first term is expressed in terms of Bessel functions whose properties are known and which can be computed. Plots of $A_q(t)$ from equation (**32**) are shown later in this paper when a discussion of the formulas and their physical significance is taken up.

Formula (**32**) is quite general in that all four line constants (R. L. C. G.) are finite and unrestricted in value. Certain particular cases corresponding to particular values of or relations among the line constants are of interest and importance.

1. If the leakage conductance $G = 0$, then, by reference to formulas (**17**) and (**22**), $\eta = \zeta$ and (**32**) reduces to

$$A_q(t) = \sqrt{\frac{C}{L}}\,\epsilon^{-\eta z}\int_0^z J_{2q}(z_1)\cdot I_0(\zeta\sqrt{z^2 - z_1^2})\cdot d\,z_1 \tag{33}$$

which is the solution for the *leakageless line* ($G = 0$)

2. If $\frac{R}{L} = \frac{G}{C}$, $\zeta = 0$, (32) reduces to

$$A_q(t) = \sqrt{\frac{C}{L}}\int_0^z \epsilon^{-\eta z_1}\cdot J_{2q}(z_1)\cdot d\,z_1 \tag{34}$$

which is the solution for the *quasi-distortionless line*. This name is, of course, suggested and justified by the fact that the corresponding smooth line (constants continuously distributed), when so proportioned that $\frac{R}{L} = \frac{G}{C}$, is exactly distortionless. We shall see later that the quasi-distortionless line, while not strictly distortionless, is characterized by a minimum of distorting action.

(3) If the line is non-dissipative $R = G = 0$, and (**32**) reduces to:

$$A_q(t) = \sqrt{\frac{C}{L}}\int_0^z J_{2q}(z_1)\cdot d\,z_1 \tag{35}$$

which is the solution for the *non-dissipative* artificial line made up of pure inductance and capacity only. This solution applies also to the non-dissipative loaded cable*

---

*See Note 7.

(4) If $L = G = 0$, the solution reduces to:

$$A_q(t) = \frac{2}{R}\, \epsilon^{-\frac{2t}{RC}} \cdot I_q\left(\frac{2t}{RC}\right) \tag{36}$$

which is the solution for the *artificial cable.* $I_q\left(\frac{2t}{RC}\right)$ is the Bessels function of order q and pure imaginary argument $\frac{2t}{RC}$. This solution is derivable from (**33**) by letting L approach zero as a limit; as a matter of fact, however, it was derived directly from the expansion theorem solution. The detailed solution is omitted, however, as involving nothing of special interest.

Before entering into a discussion of these formulas and an exposition of their nature and properties we shall first take up the problem of the smooth (continuously loaded) line and derive the solutions for both current and voltage.

IV. Solution for the Smooth or Continuously Loaded Line

The artificial line whose solution was derived in the preceding section becomes, in the limit, the corresponding smooth or continuously loaded line, if we keep the total value of the line constants finite and constant and let the number of sections approach infinity as a limit. Hence the solution for the smooth line is derivable by purely mathematical processes from the solution given by formula (**32**) for the artificial line. In this section we shall, however, derive the solution directly by aid of the expansion theorem. We shall find that the process of evaluation is precisely analagous, step by step, to that employed in the preceding section.

The symbolic formula for the current I_x at point x of a line of length s, short-circuited at $x = s$, is well known to be

$$I_x = \sqrt{\frac{C\,p + G}{L\,p + R}} \cdot \frac{\cosh \gamma\,(s - x)}{\sinh\,(\gamma\,s)} \cdot \epsilon^{pt}$$

$$= (C\,p + G) \cdot \frac{\cosh \gamma\,(s - x)}{\gamma \cdot \sinh\,(\gamma\,s)}\, \epsilon^{pt} \tag{37}$$

where R, L, C, and G are the values per unit length of the continuously distributed line constants and

$$\gamma = \sqrt{\{\,(L\,p + R)\,(C\,p + G)\,\}}$$

we write

$$\frac{1}{\sqrt{LC}} = v$$

$$\frac{R}{2L} + \frac{G}{2C} = \rho \tag{38}$$

$$\frac{R}{2L} - \frac{G}{2C} = \sigma$$

whence

$$\frac{G}{C} = \rho - \sigma$$

$$\gamma = \frac{1}{v} \sqrt{(p+\rho)^2 - \sigma^2} \tag{39}$$

Writing down the expansion theorem solution of (**36**) in accordance with formula (**3**) we get, after some easy simplifications,

$$\begin{aligned} A_x(t) = {} & C \sum \frac{\cosh(\gamma_m x)}{s \cdot \gamma_m \cdot \left(\dfrac{d\,\gamma_m}{d\,p_m}\right)} \cdot \epsilon^{p_m t} \\ & + (\rho - \sigma) \int_0^t dt_1 \cdot C \sum \frac{\cosh(\gamma_m x)}{s \cdot \gamma_m \cdot \left(\dfrac{d\,\gamma_m}{d\,p_m}\right)} \epsilon^{p_m t_1} \end{aligned} \tag{40}$$

$A_x(t)$ is, of course, the value of the current at point x and time t when a unit d-c. voltage is applied to the line at $x = 0$ and $t = 0$. We observe that the second term of (**40**) (which vanishes with the leakage), is merely the progressive integral of the first term, so that our problem is to evaluate the expansion formulated by the first term. p_m and γ_m are given by the determinantal equation

$$\sinh(\gamma s) = 0$$

$$\gamma_m = i\,m\,\frac{\pi}{s} \qquad m = 0, 1, 2, \ldots \infty$$

$$p_m = -\rho \pm i\,v \sqrt{\left(m\,\frac{\pi}{s}\right)^2 - \frac{\sigma^2}{v^2}}$$

For convenience we write

$$\begin{aligned} v\,t &= y \\ \rho/v &= a \\ \sigma/v &= b \end{aligned} \qquad (41)$$

and let the length of the line s approach infinity. By processes identical with those developed in the preceding section, the summations in (**40**) are thereby converted into definite integrals and we get without difficulty:

$$A_x(t) = \sqrt{\frac{C}{L}}\,\epsilon^{-ay}\cdot\frac{2}{\pi}\int_0^\infty \frac{\cos(x\lambda)\cdot\sin\left(y\sqrt{\lambda^2-b^2}\right)}{\sqrt{\lambda^2-b^2}}\,d\lambda \qquad (42)$$

$$+ (a-b)\int_0^y d\,y_1\cdot\sqrt{\frac{C}{L}}\,\epsilon^{-ay_1}\cdot\frac{2}{\pi}$$

$$\int_0^\infty \frac{\cos(x\lambda)\cdot\sin\left(y_1\sqrt{\lambda^2-b^2}\right)}{\sqrt{\lambda^2-b^2}}\,d\lambda$$

where now $A_x(t)$ is equal to the current at point x and time t of the *infinitely long* line. Observe that the second term is the progressive integral of the first.

We set

$$F(y) =$$

$$\sqrt{\frac{C}{L}}\,\epsilon^{-ay}\cdot\frac{2}{\pi}\int_0^\infty \frac{\cos(x\lambda)\cdot\sin\left(y\sqrt{\lambda^2-b^2}\right)}{\sqrt{\lambda^2-b^2}}\,d\lambda \qquad (43)$$

whence

$$A_x(t) = F(y) + (a-b)\int_0^y F(y_1)\cdot d\,y_1 \qquad (44)$$

and proceed with the evaluation of infinite integral factor of $F(y)$. We observe that

$$F(y) = \sqrt{\frac{C}{L}}\,\epsilon^{-ay}\int_0^y \Phi(y_1)\cdot d\,y_1 \qquad (45)$$

where

$$\Phi(y_1) = \frac{2}{\pi}\int_0^\infty \cos(x\lambda)\cdot\cos\left(y_1\sqrt{\lambda^2-b^2}\right)\cdot d\lambda \tag{46}$$

The key to the solution of this infinite integral is furnished by the known value of the infinite integral

$$\frac{2}{\pi}\int_0^\infty \frac{\sin(y\lambda)}{\lambda}\,d\lambda$$

which is a discontinuous function of value -1 for $y < o$ and $+1$ for $y > o$.

Returning to (**46**) expand the second cosine function in the usual power series, and precisely as in the preceding section, subsequently expand $(\lambda^2 - b^2)^m$ by the binomial theorem. We get, after arranging in powers of b,

$$\Phi(y_1) = \frac{2}{\pi}\int_0^\infty d\lambda\cdot\cos(x\lambda)\left\{\begin{aligned} &\Sigma(-1)^m\frac{y_1^{2m}\lambda^{2m}}{(2m)!} \\ &+\frac{b^2}{1!}\Sigma(-1)^m\frac{y_1^{2m+2}}{(2m+2)!}(m+1)\cdot\lambda^{2m} \\ &+\frac{b^4}{2!}\Sigma(-1)^m\frac{y_1^{2m+4}}{(2m+4)!}(m+1)(m+2)\lambda^{2m} \\ &+\dots \end{aligned}\right. \tag{47}$$

which corresponds exactly with formula (**28**) of the preceding section. By analysis precisely the same as that employed in deriving (**30**), equation (**47**) is reducible to the form,

$$\begin{aligned} \Phi(y_1) = {} & \varphi(y_1) \\ &+\frac{b^2}{1!}\cdot\frac{y_1}{2}\int_0^{y_1}\varphi(y_2)\cdot dy_2 \\ &+\frac{b^4}{2!}\cdot\frac{y_1}{2}\int_0^{y_1}\frac{y_2}{2}\,dy_2\int_0^{y_2}\varphi(y_3)\cdot dy_3 \\ &+\frac{b^6}{3!}\cdot\frac{y_1}{2}\int_0^{y_1}\frac{y_2}{2}\,dy_2\cdot\int_0^{y_2}\frac{y_3}{2}\,dy_3\int_0^{y_3}\varphi(y_4)\cdot dy_4 \\ &+\dots \end{aligned} \tag{48}$$

where

$$\begin{aligned}\varphi(y_1) &= \frac{2}{\pi}\int_0^\infty \cos(x\lambda)\cdot\cos(y_1\lambda)\cdot d\lambda \\ &= \frac{d}{d y_1}\cdot\frac{2}{\pi}\int_0^\infty \frac{\cos(x\lambda)\cdot\sin(y_1\lambda)}{\lambda}\, d\lambda \qquad (49) \\ &= \frac{d}{d y_1}\psi(y_1)\end{aligned}$$

Now the integral of (**46**) is by (**48**)

$$\begin{aligned}&\int_0^y \Phi(y_1)\cdot d y_1 \\ &= \int_0^y \varphi(y_1)\cdot d y_1 \\ &+ \frac{b^2}{1\,!}\int_0^y \frac{y_1}{2}\, d y_1 \int_0^{y_1} \varphi(y_2)\cdot d y_2 \\ &+ \frac{b^4}{2\,!}\int_0^y \frac{y_1}{2}\, d y_1 \int_0^{y_1} \frac{y_2}{2}\, d y_2\cdot\int_0^{y_2} \varphi(y_3)\cdot d y_3 \\ &+ \ldots\end{aligned}$$

which sum up to

$$\begin{aligned}\int_0^y \Phi(y_1)\cdot d y_1 &= \int_0^y \varphi(y_1)\cdot I_0\left(b\sqrt{y^2 - y_1^2}\right) d y_1 \\ &= \int_0^y \frac{d}{d y_1}\psi(y_1)\cdot I_0\left(b\sqrt{y^2 - y_1^2}\right)\cdot d y_1 \qquad (50)\end{aligned}$$

Now $\psi(y_1)$ is a discontinuous function which is zero before, unity after $y_1 = x$. Hence the definite integral of (**50**) is identically zero for values of the upper limit $y < x$, and its total contribution is entirely concentrated at $y_1 = x$.
Hence

$$\begin{aligned}\int_0^y \Phi(y_1)\cdot d y_1 &= 0 \text{ for } y = v\,t < x \\ &= I_0\left(b\sqrt{y^2 - x^2}\right) \text{ for } y \geq x \qquad (51)\end{aligned}$$

Hence by (**44**) and (**45**) our solution is:

$$A_x(t) = 0 \qquad \text{for} \qquad y = vt < x$$

$$= \sqrt{\frac{C}{L}} \cdot \epsilon^{-ay} \cdot I_0\left(b\sqrt{y^2 - x^2}\right)$$

$$+ (a - b) \int_x^y \sqrt{\frac{C}{L}}\, \epsilon^{-ay_1} \cdot I_0\left(b\sqrt{y_1^2 - x^2}\right) \cdot d\,y_1 \qquad \text{for} \quad y \geq x \qquad (\mathbf{52})$$

which is the final solution.

In the foregoing solution, we have dealt with discontinuous functions and doubtless some of the processes involved require further examination and justification from a purely mathematical standpoint. The final solution is, however, quite correct, and may be verified as the limit of formula (**32**); a solution to whose derivation no exception can be taken.

Special cases of the general solution will now be considered. If leakage is absent $G = 0$ and $a = b$ whence

$$A_x(t) = 0 \qquad \text{for} \qquad vt = y < x$$

$$= \sqrt{\frac{C}{L}}\, \epsilon^{-ay} \cdot I_0\left(b\sqrt{y^2 - x^2}\right) \qquad \text{for } y \geq x \qquad (\mathbf{53})$$

$$= \sqrt{\frac{C}{L}}\, \epsilon^{-\rho t} \cdot I_0\left(\rho\sqrt{t^2 - \frac{x^2}{v^2}}\right) \quad \text{for} \quad t \geq \frac{x}{v}$$

which is the solution for the *leakageless line*.* While the general solution (**52**) is original, the special solution (**53**) was previously derived by Heaviside by his operational method.

If $\frac{R}{L} = \frac{G}{C}$, $b = 0$ and the solution reduces to

$$A_x(t) = 0 \qquad \text{for } y < x$$

$$= \sqrt{\frac{C}{L}}\, \epsilon^{-ax} \qquad \text{for } y \geq x \qquad (\mathbf{54})$$

which is the solution for the *distortionless* line.

If $R = G = 0$, $a = b = 0$ and

$$A_x(t) = 0 \qquad \text{for } y < x$$

$$= \sqrt{\frac{C}{L}} \qquad \text{for } y > x \qquad (\mathbf{55})$$

---

*See Note 8.

which is the solution for the non-dissipative line.

In all cases the current is propagated with a true finite velocity $v = \frac{1}{\sqrt{LC}}$ and is identically zero until $t = \frac{x}{v}$. When we come to examine the solution for the artificial line we shall see that there is no true velocity of propagation in an exact sense, but that there is a *virtual* velocity $\frac{1}{\sqrt{LC}}$ just as in the corresponding smooth line.

If $L = 0$, the solution is derivable from (**52**) by the method of limits. It is

$$A_x(t) = \sqrt{\frac{C}{\pi R}} \cdot \frac{\epsilon^{-\frac{RCx^2}{4t}}}{\sqrt{t}} \cdot \epsilon^{-\frac{G}{C}t}$$

$$+ \frac{G}{C}\sqrt{\frac{C}{\pi R}} \int_0^t \epsilon^{-\frac{G}{C}t_1} \frac{\epsilon^{-\frac{RCx^2}{4t_1}}}{\sqrt{t_1}}\, dt_1 \qquad (\mathbf{56})$$

which is the solution for the *leaky cable.* Observe that here we have no finite velocity of propagation owing to the absence of inductance.

If the leakage G as well as the inductance L is zero, we get

$$A_x(t) = \sqrt{\frac{C}{\pi R}} \; \frac{\epsilon^{-\frac{RCx^2}{4t}}}{\sqrt{t}} \qquad (\mathbf{57})$$

which is the Kelvin-Heaviside solution.

IV A. Solution for the Voltage Formulas for Smooth Line

Of almost equal importance and interest are the formulas for the voltage existing at any point x and time t on a smooth line of infinite length when a unit voltage is applied to the line at $x = 0$ and $t = 0$. The corresponding formulas for the artificial line are deducible from the current or indicial admittance formulas, but they are of complicated form and their consideration is omitted for the present.

We denote by $V_x(t)$ the voltage at point x and time t when a unit voltage is applied to the line at $x = t = 0$, and start with the differential equation of the line,

$$C \frac{d}{dt} V + G V = - \frac{d}{d x} I$$

where V and I are perfectly general. This equation may be integrated in the form:

$$V = - \frac{1}{C} \quad \epsilon^{-(\rho-\sigma)\,t} \int_0^t \epsilon^{(\rho-\sigma)\,\lambda} \cdot \quad \frac{d}{d x} I\,(\lambda) \cdot d\,\lambda$$

If $I\,(t)$ is the current due to unit d-c. voltage applied at $x = t = 0$ it is of course $A_x\,(t)$ as given by formula (**52**) and the corresponding voltage is $V_x\,(t)$. Substituting $A_x\,(\lambda)$ as given by (**52**) for $I\,(\lambda)$ in the integral and paying due regard to the discontinuous character of $A_x\,(\lambda)$ we get:

$$\begin{aligned} V_x\,(t) &= 0 \qquad \text{for } v\,t < x \\ &= \epsilon^{-ax} \\ &\quad + b^2\,x \int_x^y \frac{\epsilon^{-ay_1} I_1\,(b\,\sqrt{y_1^2 - x^2})}{b\,\sqrt{y_1^2 - x^2}}\,dy_1 \end{aligned} \tag{58}$$

which is the required solution. $I_1\,(b\,\sqrt{y^2 - x^2})$ is the Bessel function of order unity and pure imaginary argument $i\,b\,\sqrt{y^2 - x^2}$ tables of which exist and whose computation is quite simple.

If leakage is absent, $G = 0$ and $b = a$ in the above formula. In this case the solution may be conveniently written,

$$\begin{aligned} V_x\,(t) &= 0 \qquad \text{for } T < A \\ &= \epsilon^{-A} + A \int_A^T \frac{\epsilon^{-T_1} \cdot I_1\,(\sqrt{T_1^2 - A^2})}{\sqrt{T_1^2 - A^2}}\,d\,T_1 \end{aligned} \tag{59}$$

where

$$A = \frac{R}{2} \sqrt{\frac{C}{L}}\,x$$

$$T = \frac{R}{2\,L}\,t = \rho\,t$$

If $L = 0$ but the other three line constants are finite, the formula becomes, by approaching the limit $L = 0$,

$$V_x\,(t) = x\,\sqrt{\frac{R\,C}{4\,\pi}} \int_0^t \frac{\epsilon^{-\frac{G}{C}t_1} \cdot \epsilon^{-\frac{R\,C\,x^2}{4\,t_1}}}{t_1^{3/2}}\,d\,t_1 \tag{60}$$

which is reducible to the form

$$V_x(t) = \epsilon^{-\alpha x} - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\frac{R C x^2}{4 t}}} \epsilon^{-\left(\lambda^2 + \frac{\alpha^2}{4 \lambda^2}\right)} \cdot d\lambda \qquad (61)$$

where $\alpha = \sqrt{R G}$

These are the formulas for the *leaky cable.*

If finally both G and L are zero

$$V_x(t) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\frac{R C x^2}{4 t}}} \epsilon^{-\lambda^2} \cdot d\lambda \qquad (62)$$

a formula which was first derived by Heaviside.

V. Approximate Solution for Loaded Cable

The loaded cable is defined as a non-inductive, leakageless line ($L = G = 0$) with load coils of pure inductance L inserted in series with the lines at periodic intervals. The exact solution for this structure has not yet been worked out, as it presents a problem of enormous complexity incapable of treatment by the direct methods employed in dealing with the smooth and artificial lines in the preceding section of this paper. The writer has applied a functional method of treatment, involving integral equations, to the problem which gives good promise of yielding a formal solution. This investigation has, however, not been completed, and as yet gives little hope of resulting in a practical working solution.

Fortunately, however, the close relationships obtaining between the loaded cable and the corresponding smooth and artificial lines has enabled me to arrive at a quasi-empirical solution, which very closely represents the facts, although it is not functionally exact.

To arrive at this solution, we remark that physically the loaded cable, as defined above, is intermediate between the corresponding smooth and artificial lines, and its solution must to a certain extent share the properties of the known solutions of the corresponding lines. More exactly, the required solution must satisfy the following conditions:

1. It must be asymptotic to the solution of the correspond-

ing smooth line as t approaches infinity. That is, it must approach formula (**53**) and ultimately (**57**) as t approaches infinity.

2. It must degenerate into formula (**35**) for the non-dissipative artificial line as the distributed resistance of the cable approaches zero.
3. It must degenerate into formula (**57**) for the smooth cable as the load coil inductance L approaches zero.
4. It must degenerate into formula (**53**) for the smooth line as the load coil inductance L and the distributed resistance R and capacity C per section all approach zero while the number of sections approach infinity.
5. It must give the correct virtual velocity of propagation of the structure and correctly formulate the steady state characteristics in accordance with formula (**13**).

Such a function is the following:

$$\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\cdot I_0\left(\sqrt{(\rho t)^2-A^2}\right)\cdot\frac{\epsilon^{hA}}{\operatorname{sech}\left(\frac{hA}{2q}\right)}\int_0^{2vt}\epsilon^{-\lambda\cdot\sinh\left(\frac{hA}{2q}\right)}\cdot J_{2q}(\lambda)\cdot d\lambda \qquad \mathbf{(63)}$$

Where in accordance with previous notation

$$\rho=\frac{R}{2L}\text{ and }\quad v=\frac{1}{\sqrt{LC}}\quad\text{ and }\quad A=\frac{Rq}{2}\sqrt{\frac{C}{L}}$$

This is the required quasi-empirical formula for the current in the q^{th} section or load coil of a loaded cable in response to a unit d-c. voltage applied at mid-load position at time $t=0$. R, C, and L, are the constants *per section* of the structure. The first two being distributed and the last concentrated in the load coil. For values of R. L. and C. encountered in practise: formula (**62**) may be replaced by

$$\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\cdot I_0\left(\sqrt{(\rho t)^2-A^2}\right)\cdot\epsilon^{hA}\cdot\int_0^{2vt}\epsilon^{-\frac{hA}{2q}\lambda}J_{2q}(\lambda)\cdot d\lambda \qquad \mathbf{(64)}$$

The symbol h which disappears in the first four limiting cases listed above, is a numeric. When assigned the value $\frac{1}{2}$, formula (**62**) gives the steady state characteristics with a good degree of approximation.

The foregoing formulas, while not functionally exact, and which are indeed, empirical, represent the phenomena with sufficient exactness for engineering purposes and have been verified as closely as could be expected by oscillographic tests. The physical character of the solution will be discussed and illustrated by curves in the succeeding section.

VI. Numerical Solution of Formulas and Discussion of Curves

By virtue of the mathematical analysis of the preceding sections we are now in possession of a fairly systematic formulation of the general problem, and in particular of the formulas which determine the progress and distortion of electric waves in transmission systems. While the phenomena are reduced to their simplest possible formulation they are still so complex and the line constants are involved in such a complicated manner that an understanding of the inner meaning of the solutions requires that analysis be superseded at this stage by a systematic and comprehensive program of computation and study of the resulting numerical solutions, when plotted in the form of curves. Such a systematic study is beyond the scope of the present paper and would indeed occupy a whole treatise in itself. We shall, however, exhibit a representative series of numerical solutions in the form of curves, and briefly discuss the physical character of the phenomena which they represent.

Restricting attention to the line formulas (32)-(64,) their evaluation depends on the numerical computation of Bessel functions and the integration of definite integrals. The latter operation presents no difficulties and the integrations are easily and accurately effected, once the integrands are computed and plotted as time functions, by means of a Coradi integraph which traces the progressive values of the integrals. Fortunately, also, the Bessel functions, which are of such great importance in so many branches of mathematical physics, have been the subject of exhaustive study and tabulation. While no tabulations exist for the high orders of functions which we require, their computation is effected without any serious difficulty from the asymptotic formulas of Nicholson.

We shall first consider the artificial line formulas (32)-(36) and since one of the prime objects of our investigation is an examination of the correspondence existing between the artificial and smooth line, the corresponding functions for the smooth line are plotted to the same scale on the same sheet. We shall have frequent occasion in discussing the artificial and the periodically loaded lines to refer to the "corresponding smooth line," which we define as the line having continuously distributed constants of the same aggregate value as the lumped constants of the artificial line. Thus if R, L, C, G, denote the constants *per section* of the artificial or periodically loaded line and the current in the qth section is under consideration, the corresponding smooth line has constants R, L, C, G per *unit length* and the current at point $x = q$ is the corresponding current.

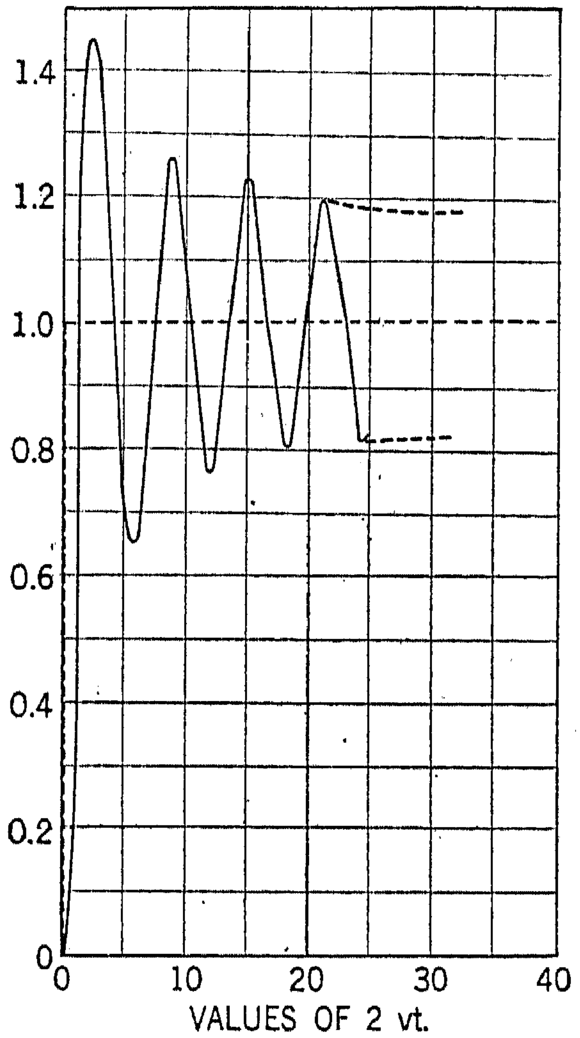


FIG. 1—[COMPUTED FROM FORMULA (35)] CURRENT ENTERING LOADED CABLE

Non-dissipative: $R = G = O$. Multiply ordinates by $\sqrt{C/L}$

Figs. 1, 2, 3 and 4 are graphs of formula (**35**) for $q = 0$, 200, 1000 and 2000 respectively and thus represent the current in the initial, the 100th, the 500th and the 1000th sections of the non-dissipative artificial line or periodically loaded cable. It will be understood that the line is infinitely long and that a unit d-c. voltage applied to the line at mid-load position and at time $t = 0$ is assumed. A study of the non-dissipative line, which of course is ideal and unrealizable, is nevertheless of practical importance because in this type of line the effect of the discontinuous loading of the periodically loaded cable is isolated and thereby exhibited in the clearest possible manner.

The dotted curves represent the current in the corresponding smooth line as computed from formula (**55**). For the smooth line, the current, as we see, is discontinuous, being identically zero for a time $v\,t = q$ and having an instantaneous jump to its final value $\sqrt{\frac{C}{L}}$ at $v\,t = q$. The current in the artificial line differs from that in the corresponding smooth line in three important respects; (1) the absence of the abrupt discontinuous wave front; (2) the presence of superposed oscillations and (**3**) *the absence of a true finite velocity of propagation.* It will be observed, however, that the current in any section is negligibly small or even sensibly zero until $v\,t = q$, so that the current is propagated with a *virtual* velocity $\frac{1}{\sqrt{L\,C}}$ per section. The presence of a well marked wave front is also evident although this is not abrupt, as in the smooth line. The effective slope of the wave front becomes smaller as the current wave travels out on the line decreasing noticeably as the number of sections is increased. When the number of sections becomes large,

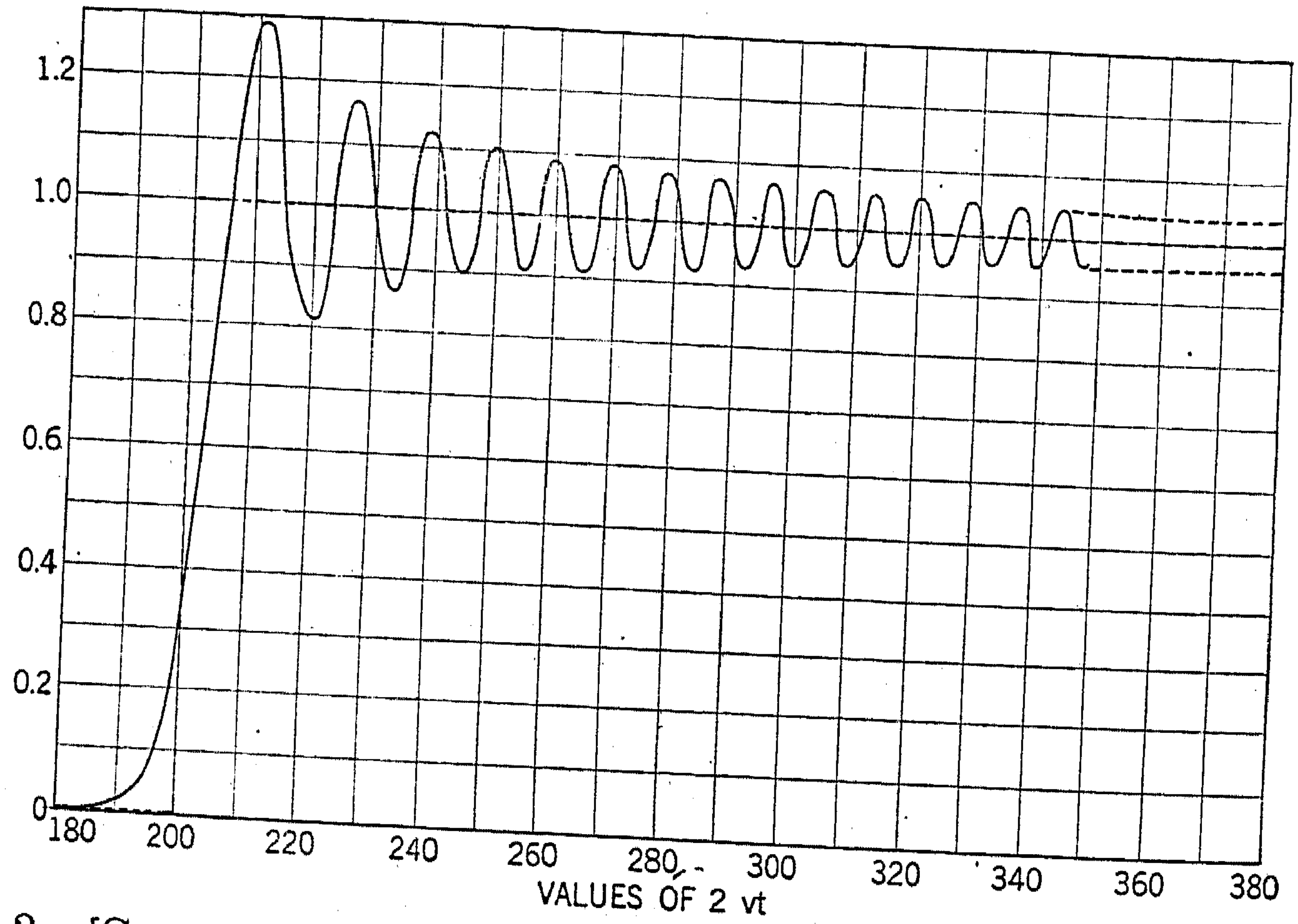


FIG. 2—[COMPUTED FROM FORMULA (35)] CURRENT IN 100TH SECTION OF ARTIFICIAL LINE

Non-dissipative: $R = G = 0$. Multiply ordinates by $\sqrt{C/L}$

however, the decrease in the slope is not rapid, being in the 500th section about 60 per cent of that in the 100th section.

The superposed oscillations are of interest. These are initially of a frequency depending upon and decreasing with the number of sections, q, but in all sections ultimately attaining the frequency $\frac{1}{\pi\sqrt{LC}} = \frac{v}{\pi}$ which is the critical or cut-off frequency of the line, above which steady-state currents are attenuated during transmission and below which they are unattenuated. When $v\,t$ is large compared with q the amplitude of these oscillations becomes $\sqrt{\frac{1}{\pi v t}}$ so that they ultimately die away and the current approaches the value $\sqrt{\frac{C}{L}}$ for all sections. The current in the loaded cable is thus asymptotic to the current in the corresponding smooth line and oscillates about it with diminishing amplitude and increasing frequency.

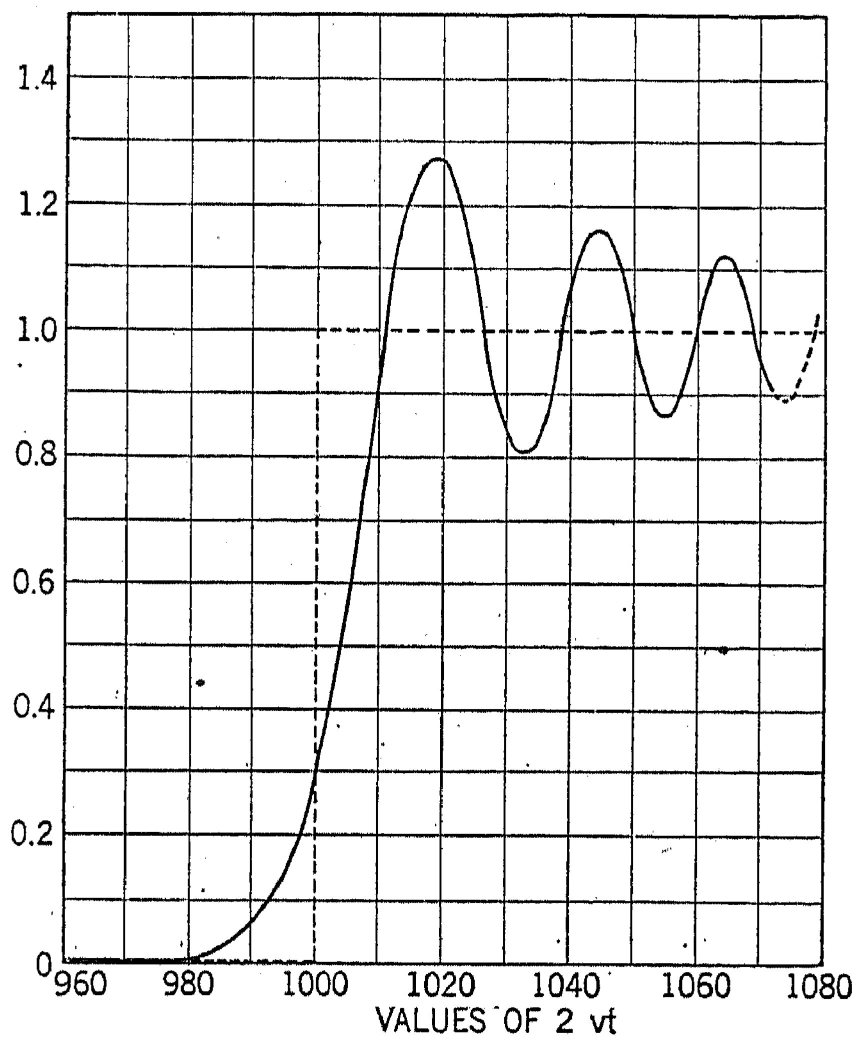


FIG. 3—[COMPUTED FROM FORMULA (35)] CURRENT IN 500TH SECTION OF ARTIFICIAL LINE

Non-dissipative: $R=G=O$. Multiply ordinates by $\sqrt{C/L}$

Since the abscissas of these curves represent values of

$2vt = \dfrac{2}{\sqrt{LC}}\, t$, and the ordinates are to be multiplied by $\sqrt{\dfrac{C}{L}}$ to translate into actual values, the curves are of universal application for all values of the constants L and C.

The additional effect of resistance in the artificial line is exhibited in Figs. 5, 6 and 7 which are computed from formula (**33**) for $q = 100$. They therefore represent the current in the 100th section of an artificial line without leakage for three different values of R/L per section. The current functions (or indicial admittances) for the corresponding smooth lines, computed from formula (**53**) are plotted on the same sheet. Curves 5, 6 and 7 correspond to values of $\dfrac{R}{2}\sqrt{\dfrac{C}{L}}\, q$ of 2.65, 5.29 and 14.75 respectively; $\dfrac{R}{2}\sqrt{\dfrac{C}{L}}\, q$ is the exponential attenuation factor and an examination of the curves shows that, for the smooth line, the discontinuous wave front is attenuated by this factor during transmission. For values of $\dfrac{R}{2}\sqrt{\dfrac{C}{L}}\, q$ greater than two, the tail of the wave exceeds in

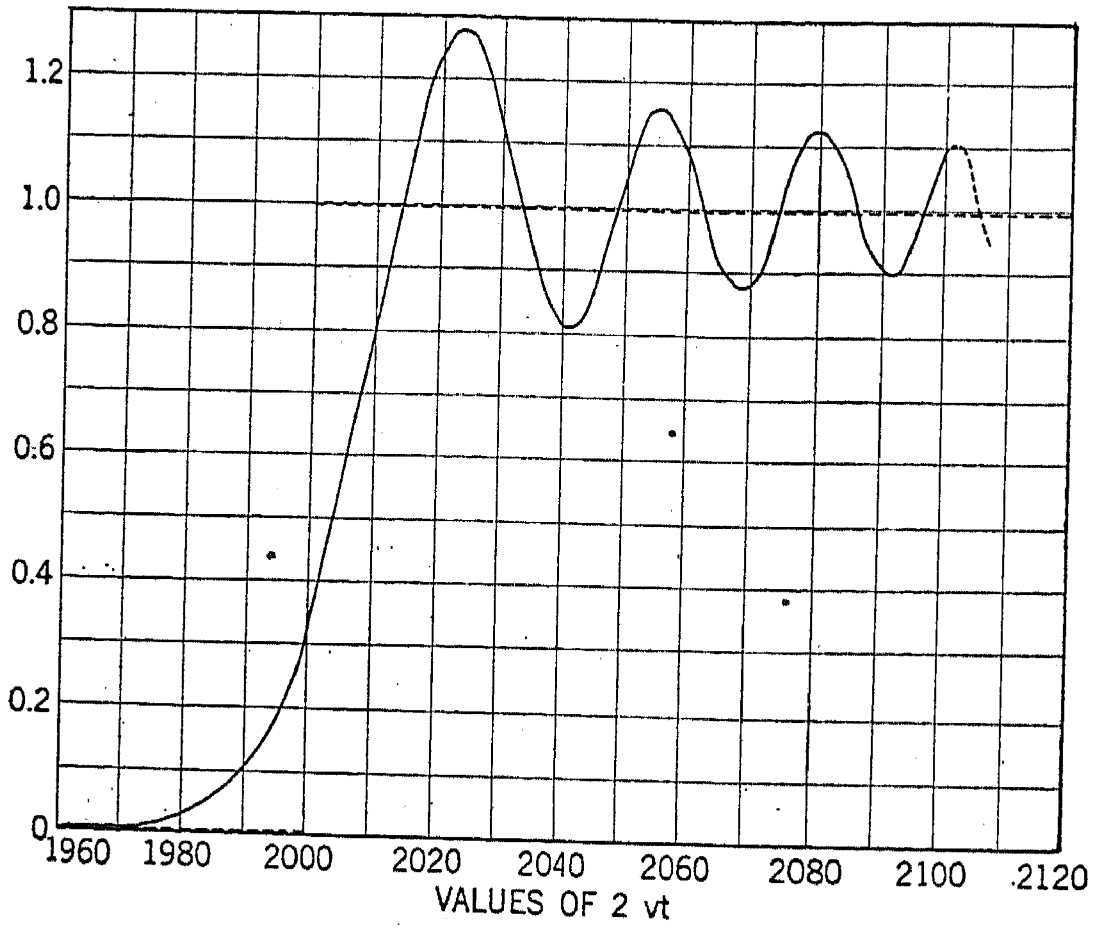


FIG. 4—[COMPUTED FROM FORMULA (35)] CURRENT IN 1000TH SECTION OF ARTIFICIAL LINE

Non-dissipative: $R = G = 0$. Multiply ordinates by $\sqrt{C/L}$

amplitude the head. For large values of $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ (see curve 7) the head of the wave is insignificant compared with the tail. It should be observed that curves 5, 6 and 7 are plotted in a scale to exhibit the character of the wave in the neighborhood of the head and do not show the way the tail builds up to its maximum and ultimately dies away. The tail of the wave builds up more and more slowly as $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ is increased and ultimately dies away in accordance with the law $\sqrt{\frac{C}{\pi R t}}\cdot\epsilon^{-\frac{RC}{4t}q^2}$. This will be recognized as the non-inductive cable formula (57) so that the line ultimately behaves like the cable to d-c. impressed voltages. Its behavior to alternating voltages is entirely different, however, as may be inferred from the difference in the curves in the neighborhood of the wave head.

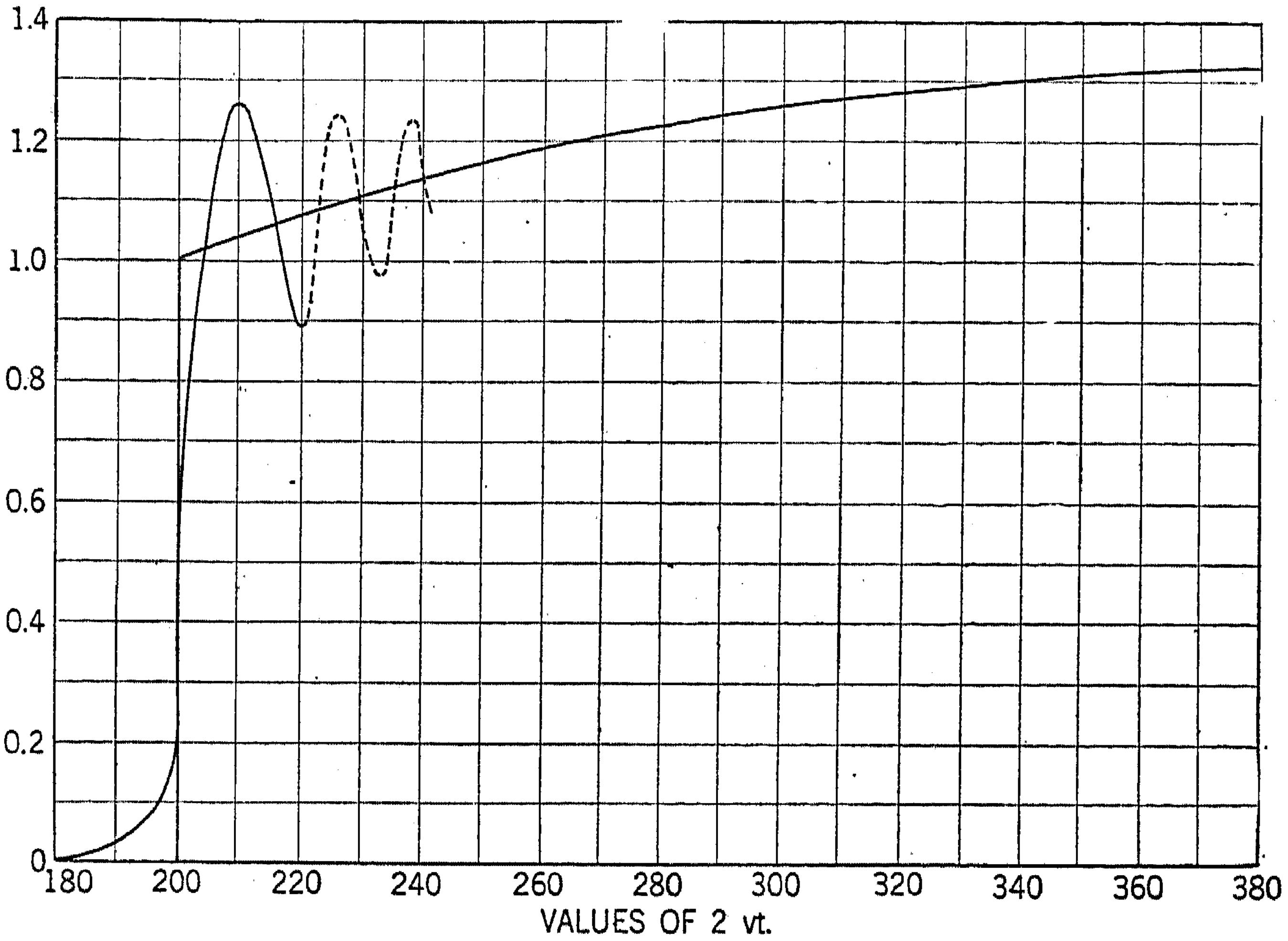


FIG. 5—[COMPUTED FROM FORMULA (33)] CURRENT IN 100TH SECTION OF ARTIFICIAL LINE

Line constants: $R/2\sqrt{C/L}=0.02646$. Multiply ordinates by $\epsilon^{-2.646\sqrt{C/L}}$

$G=0$

As regards the artificial line curves, we see that, precisely as in curves 1, 2, 3 and 4, they oscillate about the corresponding

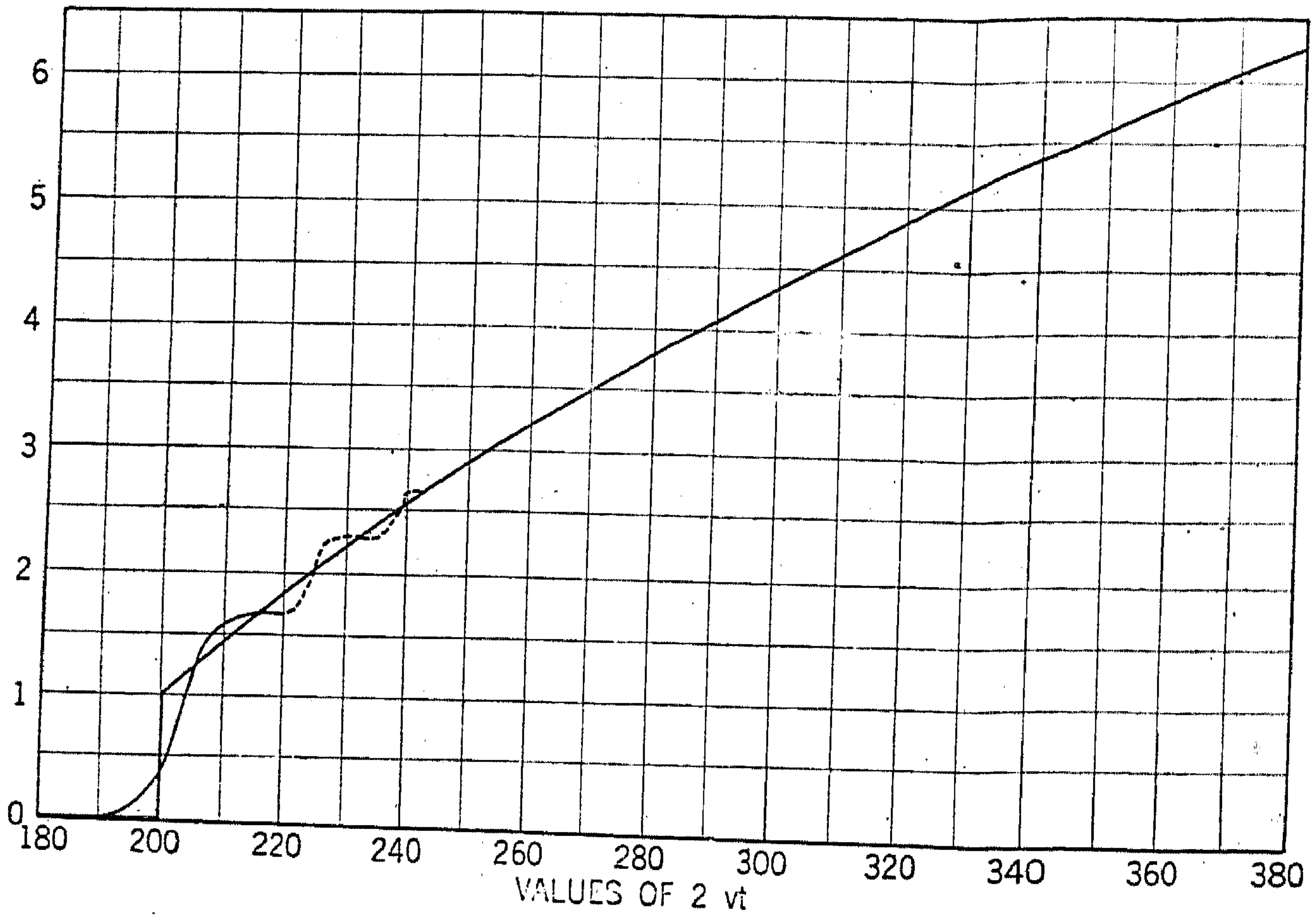


FIG. 6—[COMPUTED FROM FORMULA (33)] CURRENT IN 100TH SECTION OF ARTIFICIAL LINE

Line constants: $R/2\sqrt{C/L}=0.05291$. Multiply ordinates by $e^{-5.291}\sqrt{C/L}$

$G=0$

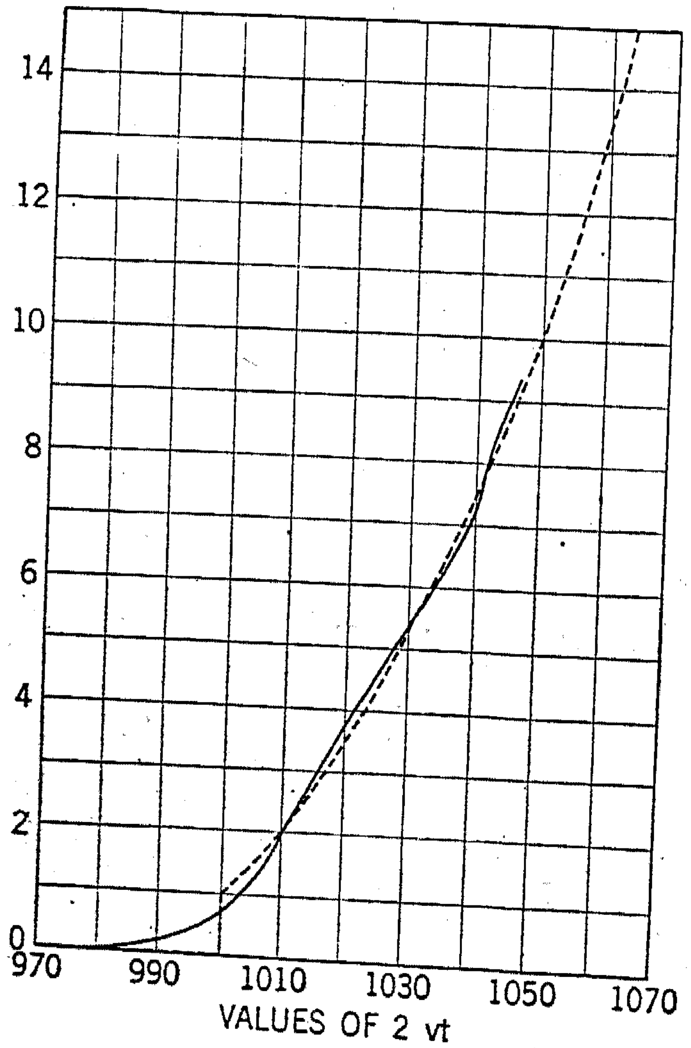


FIG. 7—[COMPUTED FROM FORMULA (33)] CURRENT IN 500TH SECTION OF ARTIFICIAL LINE

Line constants: $R/2\sqrt{C/L}=0.0295$. Multiply ordinates by $e^{-14.75}\sqrt{C/L}$

$G=0$

smooth line curves and ultimately coalesce therewith. They are distinguished, also, by the absence of the abrupt wave head and the presence of the superposed oscillations. The same virtual velocity per section $\frac{1}{\sqrt{L C}}$ exists independently of the resistance but the superposed oscillations diminish in relative amplitude as $\frac{R}{2} \sqrt{\frac{C}{L}}\, q$ is increased, as is clear from a comparison of curves 5, 6 and 7.

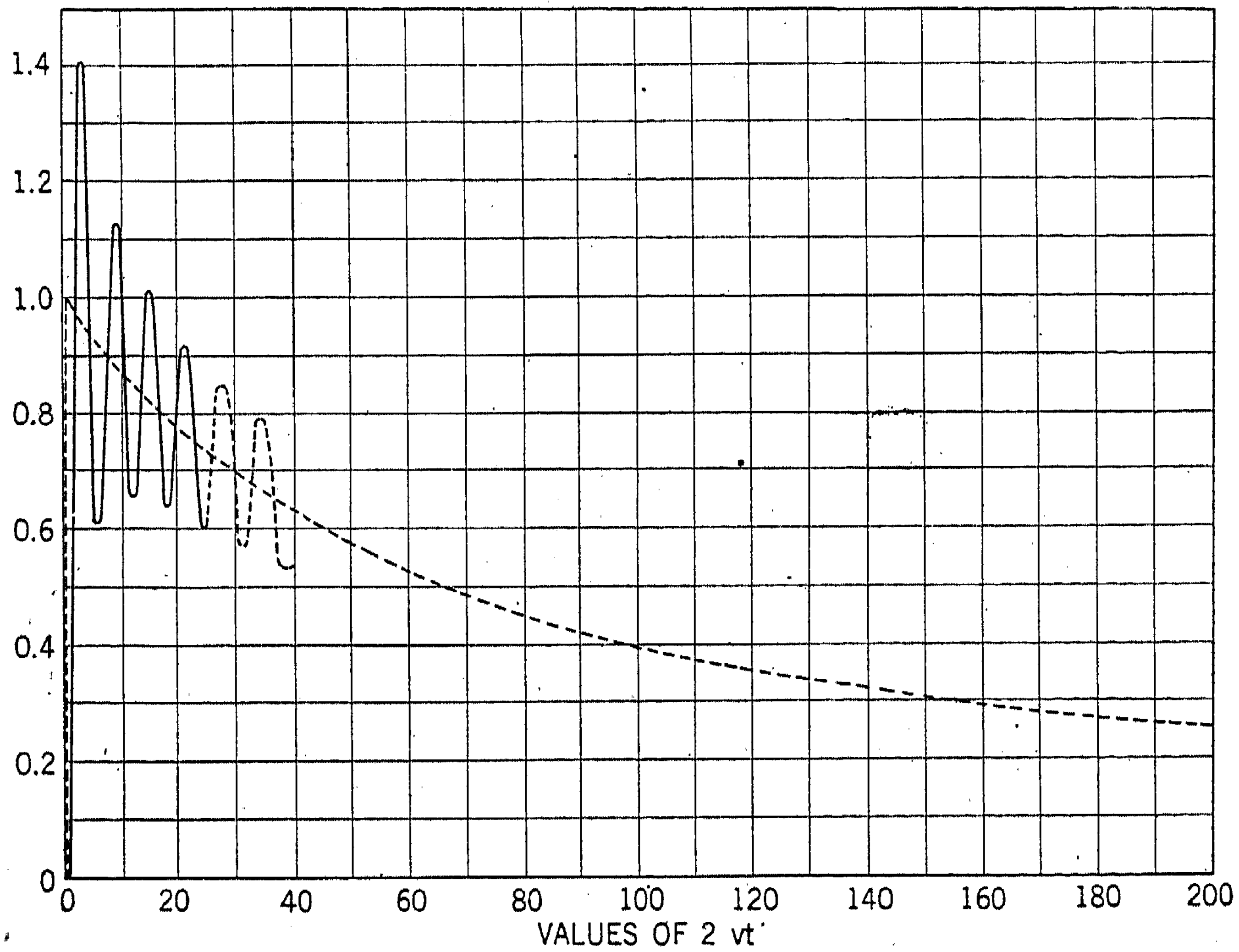


FIG. 8—[COMPUTED FROM FORMULA (62)] CURRENT ENTERING LOADED CABLE

Cable constants: $R/2 \sqrt{C/L} = 0.02646$. Multiply ordinates by $\sqrt{C/L}$
$G = 0$

Some typical loaded cable curves computed from formula (**63**) are shown in Figs. 8, 9, 10 and 11. Curves 8 and 9 are plots of the current entering the cable (corresponding to a unit d-c. voltage impressed at mid-load position) for two different values of $\frac{R}{2} \sqrt{\frac{C}{L}}$ (0.01323 and 0.02646 respectively) while curves 10 and 11 show the current in the 100th load coil for two different values of $\frac{R}{2} \sqrt{\frac{C}{L}}\, q$ (2.65 and 5.291 respectively). The corresponding smooth line current curves are shown in dotted lines. The difference between the curves for the loaded cable and the corresponding artificial line is quite small except for fairly large values of $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ (greater than three) and consists in a slower damping out of the superposed oscillations. This is precisely the kind of effect we should expect when we recall the fact that the attenuation of alternating currents for large values of $\frac{R}{2}\sqrt{\frac{C}{L}}\,q$ is much smaller in loaded cables than in artificial lines. At the same time it must be remembered that the loaded cable under consideration is ideal in that the loading coils are assumed as pure inductances without resistance. In practise the resistance of the loading coils would modify the current curves, which are actually (as has been shown by oscillographic tests) intermediate in character between the theoretical curves for the artificial line and the ideal periodically loaded cable.

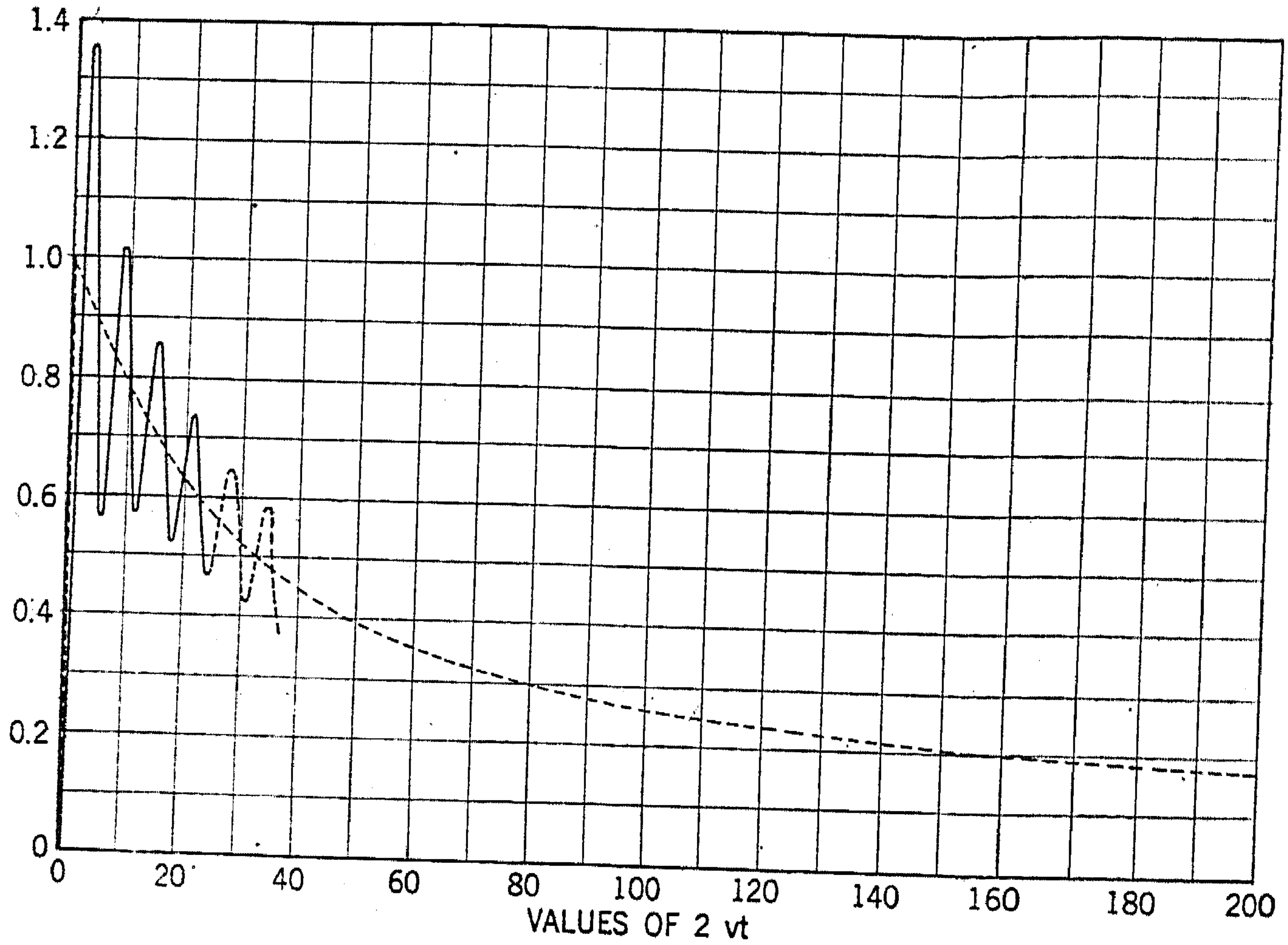


FIG. 9—[COMPUTED FROM FORMULA (62)] CURRENT ENTERING LOAD CABLE

Cable constants: $R/2\ \sqrt{C/L}=0.529$. Multiply ordinates by $\sqrt{C/L}$

$G=0$

The effect of leakage in the artificial line (when so proportioned as to make $\frac{G}{C}=\frac{R}{L}$) is shown in Figs. 12 and 13

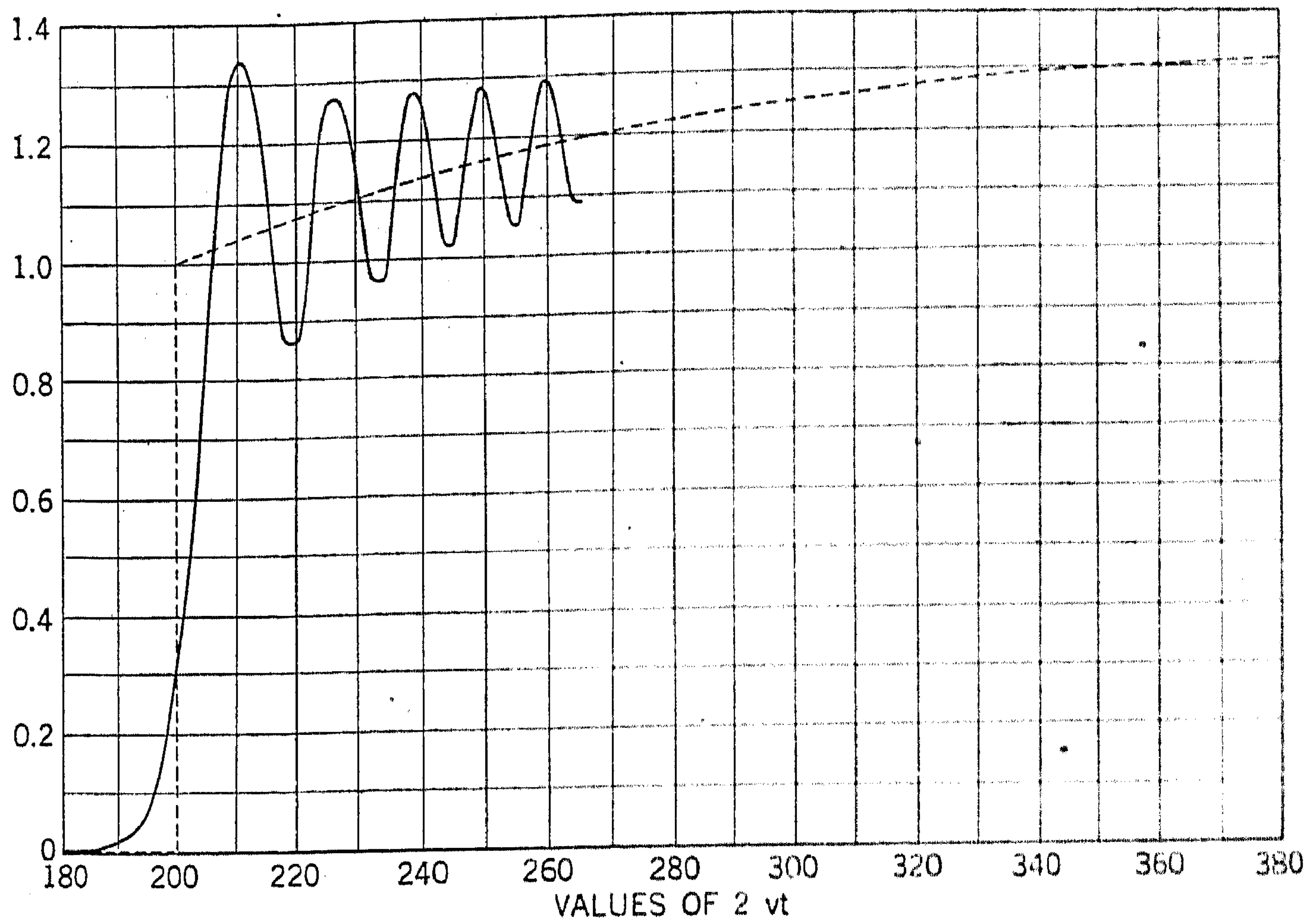


FIG. 10—[COMPUTED FROM FORMULA (162)] CURRENT IN 100TH SECTION OF LOADED CABLE

Cable constants: $R/2\ \sqrt{C/L}=0.02646$. Multiply ordinates by $e^{-2.646\sqrt{C/L}}$
$G=0$

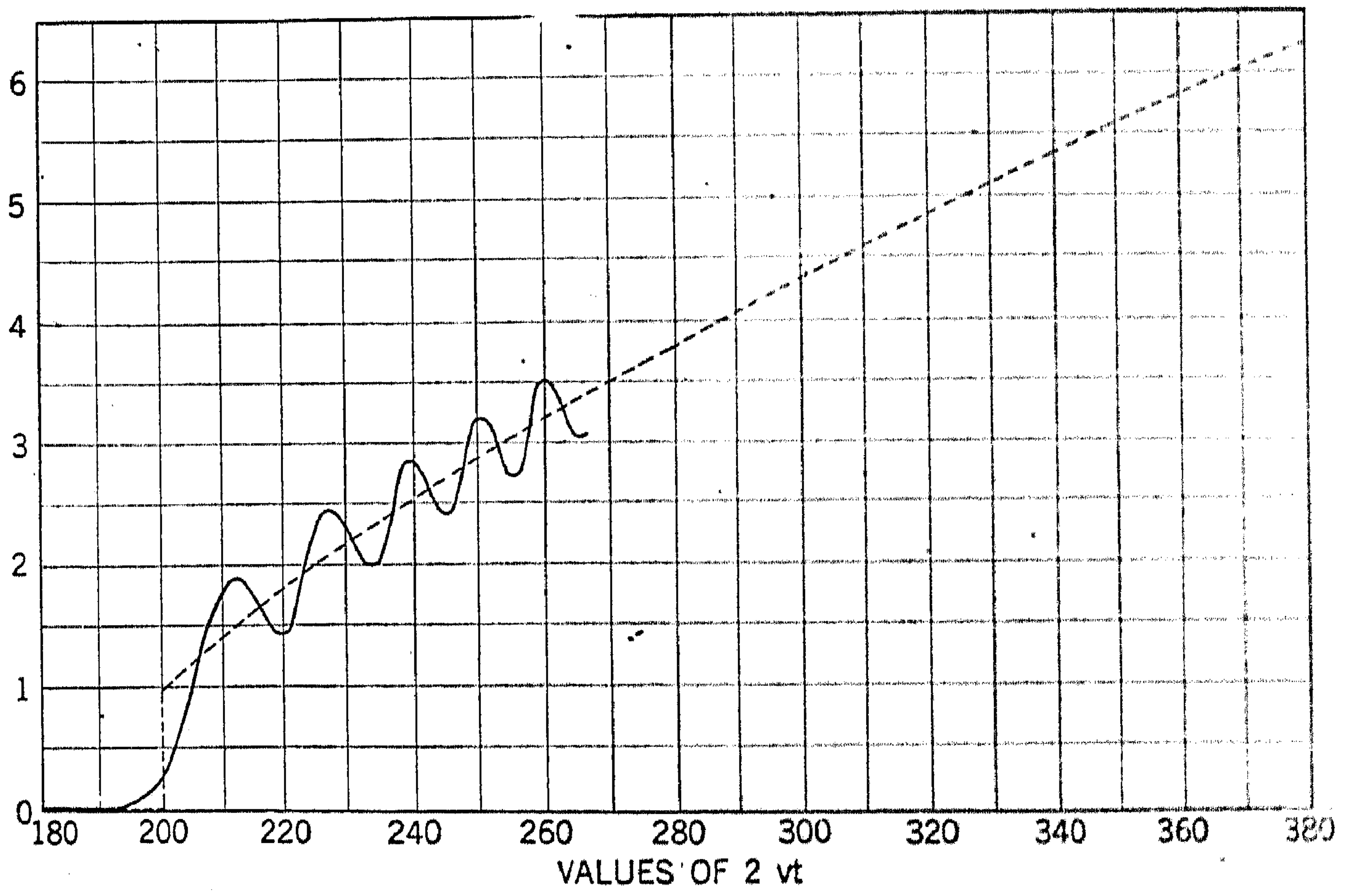


FIG. 11—[COMPUTED FROM FORMULA (62)] CURRENT IN 100TH COIL OF LOADED CABLE

Cable constants: $R/2\ \sqrt{C/L}=0.05291$. Multiply ordinates by $e^{-5.291\ \sqrt{C/L}}$
$G=0$

which are computed from formula (**34**) for the quasi-distortionless artificial line. Curve 12 exhibits the current in the 100th and curve 13 in the 500th section. The corresponding smooth lines are, of course, exactly distortionless. The artificial lines while not strictly distortionless, are characterized, as compared with the same lines without leakage, by the reduction of the tail of the wave to the same magnitude as the head, and a rapid damping out of the superposed oscillations.

The effect of leakage on the smooth line curve is exhibited in Fig. 14 which is computed from formula (**52**). The final direct-current value is indicated by the dotted line. Comparison of Fig. 14 with preceding curves shows the effect of the leakage in reducing the tail and relatively enlarging the head of the wave.

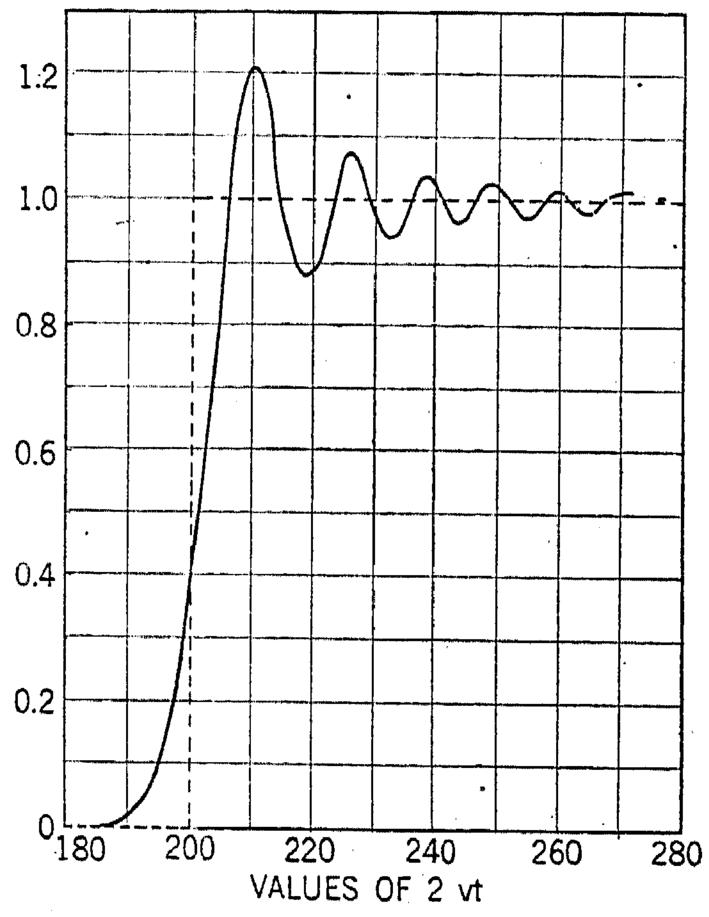


FIG. 12—[COMPUTED FROM FORMULA (34)] CURRENT IN 100TH SECTION OF ARTIFICIAL LINE

Line constants: $R/2\sqrt{C/L}=0.02646$. Multiply ordinates by $\epsilon^{-5.291}\sqrt{C/L}$

$R/L=G/C$

The curves of Figs. 15 and 16, computed from formulas (**53**) and (**57**) respectively are plots of the current waves in the smooth line of attenuation $A = 10$, and in the non-inductive cable of the same total resistance and capacity. The curves are plotted to the same argument $\frac{4\,t}{R\,C \times 2}$ in order to facilitate comparison. Referring to Fig. 15, the large value of the attenuation makes the discontinuous head of the wave quite negligible in comparison with the tail, and it will be seen that the head of the wave is not distinguishable on the scale to which the ordinates are drawn. In fact for lines of this or greater order of attenuation the inductance plays a quite minor role when the applied voltage is uniform except that its presence results in a finite velocity of propaga-

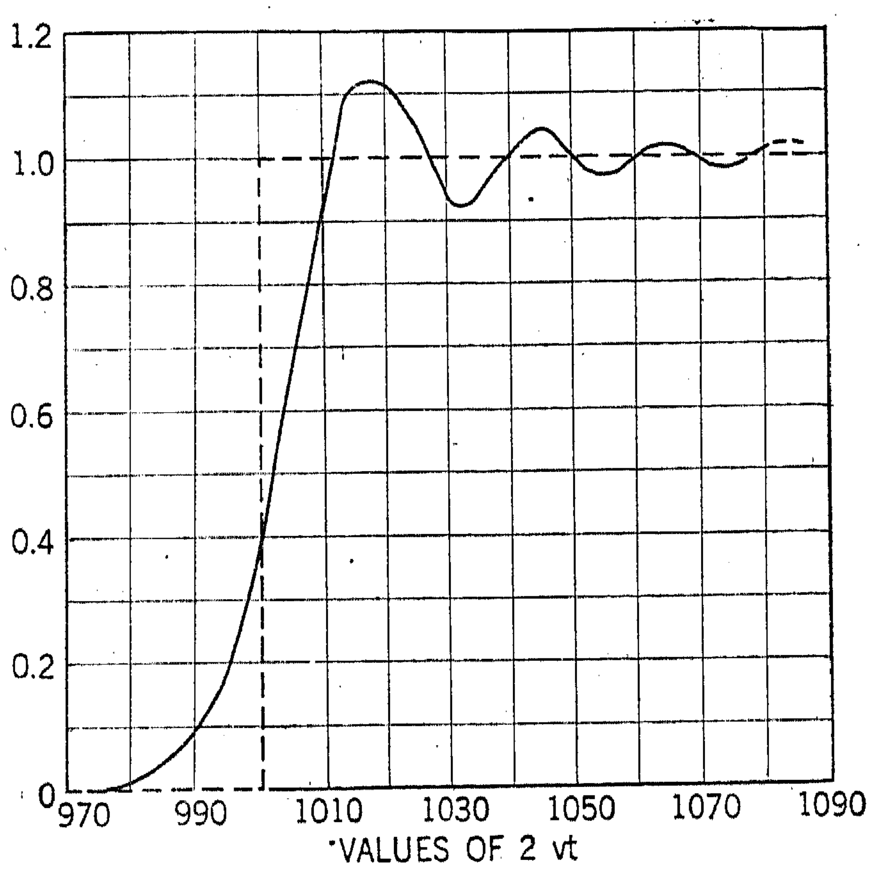


FIG. 13—[COMPUTED FROM FORMULA (34)] CURRENT IN 500TH SECTION OF ARTIFICIAL LINE

Line constants: $R/2 \sqrt{C/L} = 0.02646$. Multiply ordinates by $\epsilon^{-26.46\sqrt{C/L}}$

$R/L = G/C$

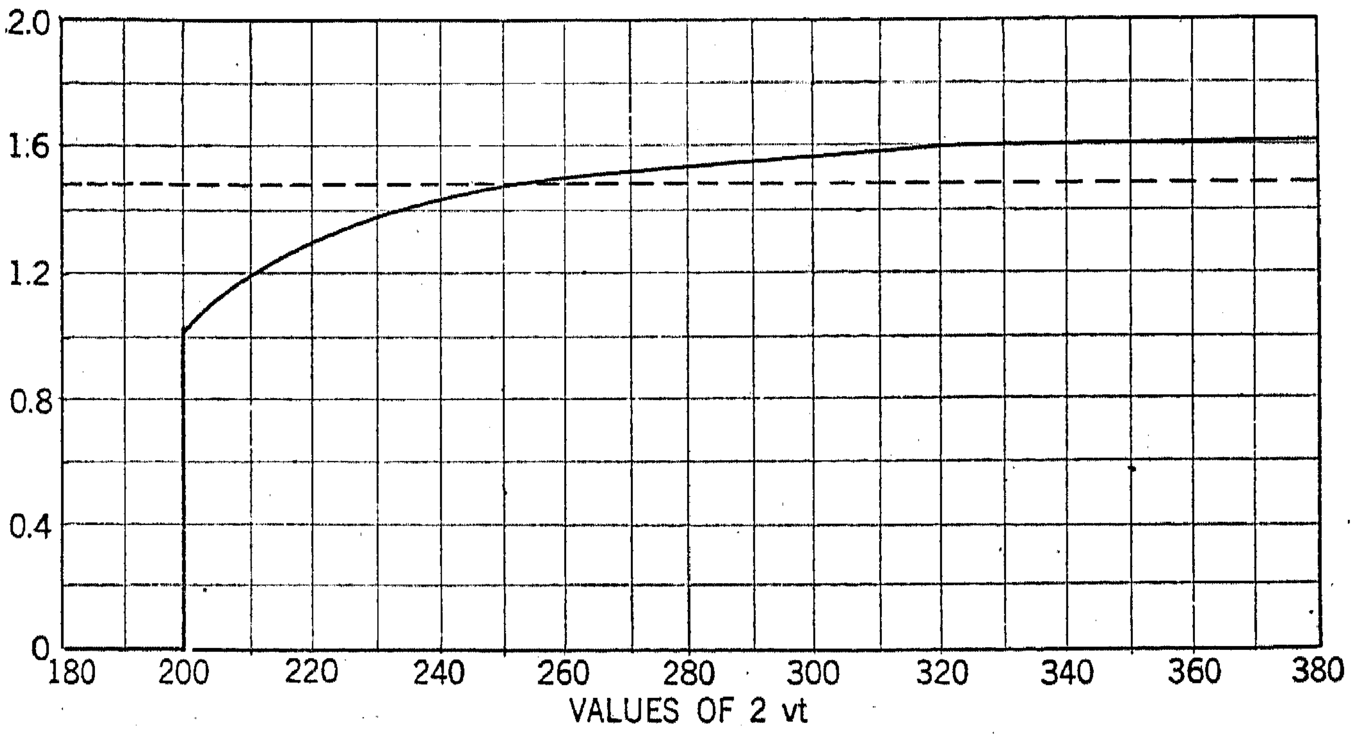


FIG. 14—[COMPUTED FROM FORMULA (52)] CURRENT AT POINT $X = 100$ OF CONTINUOUSLY LOADED LINE

Line constants: $R/2 \sqrt{C/L} + G/2 \sqrt{L/C} = 0.0706$. Multiply ordinates by $\epsilon^{-7.06\sqrt{C/L}}$

$R/2 \sqrt{C/L} - G/2 \sqrt{L/C} = 0.0353$

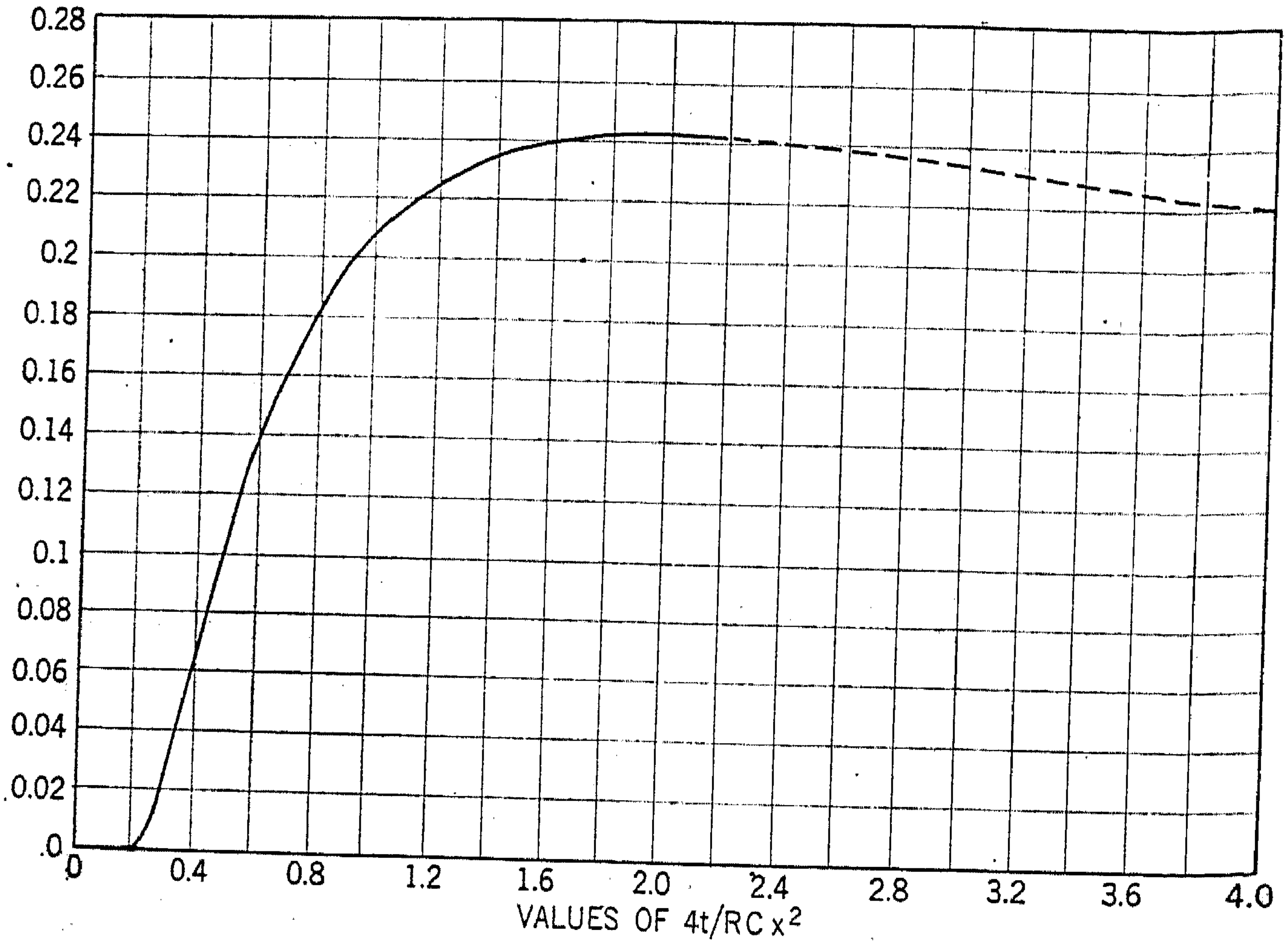


FIG. 15—[COMPUTED FROM FORMULA (53)] CURRENT AT POINT X OF SMOOTH LINE

Line constants: $R/2\ \sqrt{C/L}\ X = 10$ Multiply ordinates by $2/RX$

$G = 0$

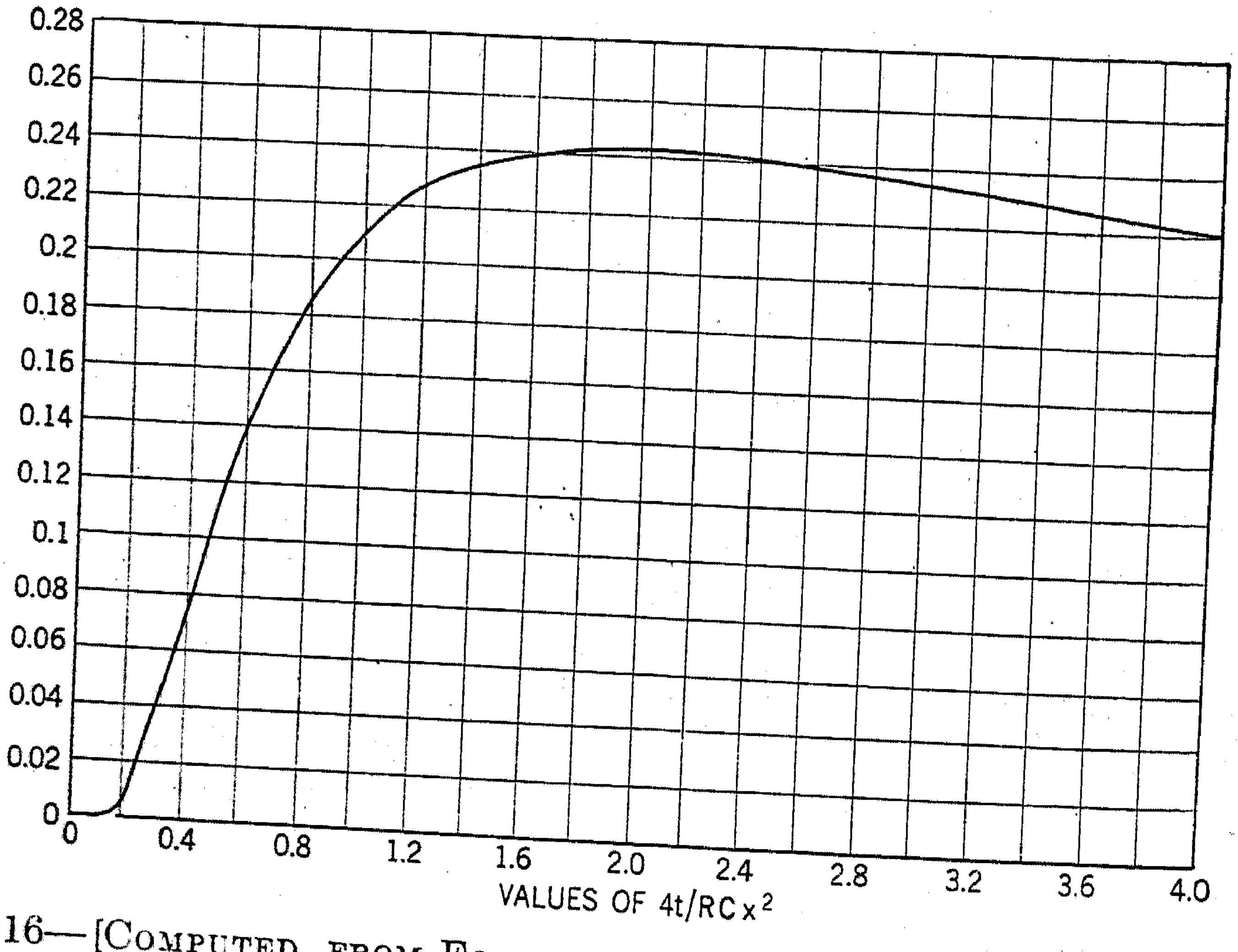


FIG. 16—[COMPUTED FROM FORMULA (57)] CURRENT AT POINT X O SMOOTH CABLE (R, C)

Multiply ordinates by $2/RX$

tion, whereas in the non-inductive cable velocity is infinite. Otherwise the current waves, as may be seen by comparison of Figs. 15 and 16, are almost identical.

The artificial cable behaves very closely like the corresponding smooth non-inductive cable if the number of sections is not too small. This may be shown quite simply from formulas (**36**) and (**57**). If we set

$$\frac{4\,t}{R\,C\,x^2} = \frac{4\,t}{R_0\,C_0} = u$$

formula (**57**) becomes

$$\begin{aligned} A_x\,(t) &= \frac{2}{\sqrt{\pi}\,R_0}\;\frac{1}{\sqrt{u}}\,\epsilon^{-1/u} \\ &= \frac{2}{\sqrt{\pi}\,R_0}\cdot\frac{1}{\sqrt{u}} \\ &\quad \left\{ 1 - \frac{1}{1\,!\,u} + \frac{1}{2\,!\,u^2} - \frac{1}{3\,!\,u^3}\;\cdots\cdots \right\} \end{aligned} \qquad \textbf{(a)}$$

Here, of course, R_0 and C_0 denote the total value of the line constants in the length x of the line.

If the corresponding artificial line is made up of q sections, the constants per section are R_0/q and C_0/q and formula (**36**) becomes,

$$A_q\,(t) = \frac{2\,q}{R_0}\,\epsilon^{-\frac{q^2\,u}{2}}\cdot I_q\left(\frac{q^2\,u}{2}\right)$$

Expanding the Bessel function in its asymptotic form:

$$\begin{aligned} A_q\,(t) = \frac{2}{\sqrt{\pi}\,R_0}\;\frac{1}{\sqrt{u}} & \left\{ 1 - \frac{1}{1\,!\,u}\left(1 - \left(\frac{1}{2\,q}\right)^2\right) \right. \\ & + \frac{1}{2\,!\,u^2}\left(1 - \left(\frac{1}{2\,q}\right)^2\right)\left(1 - \left(\frac{3}{2\,q}\right)^2\right) \\ & - \frac{1}{3\,!\,u^3}\left(1 - \left(\frac{1}{2\,q}\right)^2\right)\left(1 - \left(\frac{3}{2\,q}\right)^2\right) \\ & \qquad \left(1 - \left(\frac{5}{2\,q}\right)^2\right)\cdots\cdots \\ & \left. + \;\cdots\cdots\cdots \right. \end{aligned} \qquad \textbf{(b)}$$

Comparison of (**a**) and (**b**) shows that, except for very small values of the argument u the artificial cable simulates the corresponding smooth cable quite closely if q is not too small. The departure between the two formulas decreases very rapidly as either u or q is increased in value. We observe, therefore, that the non-inductive cable admits of mush closer simulation by an artificial structure than does the transmission line which includes inductance as well as resistance and capacity.

The curves of Figs. 17, 18 and 19 exhibit the voltage waves on smooth lines, curves 17 and 18 being computed from formula (**59**) while curve (19) is plotted from formula (**58**) and exhibits the additional effect of leakage. Referring to Figs. 17 and 18 the voltage is identically zero until $T = A$ or $v\,t = x$, and has an instantaneous jump to the value ϵ^{-A}, after which it builds up slowly to its final value, unity. When leakage is present the wave head has likewise an instantaneous jump at $v\,t = x$, and thereafter builds up to its final value (shown by the dotted line in Fig. 19) $\epsilon^{-x\sqrt{RG}}$. The constants corresponding to curve 19 are so chosen as to make the line almost but not quite distortionless, the ratio ρ/σ being 5.

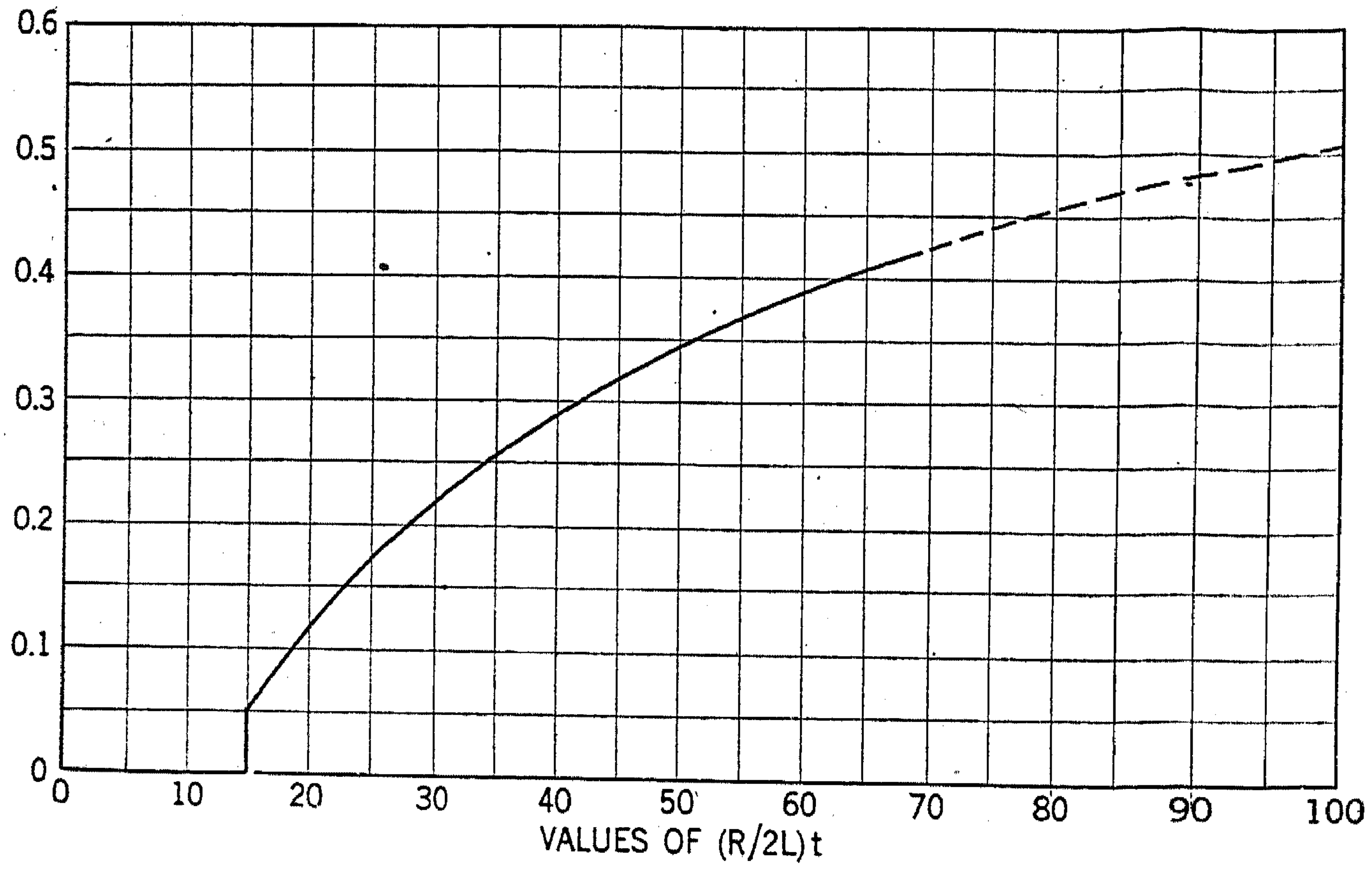


FIG. 17—[COMPUTED FROM FORMULA (59)] VOLTAGE ON CONTINUOUSLY LOADED LINE

Line constants: $R/2\sqrt{C/L}\,X = 3$
$G = 0$

The preceding curves all represent the current or voltage waves in response to a unit d-c. voltage applied to the line. Figs. 20 and 21 show how the current starts to build up when the line is excited by alternating voltages of two different frequencies. The line is the non-dissipative periodically loaded

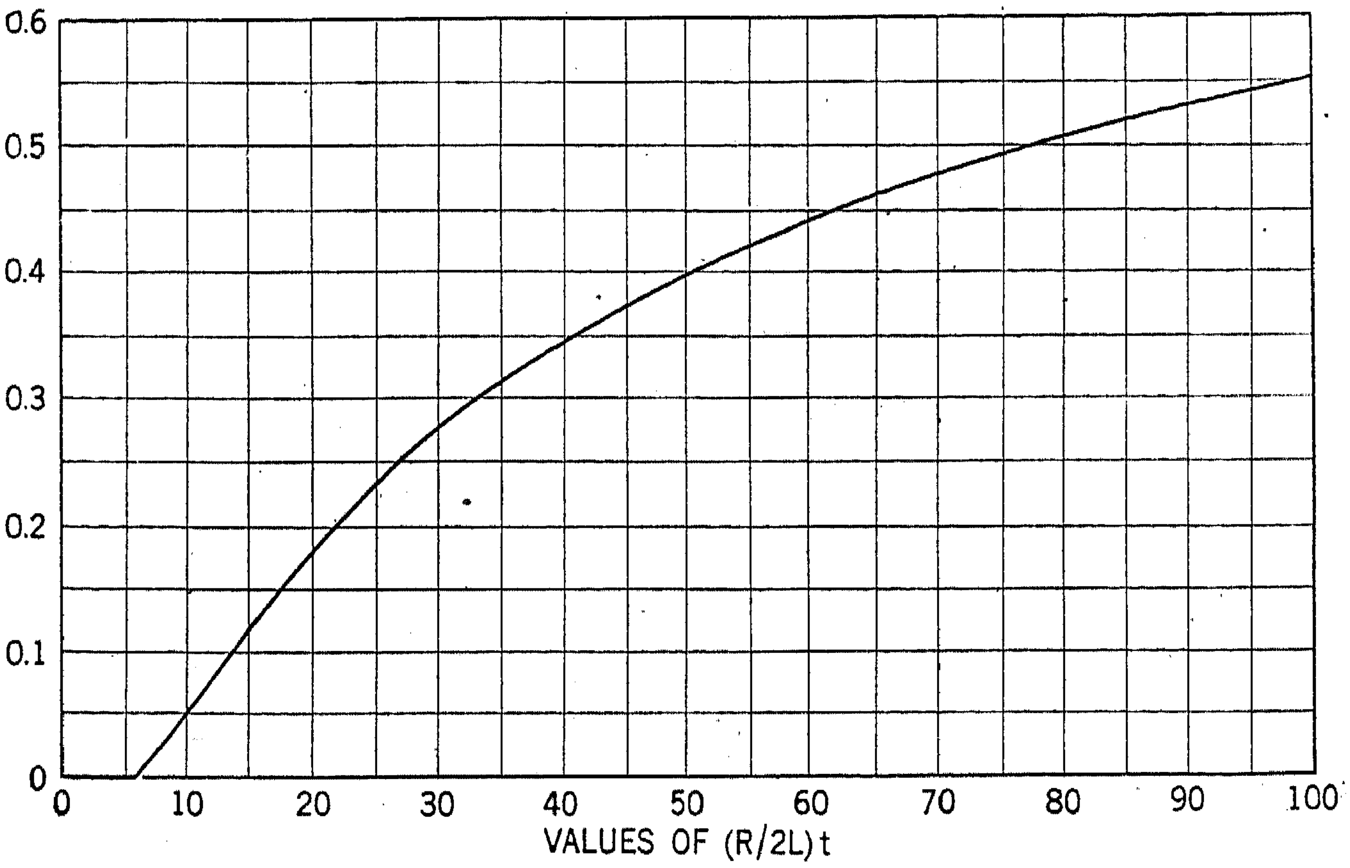


FIG. 18—[COMPUTED FROM FORMULA (59)] VOLTAGE ON CONTINUOUSLY LOADED LINE

Line constants: $R/2\sqrt{C/L}\,X=6$
$G=0$

cable and the current is that existing in the 1000th section. Observe that in Fig. 21 the current has not yet attained ten per cent of its final steady state value, so that in this case the

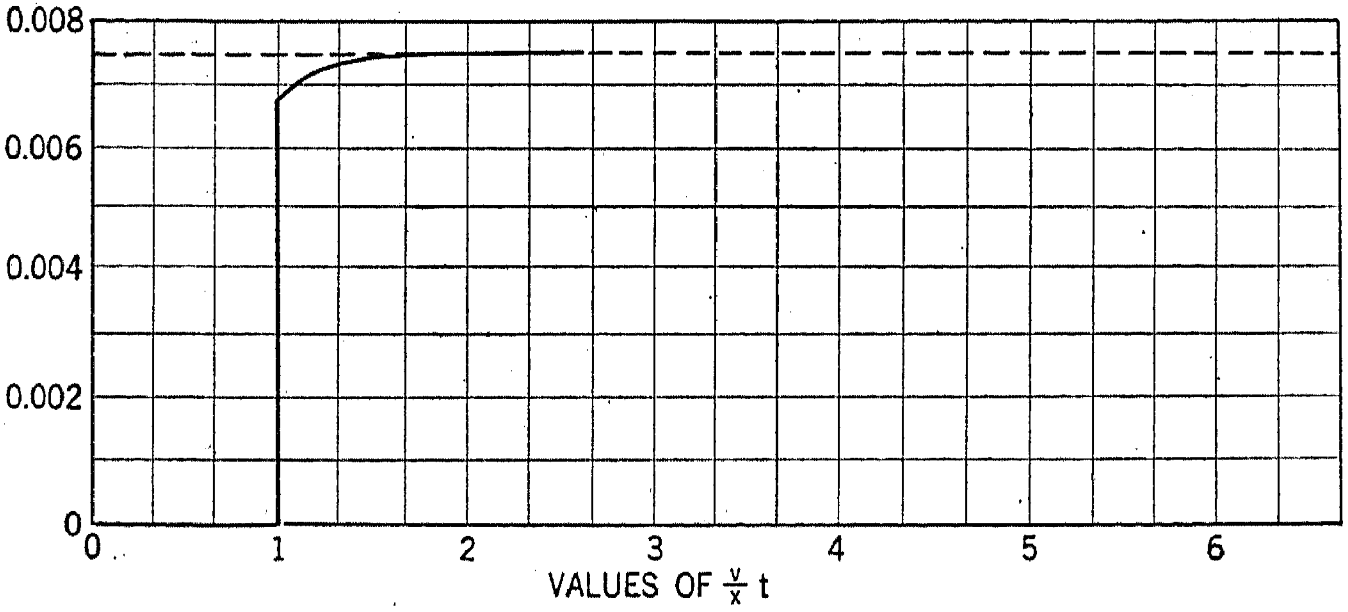


FIG. 19—[COMPUTED FROM FORMULA (58)] VOLTAGE ON CONTINUOUSLY LOADED LINE

Line constants: $R/2\sqrt{C/L}\,X=3$
$G/2\sqrt{L/C}\,X=2$

building up is very slow. These curves have been computed by special methods applicable to the non-dissipative loaded cable, which have proved considerably more rapid than the mechanical evaluation by the Coradi harmonic analyser as described in Section I. These special methods, however, are restricted to the non-dissipative loaded cable, and are therefore not of sufficient general interest to warrant their discussion here.

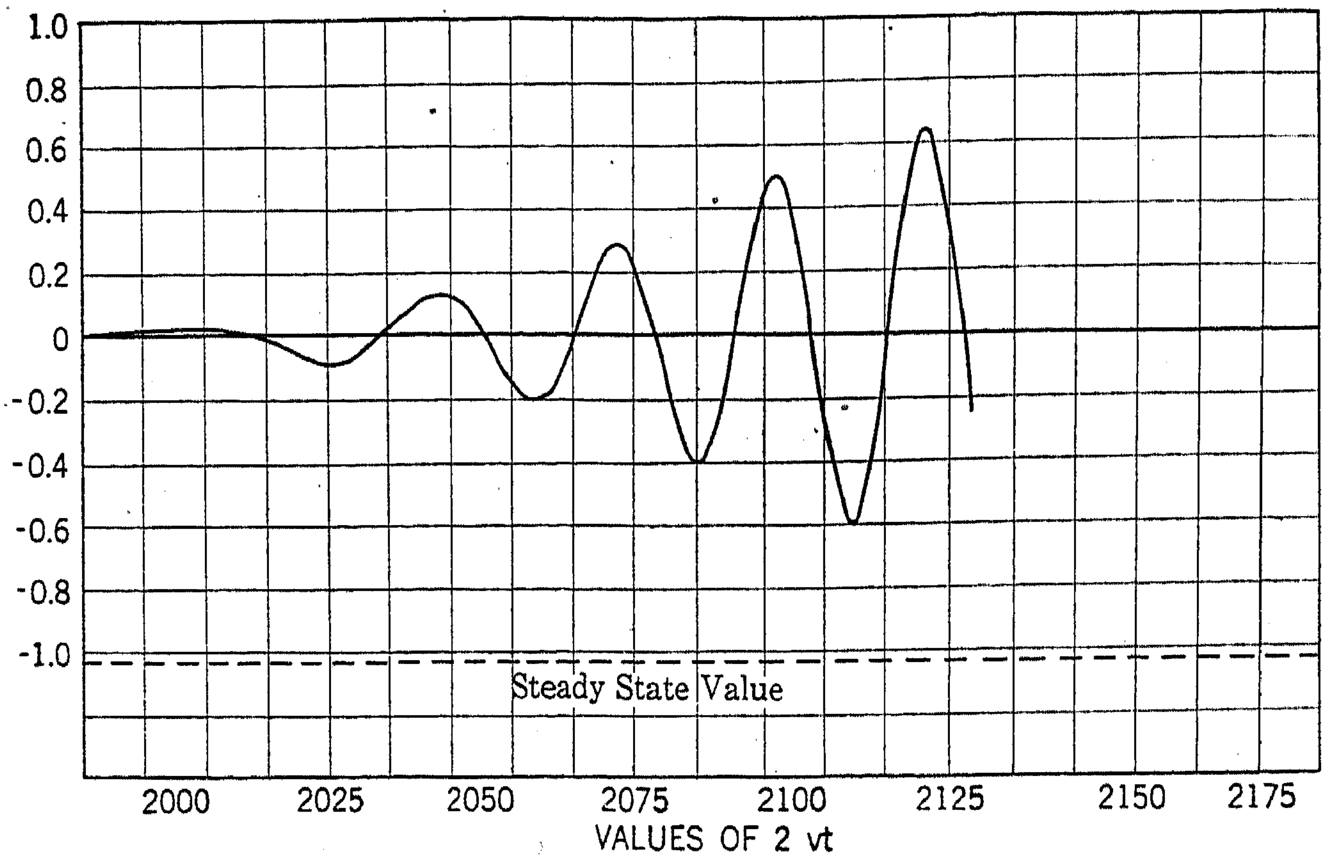


FIG. 20—CURRENT IN 100TH SECTION OF LOADED CABLE IN RESPONSE TO VOLTAGE COS pt

Non-dissipative: $R=G=0$. $f/fc=0.25$. Multiply ordinates by $\sqrt{C/L}$

The foregoing discussion is merely suggestive and makes no pretense to more than point out briefly the prominent outstanding facts. It is hoped, however, that the discussion is full enough to show that a real interpretation and understanding of the complex phenomena of wave propagation is deducible from the formulas derived in this paper.

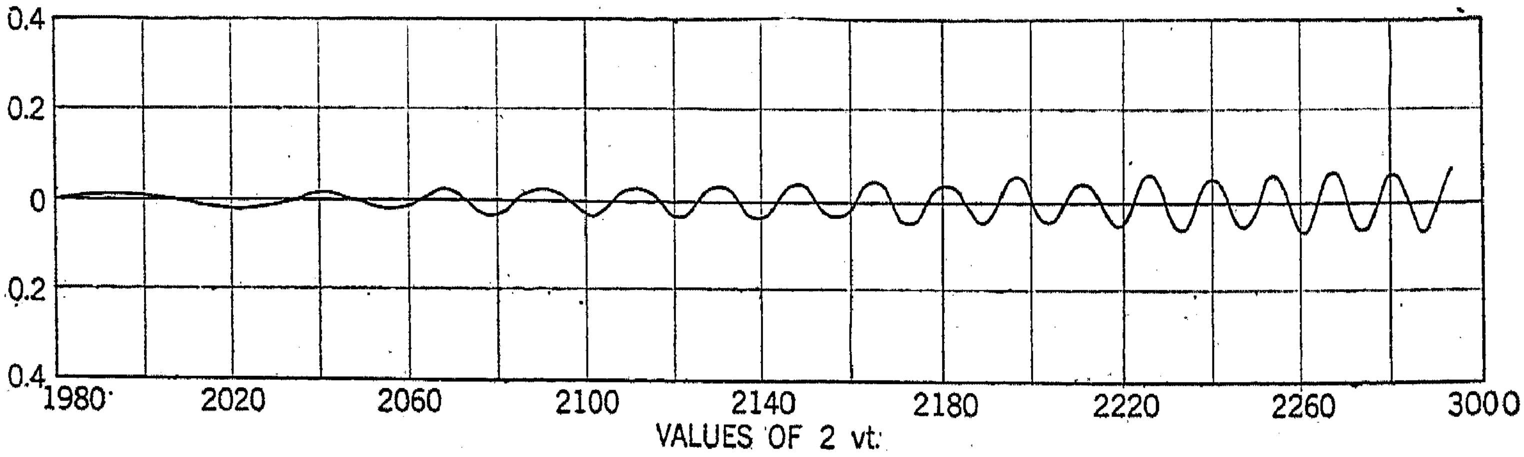


FIG. 21—CURRENT IN 1000TH SECTION OF LOADED CABLE IN RESPONSE TO VOLTAGE COS pt

Non-dissipative: $R=G=0$. $f/f_c=0.6283$. Steady state amplitude $=1.286$

VII. Application of Synthetic Method to Transmission Lines with Terminal Impedances

The synthetic method, developed and discussed in a previous part of this paper, is adapted to determine the effect of terminal apparatus in transmission systems, and indeed this is one of its chief uses. We shall now apply the method to the solution of a practical problem, and since we are concerned primarily with the method of treatment and not with detailed results, a relatively simple problem will be chosen to illustrate the method and its possibilities. A statement of the problem is as follows:

A smooth transmission line of given length and given constants has a unit d-c. voltage applied at $x = t = 0$, while the other end of the line is closed by the primary of a transformer the secondary of which is closed by a resistance R_2 of given value. What is the current in the secondary of the transformer; or in other words, what is the transfer indicial admittance of the system?

We denote the length of the line by s; its constants per unit length by R, L, and C (leakage is assumed absent); the primary and secondary inductances of the transformer (assumed as unity ratio and without magnetic leakage) by L_1; the unknown line current and voltage at $x = s$ by $I_s(t)$ and $V_s(t)$ respectively. We also denote by $A_1(t)$ and $A_2(t)$ the driving point and transfer indicial admittances of the line per se. These are then equal to the currents existing at $x = 0$ and $x = s$ respectively when a unit d-c. voltage is applied to the line at $x = 0$ and $t = 0$. By aid of formulas (**15**) and (**53**), $A_1(t)$ and $A_2(t)$ are given by

$$A_1(t) =$$

$$\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\left\{ I_0(\rho t) + 2I_0\left(\rho\sqrt{t^2 - \left[\frac{2s}{v}\right]^2}\right) + 2I_0\left(\rho\sqrt{t^2 - \left[\frac{4s}{v}\right]^2}\right) + \dots \right\} \qquad \textbf{(a)}$$

$$A_2(t) = 2\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\left\{ I_0\left(\rho\sqrt{t^2 - \left[\frac{s}{v}\right]^2}\right) + I_0\left(\rho\sqrt{t^2 - \left[\frac{3s}{v}\right]^2}\right) + \dots \right\} \qquad \textbf{(b)}$$

In these formulas, I_0 denotes the Bessel function of order zero, and the successive terms of the series come into existence at the time when their arguments pass from imaginary to real values. For smaller values of the time they are to be regarded as identically zero since they represent successive reflected waves.

Now the line, regarded by itself, is actuated by a unit d-c. voltage applied at $x = 0$ and a voltage $V_s(t)$ applied at $x = s$ by formula (**16**) the current $I_s(t)$ existing at $x = s$ is given by

$$A_2(t) - \frac{d}{dt}\int_0^t V_s(\lambda) \, . \, A_1(t-\lambda) \cdot d\lambda \qquad (\mathbf{c})$$

Now this same current is that which enters the primary of the transformer which is connected across the line at $x = s$. We can therefore express this current as that entering the terminal apparatus in response to the voltage $V_s(t)$; it is then

$$\frac{1}{R_2} V_s(t) + \frac{1}{L_1}\int_0^t V_s(\lambda) \cdot d\lambda \qquad (\mathbf{d})$$

Equating (**c**) and (**d**) we get after simplification

$$V_s(t) = \left[\frac{R_2}{1 + R_2 A_1(0)}\right] .$$

$$\left[A_2(t) - \int_0^t V_s(\lambda)\left\{A'_1(t-\lambda) + \frac{1}{L_1}\right\} \cdot d\lambda\right] \qquad (\mathbf{e})$$

which is the integral equation in $V_s(t)$. Since the voltage across the secondary of the transformer is $V_s(t)$ also, the secondary current is simply

$$\frac{1}{R_2} V_s(t)$$

The numerical integration of this equation can, of course, be effected only when the numerical values of the constants of the problem are assigned. We take the following values.

$R_2 = A_1(0) = \sqrt{\frac{C}{L}} = 1935$ ohms

$L_1 = L_2 = 0.4$ henrys

$\frac{R}{2L} = \quad = 292$

$v = \quad = 1.105 \times 10^4$

$s = 100$

The results of the numerical integration of the equation are shown in Fig. 22 which is a plot of the voltage existing across the end of the line and hence is directly proportional to the current in the secondary of the terminal transformer. In this particular case the effect of the secondary reflections is insignificant and the voltage sensibly zero before the secondary reflected wave has arrived.

As a second example of the synthetic method we shall develop the solution for the voltage existing at the terminals of an infinitely long non-inductive cable when a unit d-c. voltage is applied at time $t = 0$ to the cable through a condenser C_0. This problem is not only of some practical importance but is of very great theoretical interest as well in that we are led not only to the formal solution but also to an asymptotic solution identical with that gotten by Heaviside by his remarkable "fractional differentiation." We shall have occasion, therefore, to discuss briefly Heaviside's mode of solution in this connection.

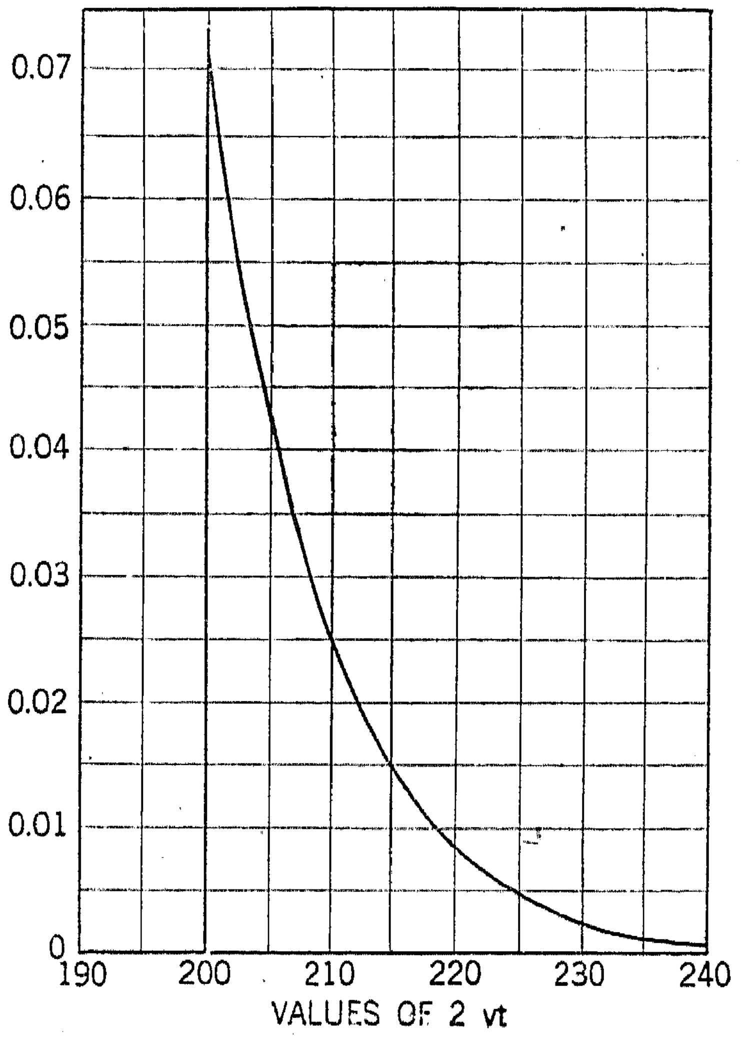


FIG. 22—[COMPUTED FROM FORMULA (63)] VOLTAGE ACROSS TERMINAL IMPEDANCE ON SMOOTH LINE. FOR DATA SEE SECTION VII

To proceed with the problem let R and C be the constants per unit length of the cable and C_0 the terminal condenser capacity. By formula (57) the indicial admittance of the cable is $\sqrt{\frac{C}{\pi R t}}$ so that, if $V(t)$ denote the required terminal voltage, the current entering the cable is, by formula (**11**),

$$\sqrt{\frac{C}{\pi R}} \cdot \frac{d}{dt} \int_0^t d\lambda \cdot \frac{V(\lambda)}{\sqrt{t-\lambda}}$$

But since this current is likewise equal to C_0 times the time rate of increase of the voltage $1 - V(t)$ which exists across the condenser, we have

$$C_0 \frac{d}{dt}\left(1 - V(t)\right) = \sqrt{\frac{C}{\pi R}} \frac{d}{dt} \int_0^t d\lambda \frac{V(\lambda)}{\sqrt{t-\lambda}}$$

whence

$$V(t) = 1 - \sqrt{h} \int_0^t d\lambda \frac{V(\lambda)}{\sqrt{t-\lambda}} \qquad (\mathbf{a}_1)$$

where $$\sqrt{h} = \frac{1}{C_0} \sqrt{\frac{C}{\pi R}}$$

which is the characteristic integral equation of our problem.

The solution of this integral equation in the form of an asymptotic series can be made to depend on Dirichlet's formula,

$$\int_\lambda^t \frac{d\lambda_1}{\sqrt{(t-\lambda_1)(\lambda_1-\lambda)}} = \pi$$

To apply this formula to our problem, multiply the integral equation (to argument λ) by $\frac{1}{\sqrt{t-\lambda}}$ and integrate between the limits 0 and t; this gives

$$\int_0^t d\lambda \frac{V(\lambda)}{\sqrt{t-\lambda}}$$

$$= 2\sqrt{t} - \sqrt{h} \int_0^t d\lambda \cdot V(\lambda) \int_\lambda^t \frac{d\lambda_1}{\sqrt{(t-\lambda_1)(\lambda_1-\lambda)}}$$

$$= 2\sqrt{t} - \pi\sqrt{h} \int_0^t V(\lambda)\, d\lambda \qquad (\mathbf{b}_1)$$

by Dirichlet's formula.

By aid of this equation, the integral equation is reducible to the form,

$$V(t) - \pi h \int_0^t V(\lambda) \cdot d\lambda = 1 - 2\sqrt{h t} \qquad (\mathbf{c}_1)$$

the formal solution of which is

$$V(t) = \epsilon^{\pi h t} - \sqrt{h} \int_0^t d\lambda \cdot \frac{\epsilon^{\pi h \lambda}}{\sqrt{t - \lambda}}$$

This is the formal solution of the problem and might have been gotten directly from the original integral equation by the method of successive substitutions. The advantage of the method followed here is, however, that we have succeeded in reducing the original integral equation to the following differential equation which results from differentiating c_1 with respect to t:

$$V(t) - \frac{1}{\pi h} V'(t) = \frac{1}{\pi \sqrt{h t}}.$$

Now this differential equation admits of easy asymptotic solution by known methods; perhaps the simplest is to assume a solution of the form

$$V(t) \sim \frac{1}{\sqrt{t}} \left\{ a_0 + \frac{a_1}{t} + \frac{a_2}{t^2} + \frac{a_3}{t^3} + \dots \right\}$$

and identify the unknown coefficients by substitution in the differential equation and equation of like powers of t. Proceeding in this manner we get without any trouble the required asymptotic expansion:—

$$V(t) \sim \frac{1}{\pi \sqrt{h t}} \cdot \left\{ 1 - \left(\frac{1}{2 \pi h t} \right) + 1.3 \left(\frac{1}{2 \pi h t} \right)^2 - 1.3.5 \left(\frac{1}{2 \pi h t} \right)^3 \cdot + \dots \right\} \quad (\mathbf{d}_1)$$

This expansion, although it does not formally satisfy the original integral equation of the problem is a true asymptotic solution, and may be used for numerical computation with an error less than the last term included. Its convenience for large values of the argument is obvious, and it shows by mere inspection that $V(t)$ ultimately approaches zero in accordance with the law $\frac{1}{\pi \sqrt{h t}}$.

Heavisides operational solution is arrived at as follows: If we go back to the original differential equations of the problem and replace $\frac{d}{dt}$ by p, we get the symbolic formula

$$V = \frac{\sqrt{\frac{R}{C\,p}}}{\sqrt{\frac{R}{C\,p}} + \frac{1}{C_0\,p}}$$

Rearranging and introducing the symbol

$$\sqrt{h} = \frac{1}{C_0}\sqrt{\frac{C}{\pi\,R}}$$

this may be written

$$V = \frac{\sqrt{\frac{p}{\pi\,h}}}{1+\sqrt{\frac{p}{\pi\,h}}} = \sqrt{\frac{p}{\pi\,h}}\left(1+\sqrt{\frac{p}{\pi\,h}}\right)^{-1}$$

Expanding by the Binomial theorem and rearranging we have symbolically

$$V = \left\{1+\frac{p}{\pi\,h}+\left(\frac{p}{\pi\,h}\right)^2+\ldots\right\}\sqrt{\frac{p}{\pi\,h}} - \left\{\frac{p}{\pi\,h}+\left(\frac{p}{\pi\,h}\right)^2+\ldots\right\}$$

This equation, as it stands is quite without meaning. Nevertheless Heaviside deduces the correct asymptotic solution of the problem from the symbolic formula by the following empirical rule: Discard all integral powers of the operator p, identify $\sqrt{p}$ as $\frac{1}{\sqrt{\pi\,t}}$ and replace any p^k in the first bracketted series by $\frac{d^k}{dt^k}$; then,

$$V(t) = \left\{1+\frac{1}{\pi\,h}\,\frac{d}{dt}+\frac{1}{(\pi\,h)^2}\,\frac{d^2}{dt^2}+\ldots\right\}\frac{1}{\pi\,\sqrt{h\,t}}$$

$$= \frac{1}{\pi\sqrt{h\,t}}\left\{1-\left(\frac{1}{2\,\pi\,h\,t}\right)\right.$$

$$\left.+\,1.3\left(\frac{1}{2\,\pi\,h\,t}\right)^2-\,.\,.\,.\right\}$$

which agrees with the asymptotic expansion derived above.

Compared with the foregoing operational solution, the asymptotic expansion from the integral equation of the problem may appear cumbersome and involved, and it must be admitted that it entirely lacks the extraordinary directness and simplicity of the Heaviside solution. On the other hand there is this to be said for the solution from the integral equation; it is rigorous and demonstrably correct, whereas Heaviside's remarkable method was as he professes, the outcome of experiment and still lacks the logical demonstration which we are accustomed to require of our mathematical solutions. Again it must be remembered that the chief difficulty lies not so much in deriving as in subsequently computing and interpreting the solution. From this standpoint a series solution leaves much to be desired since its behavior and significance can be arrived at only by direct numerical computation. When, however, the solution is defined by an integral equation, we can not only always derive the formal series solution, and the asymptotic expansion (where such exists), but a study of the integral equation as it stands in many cases gives an insight into the behavior of the function without computation. Furthermore its numerical computation is usually far less laborious than is the computation of the equivalent series solution.

As a final example of the derivation of the asymptotic expansion from the integral equation of the problem we shall require the current at distance x from the sending end of the cable considered above. By formulas (**11**) and (**57**) the required current $I\ (t)$ is given by

$$\sqrt{\frac{C}{\pi R}}\ \frac{d}{dt}\int_0^t d\,\lambda\cdot V\,(t-\lambda)\cdot\frac{\epsilon^{-a/\lambda}}{\sqrt{\lambda}}$$

where $V\ (t)$ is defined by equation ($\mathbf{a}_1$) and $a\ =\ \dfrac{R\,C\,x^2}{4}$

Consequently, as may be shown without any trouble, the integral equation which defines the required current is

$$I\,(t) = C_0\,\sqrt{h}\,\frac{\epsilon^{-a/t}}{\sqrt{t}} - \sqrt{h}\int_0^t d\,\lambda\,\frac{I\,(\lambda)}{\sqrt{t-\lambda}} \qquad (\mathbf{a}_2)$$

where, as above, $\sqrt{h} = \frac{1}{C_0}\sqrt{\frac{C}{\pi\,R}}$

Multiplying this equation (to argument λ) by $\frac{1}{\sqrt{t-\lambda}}$, integrating with respect to λ from o to t, and applying Dirichlet's formula, we get:—

$$\int_0^t d\,\lambda\cdot\frac{I\,(\lambda)}{\sqrt{t-\lambda}} = C_0\,\sqrt{h}\int_0^t d\,\lambda\,\frac{\epsilon^{-a/\lambda}}{\sqrt{t\,\lambda-\lambda^2}} - \pi\,\sqrt{h}\int_0^t I\,(\lambda)\cdot d\,\lambda \qquad (\mathbf{b}_2)$$

and finally, from $(\mathbf{a}_2)$ and $(\mathbf{b}_2)$,

$$I\,(t) - \pi\,h\int_0^t I\,(\lambda)\cdot d\,\lambda = C_0\,\sqrt{h}\,\frac{\epsilon^{-a/t}}{\sqrt{t}} - C_0\,h\int_0^t d\,\lambda\,\frac{\epsilon^{-a/\lambda}}{\sqrt{t\,\lambda-\lambda^2}} \qquad (\mathbf{c}_2).$$

But

$$\int_0^t d\,\lambda\,\frac{\epsilon^{-a/\lambda}}{\sqrt{t\,\lambda-\lambda^2}} = \sqrt{a\,\pi}\int_0^t d\,\lambda\cdot\frac{\epsilon^{-a/\lambda}}{\lambda^{3/2}}$$

$$= \pi - 2\sqrt{\frac{a.\pi}{t}}\left\{1 - \frac{1}{3\cdot 1\,!}\left(\frac{a}{t}\right) + \frac{1}{5.2\,!}\left(\frac{a}{t}\right)^2 + \ldots\right\} \qquad (\mathbf{d}_2)$$

and

$$\frac{\epsilon^{-a/t}}{\sqrt{t}} = \frac{1}{\sqrt{t}}\left\{1 - \frac{1}{1\,!}\left(\frac{a}{t}\right) + \frac{1}{2\,!}\left(\frac{a}{t}\right)^2 \ldots\right\} \qquad (\mathbf{e}_2)$$

Substitution of ($\mathbf{d}_2$) and ($\mathbf{e}_2$) in ($\mathbf{c}_2$) and rearrangement gives

$$\pi h \int_0^t I(\lambda) \cdot d\lambda - I(t) = \pi h C_0$$

$$- C_0 \sqrt{\frac{h}{t}} \left\{ \begin{array}{l} \left(1 + \frac{2}{1} \sqrt{a h \pi} \right) \\ - \frac{1}{1!} \left(\frac{a}{t} \right) \left(1 + \frac{2}{3} \sqrt{a h \pi} \right) \\ + \frac{1}{2!} \left(\frac{a}{t} \right)^2 \left(1 + \frac{2}{5} \sqrt{a h \pi} \right) \\ - \frac{1}{3!} \left(\frac{a}{t} \right)^3 \left(1 + \frac{2}{7} \sqrt{a h \pi} \right) \\ \quad \cdots \end{array} \right. \qquad (\mathbf{f}_2)$$

The asymptotic expansion of this equation can now be derived without any difficulty; as the expression becomes somewhat complicated we shall not however, carry out the solution in detail. From the equation, as it stands, it is clear that the total charge passing any point x is simply C_0 and that $I(t)$ ultimately becomes

$$\frac{1}{2\pi} \frac{C_0}{t\sqrt{h\,t}} \left(1 + 2\sqrt{a h \pi}\right)$$

To enter upon more advanced applications of the synthetic method is beyond the scope of the present paper. The simple examples dealt with above, will, it is hoped, serve to indicate the appropriate treatment and the wide range of problems to which the method is applicable.

NOTE 1

Owing to the very great importance of mono-periodic currents, the formulas appropriate to the sinusoidal e.m.f. $\sin(p\,t + \theta)$ have been the subject of particular study. While formula (**12**) is fundamental and its evaluation by aid of the harmonic analyzer is usually indicated, the derived formulas given below are of value in particular cases, especially when a knowledge of the behavior of the system to very low or very high frequencies is required.

If the cosine and sine functions in the intergrand of formula (**12**) are expanded in the usual convergent power series, it is easy to show that we get by repeated partial integrations and some easy simplifications

$$I = \sin\theta .\ u(t) + \cos\theta \cdot v(t) \qquad \text{(a)}$$

where

$$u(t) = A(t) - p^2 \int_0^t \int_0^\lambda A(\lambda_1) \cdot d\lambda\, d\lambda_1$$

$$+ p^4 \int_0^t \int_0^\lambda \int_0^{\lambda_1} \int_0^{\lambda_2} A(\lambda_3)\, d\lambda\, d\lambda_1\, d\lambda_2\, d\lambda_3 \qquad \textbf{(b)}$$

$$-p^6 \int_0^t \cdots \int_0^{\lambda^4} A(\lambda_5) d\lambda \ldots d\lambda_5$$

$$\ldots\ldots$$

and

$$v(t) = p \int_0^t A(\lambda)\, d\lambda - p^3 \int_0^t \int_0^\lambda \int_0^{\lambda_1} A(\lambda_2) d\lambda\, d\lambda_1 d\lambda_2$$

$$+ p^5 \int_0^t \cdots \int_0^{\lambda_3} A(\lambda_4)\, d\lambda \ldots d\lambda_4 \qquad \textbf{(c)}$$

$$- p^7 \int_0^t \cdots \int_0^{\lambda_5} A(\lambda_6)\, d\lambda \ldots\, d\lambda_6$$

$$\ldots\ldots$$

Both of these series are absolutely convergent for all values of p, but their direct computation is practicable only for relatively small values of p. In the limit $p = 0$, $u(t)$ reduces to $A(t)$ and $v(t)$ to zero as we should expect.

The functions $u(t)$ and $v(t)$ are obviously connected by the relation,

$$v(t) = p \int_0^t u(\lambda) . d\lambda \qquad \textbf{(d)}$$

so that $v(t)$ is calculable from $u(t)$ by mechanical integration.

Furthermore it is easy to show from formula (**b**) that $u(t)$ satisfies the integral equation

$$u(t) = A(t) - p^2 \int_0^t \int_0^\lambda u(\lambda_1) . d\lambda\, d\lambda_1$$

which can be written (by reversing the order of integration of the multiple integral)

$$u(t) = A(t) - p^2 \int_0^t (t - \lambda) . u(\lambda) . d\lambda \qquad \textbf{(e)}$$

The integral equation is now in canonical form, and in some problems its numerical integration is simpler than the mechanical evaluation of formula (**12**). For a brief account of integral equations and the methods for their formal and numerical solution, see Note 3.

The foregoing formulas are often useful when we wish to investigate the behavior of the system to applied voltages of low frequency; when, on the other hand, the applied frequency is high the following formula is applicable and is of considerable value in interpreting the ultimate character of the *transient distortion:*

If the applied voltage be $\sin(p\,t+\theta)$ *and if* p *be greater than* (p_m) *where* p_m *is the greatest root of the determinantal equation* $Z(p) = 0$, *then the transient distortion, as defined by formula* (**14**), *is formulated by the convergent series:*

$$\cos\theta\,.\,g(t) + \sin\theta\,.\,h(t)$$

where

$$g(t) = \frac{1}{p}A^{(1)}(t) - \frac{1}{p^3}A^{(3)}(t) + \frac{1}{p^5}A^{(5)}(t)\ldots.$$

$$h(t) = \frac{1}{p^2}A^{(2)}(t) - \frac{1}{p^4}A^{(4)}(t) + \frac{1}{p^6}A^{(6)}(t)\ldots.$$

and

$$A^{(n)}(t) = \frac{d^n}{dt^n}A(t)$$

The proof of this last formula and the sufficient condition for its convergence are easily established. If the frequency is very high it is clear that the transient distortion becomes ultimately,

$$\frac{1}{p}A'(t)$$

and therefore, if the frequency be high enough the *transient distortion in all systems is inversely proportional to the applied frequency,* provided that the characteristic oscillation frequencies have a finite upper limit. Furthermore, even when divergent, series of the above form, in many cases, constitute asymptotic expansions which are valid for numerical computation.

NOTE 2

From our restricted standpoint an asymptotic expansion is defined as an expansion of a function in inverse powers of the argument. The theory of asymptotic expansions occupies an important place in modern analysis. With the questions of divergence, convergence and summability to which pure analysis is largely restricted, we are not concerned, but rather with methods of asymptotic expansions, regarding which knowledge is quite fragmentary. For a good brief discussion of asymptotic expansions from the point of view of analysis, the reader is referred to Whittaker and Watson—"A Course in Modern Analysis," Chap. VIII.

When the system under consideration consists of a finite number of branches or meshes, the asymptotic expansion of the function of $\frac{1}{Z(p)}$ is always possible. A general rule for the expansion is as follows: Replace p by $\frac{1}{y}$ in $\frac{1}{Z(p)}$ and write

$$\frac{1}{Z(p)} = \frac{1}{Z\left(\frac{1}{y}\right)} = \varphi(y)$$

Then expand $\varphi(y)$ in the usual Taylor's series

$$\varphi(y) = \varphi(o) + \varphi'(o) \cdot \frac{y}{1!} + \varphi''(o) \cdot \frac{y^2}{2!}$$

$$= \Sigma\, \varphi^{m}(o) \cdot \frac{y^m}{m!}$$

Then the required asymptotic expansion is given by

$$\frac{1}{Z(p)} \sim \Sigma\, \varphi^m(o) \cdot \frac{1}{m!\, p^m}$$

In many problems, however, much shorter ways of expanding suggest themselves, and often the binomial expansion can be directly applied.

NOTE 3

In this connection the writer takes the opportunity of calling attention to a formula, as yet unpublished, which was derived by Mr. T. C. Fry of the Western Electric Company in Nov. 1916. Starting with the Fourier integral which formulates the current in response to an applied voltage $f(t)$ and applying a theorem of Hermites' thereto, Mr. Fry arrived at the original formula (adopting the notation of the present paper)

$$I = \int_0^t f(\lambda)\, d\lambda \,.\, \Sigma \frac{\epsilon^{p_m(t-\lambda)}}{Z'(p_m)}$$

Mr. Fry restricted consideration to systems for which $\frac{1}{Z(\infty)} = A(o) = o$; for this case it is equivalent to formula (**10**) of this paper.

The writer has derived the power series solution from the Fourier integral formulation by asymptotically expanding $\frac{1}{Z(p)}$ in the integrand and then evaluating the infinite integrals by the calculus of residues.

Since this paper was written the author has been privileged to see some more recent work, as yet unpublished, of Mr. Fry's, in which he has secured results of great originality and value by attacking the Fourier integral by the methods of advanced analysis.

NOTE 4

The integral equation, known as Fredholm's equation is of the following form

$$v(t) = f(t) + \int_b^a v(\lambda) \cdot u(t, \lambda) \cdot d\lambda$$

where $f(t)$ and $u(t, \lambda)$ are known functions. The type of integral equation with which we are concerned is less general in form:

$$v(t) = f(t) + \int_0^t v(\lambda) \cdot u(t - \lambda) \cdot d\lambda \qquad (\mathbf{a})$$

differing from the general equation in that the upper limit of integration is variable and in that the kernel or nucleus of the equation is a function of $t - \lambda$ instead of being a perfectly arbitrary function of t and λ. When the upper limit of integration is variable the equation is usually called Voltenra's equation in honor of the Italian mathematician who has studied it extensively.

The formal solution of this equation is gotten by successive or repeated substitution of the right hand side of the equation for $v(t)$ in the integral. It is

$$v(t) = f(t) + \int_0^t u(t-\lambda) \cdot f(\lambda) \cdot d\lambda$$
$$+ \int_0^t u(t-\lambda) \cdot \int_0^\lambda u(\lambda - \lambda_1) \cdot f(\lambda_1) \cdot d\lambda \, d\lambda_1$$
$$+ \int_0^t u(t-\lambda) \int_0^\lambda u(\lambda - \lambda_1) \int_0^{\lambda_1} u(\lambda_1 - \lambda_2) \, f(\lambda_2) \, d\lambda \, d\lambda_1 \, d\lambda_2$$
$$+ \dots$$

Pure analysis is concerned, of course, not with the numerical interpretation of this solution but with questions as to the uniqueness, convergence, etc. and it has been shown that the solution is not only unique but absolutely convergent.

Except, however, for very simple forms of the functions $f(t)$ and $u(t)$ the formal solution given above is quite useless for our purpose, which must always be to arrive at interpretable and computable results. Indeed our conception of a solution is quite distinct from that of pure analysis, and our tests must always be pragmatic.

If neither $f(t)$ or $u(t)$ involve fractional powers of t the *power series solution* can be gotten from (a) by repeated differentiations as follows:

From (a) we have:

$$v(o) = f(o)$$

$$v'(t) = \frac{d}{dt} v(t) = f'(t) + u(o) \cdot f(t) + \int_0^t v(\lambda) \cdot u'(t-\lambda) \cdot d\lambda$$

$$v'(o) = f'(o) + u(o) \cdot v(o)$$

Proceeding with repeated differentiations we get

$$v''(o) = f''(o) + u(o) \cdot v'(o) + u'(o) \cdot v(o)$$
$$v'''(o) = f'''(o) + u(o) \cdot v''(o) + u'(o) \cdot v'(o) + u''(o) \cdot v(o)$$

Hence $v(o)$, $v'(o)$, $v''(o) \dots v^{(m)}(o)$ are all expressible in terms of $f(t)$ and $u(t)$ and their derivatives. Hence the solution is

$$v(t) = v(o) + v'(o) \cdot \frac{t}{1!} + v''(o) \cdot \frac{t^2}{2!} + -$$

$$= \sum v^{(m)}(o) \cdot \frac{t^m}{m!}$$

where $v^m(o)$ is gotten from (*b*).

If either $u(t)$ or $f(t)$ contains fractional powers of t, the solution can be gotten by *assuming* a series solution of the form

$$v(t) = \Sigma\, a_m t^m + \sqrt{t}\, \Sigma\, b_m t^m$$

substituting in the integral equation, integrating and then determining the unknown coefficients of the series solution by direct equation of like powers of t.

Corresponding to every series solution of the form

$$\Sigma\, a_m t^m + \sqrt{t}\, \Sigma\, b_m t^m$$

Heaviside, by his operational treatment, arrives at an asymptotic solution of the form

$$\Sigma \frac{c_m}{t^m} + \frac{1}{\sqrt{t}} \Sigma \frac{d_m}{t^m}$$

which, although divergent, may be used for computation, in spite of the fact, which may be easily proved, that the latter series does not satisfy the integral equation and is therefore not a formal solution of the problem. We shall show later that the asymptotic solutions are derivable from the integral equations of the problem, and thus verify Heaviside's remarkable results.

The numerical integration of the integral equation (**a**) is carried out as follows: We remark that the integral appearing in (**a**) is the limit of the summation:

$$\begin{aligned}
&[v(o) \cdot u(t) + v(\Delta t) \cdot u(t - \Delta t)]\, \Delta t/2 \\
&+ [v(\Delta t) \cdot u(t - \Delta t) + v(2\Delta t) \cdot u(t - 2\Delta t)] \\
&\qquad\qquad\qquad\qquad\qquad\qquad \Delta t/2 \\
&+ \cdots \\
&+ [v(t - \Delta t) \cdot u(\Delta t) + v(t) \cdot u(o)] \cdot \Delta t/2 \\
&= v(o) \cdot u(t) \cdot + v(t) \cdot u(o) \Delta \cdot t/2 \\
&+ \Delta t \sum_{1}^{n-1}{}_m\, v(m\Delta t) \cdot u(t - m\Delta t)
\end{aligned}$$

When $t = n\Delta t$

Equation (**a**) then is the *limit* of the expression

$$\left[1 - u(o)\cdot\frac{\Delta t}{2}\right] v(t) = f(t) + v(o)\cdot u(t)\cdot\frac{\Delta t}{2} + \Delta t \sum_{1}^{n-1}{}_{m}\; v(m\Delta t)\cdot u(t - m\Delta t)$$

When $n\Delta t = t$.
or

$$v(t) = \frac{1}{1 - u(o)\cdot\frac{\Delta t}{2}} \left\{ \begin{array}{l} f(t) + f(o)\cdot\frac{\Delta t}{2}\cdot u(t) \\ + \Delta(t) \sum_{1}^{n-1}{}_{m}\, v(m\Delta t)\cdot u(t - m\Delta t) \end{array} \right.$$

If we now call $t = t_1$; $2t = t_2$; . . .
$n\Delta t = t_n$ we have:—
$v(o) = f(o)$

$$v(t_1) = \frac{1}{1 - u(o)\cdot\frac{\Delta t}{2}} \left\{ f(t_1) + f(o)\cdot u(t_1)\cdot\frac{\Delta t}{2} \right.$$

$$v(t_2) = \frac{1}{1 - u(o)\cdot\frac{\Delta t}{2}} \left\{ \begin{array}{l} f(t_2) + f(o)\cdot u(t_2)\cdot\frac{\Delta t}{2} \\ + v(t_1)\cdot u(t_1)\cdot\Delta t \end{array} \right.$$

$$v(t_3) = \frac{1}{1 - u(o)\cdot\frac{\Delta t}{2}} \left\{ \begin{array}{l} f(t_3) + f(o)\cdot u(t_3)\cdot\frac{\Delta t}{2} \\ + \Delta t\,\{v(t_1)\cdot u(t_2) + v(t_2)\cdot u(t_1)\} \end{array} \right\}$$

. . . . .

In each of these equations the factors appearing on the right hand side are known from the preceding equations, and we thus can compute $v(t)$ for any required value of t.

The time element Δt should be chosen with reference to the character of the known function involved in the integral equation; that is, it should be so small as to make the curvature, or second differences of these functions small within the interval Δt. The precision of this method increases, of course, as the time interval Δt is diminished and can be made as exact as we please by taking Δt sufficiently small.

The method has been tried out pretty thoroughly in a series of problems and works well with a precision quite sufficient for practical purposes. The labor is very much less than that involved in a computation of a power series solution; and the accuracy of the results is readily checked.

NOTE 5

The steady-state properties and characteristics of periodic structures (of which the artificial line is a particular case) were investigated by Dr. G. A. Campbell, Research Engineer of the American Telephone and Telegraph Company, who discovered their extremely interesting and important action as "wave-filters," due to their property of transmitting with small attenuation steady-state currents of all frequencies lying within a continuous range or ranges of frequency (which may be termed transmission bands) while sharply attenuating all currents of frequencies outside these transmission bands. For a fairly full account of the wave filter and its theory, see U. S. patent No. 1,227,113 to G. A. Campbell.

NOTE 6

The Bessel function of the first kind, $J_n(Z)$ and $I_n(Z)$ are defined, when n is a positive integer, by the absolutely convergent series:

$$J_n(Z) = \left(\frac{Z^n}{2^n \cdot n\,!}\right) \cdot \left\{ \begin{aligned} &1 - \frac{Z^2}{2\,(2\,n+2)} + \frac{Z^4}{2.4\,(2\,n+2)\,(2\,n+4)} + \cdots \\ &- \frac{Z^6}{2.4.6\,(2\,n+2)\,(2\,n+4)\,(2\,n+6)} - + \cdots \end{aligned} \right.$$

$$\begin{aligned} I_n(Z) &= i^{-n} J_n(i\,Z) \\ &= \left(\frac{Z^n}{2^n \cdot n\,!}\right) \cdot \left\{ \begin{aligned} &1 + \frac{Z^2}{2\,(2\,n+2)} + \frac{Z^4}{2.4\,(2\,n+2)\,(2\,n+4)} \\ &+ \frac{Z^6}{2.4.6\,(2\,n+2)\,(2\,n+4)\,(2\,n+6)} + \cdots \end{aligned} \right. \end{aligned}$$

The terms of $J_n (Z)$ are alternating in sign while those of $I_n (Z)$ are all positive.

For values of Z very large compared with n, the functions can be computed from the asymptotic formulas:

$$J_n (Z) = \sqrt{\frac{2}{\pi Z}} \left\{ P_n . \cos \left(\frac{2n+1}{4} \pi - Z \right) + Q_n . \sin \left(\frac{2n+1}{4} \pi - Z \right) \right\}$$

where

$$P_n = 1 - \frac{(4n^2-1)(4n^2-9)}{2!\,(8Z)^2} + \frac{(4n^2-1)(4n^2-9)(4n^2-25)(4n^2-49)}{4!\,(8Z)^4} \dots$$

$$Q_n = \frac{4n^2-1}{8Z} - \frac{(4n^2-1)(4n^2-9)(4n^2-25)}{3!\,(8Z)^3}$$

and

$$I_n (Z) = \frac{\epsilon^Z}{\sqrt{2\pi Z}} \left\{ 1 - \frac{4n^2-1}{1!.2^3 Z} + \frac{(4n^2-1)(4n^2-9)}{2!.2^6.Z^2} \dots \right.$$

For the large order of functions with which we are mainly concerned these asymptotic expansions are useless for values of argument of practical importance, since we are concerned with the behavior of the functions in the neighborhood of $Z = n$. The computation of the functions is carried out without much difficulty from the valuable asymptotic formulas given by J. W. Nicholson in a series of papers appearing in the *Philosophical Magazine* (Dec. 1907, August 1908, July 1909, Feb. and Dec. 1910). These formulas are too long to reproduce here and the original papers should be consulted.

The properties of the integral function $\int_0^Z J_n (Z_1)\, d z_1$, are of course determined by the corresponding properties of the Bessel function appearing in the integrand; it does not appear, however, that the integral function has been the subject of any extensive study in spite of its extremely interesting properties. A few of these properties which are of importance for our purposes are:

(1) $\int_0^\infty J_n(z)\,dz = 1$ for all values of n.

(2) For values of z very large compared with n,

$$\int_0^Z J_n(z_1)\,dz_1 \sim 1 - \sqrt{\frac{2}{\pi}} \int_t^\infty \cos\left(z_1 - \frac{2n+1}{4}\pi\right) \frac{dz_1}{\sqrt{z_1}}$$

the integral term of which can be resolved into Fresnel integrals.

(3) For large values of n, the function $\int_0^Z J_n(z_1)\,dz_1$ is insignificant for values of Z less than n. From a mathematical standpoint, this property explains the virtual velocity of propagation of the artificial line.

(4) Limit $n \rightarrow \infty \left[\int_0^{\lambda n} J_n(z_1)\,dz_1\right] \begin{array}{ll} = o, & \lambda < n \\ = 1, & \lambda > n \end{array}$

This relation enables us to derive the solution for the smooth line as the limiting case of the artificial line, as the aggregate value of the constant is kept finite and the number of sections approaches infinity.

NOTE 7

The correctness of the solution formulated by (**35**) may be verified by deriving the corresponding steady-state formula by aid of equation (**13**). If a voltage $\sin(pt + \theta)$ is applied at midload position to an infinitely long periodically loaded cable the steady-state or periodic current in the *qth* load-coil is given by substitution of (**35**) in (**13**); it is

$$\sqrt{\frac{C}{L}} \left\{ \begin{array}{l} \sin(pt+\theta) \int_0^\infty \cos(w\lambda) \cdot J_{2q}(\lambda) \cdot d\lambda \\ -\cos(pt+\theta) \int_0^\infty \sin(w\lambda) \cdot J_{2q}(\lambda) \cdot d\lambda \end{array} \right.$$

When $w = \dfrac{p}{p_c} = \dfrac{p}{2v} = \dfrac{p}{2}\sqrt{LC}$

p_c indicates $2\pi f_c$ where f_c is the "critical or cut-off frequency of the line.

The infinite integrals have been evaluated, they are (see Nielsen—*Handbuch der Cylinderfunctionen,* p. 195.)

$$\int_0^\infty \cos(w\lambda)\cdot J_{2q}(\lambda)\cdot d\lambda = \frac{\cos(2q\sin^{-1}w)}{\sqrt{1-w^2}} \qquad w < 1$$

$$= -\frac{\sin q\pi}{\sqrt{w^2-1}}(w-\sqrt{w^2-1})^{2q} \qquad w > 1$$

$$\int_0^\infty \sin(w\lambda)\cdot J_{2q}(\lambda)\cdot d\lambda = \frac{\sin(2q\sin^{-1}w)}{\sqrt{1-w^2}} \qquad w < 1$$

$$= \frac{\cos q\pi}{\sqrt{w^2-1}}(w-\sqrt{w^2-1})^{2q} \quad w > 1$$

Hence, if $w < 1$, $p < p_c$, the frequency is less than the critical frequency and

$$|I_q| = \sqrt{\frac{C}{L}}\;\frac{1}{\sqrt{1-w^2}}$$

If $w > 1$ the frequency is above the critical frequency and

$$|I_q| = \sqrt{\frac{C}{L}}\;\frac{1}{\sqrt{w^2-1}}\left(\frac{1}{w+\sqrt{w^2-1}}\right)^{2q}$$

which agree exactly with the results gotten by the usual steady state or periodic theory, and show the critical character of the frequency $\frac{p_c}{2\pi}$.

NOTE 8

From formulas (**12**) and (**53**) we can arrive at some important deductions regarding the relations obtaining between the frequency of the applied voltage and the character of the transient distortion on the smooth line.

If the applied e.m.f. is taken as sin $(p\,t + \theta)$ (applied at

$t = o$ and $x = o$), the current at point x on the line is by (**12**) and (**53**)

$$I_x = 0 \qquad \text{for } v\,t < x$$

$$I_x = \sqrt{\frac{C}{L}}\;\epsilon^{-ax}\sin\left(p\left[t - \frac{x}{v}\right] + \theta\right)$$

$$+\sqrt{\frac{C}{L}}\,\epsilon^{-ax}\left\{\begin{array}{l} \sin\,(p\,t+\theta)\displaystyle\int_x^{vt}\cos\left[\frac{p}{v}\,\lambda\right]\frac{d}{d\,\lambda}\,\varphi\,(\lambda)\,d\,\lambda \\ -\cos\,(p\,t+\theta)\displaystyle\int_0^{vt}\sin\left[\frac{p}{v}\,\lambda\right]\frac{d}{d\,\lambda}\,\varphi\,(\lambda)\,d\,\lambda \end{array}\right.$$

where

$$\varphi\,(\lambda) = \epsilon^{-a(\lambda - x)}\,I_0\,(a\sqrt{\lambda^2 - x^2})$$

Inspection of this formula shows that the first term is the final steady state provided the frequency of the applied voltage is high enough to justify the employment of the usual approximate formulas for the steady-state characteristic impedance and propagation factor. The integral terms represent transient distortion. To a first order of approximation the integral terms are of an order of magnitude

$$\frac{1}{p}\cdot\frac{R}{2\,L}\cdot(a\,x - 2)$$

provided $a\,x > 2$.

The building up of the current wave to its final steady state value may therefore be described as follows:

If a voltage of frequency f is applied to a smooth transmission line the current at distance x out on the line is identically zero for a time interval x/v after the application of the voltage; at the end of this time interval it has an instantaneous jump to its final steady state value, but is thereafter subjected to a transient distortion the magnitude of which is roughly *inversely proportional to the applied frequency*, and the sensible duration of which diminishes with the applied frequency. The transient distortion is negligibly small for $a\,x < 2$, while for large values of $a\,x$ it increases rapidly at a more than linear rate with the product $R/L\,a\,x$. If the applied frequency is made indefinitely great the transient distortion approaches zero both in magnitude and duration.

From formulas (**15**) and (**53**) the complete wave solution for the leakageless line of length s, short-circuited at $x = s$, and actuated by a unit voltage applied at $x = o$ and $t = o$, is

$$A_x(t) = \sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\,.$$

$$\left\{\begin{aligned} & I_0\left(\rho\sqrt{t^2-\left[\frac{x}{v}\right]^2}\right) \\ & \quad + 2\sum_1^\infty{}_m\, I_0\left(\rho\sqrt{t^2-\left[\frac{2\,m\,s+x}{v}\right]^2}\right) \\ & \quad + 2\sum_1^\infty{}_m\, I_0\left(\rho\sqrt{t^2-\left[\frac{2\,m\,s-x}{v}\right]^2}\right) \end{aligned}\right. \qquad (\mathbf{a})$$

This solution will be discussed with particular reference to the velocity of transmission regarding which there seems to exist considerable confusion of thought or at any rate looseness of statement. The case of the leakageless line is chosen purely for the sake of simplicity and the following remarks apply also to the general case when leakage is present.

Now in the light of the derivation of formula (**53**) it is to be understood that the terms of formula (**a**) come into existence not at time $t = o$ but *only as the arguments of the* I_0 *functions pass from imaginary to real values.* That is, while all the terms are written down in formula (**a**), they come into existence at successive intervals of time.

Thus for $o < t < x/v$, $A_x(t)$ is identically zero, and at $t = x/v$ the direct wave (which alone exists on the infinitely long line) arrives at x, and it alone exists there until $t = \dfrac{2\,s-x}{v}$ when the first reflected wave arrives. At a subsequent time $t = \dfrac{2\,s+x}{v}$ the second reflected wave arrives and so the final steady state is built up. Analytically,

$$A_x(t) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 0 < t < x/v$$

$$A_x(t) = \sqrt{\frac{C'}{L}}\, \epsilon^{-\rho l}\, I_0\left\{\rho\sqrt{t^2-\left(\frac{x}{v}\right)^2}\right\} \ldots\ldots\ldots\ldots$$

$$x/v < t < \frac{2s-x}{v}$$

$$A_x(t) = \sqrt{\frac{C}{L}}\, \epsilon^{-\rho t}\left\{ I_0\left(\rho\sqrt{t^2-\left[\frac{x}{v}\right]^2}\right)\right.$$

$$\left. + 2\, I_0\left(\rho\sqrt{t^2-\left[\frac{2\,s-x}{v}\right]}\right)\right. \quad \frac{2\,s-x}{v} \cdot < t < \frac{2\,s+x}{v}$$

. . . . . .

Not until $t = \infty$ do all the terms finally come into existence.

Now the outstanding deduction from this interpretation of formula (**a**) is that it represents a series of waves which are propagated with finite velocity $x = \frac{1}{\sqrt{L\,C}}$ which is independent of the dissipation occurring during transmission whether due to line resistance or leakage. Consequently formula (**a**) is the wave solution; the first term represents the direct wave, which alone exists on the infinitely long line; the second summation represents a series of secondary waves reflected at $x = s$ and the first summation a series of secondary waves reflected at $x = o$. Each wave is represented by the product of an exponential term and a Bessel function, the latter formulating the distortion which the wave suffers during transmission, and which is absent on the non-dissipative or distortionless line. The foregoing discussion of the wave solution holds for all types of applied voltages except that the character of the distortion of the wave during transmission is modified in accordance with formula (**11**). It is also applicable to all types of terminal arrangements, due regard being had to the fact that the form of the reflected waves is modified by the character of the terminal apparatus. In no case, however, is the velocity of transmission of the series of waves modified by the type of applied voltage or character of the end terminations.

Now in accordance with the method of derivation of formula (**a**) it must be equivalent to the expansion (**40**) (setting $G = o$) and this expansion will accordingly be investigated. After easy simplification formula (**40**) with G put equal to zero may be written,

$$A_x(t) = 2\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\cdot\sum_1^{\infty}{}_m\,\frac{\cos\left[m\cdot\frac{x\,\pi}{s}\right]}{s\sqrt{\left[m\,\frac{\pi}{s}\right]^2-\left[\frac{\rho}{v}\right]^2}}$$

$$\cdot\sin\left(v\sqrt{\left[m\,\frac{\pi}{S}\right]^2-\left[\frac{\rho}{v}\right]^2 t}\right)+\frac{1}{R\,s}\left(1-\epsilon^{-\frac{R}{L}r}\right) \qquad (\mathbf{b})$$

$$=\sqrt{\frac{C}{L}}\,\epsilon^{-\rho t}\,\Sigma\,m$$

$$\left\{\begin{aligned}&\sin\left(m\,\frac{v\,\pi}{s}\sqrt{1-h^2_m}\left[t-\frac{x}{v\sqrt{1-h^2_m}}\right]\right)\\+&\sin\left(m\,\frac{v\,\pi}{s}\sqrt{1-h^2_m}\left[t+\frac{x}{v\sqrt{1-h^2_m}}\right]\right)\end{aligned}\right\}$$

$$\frac{1}{\pi\,m\sqrt{1-h^2_m}}+\frac{1}{R\,s}\left(1-\epsilon^{-\frac{R}{L}t}\right) \qquad (\mathbf{c})$$

where $h_m = \frac{1}{m}\,\frac{\rho\,s}{\pi\,v}$

This type of solution is often referred to or regarded as a wave solution; in fact, however, it is a formulation in terms of normal or characteristic *vibrations* and in a more precise terminology is a normal function or vibrational solution. Each term of (**b**) is the product of a cosine function of space and a sine function of time which may be thrown into the form of (**c**) in which the individual terms appear to represent propagated waves. The propagation is only apparent, however, since each term of (**c**) is instantaneously produced on the spot at $t = o$ and is not *propagated* as are the component waves of the wave solution (**a**). The instantaneously produced terms of (**c**), however, combine for a definite time x/v to annul each other so that the summation is identically zero until $t = \frac{x}{v}$

the solution is therefore mathematically correct but physically artificial and deceptive. The most serious confusion of thought results from a tendency to interpret the summation (**c**) from a consideration of a single term and from the prevalent error of regarding the individual characteristic *vibrations* as *propagated waves.* Thus by inspection of (**c**) each term has an *apparent* velocity of phase propagation $v\sqrt{1-h^2_m}$ whereas the actual velocity of propagation of the wave is identically v. From this viewpoint the velocity of propagation of the vibrations *appears* to increase with the frequency of the vibrations up to the limiting value v. $v\sqrt{1-h^2_m}$ is merely the *apparent velocity of phase propagation* of the characteristic vibration and only in the case of the non-dissipative and distortionless line (where $h_m = o$) is the apparent velocity of phase propagation equal to v which is the true "speed of propagation of the electric phenomenon in the circuit." The increase of $v\sqrt{1-h^2_m}$ with the frequency of the characteristic vibration does not indicate an increased speed of propagation with frequency but merely a *change of form* or *distortion* of the wave during transmission.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 21, 1919.

Copyright 1919. By A. I. E. E.

TELEPHONE CIRCUITS WITH ZERO MUTUAL INDUCTION

BY WILLIAM W. CRAWFORD

ABSTRACT OF PAPER

The paper deals with the reduction of inductive interference in telephone circuits.

Various relative positions of two or more circuits, in which the mutual inductance is zero, and the mutual capacitance unbalance is approximately zero, are discussed. The most important case is that when the two wires of one circuit occupy opposite ends of one diagonal of a square, and the other circuit, the ends of the other diagonal. A phantom formed on circuits so related is nearly immune from external induction.

Several forms of construction embodying this arrangement, and built largely with standard parts, are illustrated. In these forms, about one third of the inductive relations of each physical circuit are with circuits in a position for approximately zero mutual inductance. The form of phantom transposition used automatically balances the circuits not so related.

Calculations and tentative designs are presented to show that the use of these forms of construction will give greatest refinement of balance against induction from power circuits, and possibly also against cross-talk, increased flexibility in coordinating with the variations in exposure to power circuits, a simplification of the transpositions system, fewer transposition poles and transpositions, and when desired, the realization of a part of these advantages with the lead compressed into less than the normal space. The improvement is greatest in the phantoms.

Adverse factors are the increased sensibility to cross-talk due to variations in sag, and the slightly increased capacitance of the phantoms.

The cost of initial construction of the proposed forms should not be materially higher than for the present standard form. The cost of conversion of an existing lead to one of the proposed forms will be considerably greater than that of re-transposing the lead according to present practises.

I. GENERAL

THE method of transposition as applied at present in telephone practise involves balancing against each other fairly large inductive effects in adjacent sections of the circuits. Mr. Harold S. Osborne has in a recent paper[1] named various influences which render it impossible to make the opposing effects completely annul each other. By proper design the residual unbalances are ordinarily reduced to a point which gives satisfactory operation. The design of the transposition system is, however, subject to severe limitations in the case where the telephone line is to parallel electric power circuits. Particularly, it is difficult to obtain proper co-ordination between the transpositions in the telephone circuits and the points where changes take place in the degree of exposure to power circuits.

There are certain relative positions of two or more circuits composed of parallel wires, in which the electrostatic and electromagnetic co-efficients of mutual induction are continuously zero. Between circuits so positioned not only should the residual effects disappear, but there should be no need for transposition. It seems worth while to inquire how far the employment of such arrangements will go toward giving greater refinement of balance, increasing the flexibility of design, and rendering usable for telephone leads locations which are not permissible in present practise. The idea involved is a very old one[2], and the aim of this paper is to show that it has certain merits in connection with modern practise which it did not have before the advent of the phantom circuit.

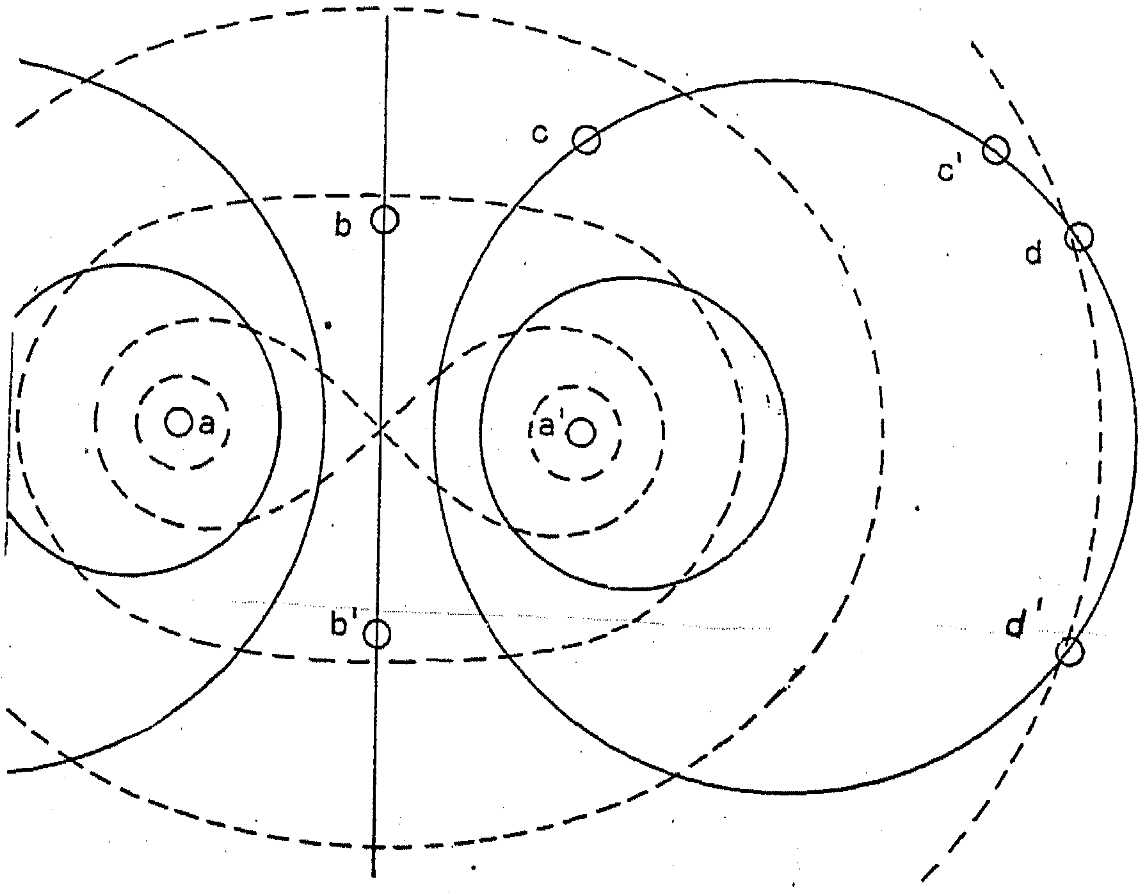


Fig. 1

In Fig. 1 the solid lines represent the electrostatic equipotential surfaces due to equal and opposite charges on the

---

1. Trans. A. I. E. E., 1918, p. 897.
2. See A. V. Abbott, Electrical Transmission of Energy, 1898, Art. 52.

parallel wires $a\,a'$. Placing the two wires of the circuit $c\,c'$ in the same equipotential surface as shown, subjects both wires to the influence of the same potential, there is no tendency to induce a current around the circuit $c\,c'$. If, however, a and a' have like charges, the equipotentials are as shown by the dotted lines, c and c' are unequally affected.

A like situation arises with respect to electromagnetic induction. The lines of magnetic force due to a current out a and back a', or vice versa, follow the same curves as shown by the solid lines in Fig. 1. For such currents there is no mutual inductance between $a\,a'$ and $c\,c'$. But for currents travelling in the same direction along a and a', the lines of force follow the dotted curves, they link between c and c', and induce a current around that circuit.

The voice currents in the physical circuits travel around the circuits, but the phantom voice currents, and some classes of stray currents due to induction from outside sources, travel along both wires in the same direction. Therefore the arrangement of Fig. 1 would tend to eliminate cross-talk between the physical circuits $a\,a'$ and $c\,c'$, but not cross-talk between these circuits and a phantom set up on them, nor the tendency of stray lengthwise currents in one physical circuit to render the other physical circuit noisy. The latter effects are removed by transposing both circuits alike.

The same is more or less true of every mutually non-inductive relative position of the two circuits except that shown by $a\,a'$ and $b\,b'$, Fig. 1. Wires $b\,b'$ lie in the same electrostatic equipotential, or link zero flux, for either like or unlike charges or currents on $a\,a'$. The circuit $a\,a'$ is also balanced for currents or charges in $b\,b'$. A circuit in the position $d\,d'$ is balanced with respect to $a\,a'$, but $a\,a'$ is only partially balanced with respect to $d\,d'$.

If a phantom is set up on the circuits $a\,a'$ and $b\,b'$ the, phantom is very nearly immune from external fields. The average effect of an external field on wires a and a' will approximately balance that on wires b and b', unless the field is very non-uniform. There is, moreover, no mutual induction between the phantom and its own side circuits—a marked contrast to present standard practise.

The above statements are inaccurate to the extent that the affected wires may have unequal capacities to ground, so that equal potentials induce unequal charges, and that the electrostatic field is distorted by the presence of other wires on the lead, which behave as if grounded[3]. Consequently, there is no single relative position which will in all cases give perfect balance for both electrostatic and electromagnetic effects. Such transpositions as are necessary for other reasons, are sufficient to eliminate also these effects.

II. Application to Complete Leads

The practicability of applying the proposed group arrangement in practise will be discussed in terms of the forms of construction shown in Figs. 2 to 9 inclusive.

In form *A*, Fig. 2, the cross-arm spacing is made equal to the pin spacing. Wires 1 and 12 form a physcial circuit, as do the wires 2 and 11, all the other wires are arranged in similar groups. There is a considerable mutual induction between wires 1-12 and 4-13, also between 2-11 and 3-14. Between 1-12 and 3-14, 5-16, etc., there is an approach to the same relation as between $a\,a'$ and $c\,c'$, Fig. 1.

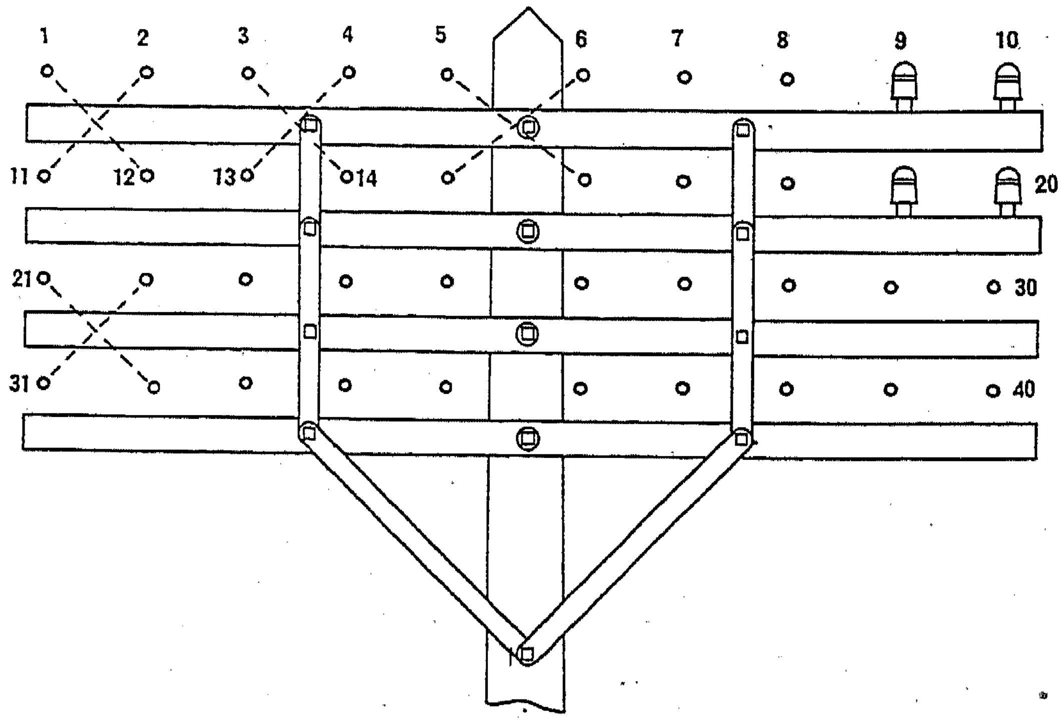


Fig. 2—Form *A*

There being no necessity of transposing the two physical circuits of a group relatively to each other, each group is treated as a unit in transposing relatively to the other groups. Special insulators or pins are not necessary to make a transposition. The wires are merely rotated around the quadrangle, for example, 1 replacing 2, 2 replacing 12, 12 replacing 11, and 11 replacing 1. For a physical transposition this rotation is continued in two spans, whereupon the two wires of each circuit are interchanged, see Fig. 3.

---

[3] Loc. cit. page 925.

As shown in Table III, there is some mutual inductance between phantoms, though this is smaller than it is in the present system, and falls off much more rapidly with distance. The amount is sufficient to require transposition of the nearest phantoms. For a phantom transposition, the wires are rotated in only one span. This method transposition is the same as that now used in polyphase power circuits.

At the center of the span, the distance between the wires is reduced to $1/\sqrt{2}$ of its value at the poles, see dotted square in Fig. 3.

If existing crossarms are used to construct form *A* or *C*, the central group is not quite equilateral, there being a greater space between the pole pins. The chief effect of this distortion is to introduce into this one group a mutual inductance between the side circuits, about equal to that between a phantom and each of its side circuits in present practise, see Tables I and III. The phantom transpositions, by interchanging the pin positions of the side circuits, balance out this mutual inductance. Considerations of type unbalance between the side circuits may require transposing the phantom of this group to a pattern having more frequent transpositions than would be required for the benefit of the phantom alone.

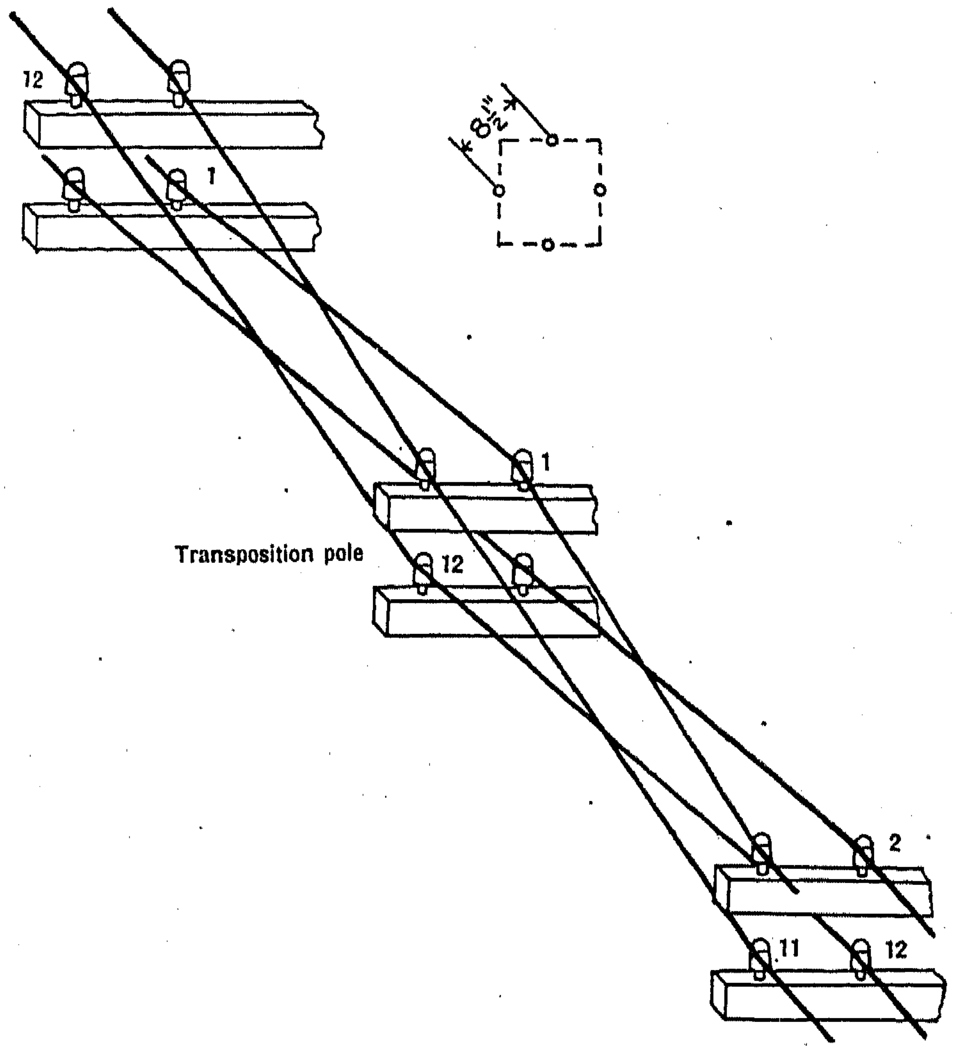


Fig. 3—Physical Transposition

In form *B*, Fig. 4, the use of present standard crossarm spacing, as well as standard pin spacing, is contemplated. The only alteration required to convert a present type lead into form *B* would be to re-transpose, and re-connect the jumpers at the ends of the line. The mutual inductance between the side circuits, and the susceptibility of the side circuits to external induction appear, however, to be prohibitive, see Tables II and VII.

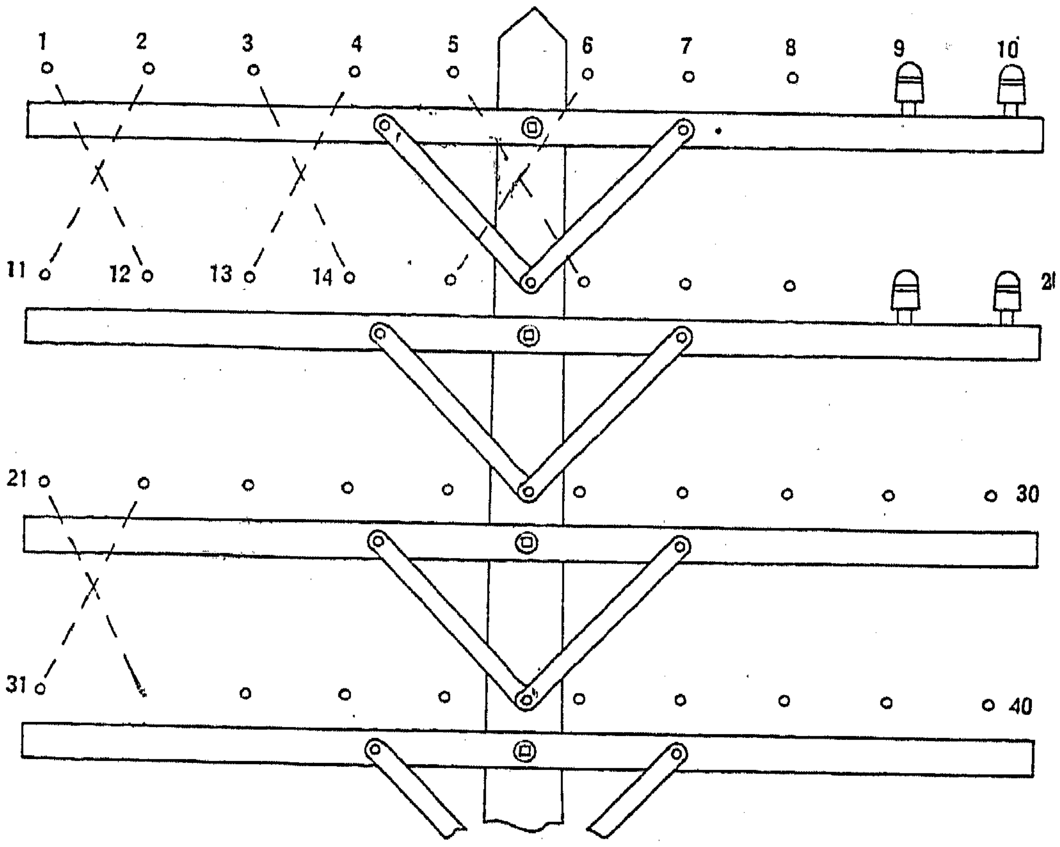


FIG. 4—FORM *B*

Form *C*, Fig. 5, resembles form *A* except that the distance between the various horizontal rows of groups is increased, giving electrical characteristics somewhat better than form *A*, and better accessibility of the wires.

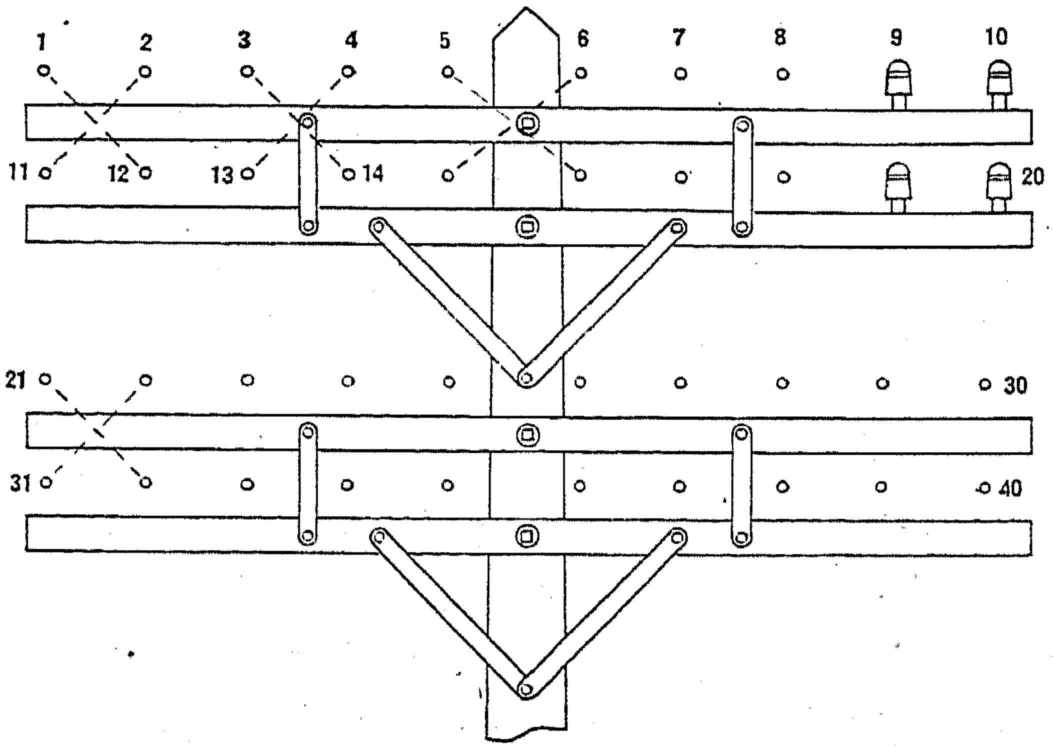


FIG. 5—FORM *C*

In form *D*, Fig. 6, the aim is to provide still better conditions as to freedom from induction. The unsymmetrical center group is removed, and the spacing between the other groups is increased.

Form *E*, Fig. 7, is obtained by adding to a lead, constructed in accordance with present practise, four pairs of wires per crossarm, the added circuits being supported in vertical planes on special duplex pins or brackets. The necessity of having the lower wires clear the crossarm makes the groups unsymmetrical, but without introducing mutual inductance between the side circuits. The other inductive relations are affected as shown in Table I. The pole-pin wires may be reserved for non-phantomed circuits, or phantomed by the method now in use.

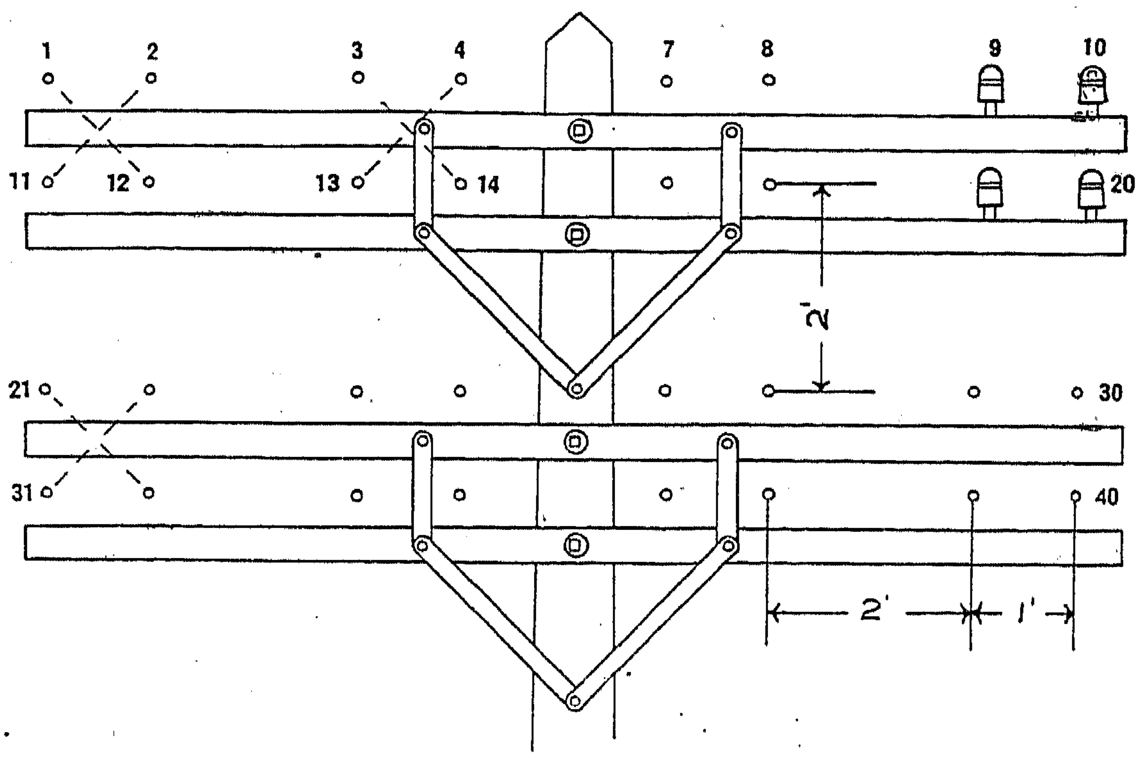


Fig. 6—Form *D*

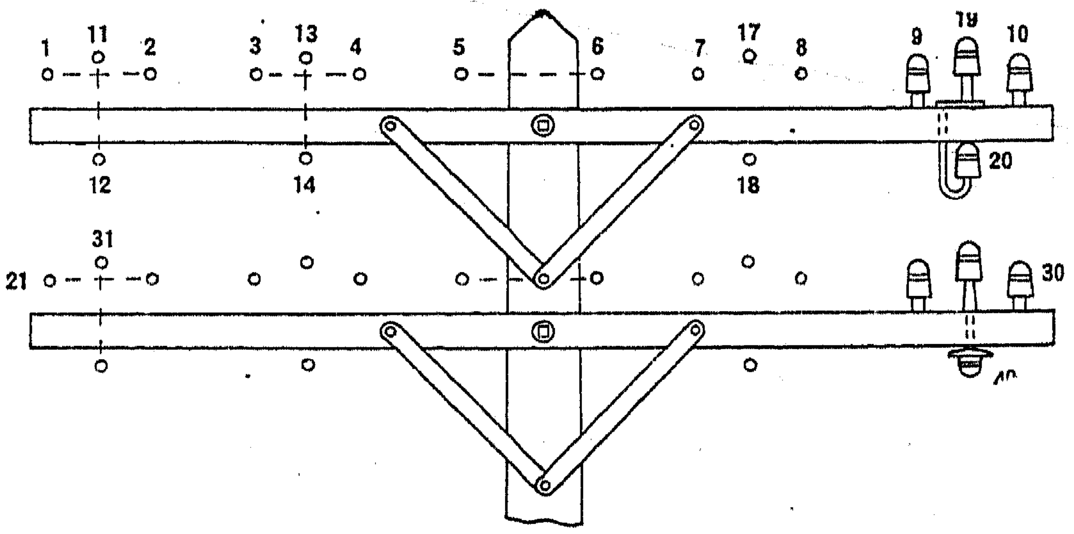


Fig. 7—Form *E*

One or more form *E* groups may be incorporated in a lead which is otherwise standard, giving proper attention to the co-ordination of the transpositions of the two different styles of construction. The form *A* group may also be constructed with duplex pins or brackets, in which case it becomes feasible to use this form in the same way as the form *E*.

Form *F*, Fig. 8, is an idealised form *E*, in which the groups are symmetrical. Form *G*, Fig. 9, bears the same relation to form *F* that form *D* does to form *A*. On account of the small clearance between wires in form *F*, and *G*, standard crossarms cannot be used, and some special form of construction would have to be devised.

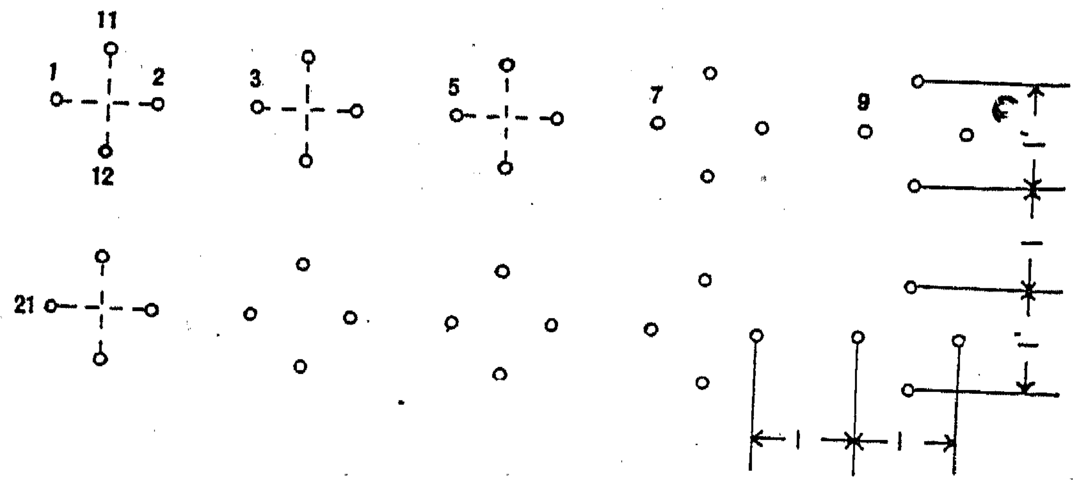


Fig. 8—Form *F*

Certain other designs, in which the form *A* group is alternated with the form *E* group, or in which the different horizontal rows of groups have different spacings, give zero mutual inductance between some of the adjacent phantoms. The possibilities presented by these designs have not yet been fully worked out.

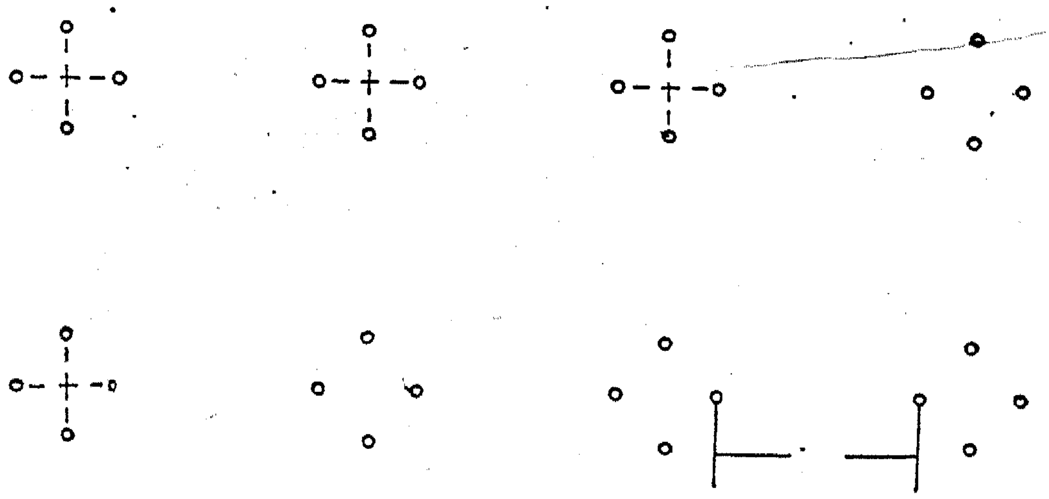


Fig. 9—Form *G*

III. Comparisons

In comparing the susceptibility to induction of the various forms it has unfortunately been possible to take account only of the mutual inductance, and not of the capacitance unbalances, because of the great labor required to compute the latter[4]. Probably the two effects will vary nearly enough in the same proportion, as between different forms, so that the comparisons on a basis of inductance alone, if used with due caution, will not give false conclusions. Values of mutual inductance are given in Tables I, II, and III.

---

[4] Loc. cit. page 902.

In form *A*, the maximum value of mutual inductance (that between circuits related like 1-12 and 4-13) is about equal to that between a phantom and one of its own side circuits, or between two vertically adjacent horizontal type phantoms, in present practise. Each physical circuit other than the pole-pin circuits in form *A* has two or three such relations. The pole-pin circuits have one additional large value. In the standard form, the large value comes only once into each physical circuit, but three or four times into each phantom, except the pole-pin circuits and phantom. Form *A* therefore decreases the induction between phantoms at the expense of the physical circuits.

In form *C*, the maximum value of mutual inductance is the same as in form *A*, but this value occurs less frequently.

Forms *D*, *F*, and *G* give on the whole a very great reduction in mutual inductance. Forms *F* and *G* are interesting from the large number of values of zero mutual inductance which they afford. Most of these are due to the relation $a\,a'$ to $d\,d'$, Fig. 1.

Form *E* has a large value of mutual inductance, about equal to that in the standard form, which comes once into each phantom, and once each into half the side circuits.

A summation was obtained of the inductive effects on each circuit of all the other circuits (Table IV and V), showing that the phantoms in the proposed forms are on the average about one tenth as sensitive to cross-talk as are the side circuits. In the standard form the phantoms are twice as sensitive as the side circuits. Although the sensitivity of the side circuits is increased in form *A*, the reduction of the sensitivity of the phantoms is sufficient to make the average for all circuits less than in the standard form, despite the fact that in form *A* a given number of circuits are compressed into half the normal space. Such a compression of the standard form would result in considerable increases in the mutual inductance values, with consequent increase of the sensitivity to cross-talk. In form *C* the sensitivity of the side circuits is about the same as in the standard form, the improvement of the phantoms is clear again. In the other forms, the sensitivities of side circuits as well as phantoms, are decreased.

Corresponding figures (Table VII) for the sensitivity to induction from power circuits show a similar result. In forms *A*, *C*, and *D*, the sensitivity of the side circuits is increased because in these forms the distance between the two wires of a side circuit is $\sqrt{2}$ times what it is in present practise. The sensitivity of the phantoms is, however, negligible, except when the disturbing circuit is very close to the telephone circuit.

On account of the shorter average distance between the wires in a group in the proposed forms, it is to be expected that the effect of accidental displacements of the wires from their correct positions will be greater than in standard practise. The unbalance produced will in each case be greatest between the circuits forming one group, and the comparison is made only on the basis of such circuits. (Table VI).

For horizontal displacements, the various forms have nearly the same sensitivity as standard construction. Horizontal displacements should be rare, because the horizontal spacing is fixed by the pin locations.

The effective vertical spacing depends on the sag, and this is likely to vary. For displacements as small as the value assumed for comparison (root-of-mean-square displacement, 0.1 foot, or 0.03 meter) the effect is practically nil in the standard form, but in all the proposed forms, it is appreciable. This is a distinct disadvantage, the importance of which can be determined only by the aid of data from actual construction. Comparison with the values in Table V appears to show that unless the r. m. s. value of displacement is considerably greater than the value assumed, the total unbalance due to these displacements will not be more serious than the unbalance which occurs in standard practise due to the transpositions being located at the nearest poles, instead of at the theoretically correct points. The tendency of the proposed forms being to reduce the effect of inexact location of the transpositions, the total effect of all irregularities should not be much greater than in present practise.

Keeping the crossarms carrying wires in the same groups accurately parallel is evidently essential in forms *A*, *C*, and *D*, the method of bracing illustrated in these forms is intended to provide for this.

The effect of displacements of the wires on the induction from power circuits is the same in the proposed forms as in the standard form.

Number of Transpositions and Number of Transposition Patterns. Certain comparisons are fairly obvious between the number of transposition patterns (types) and the total number of transpositions required in the standard and proposed forms.

The condition of zero mutual induction between a phantom and its side circuits does not mean that the three circuits can be transposed after the same pattern. If that were done, each time a side circuits reversed, it would move to the other pin position, for a given pin position it would always have the same polarity, it would not be balanced with respect to external induction. This fact is exemplified in the standard *E*-section[4] in circuits 5-6 and 15-16, which have zero mutual inductance with the phantom, nevertheless, they are transposed to different patterns from the phantom (Table IX).

If each circuit is to have its own transposition pattern, thirty patterns are required for the commonly used physical and phantom circuits on the first four arms in the standard form. In the proposed forms, each pair of side circuits being transposed as a unit, only twenty patterns would be required.

In the *E*-section, 16 types of transposition pattern are used, and mutual inductance and capacitance unbalances as large as 5.44 microhenries per kilometer, and 3.0 micro-microfarads per kilometer, respectively, are allowed. Adopting these limits for the proposed forms permits a still further reduction in the number of patterns, and in the number of transpositions, required.

The effect of simultaneous phantom transpositions in two groups is to reverse the mutual inductance between the circuits whose planes are at right angles, like 1-12 and 4-13, Figs. 2, 5, or 6. Only the circuits related like 1-12 and 3-14 are left unbalanced, and the mutual inductance between these is small. Whenever the mutual inductance of two phantoms is small enough so that they can be transposed to the same pattern, the same is true of the physical circuits.

Fig. 10 shows a transposition system which utilized this fact. Four types of group transposition are used, denoted by I, II, III, IV. There are only sixteen transposition poles per section. The procedure in design was as follows:

Physical transpositions were inserted in all the groups at the odd-numbered transposition poles, balancing the physical circuits for outside induction every eighth section. Next, phantom transpositions were inserted at the poles 2, 6, 10, and 14. This balances the phantoms every quarter section for sources of induction which are too near for the phantoms to be immune due to their form. As already stated, these phantom transpositions greatly reduce the unbalance between physical circuits. Poles 4, 8, and 12 remain for balancing out the induction between phantoms and the residual induction between physical circuits in the four groups. Only two phantom types are used, leaving the diagonally adjacent phantoms untransposed. It is then possible to obtain complete balance of the four adjacent pairs of physical circuits. There are several ways of doing this, one of which is shown. It is also possible to balance the four phantoms, leaving a slight residual unbalanced in the diagonally adjacent pairs of physical circuits.

---

[4] Loc. cit. page 911, Fig. 5.

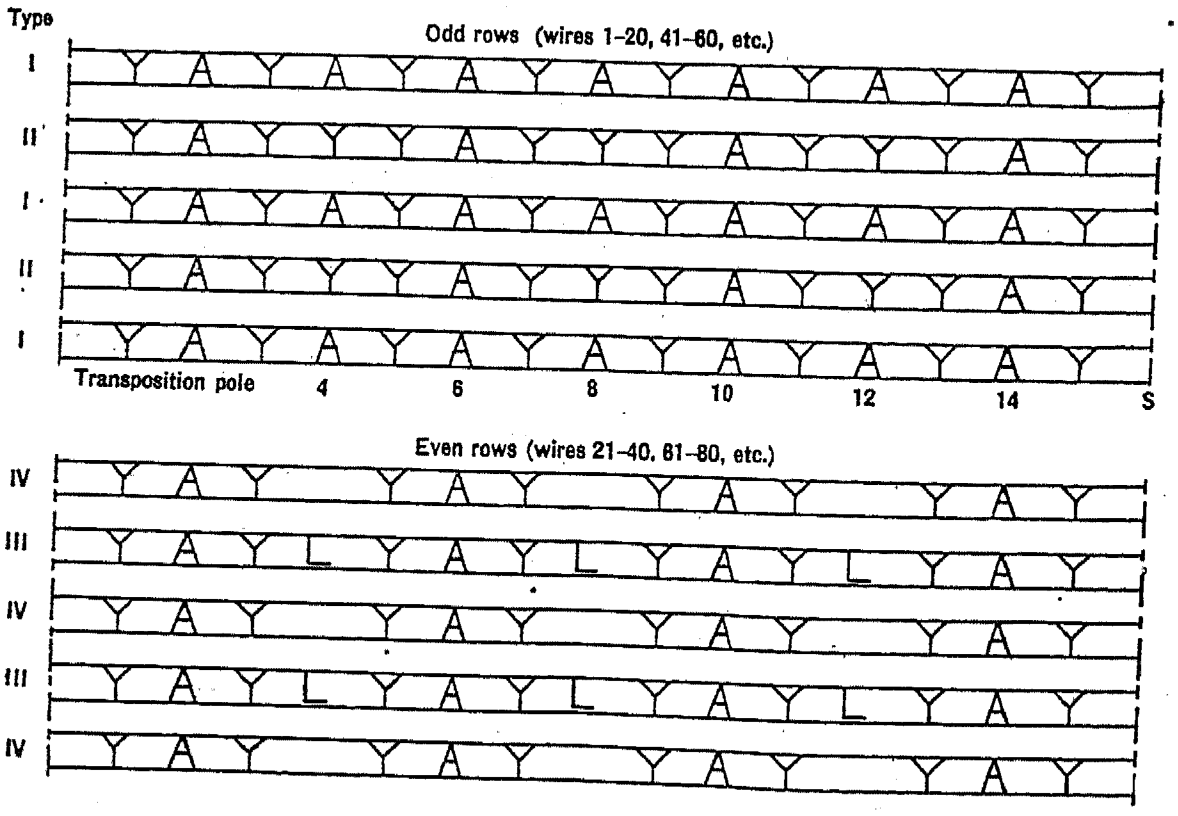

Type
Odd rows (wires 1-20, 41-60, etc.)
I
II
I
II
I
Transposition pole
4
6
8
10
12
14
S
Even rows (wires 21-40, 61-80, etc.)
IV
III
IV
III
IV

Fig. 10

Y—physical transposition
A—phantom transposition, right-handed
L—phantom transposition, left-handed

The largest values of unbalanced mutual inductance in Fig. 10 are as follows:

Form	*A*	*C*	*D*	*F*	*G*
Diagonally adjacent groups					
Phantoms	4.77	0.76	0.93	1.17	0.23
Alternate groups in the same horizontal row					
Phantoms	1.17	1.17	0.23	0.29	0.06
Physical circs.	1.55	1.55	0.31	0.40	0.08

Limits in the standard *E*-section
Inductance unbalance, 5.44, capacitance unbalance, 3.0

The inductance unbalance is within the limit even for form

A. The limiting value of capacitance balance in the *E*-section, however, occurs between circuits which are separated by a large number of intervening wires, the screening action of which has a large effect in reducing the capacitance unbalance. There will be very little screening action between the diagonally adjacent groups in the proposed forms, hence, in form *A* the capacitance unbalance between the diagonally adjacent phantoms will be much too large if Fig. 10 is used. In the other forms, however, the inductance unbalances are so far below the limit that it is probable that the capacitance unbalances are not excessive, and Fig. 10 can be used.

Fig. 11 shows a system in which the mutual inductance between the diagonally adjacent phantoms is eliminated. The method of design was the same as in Fig. 10, except that physical balance was obtained between horizontally and vertically adjacent groups at the outset, giving the necessary added flexibility to permit transpositions on poles 4, 8, and 12 to completely balance the four groups. In this case, the maximum unbalance is between alternate groups in the same horizontal row. Fig. 11 is apparently suitable for use with form *A*. By using a slightly larger number of transpositions, with the same number of transposition poles, non-adjacent groups can be completely balanced if desired.

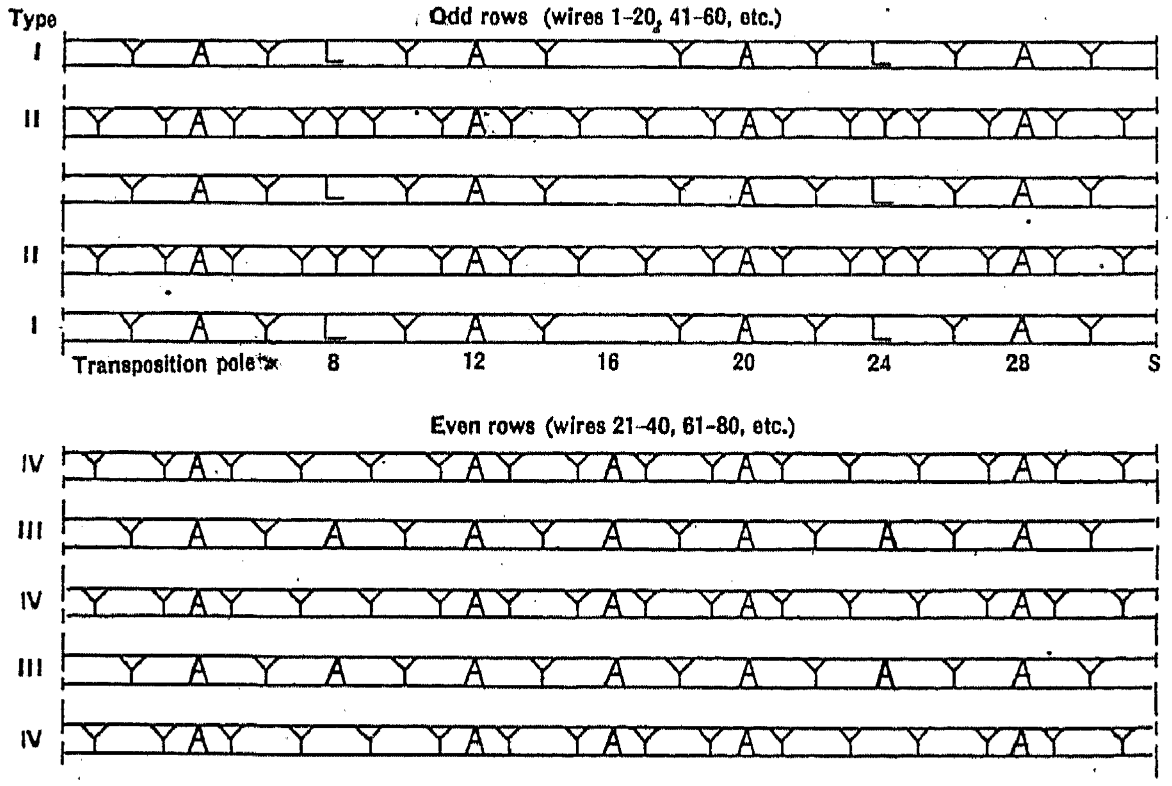

Type
Odd rows (wires 1-20, 41-60, etc.)
I
II
II
I
Y A Y L Y A Y Y A Y L Y A Y
Y YAY YYY YAY Y Y YAY YYY YAY Y
Transposition poles 4 8 12 16 20 24 28 S
Even rows (wires 21-40, 61-80, etc.)
IV
III
IV
III
IV
Y YAY Y Y YAY YAY YAY Y Y YAY Y
Y A Y A Y A Y A Y A Y A Y A Y

FIG. 11

Y—physical transposition
A—phantom transposition, right-handed
L—phantom transposition, left-handed

The extension of Figs. 10 and 11 to more than forty wires is a matter only of repeating the same system, additional transposition poles are not necessary.

These designs were not obtained by a systematic study of all the possibilities, but rather, by a logical process from the assumed conditions, to which method the proposed form of construction appears to be favorable. The designs are of a tentative nature, and are intended merely as illustrations of the possibilities of the proposed forms.

In standard practise, a distinction is drawn between the effect of distant and near disturbing circuits, on account of the effect of the phantom transpositions in shifting the pin positions of the physical circuits. In Figs. 10 and 11, this effect does not need to be considered, since each section of physical circuit between two phantom transpositions is completely balanced for external induction. The distinction between near and distant circuits is in the proposed forms based on the phantoms, the balance of which with respect to distant circuits is not dependent on transpositions.

COMPARISONS OF DESIGNS FOR TRANSPOSITION SECTIONS

	Standard *E*-section	Proposed	
		Fig. 10	Fig. 11
No. of transposition poles per section			
First 40 wires	32	16	32
Additional wires	64	16	32
No of transpositions per section, 40 wires	288	141	179
Wires cut	876	564	716
Balanced to distant non-transposed circuits, every	Eighth-section.	Eight-section.	Eighth-section.
	Eighth-section.	Eighth-section.	Eighth-section.
Balanced to near non-transposed circuits, every	Half-section.	Quarter-section.	Quarter-section.
Balanced to distant 3-phase power circuits for	Two successive half-mile barrels. 3, 6, or 12-mile barrels.	Single half-mile barrels. 3, 6, or 12-mile barrels.	Two successive half-mile barrels. 3, 6, or 12-mile barrels.
Balanced to near 3-phase power circuits for	Quarter-mile barrels, 12-mile barrels.	Quarter or half-mile barrels, 6 or 12-mile barrels.	Quarter-mile barrels, 6 or 12-mile barrels.
Neutral points for discontinuities of telephone lead.	*S*–poles	*S*–poles	*S*–poles, half-section points.

Balance for a half-section, equal to Fig. 10 for a whole section.

The permissible locations for discontinuities of the telephone lead depend a good deal on the form of construction. For example, a form *D* lead transposed by Fig. 10 or 11 could be discontinued at a half-section, or even at a quarter-section point, with only a very slight resultant unbalance.

Provision for Non-Phantomed Circuits. Where non-phantomed circuits occur in pairs, they may be treated in the same manner as phantomed circuits, except, possibly, in the case of circuits which have transmitters of large output, in which the unbalance due to sag variations may make it undesirable to associate the circuits in groups of the proposed form. Each single non-phantomed circuit will have to occupy the space appropriate for a group. If there are only a few such circuits, the loss of space will be small, for a larger number, one or more crossarms of standard construction may be incorporated in the lead.

Effects of the Constants of the Circuits. The capacitance of the phantoms is about 30 per cent greater in the proposed forms than in present practise, and the inductance correspondingly less. The change in the constants of the side circuits is very slight.

Cost. On account of the necessity of re-spacing the crossarms, or of providing special insulator supports, the cost of converting from the standard form to one of the proposed forms is evidently in general considerably greater than that of re-transposing a standard form lead. The cost of converting to form *E* with an increased number of circuits may in some cases be less than that of providing the additional circuits by the standard method.

For initial construction of forms *A* or *C* there would apparently be no increased cost for materials, and a saving in the cost of transposing. Form *D* presents a slight increased cost of materials, due to the increased number of crossarms per circuit-mile.

IV. CONCLUSIONS

The anticipation that the use of the proposed forms of telephone line construction and transposition under exposed-line conditions will give greater refinement of balance and increase the flexibility in co-ordinating with transpositions and discontinuities of the disturbing circuits, particularly, when the telephone circuits are phantomed, appears to be justified.

The proposed forms appear to be especially well adapted for use on joint pole lines and in other cases of severe or irregular exposure. Obviously, they affect only the particular phases of inductive interference for which transposition is a remedy. The worst effect of severe exposure to power circuits is that of the high longitudinal electromotive forces and currents induced in the telephone circuits when an adjacent power circuit operates normally with a ground return, or is accidentally grounded[5]. The increased balance of the open-wire circuits afforded by the proposed forms should permit operating with larger values of longitudinal currents than in the standard form, except where the limiting value of the current or e. m. f. is determined by conditions of safety, or by the balance of the terminal apparatus.

Present-type phantoms are often used as parts of longer lines, despite the fact that they are considerably more susceptible to noise and cross-talk than are the physical circuits, and that a given amount of noise is of greater importance in the longer circuit. The proposed forms in which the phantoms are relatively immune from disturbances, bring the desirable qualities of noiselessness and good transmission together in the circuits where they are most needed.

Form *A* is for use in new construction where space limitations are extreme.

Form *C* permits an appreciable saving of space, with a good degree of balance, a saving in the number of transpositions and transposition poles, and increased flexibility in the location of discontinuities in power circuits, as compared to the standard *E*-section.

Form *D* gives a considerable reduction of type unbalance and unbalance due to unequal transposed lengths, with a slight increase of cost, as compared to form *C*.

Partial or complete conversion to form *E* recommends itself when it is desired to route one or more high-grade phantom circuits through locally adverse conditions, or where it is desired to add a few circuits to the lead under conditions of limited space. Partial conversion to form *A*, *C*, or *D*, may also be effected; in these forms, the phantoms have better balance than in form *E*. Such conversion is of course conditional on the ability of the crossarms to support the additional weight.

Forms *F* and *G* promise the highest degree of balance of all the forms shown, but would probably not be considered in the present state of the art on account of the special construction required.

---

[5] "Transposition and Relative Location of Power and Telephone Wires," P. M. Lincoln, TRANS. A. I. E. E., 1903.

"Inductive Effects, etc.," H. S. Warren, TRANS. A. I. E. E., 1918, p. 503.

V. NUMERICAL VALUES AND METHODS OF CALCULATION

a. *Mutual Inductance.* The calculations were made on the basis of a spacing between the pole pins equal to that between the other pins, except in forms *D* and *G*, where the distance between groups was made uniform. The order of magnitude of the differences due to these assumptions is indicated in Table I. Where the differences are large enough to affect the comparisons appreciably, allowance has been made for them.

From the values given in Tables I, II and III, the mutual inductances of most of the other pairs of circuits can be found, by determing to which of the circuits given in the table, their relative location is similar.

TABLE I

EFFECT OF ASSUMPTIONS AS TO SPACING

Circuits	Mutual inductance, microhenries per kilometer	
	Actual	Assumed
Standard Form		
1–2 and 5–6	−16.00	−12.92
1–2 and 9–10	−2.92	−3.16
Forms *A* and *C*		
5–16 and 6–15	−115.08	0
Phants. 3–14–4–13 and 5–16–6–15	−19.38	−17.86
Phants. 3–14–4–13 and 7–18–8–17	−0.84	−1.17
Forms *E* and *F*	Form *E*	Form *F*
1–2 and 13–14	−15.58	0
Phant. and its own vertical side	122.38	0
Phants. 1–2–11–12 and 3–4–13–14	9.87	4.77
Phants. 1–2–11–12 and 21–22–31–32	−0.40	4.77

b. *Summated Effects of Unequal Transposed Lengths and Type Unbalance.* The summation of these effects was obtained as follows:

Assume a given circuit to be affected by equal values of current I and equal unbalanced lengths l in all the other circuits. The voltages induced will be

$$E_2 = -jM_2\,\omega_2\,I_2\,l$$
$$E_3 = -jM_3\,\omega_3\,I_2\,l$$
etc.

and if the frequencies are nearly equal, the energies induced in the first circuit by the various other circuits will be proportional to

$$E_2^2,\ E_3^2,\ \text{etc.}$$

The total energy will be proportional to ΣE^2, or, since $j I \omega l$ is assumed to be constant, to ΣM^2. ΣM^2 is therefore a measure of the total sensitivity of the circuits to electromagnetic induction from the other circuits. For a complete result, of course, a like value for electrostatic induction should be obtained.

TABLE II

MUTUAL INDUCTANCES BETWEEN PHYSICAL CIRCUITS

Microhenries per Kilometer.

Form of construction Present		Proposed A		B	D	F		G
Effect on physical circuit 1–2		1–12		1–12	1–12	1–2		1–2
Of Circ.	M	Of Circ.	M	M	M	Of Circ.	M	M
3–4	−57.54	2–11	0	277.3	0	11–12	0	0
5–66	−12.92	3–14	22.31	140.2	4.82	3–4	−57.54	−23.55
7–8	−5.64	4–13	102.17	−81.1	44.63	13–14	0	0
9–10	−3.16	5–16	1.55	38.7	....	5–6	−12.92	−5.63
11–12	44.63	6–15	25.03	57.5	....	15–16	0	....
13–14	1.55	7–18	0.31	17.1	0.31	7–8	−5.64	−5.64
15–16	−5.92	8–17	11.11	60.4	11.11	17–18	0	0
17–18	−4.02	9–20	0.09	9.5	0.07	9–10	−3.16	−2.47
19–20	−2.61	10–19	6.26	15.3	4.93	19–20	0	0
21–22	12.12	21–32	22.31	−32.5	4.82	21–22	44.63	21.08
23–24	6.07	22–31	−102.17	−69.7	−44.63	31–32	0	0
25–26	4.03	23–34	−57.54	−57.5	−23.55	23–24	1.55	0.30
27–28	−1.54	24–33	0	−28.4	0	33–34	−25.04	−11.11
29–30	−1.49	25–36	−16.25	−25.6	....	25–26	−5.92	......
31–32	5.48	26–35	11.99	1.4	....	35–36	−8.01	....
33–34	3.98	27–38	−6.92	−9.3	−6.17	27–28	−4.02	−2.66
35–36	1.52	28.37	8.00	7.8	5.33	37–38	−3.00	−3.56
37–38	0.01	29–40	−2 72	−3.2	−2.66	29–30	−2.61	−1.78
39–40	−0.56	30–39	5.19	7.6	3.55	39–40	−1.38	−1.25
Effect on		2–11		....	2–11	11–12		11–12
		Of	M		M	Of	M	M
		26–35	15.69	....	....	19–20	3.12	1.78
		28–37	6.06	....	7.05	25–26	6.06	....
		30–39	2.81	....	2.67	27–28	....	5.48

Form *C*. First row like Form *A*. Second row:

Effect on	1–12		1–12		2–11	
	Of	M	Of	M	Of	M
	21–32	4.82	26–35	4.46	24–33	26.83
	22–31	−44.63	27–38	−7.17	28–37	7.05
	23–34	−30.15	28–37	5.33	26–35	14.84
	24–33	−11.78	29–40	−3.60	30–39	3.61
	25–36	−15.93	30–39	4.13		

ΣM^2 may be taken for each pair of circuits on the lead, and the average per circuit or per group taken as representing the whole system.

The induced e. m. f. or current due to type unbalance is, for a given relative transposition type, also proportional to the co-efficient of mutual induction. ΣM^2 may therefore be considered to represent in a measure the total effect of type unbalance, although the comparison will be affected to some extent by the practise of reducing the effect of large values of

TABLE III

MUTUAL INDUCTANCE BETWEEN PHANTOMS AND BETWEEN PHANTOMS AND PHYSICAL CIRCUITS

Microhenries per Kilometer

Form of construction Present		Proposed *A*	*D*	*F*	*G*		
Effect on phantom 1-2-3-4		1-12-2-11			1-2-11-12		
Of Phantom	*M*	Of Phantom	*M*	Of Phantom	Of Phantom	*M*	*M*
5–16	–60.02	3–13	–17.86	–3.64	3–14	4.77	0.93
7–10	–24.75	5–15	–1.17		5–16	0.29	
11–14	117.34	7–17	–0.23	–0.23	7–18	0.06	0.06
17–20	–18.88	9–19	–0.08	–0.04	9–20	0.03	0.01
21–24	41.35	21–31	–17.86	–3.64	21–32	4.77	0.93
25–36	–19.90	23–33	4.77	0.93	23–34	-1.17	-0.23
27–30	–5.34	25–35	0.21		25–36	–0.05	
31–34	20.38	27–37	–0.05	0.04	27–38	0.02	–0.01
37–40	2.28	29–39	–0.03	–0.01	29–40	0.01	0.003
Of Phy. Circ.		Of Phy. Circ.			Of Phy. Circ.		
1–22	109.86	1–12	0	0	1–2	0	0
3–4	109.86	2–11	0	0	11–12	0	0
5–6	–58.78	3–14	–29.39	–8.12	3–4	14.32	3.96
7–8	–28.77	4–13	-29.39	–8.12	13–14	0	0
9–10	–8.51	5–16	–3.31		5–6	1.62	
11–12	47.78	6–15	–3.31		15–16	0	
13–14	47.78	7–18	–0.95	–0.95	7–8	0.46	0.46
15–16	–7.66	8–17	–0.95	–0.95	17–18	0	0
17–18	–9.97	9–20	–0.40	–0.28	9–10	0.20	0.14
19–20	–6.54	10–19	–0.40	–0.28	19–20	0	0
21–22	19.28	21–32	–29.39	–8.12	21–22	0	0
23–24	19–28	22–31	29.39	8.12	31–32	–14.32	-3.96
25–26	5.45	23–34	14.32	3.96	23–24	–3.31	–0.95
27–28	–1.84	24–33	0	0	33–34	3.31	0.95
29–30	–3.02	25–36	1.78		25–26	0.18	
31–32	9.79	26–35	–2.45		35–36	-1.08	
33–34	9.79	27–38	0.18	0.54	27–28	0.22	0.06
35–36	5.42	28–37	–1.08	–0.75	37–38	0.32	0.32
37–38	1.38	28–40	–0.03	0.06	29–30	0.14	0.07
39–40	-0.62	30–39	–0.50	–0.32	39–40	0.12	0.10

M by more frequent transposition, which practise may be favored by one form of construction more than another.

TABLE IV

VALUES OF ΣM^2 BY CIRCUITS

$$\frac{\text{Microhenries}}{\text{Kilometer}} \div 10{,}000$$

Standard Form							
Wires	1–2		3–4	5–6	7–8		9–10
Sides	2.10		2.89	1.66	2.89		2.10
Phants		5.51		4.01		5.51	
Wires	11–12		13–14	15–16	17–18		19–20
Sides	2.54		3.39	1.87	3.39		2.54
Phants		7.51				7.51	

For the lower two cross-arms, use the same values in the reverse order.

Form *A*					
Wires	1–12	3–14	5–16	7–18	9–20
Sides	2.84	4.22	5.64	4.22	2.70
Wires	2–11	4–13	6–15	8–17	10–19
Sides	2.70	4.22	5.64	4.22	2.84
Phants	0.44	0.67	0.67	0.67	0.44

For the second row of groups, put 21–32 for 2–11, 22–31 for 1–12, etc.

Form *D*					
Wires	1–12	3–14		7–18	9–20
Sides	0.50	0.74		0.75	0.49
Wires	2–11	4–13		8–17	10–19
Sides	0.49	0.75		0.74	0.50
Phant	0.034	0.050		0.050	0.034

For the second row of groups, put 21–32 for 2–11, 22–31 for 1–12, etc.

Form *F*					
Wires	1–2	3–4	5–6	7–8	9–10
Sides	0.65	1.06	1.08	1.06	0.65
Wires	11–12	13–14	15–16	17–18	19–20
Sides	0.64	0.80	0.92	0.80	0.64
Phants	0.049	0.073	0.074	0.073	0.049

Second row of groups, same values.

Form *G*					
Wires	1–2	3–4		7–8	9–10
Sides	0.120	0.188		0.188	0.120
Wires	11–12	13–14		17–18	19–20
Sides	0.120	0.175		0.175	0.120
Phants	0.004	0.006		0.006	0.004

Second row of groups, same values.

The values of ΣM^2 in Tables IV and V are found for a 40-wire lead except in forms *D* and *G*, where 32 wires are assumed.

The values for form *C* in Table V were estimated by combining the values in Table IV for forms *A* and *D*.

The improvement in forms *F* and *G* over *A* and *D* respectively shown in Table V, is found to be due primarily to the reduced size of the square on which the wires are spaced. Like results could be obtained in forms *A* and *D* by a corresponding reduction in the minimum distance between wires.

c. *Effect of Accidental Variation in Spacing.* A given displacement of one wire in a group in general changes the mutual inductances between the side circuits and between each side circuit and the phantom. If M represents a change of mutual inductance, the sensitivity of the group to these effects may be expressed as $(\Sigma M)^2$, where the summation is taken over the three circuits and the two inductive relations for each.

TABLE V

Average $\Sigma M^2 \div 10{,}000$

Form	Physical circuits	Phantom circuits	Per group
Standard	2.52	6.01	11.05
A	3.92	0.58	8.42
C	2.82	0.42	6.06
D	0.62	0.042	1.28
F	0.83	0.0644	1.72
G		0.005	0.307

The effects of opposite directions of displacement are averaged. In the standard form, the effect depends on whether wire 1 or 2 is displaced, the values given are averages for displacements of the two wires.

In forms *E*, *F*, *G*, for the effect of displacements of wires 11–12, interchange columns 2 and 3. The average for vertical displacements of any one of the four wires is 0.238 per group.

The reasoning in arriving at the relative importance of sag variation and other sources of unbalance was as follows:

On the assumption that the displacements follow the probability law for errors, the resultant value of M for a considerable number of spans will be the square root of the sum of the squares of the values for the individual spans. The value of $(\Sigma M)^2$ for n spans will then be n times the average value for one span. Dividing the value of ΣM^2 for a group in the standard form, viz, 11.05 by the values of $(\Sigma M)^2$ in Table VI therefore gives the number of spans in which the displacement effect must occur to be the equivalent of one span of unbalanced length in the standard form. This is 91 spans for forms *A*, *C*, and *D*, and 46 spans for forms *E*, *F*, and *G*. In the *E*-section, the frequently-used transposition patterns have only 14 to 40 spans per transposition. At each of these transpositions there is a liability to one span unbalance, assuming that the transposition have not been set exactly at the correct points. In a long line these unbalances will add by the same law as the displacement unbalances and will produce a greater total unbalance than that due to sag variation unless the r. m. s. displacement is considerably larger than the value assumed. The value of $(\Sigma M)^2$ due to displacements of all four wires appears to be about the same as that for dis-

TABLE VI

EFFECT OF DISPLACEMENTS

Standard of Comparison, effect of displacing one wire 3 cm. (0.1 foot)

Values of $(\Sigma M)^2 \div 10{,}000$

Effect on circuit	Circuit of which one wire is displaced, marked Horizontal displacement	Vertical displacement
Standard Form		
1-2	0.043	Less than 0.001
3-4	0.015	"
Phant	0.034	"
Group	0.092	"
Forms *A*, *C*, *D*		
1-12	0.051	0.051
2-11	0.051	0.051
Phant	0.020	0.020
Group	0.122	0.122
Forms *E*, *F*, *G*		
1-2	0.040	0.158
11-12	0	0.198
Phant	0.040	0.040
Group	0.080	0.396

placements of only one wire, due to the large number of cases in which the displacements of two or more wires compensate each other.

The effect of turning corners is to reduce the horizontal spacing without affecting the vertical spacing. A 52 deg. turn (Fig. 12) is required to reduce the horizontal spacing by 10 per cent. in a whole span. The value of $(\Sigma M)^2$ due to this distortion is zero, within the group, for standard construction, and for forms *E*, *F*, and *G*. In forms *A*, *C*, and *D*, it is as follows:

Per physical circuits	0.177
Per phantom circuit	0
Per group	0.354

The effect is always in the same direction until a phantom transposition is reached, when it is reversed. Consequently the total effect varies as the square of the number of corners. There would have to be an unbalance equivalent to five or six 52 deg. turns of the style shown in Fig. 12, between successive phantom transpositions, in order to equal the unbalance of $\Sigma M^2 = 11.05$ which may occur in standard practise due to transposition poles not being located at the exactly correct points.

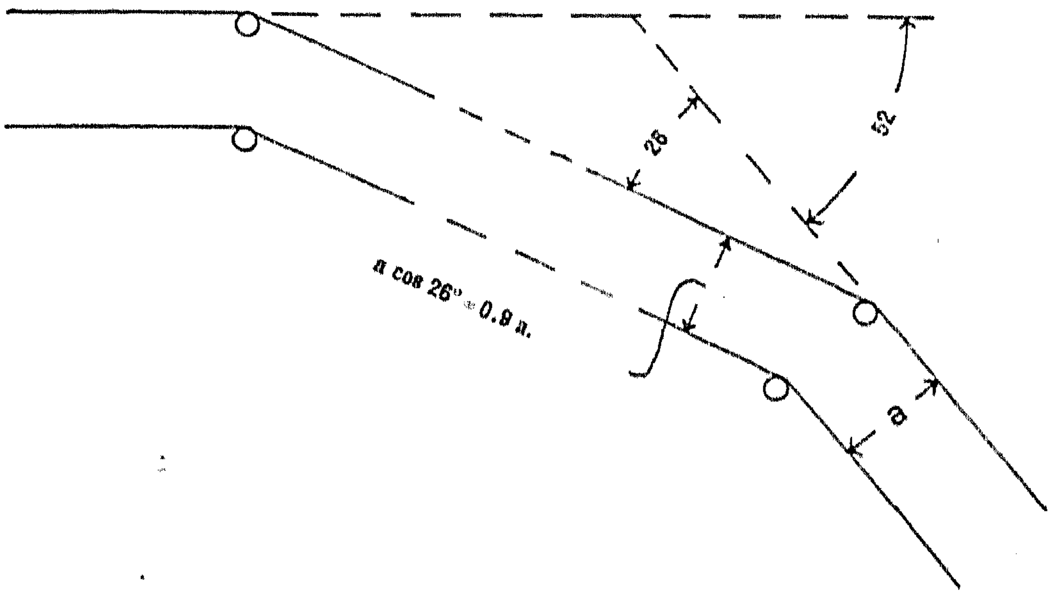


FIG. 12

d. *Induction from Power Circuits.* Following a reasoning analogous to that for cross-talk, the sensitivity of a given circuit to induction from a power circuit is taken as proportional to average E^2, where E is the voltage induced in the telephone circuit by a standard value of I in the power circuit, and the average is taken for every relative direction of the power circuit from the telephone circuit. Assuming a constant frequency, average E^2 is proportional to average ϕ^2, where ϕ is the flux through the telephone circuit due to unit current in the power circuit. For the effect of a single wire on a physical circuit,

$$\phi = 2\,\frac{a^2}{d^2}\cos\phi \tag{1}$$

and $$\text{avg. }\phi^2 = 2\,\frac{a^2}{d^2}\text{, approximately} \tag{2}$$

where a is the distance between the wires of the telephone circuit and d is the distance of the power wire from the center of symmetry of the telephone circuit, and θ is the angle between a and d.

TABLE VII.

ELECTROMAGNETIC INDUCTION FROM POWER CIRCUITS, SINGLE WIRE AT DISTANCE OF 1.5 METERS (5 FEET)

Form	Circuit	a feet	Avg. ϕ^2	
			Per circ.	Per group
Standard	Physical Phantom	1 2	0.08 0.32	0.48
A, *C*, *D*	Physical Phantom	V2 V2	0.16 0.0008	0.321
F, *G*	Physical Phantom	1 1	0.08 0.0002	0.160
B	Physical	V5	0.4	0.8

For a present type phantom, formulas (**1**) and (**2**) are used, assuming that the phantom is made up of two circuits in parallel For the proposed phantoms,

$$\phi = \frac{a^2}{2d^2}\cos 2\phi \tag{3}$$

$$\text{avg. }\phi^2 = \frac{a^4}{8d^4} \tag{4}$$

where a is the diagonal of the group, and d and θ are as before. Comparative values are given in Table VII.

In form *E*, the distortion of the group increases average ϕ^2 for the phantom by 0.01, making the total for the group 0.17.

The above values represent also a reduction in the amount of noise induced in a given phantom by stray longitudinal currents in the other wires of the telephone lead.

e. *Characteristics of Standard E-Section.*

In the following table, M_I is the mutual inductance, in micro-henries per kilometer, computed from Table II, and M_C the mutual capacitance unbalance, in micro-microfarads per kilometer, computed from the table given by Osborne[6]. In computing both M_I and M_C account was taken of the two pin positions of each physical circuit.

f. *Computation of Unbalances in Proposed Forms.* In Fig. 2, let the mutual inductance (or capacitance unbalance) between the physical circuits of one group (say 1–12–2–11) and another group (say 3–14–4–13) be represented as follows:

TABLE VIII.

Type	Transp. per Section	Circuits		M_I	M_C
		Physical	Phant.		
i	23	5–6	37–40	5.44	0.08
j	22	...	5–16	....	...
k	21	15–16	31–34	5.44	0.8
l	20	13–14, 27–28	.....	−4.14	2.5
m	19	29–30, 31–32	.....	−4.14	2.4
n	18	9–10, 23–24	.....	−0.14	0.6
o	17	1–2	27–30	−2.43	3.0
p	16	7–8	21–24	−2.43	3.0
A	15	11–12, 39–40	.....	−0.14	0.6
B	14	19–20, 21–22	.....	−4.14	2.6
C	13	... ...	25–36	....	...
D	12	3–4, 17–18	.....	−4.14	2.8
E	11	37–38	11–14	−2 43	2.3
F	10	35–36	7–10	5.44	0.8
G	9	25–26	1–4	5.44	0.8
H	8	33–34	17–20	−2 43	2.2

Between 1–12 and 3–14, by w
" 1–12 " 4–13, by x
" 2–11 " 3–14, by y
" 2–11 " 4–13, by z

Table IX shows the changes in these coefficients due to successive phantom transpositions of one or both groups.

The use of this table permits working out readily the degree of balance between the physical circuits when the phantoms are transposed to any given patterns. If there are three equidistant phantom transpositions in one group between the phantom transpositions of the other, there is complete balance for all the inductive relations of the physical circuits. With a few exceptions, there is balance for the physical circuits whenever one phantom has gained three transpositions on the other one, whether this gain be made in one or several transposition sections of the line. Some patterns give balance of the physical circuits in two sections. The effect of phase displacement prevents depending on the phantom transpositions to balance the physical circuits except when a satisfactory degree of balance is obtained in one section.

---

[6] Loc. cit. facing p. 902.

TABLE IX.

EFFECT OF PHANTOM TRANSPOSITIONS ON COEFFICIENTS OF INDUCTION BETWEEN CIRCUITS IN DIFFERENT GROUPS.

Circuits initially in the position	Number of Transposition:									
		1	2	3	4	1	2	3	4	
	Number of Element:									
	1	2	3	4	5	1	2	3	4	5
	Group 1–12–2–11 transposed									
	To right					To left				
1–12 and 3–14	w	y	$-w$	$-y$	w	w	$-y$	$-w$	y	w
1–12 and 4–13	x	z	$-x$	$-z$	x	x	$-z$	$-x$	z	x
2–11 and 3–14	y	$-w$	$-y$	w	y	w	w	y	$-w$	$-y$
2–11 and 4–13	z	$-x$	$-z$	x	z	z	x	$-z$	$-x$	z
	Group 3–14–4–13 transposed									
	To right					To left				
1–12 and 3–14	w	x	$-w$	$-x$	w	w	$-x$	$-w$	x	w
1–12 and 4–13	x	$-w$	$-x$	w	x	x	w	$-x$	$-w$	x
2–11 and 3–14	y	z	$-y$	$-z$	y	y	$-z$	$-y$	z	y
2–11 and 4–13	z	$-y$	$-z$	y	z	z	y	$-z$	$-y$	z
	Both groups transposed									
	Same direction					Opposite direction				
1–12 and 3–14	w	z	w	z	w	w	$-z$	w	$-z$	w
1–12 and 4–13	x	$-y$	x	$-y$	x	x	y	x	y	x
2–11 and 3–14	y	$-x$	y	$-x$	y	y	x	y	x	y
2–11 and 4–13	z	w	z	w	z	z	$-w$	z	$-w$	z

For groups with their diagonals in parallel planes, as is the case in all the forms illustrated, x is equal to y, whatever the relative position of the groups, so that simultaneous phantom transpositions of both groups in the same direction balance the x and y values. Referring to Table II and the figures, shows that when the sides of the groups are in line, w and z are small compared to x and y. If the diagonals are in line, and in most other cases, w and z are opposite in sign and nearly equal. Hence in any case the simultaneous phantom transpositions leave only a small residual unbalance, equal to $(w + z)/2$ per unit length, between the circuits whose planes are parallel.

For example, let the following groups in Form *A* have the same pattern of phantom transposition:

1–12–2–11 and 3–14–4–13. $w = 22.31$, $x = 102.17$, $y = 102.17$,
$z = 22.31$, $(w + z)/2 = 22.31$, M between phantoms, –17.86.
1–12–2–11 and 23–34–24–33. $w = -57.54$, $x = 0$, y 0,
$z = 44.63$, $(w + z)/2 = -6.46$ M between phantoms, 4.77.
1–12–2–11 and 5–16–6–15. $w = 1.55$, $x = 25.03$, $y = 25.03$;
$z = 1.55$, $(w + z)/2 = 1.55$, M between phantoms, 1.17.
1–12–2–11 and 25–36–26–35. $w = -16.25$, $x = 11.99$, $y = 11.99$.
$z = 16.69$, $(w + z)/2 = -0.28$, M between phantoms, 0.21.

These figures illustrate the statement previously made that whenever the mutual unbalance between phantoms can be neglected, the same is true of the physical circuits.

Then following table of co-efficients between groups I and IV in Fig. 10 illustrates the use of Table IX.

Transp. pole		2	4	6	8	10	12	14
1–12 to 21–32	w	z	$-x$	y	$-w$	$-z$	x	$-y$
1–12 to 22–31	x	$-y$	w	z	$-x$	y	$-w$	$-z$
2–11 to 21–32	y	$-x$	$-z$	$-w$	$-y$	x	z	w
2–11 to 22–31	z	w	y	$-x$	$-z$	$-w$	$-y$	x

The sum of each horizontal line is zero, and inspection will show that since $x = y$, the larger co-efficients, are balanced in each quarter section.

The phantom transpositions with left and right handed rotation resemble the present types 2 and 3 phantom transpositions. By using transposition insulators and brackets, types analogous to 1 and 4 could be introduced, but apparently there would be no advantage in doing so.

g. *Formulas Used.* In Fig. 1, the solid lines are circles for which the radius is:

$$R = \frac{2\,a\,q}{q^2 - 1} \qquad (5)$$

and the intercept on the X-axis is

$$x_1 = a\,\frac{q - 1}{q + 1} \qquad (6)$$

where

$$q = \left(\frac{2\,a}{r}\right)^{1-2n} \qquad (7)$$

n being the fraction of the flux, or potential difference, to be included within the given curve, and a, half the distance between wires. Curves are drawn for $n = 0.4, 0.45, 0.5, 0.55, 0.6$.

The dotted lines have the equation

$$y^2 = \pm\sqrt{(a^2 - x_1^2)^2 + 4\,a^2\,x^2} - a^2 - x^2 \tag{8}$$

When x_1 is less than a,

$$x_1^2 = a^2\left[1 - \left(\frac{a}{2\,r}\right)^{-1}\right] \tag{9}$$

When x_1 is greater than a, the sign of the second term within the brackets is reversed. The unit of flux ($n = 1$) is the amount of flux linking only one wire. For $n = 1$, the curve shaped like a Fig. 8 is obtained. Curves are drawn for $n = 0.8, 1.0, 1.2, 1.4, 1.6$.

Mutual Inductances. Physical circuits 1–2 and 3–4

$$M = \log_e \frac{d_{14}^2\, d_{23}^2}{d_{13}^2\, d_{24}^2} \tag{10}$$

where d_{14} = distance from wire 1 to wire 4
etc. etc.

Phantom 1–2–3–4 and its own side circuit 3–4.

$$M = \frac{1}{2}\log_e \frac{d_{14}^2\, d_{24}^2}{d_{13}^2\, d_{23}^2} \tag{11}$$

Phantom 1–2–3–4 and circuit 5–6

$$M = \frac{1}{2}\log_e \frac{d_{16}^2\, d_{26}^2\, d_{35}^2\, d_{45}^2}{d_{15}^2\, d_{25}^2\, d_{36}^2\, d_{46}^2} \tag{12}$$

Phantom 1–2–3–4 and Phantom 5–6–7–8

$$M = \frac{1}{4}\log_e \frac{d_{17}^2\, d_{18}^2\, d_{27}^2\, d_{28}^2\, d_{35}^2\, d_{36}^2\, d_{45}^2\, d_{46}^2}{d_{15}^2\, d_{16}^2\, d_{25}^2\, d_{26}^2\, d_{37}^2\, d_{38}^2\, d_{47}^2\, d_{48}^2} \tag{13}$$

Self-inductances. Physical circuit,

$$L = 1 + 4\log_e a/r \tag{14}$$

in which a is the distance between wires, r the radius of the wires.

Phantom circuit,

$$L = 1/4\,(L_1 + 2\,M + L_2) \tag{15}$$

in which L_1 and L_2 are the self-inductances of the component circuits 1–3 and 2–4 (or 1–4 and 2–3), and M is the mutual inductance between these circuits, the side circuits being 1–2 and 3–4.

The above formulas give mutual and self-inductances in c.g.s. units per centimeter length. For microhenries per kilometer, multiply by 100.

Capacitances. Physical circuit,

$$C = \frac{1}{4 \log_e a/r} \tag{16}$$

Phantom, standard form, horizontal type,

$$C = \frac{A' + B'}{A\,B' + B\,A'} \tag{17}$$

where $A = \log_e a/r,$ $B = \log_e 2,$
$A' = \log_e a/r - \log_e 2,$ $B' = \log_e a/r + \log_e 3/2$

Phantom, proposed form, non-distorted,

$$C = \frac{1}{2 \log_e a/r - \log_e \sqrt{2}} \tag{18}$$

These formulas give capacitances in e.s.u. per cm. length.

DISCUSSION ON "RADIO TELEPHONY" (CRAFT AND COLPITTS), "THEORY OF THE TRANSIENT OSCILLATIONS OF ELECTRICAL NETWORKS AND TRANSMISSION SYSTEMS" (CARSON) AND "TELEPHONE CIRCUITS WITH ZERO MUTUAL INDUCTION" (CRAWFORD), NEW YORK, N. Y., FEBRUARY 21, 1919.

George O. Squier: To save time, I will refer only to the conclusion as presented by the first paper. That gives an outline of the marvellous state in which the inter-communicating systems for the transmission of intelligence have been linked up, and we are apt to forget the significance of that from a national and international standpoint, unless we pay special attention to it and try to see what it portends.

The author of the first paper described the wonderful and beautiful experiments whereby he spoke as far as Honolulu and Paris, and then linked these up with the great wire system of the country. Then, to cap the climax he gave you an account of the communication which was had with the New Hampshire at sea, which was a combination of the radio system at sea and the wire system on land. And only a few days ago from an airplane, we spoke through radio to the wire system, so that we are gradually linking up all the different systems of communication. I think we are within a fairly measurable distance of the goal when, on occasion any two persons shall be able to speak with each other from any two points. That, of course, is the ultimate object.

That brings out the point of correlating these systems which are of immediate value as independent links in this chain. In this connection there is urgent need for improvement in the engineering features of the present cable system as now operated throughout the world.

During the last few months the pressure on the cables in the Pacific and Atlantic Oceans has been unprecedented, and particularly so just now, with the necessity for cabling the doings of the Peace Conference. All departments of the Government, the State Department, the Navy Department, the War Department, the Signal Corps, which is part of the War Department, and other departments realize the necessity of bringing to bear the best engineering skill of the country to try to improve cable engineering practise.

An ocean cable is a most simple sort of circuit. It reaches across the Atlantic Ocean for instance, which represents a distance of some three thousand miles, and has a capacity of something like 1000 microfarads; very little inductance and practically no leakance at all. It was one of the first accomplishments of telegraphy and is over fifty years old now. This fact gives an idea of the engineering state of the cables which are so much older than the new art we have heard about this morning, which does all these wonderful things, I will have to say to you that today the ocean cable system is operated, so far as transmitting is concerned, and very largely so far as receiving is concerned, practically the same as it was in the beginning. The Duplex is the greatest advancement which has been made since the beginning.

The present method of signaling connects the end of the cable for a given length of time to a battery and then opens the circuit and connects the cable to earth.

But since the cable has this enormous capacity, when you close the key to make a dot or dash, there is a tremendous inrush of current charging up the cable. And when you open the key it takes a considerable time for the energy thus stored to run off to ground before you can send another dot or dash. Thus for each individual letter, an appreciable time is consumed in letting that charge run off, and during that interval, the cable is not in use for the purpose of sending messages. That amount of time, it is estimated, in the case of a long cable is 40 per cent of the total time of each element of a letter, if I am not mistaken. Therefore if both ends of an Atlantic cable were operating at maximum speed for twenty-four hours a day, we can say that every day the Atlantic cable system is practically shut down forty per cent of the twenty-four hours or nine and a half hours, during which time the cable is not performing any useful work. In the life of the cable, extending over a period of fifty years, the cable has been non-producing something like forty per cent of the whole time, or twenty years.

This is in contrast with the beautiful results we have heard described this morning, and is an indictment of present cable engineering practise. What I wish to draw your attention to, is the present attitude of the Government, in trying to remedy this state of affairs, because the cables are very important from an international standpoint. The League of Nations plan will be largely dependent on an international system of signalling, including what we have heard described this morning.

As Chief Signal Officer at the War Department I will furnish any assistance that is in my power to anyone who wishes to attack any of these questions, mathematically or practically, and will turn over the use of our cable to Alaska, as far as is consistent with the public traffic on that cable for testing any improvements.

M. I. Pupin: Just twenty years ago I heard General Squier say exactly the same things about cable communication. The art has not advanced one bit and it is fortunate it has not because engineers have been engaged in other fields, notably radio telephony.

People long ago realized that you cannot do much with the cable. There it is, just like a long amphibious animal stretching from one shore to the other. No matter what you do to it it moves slowly. There is no life in it.

Now, General Squier says if any one of you will make a new, fresh start in the direction of improving the cable the Government is at your service. That is a very attractive offer, but I do not think they will get many acceptances, for the simple reason that the engineer will choose radio telegraphy and radio telephony. This art is new, it is full of life, it is full of the brightest dreams. It is the beautiful science connected with this new art which attracts us and stirs up our imagination.

Now, let me add a few things, if I can, to the history of this beautiful art. As long ago as twenty years, in 1899, I was very much interested in Peter Cooper Hewitt's work, the mercury vapor lamp, as an outsider and as a looker-on. It was a beautiful piece of work. Then, after that I took an interest in the Wehnelt tube and thought it was a beautiful piece of work, and it is; and as an outsider, unprejudiced, I compared the merits of the two inventions. I found practically no difference. You are dealing with a hot cathode and a cold anode, you are dealing with practically, under certain conditions, a semi-thermyonic current, as we call it. The thermyonic current is not new, it is older than Wehnelt tubes or the Cooper Hewitt tube. It takes us back fifty years, it was Hittorf who first showed that if you heat the cathode you can pass an electric current through a vacuum tube very easily with a few volts whereas if you have both a cold anode and a cold cathode you cannot pass through any thermyonic current or practically none.. Of course, with a little bit of exterpolation of the Hittorf theory you can infer from that that the current can pass easily through a hot cathode and a cold anode. But if you apply the electromotive force the other way, you will not get the current.

Then came the so-called Edison effect—that is specifically a case of the Hittorf tube, both the Hittorf tube and the ordinary vacuum tube,—I mean that the ordinary incandescent bulb can act as a rectifier. I must say that to me that was absolutely clear long before the tubes were employed for wireless telegraphy. I did not attach much importance to it, for reasons that I will not explain, but when the Fleming valve came out I was really surprised that a patent was granted for it, but as the Patent Office did grant the patent, I therefore assume that he had a right to it.

I think I was one of the first to realize the importance of a rectifier in wireless telegraphy. I made an electrolytic rectifier which was operated by myself and Professor Merritt of Cornell University for the purpose of rectifying Hertzian oscillations. The paper in which I described my experiments was read before the American Physical Society in November, 1899; twenty years ago. Three years after that it became known in the art as the Fessenden barreter. I asked Professor Fessenden why he called it a barreter. I told him I thought it was a rectifier. He said, No, it is not a rectifier, it does not act in accordance with the principles you mention." But it was a rectifier and it was mentioned here to day as the Fessenden rectifier by a good friend of mine, who ought to know better. If it is a rectifier then I say it is the Pupin rectifier, and if it is not, then it is the Fessenden barreter. The rectifier is very important, no doubt to day, the rectifier employed by the Western Electric Company, the American Telephone & Telegraph Company and others, the vacuum tube rectifier, is not a rectifier of that kind at all. It operates differently; it operates better, it is an entirely improved form. It needs three electrodes and the man who added the third electrode to the vacuum tube is the man who made this wonderful epoch-making invention referred to here today possible, and that was Lee de Forest. Lee de Forest read a paper before this body in 1906. I was asked to discuss that paper.

Well, I acknowledge, I did not understand at that time quite fully the importance of the paper. Perhaps the author did. However, whether he understood it quite fully or not, he did make an epoch-making invention, one which is perhaps more important than any other electrical invention made during the last twenty years. Mr. Peter Cooper Hewitt also used a third electrode in the vacuum tube, when he used mercury for his negative electrode. At that stage I said, Here is a thing which is full of the greatest promise. This is a thing which needs investigation, scientific reasearch of the most profound character. I called up J. J. Carty, of the American Telephone & Telegraph Company, knowing that the directors of that organization lay great weight upon his judgment. I said, "Carty, here is a thing that must be developed," and Carty was not slow in getting the thing installed in 1912. It is 1912 that marks the beginning of this great art, that is the time when the American Telephone & Telegraph Company took up the Peter Cooper Hewitt and Lee de Forest patents.

As the Chairman of the Committee on wireless communication between airplanes of the Committee of the National Advisory Board for Aeronautics, I was requested to find various means by which wireless communication between airplanes could be advanced. That was in 1917. Early in that year, or the latter part of 1916, I met Captain Culver, who had just returned from San Diego where he was flying and making experiments with wireless communication between airplanes and from airplanes to earth. It seemed that there was no difficulty whatever in receiving a wireless telegraphic message while in an airplane, in spite of the great noise made by the machine, provided the ears of the man in the airplane were suitably protected. His information was so encouraging that I said, "Here, Captain Culver, you ought to run over to the Western Electric Company and see if they cannot give you an outfit for wireless telephony". I told General Squier the same thing and hence the conference on May 22, 1917.

From that conference to the present time, a period of less than two years, I think that the developments of the wireless telephony art, as used in airplanes today, and as it has been used during the war, is perfectly marvellous. It shows the great usefulness of having a great organization maintaining a first class research laboratory. That is what accomplished it.

These great research laboratories are national assets, they are not merely local organizations belonging to a particular company. We are proud that the value of pure science has been recognized not only by the universities—for that is their business, they are teaching pure science—but by practical men, the men who are the directors of great industrial organizations. I am very glad from that point of view that we have these splendid results to convince these men and the directors of other industrial organizations that the best investment which a great industrial organization can make is to maintain a great research laboratory.

The idea which I have just expressed has been the guiding idea of the men who during the last three years have been working hard to organize the National Research Council, the object of which will be to co-ordinate and stimulate the scientific research of this country for the good of mankind, and particularly for the good scientific reputation and standing of this country. They will attempt to co-ordinate the work of such organizations as we have, like the great industrial research laboratories of the large manufacturers and to stimulate other industries to establish similar research laboratories. To establish points of contact between industrial laboratories and the universities and the scientific men in general, so that they will understand each other and work for the common good, that is, for the advancement of science, for the glory of mankind and for the glory of the scientific reputation of this country.

Alfred H. Cowles: I would like to criticize one point which appears in the historical part of the paper of Messrs. Craft and Colpitts. It will be noted, that it is stated on the second page of their paper in effect that Maxwell "*predicted* electromagnetic wave propagation and the identity of light and electromagnetic radiation". The point I wish to make is, that electromagnetic radiation was known to Maxwell as a phenomenon and he explained it instead of predicting it. His became an accepted theory which classified and reduced to order known facts and phenomena and not an unproven hypothesis.

In about 1879, there were but two telephone lines in the city of Cleveland, one between my home and my father's office, the other between the office of the Brush Electric Company and that of their President. I had occasion to talk over one wire and a voice came in, that was a strange voice. I was calling my father. . . . . and the voice said, "who is this." I said, "Cowles." It said, "ha ha ha, this is General Leggett, *our lines are crossed.*" The strange voice was that of General M. D. Leggett, then President of the Brush Electric Company. By the expression "our lines are crossed," he did not mean that the wires were in contact. Now there was a case where we had wireless telephony almost in the sense of today. There passed upon the wires successive alternating waves corresponding with the fundamental vibrations of the human voice, superimposed upon these waves were various harmonics due to the resonance, fixed by the form of one's organs of speech which with pitch of tone enables the distinguishing of one voice from another. All of these fundamental and supplemented concurring vibrations were in this instance transmitted from one wire to the other by Maxwell's electromagnetic waves of propagation, thus causing similar induced waves in the crossed wire that brought to me the recognizable voice of General Leggett.

Hertz did not forecast or even think of wireless telegraphy in connection with his published papers in 1888. Neither he nor his predecessors had mentioned such a thing excepting in the sense of my 1879 experience. In 1899 Nikola Tesla made his wonderful experiments at Colorado Springs. He lighted incandescent lamps in the yard about his laboratory by wireless. I had occasion several years later to read one of his papers that was presented to the Franklin Institute, at Philadelphia, February 24th 1893, and I was astounded to see in that paper a statement which I can almost remember literally. He said—"It behooves the electrical engineers of the world to give their closest attention to the proposition of wireless telegraphy to all parts of the world. It is possible and it is sure to come." That gave me an impression of Nikola Tesla's genius which I felt in later years that a great many other electrical engineers had failed to recognize, and I think it ought to be recognized today.

His work was several years before you began to hear about wireless. He had published the fact, that he had observed by aid of his delicate instruments the passage of an electric impulse from a flash of lightning and had detected its return electric echo from the far side of the earth. His instruments, to explain, had given two indications, one the flash and the other the echo. Upon measuring the difference of time between the two impulses and calculating what it should be, knowing the velocity of electromagnetic waves he found the difference of time to be that which he looked for. To be exact it was periodic echoes of stationary waves that he measured. From this he drew the conclusion that it was possible to have wireless communication to all parts of the world. All that was necessary was to set up two instruments in synchrony at any two points on the earth's surface. By the use of one give to the charge on the earth a sufficiently heavy set of impulses of a definite vibratory period in which case the other instrument would be bound to take up these impulses when in resonance with the first.

It seems to me that in this instance we had the most wonderful forecast of the real possibility of wireless. He had prior to July 1899 invented his oscillator and magnifying transmitter and had them long in use. He could take at will, according to design, the energy stored during a given length of time of flow along any wire and discharge that quantity of energy in a millionth or less part of such time. This enabled him to produce what were equivalent to flashes of lightning 100 feet long.

While Hertz in his experiments in 1888, accomplished the transmission of impulses through a stone wall, succeeded in refracting electromagnetic waves, producing interferences, etc., they were not far different from similar experiments being performed in other physical laboratories the world over. None of these experiments suggested or hinted at world wide wireless. They were insignificant when compared with that which Tesla forecasted and afterward did. Using Tesla's theory of explanation, his impulses would go through or around the earth which seemed to me perfectly logical when I first read his February 1893 paper. He viewed the earth as a charged sphere. The presence of Maxwell's electromagnetic waves were to be inferred as accompanying the phenomena that Tesla observed. His method of explaining these phenomena did not embody one point which puzzled many electricians, *viz*, the question as to how ordinary electromagnetic waves could pass around segments of the earth's circumference, yet fundamentally his theory was in harmony with the generalizations of Maxwell and could just as logically be applied to explain long distance wireless as the electric wave theory through either. The Craft and Colpitt's paper seems to me to unnecessarily differentiate as between Maxwell's electromagnetic waves and Hertzian waves which are truly one and the same thing inasmuch as what is now called the Hertzian wave in their paper was well understood before the date of Hertz's experiments.

It seems to me that from 1890 through to recent years Tesla's publications and patents were not generally understood, because of his starting from the static end, while others have begun their explanation of wireless from the dynamic portion of electrical science. I have no personal interest in this matter other than that I wish to see fair credit given to an American inventor.

Michael I. Pupin: I want to call your attention to the fact, as announced on the second page of the paper, that the author deals with electrical phenomena in which all the reactions can be expressed by first or second differential coefficients, with constant coefficients, and, therefore, it is not as general as you might suppose. For instance the equations will not apply to the theory of the induction motor, because in that theory you will find that one of the reactions is expressed in terms of a differential coefficient multiplied by a function of the time.

If you take an iron core and subject it to periodic magnetization there, too, you have the development of harmonics, because the permeability is not a constant quantity, so from that point of view the paper is limited to the so-called linear differential equations and constant coefficients. That, however, is not a serious limitation, because there are very few electrical phenomena that fall outside of that orbit, that is, so far as we know at the present time.

Mr. Carson discussed artificial lines. I gave the first discussion on artificial lines in March, 1899, and in another paper a year after that I discussed the connection of lines with periodic structures, taking special cases, but stated at that time that other combinations could be treated by the same method.

In that paper I showed how to analyze the connection of an artificial line. First, where the artificial line is capable of oscillation, and secondly where it is aperiodic, and so that in one case you have periods of oscillations from which you can compound your general solution, and in the other case you have a certain number of types of exponential functions and you can compound any complete solution by taking these types and putting them together in the proper way. As far as I know, no such study had been made prior to that time.

Afterward I wanted to show, and I did show in a paper—not published yet, although the paper is nearly twenty years old—the first case treated by Kirchhoff and also by Kelvin—what happens when we impress constant voltage on an overhead line or on a cable. I discussed that from the point of view considering the cable as an artificial line with resistance and capacity, and the overhead line as an artificial line consisting of sections of resistance, capacity and inductance. I found the general solution which, when the number of terms was raised to infinity, resulted in Kirchoff's and Kelvin's solutions, and I said, "This is a more general solution, because from this solution you can get others, whereas from others you cannot get this one." In fact, the problems of wave propagation of continuous lines can be handled by considering the problems, first, on artificial lines; but I must say that it is more difficult to handle the artificial line than it is to handle the continuous line in most cases. In some cases, however, it is easier, because for any solution in one you can get a corresponding solution in the other, whereas in the one case of the artificial line with a finite number of sections, you deal with the sum of the finite number of turns; and in the continuous line, if the limit is an artificial line of an infinitely small section, you deal with the integral.

I think in the papers of March, 1899 and June, 1900, that subject has been treated pretty exhaustively, except that I did not go into any arbitrary impressed forces. Mr. Carson does, and he does it very well, indeed. That, however, a man can do by studying the problems which have already been solved for continuous lines, and then he can guess pretty well, if he is skilled in the art, what will be the corresponding solution in the case of the artificial line.

The artificial lines, as Mr. Carson says, stand on their own legs—they are independent structures for filtering, they call them filterers in the Western Electric Company, I call them pilot conductors at Columbia University, but they are exactly the same thing. You can use these artificial lines, and the theory I published in 1899 shows that you can use them as filterers. They will let through certain frequencies and exclude very strongly and very perfectly other frequencies. You can use two structures like that so that they will transmit only certain intervals of frequencies. I have used them at Columbia University but here is the difficulty—I will tell you something that is not contained in the paper—two structures having inductance, capacity and resistance to be effective filters and to be used as such must have resistances very small in comparison with the reactances. What happens, then, is this. Any disturbance occurring in the vicinity of an impulsive nature will start oscillations in the circuit, oscillations of about the same frequency as the signaling frequency. Although you exclude other simple harmonic states from it, you are bothered by the local oscillations produced in the circuit itself, and for that reason the circuit is not as universally applicable as might be thought.

The best structure is, of course, aperiodic structure. In fact, all the work I have done in Columbia University in the line of excluding, for instance, the disturbance on a commutator employs aperiodic structures. I find that sometimes it is desirable to use a dynamo-electric machine in place of the storage battery and dry cells in wireless work because it is less expensive and needs less attention. However the commutator of the machine makes noises, and these noises have to be excluded which is done by aperiodic artifical lines. They are very much more effective on account of the absence of local oscillations. We should congratulate ourselves upon having in one little pamphlet a presentation of the mathematical apparatus that, when put together, is useful to every engineer, who essays, in particular, any investigation into high-frequency electrical work.

A. S. Dana: I should like to draw the attention of members of the Institute to two papers which may serve as an introduction to part of this paper and which were presented to the I. R. E. and A. I. E. E. by Dr. V. Bush in 1916 or 1917. The "Threshold Impedance" introduced in Dr. Bush's papers is equal to $p_m Z^1 (p_m)$ in Mr. Carson's "indicial admittance", equation (**3**).

A. Pinto: (communicated after adjournment): In the last two paragraphs of the section on "Aircraft Telephony" of the Craft-Colpitts paper an explanation is given of the principle of operation of the so-called "constant-current" system of modulation, devised by R. A. Heising. On the basis of this explanation the writers claim that the function of the "low-frequency choke" is to maintain the total current supplied by the main battery to both the modulator and the oscillator plate circuits practically constant; so that, if the telephone transmitter is spoken into, and the modulator plate current thereby increased or decreased (through the action of the modulator grid), the oscillator plate circuit will be either "robbed of current" or else "will have additional current forced through it", and thus the output of the oscillator will be changed in accordance with the speech currents.

The above seems to me both inexact and confusing. For, how could the plate circuit of the oscillator be "robbed of current" or "have additional current forced through it" unless the voltage applied to it were varied? And how could this voltage vary if the current through the "low-frequency choke" were practically constant, as claimed by the writers? On the contrary, if the amplitude of the high-frequency current generated by the oscillator is to vary, *i. e.*, if modulation is to take place, the modulator must cause to flow through the "low-frequency choke" a *varying,* and not a *constant,* current.

Thus, by speaking into the telephone transmitter a result is produced which is equivalent to varying the impedance of the modulator plate circuit. Since this is in multiple with the oscillator plate circuit, the joint impedance of the two plate circuits is varied, and hence the impedance of the entire battery circuit (which includes the "low-frequency choke") is varied. It follows that the total current supplied by the main battery (or the current flowing through the "low-frequency choke") is varied, and so is the potential drop taking place over the "low-frequency choke". This varying drop, subtracted from the constant voltage of the main battery, produces across the oscillator plate circuit a voltage which varies at audio-frequency, and which causes the amplitude of the antenna current to vary at audio-frequency and in a manner more or less corresponding to the vibrations of the telephone transmitter diaphragm.

It may be easily seen that the impedance of the "low-frequency choke" at audio-frequency must be comparable with the joint impedance of the two plate circuits. For, if it were very much higher, any variation of the joint impedance of the two plate circuits would produce such a small variation of the impedance of the entire main battery circuit that the current therein would remain practically constant, and thus produce an inappreciable change in the voltage across the oscillator plate circuit, with resultant ineffective modulation. On the other hand, if the impedance of the "low-frequency choke" were very low, a variation of the joint impedance of the two plate circuits would cause such a small change of voltage drop to take place over the "low-frequency choke" as to again produce ineffective modulation.

With reference to the names "low-frequency choke" and "high-frequency choke", applied to two coils appearing in the schematic diagram of the Heising circuit, I wish to say that it has been my experience, in connection with the training of radio officers at the Air Service School established at Columbia University, to find these two names a source of confusion and misunderstanding. I would, therefore, suggest that the "low-frequency choke" be called the "modulating coil", and the "high-frequency choke" the "feed-back coil", these two names being more nearly in accord with the functions which the two coils are called upon to perform.

V. Bush (communicated after adjournment): The very remarkable analysis presented in Heaviside's "Electromagnetic Theory" is rapidly being placed upon a more secure mathematical foundation, extended to meet present day needs, and placed in such form as to be readily available for the solution of constantly recurring, practical problems.

Heaviside obtained many results of great interest by applying the symbolic operator developed into a power series. This method is shown in the Carson paper in connection with the problem of the infinite cable with terminal condenser; where it indeed applies, as stated, with extraordinary simplicity and directness. As the author points out, however, the method has never been justified mathematically; and its use is hence attended with the same liability to missteps as are all empirical formulas. On account of the importance of this method I will attempt below to throw some light on its aspects mathematically.

In the problem stated, the operational solution is, in the notation of the paper:

$$V = \left[\frac{E}{\sqrt{\pi h}} \left\{ p^{1/2} + \frac{1}{\pi h} p^{3/2} + \frac{1}{(\pi h)^2} p^{5/2} + \dots \right\} - E \left\{ \frac{1}{\pi h} p + \frac{1}{(\pi h)^2} p^2 + \dots \right\} \right] \xi \qquad (1)$$

where p^n replaces $\frac{d^n}{d\,t^n}$, and where for convenience the discontinuous function, which is zero when $t < 0$ and the constant value E when $t > 0$, and which is operated upon, is represented here explicitly by ξ.

The interpretation of this expression, and of similar expressions where they occur in the application of this operational method, necessitates a determination of the value of

$$p^n \xi \qquad (2)$$

where n may have any value integral or fractional.

When n is a negative integer, the desired result follows by direct integrations. When n is a positive integer the matter is, as Heaviside states, not so simple as it looks. The trouble however is not mathematical, but lies in the fact that we have not defined ξ at the point $t = 0$, and hence cannot state anything in regard to its derivatives at this point. When $t > 0$ we know that expression (**2**) is zero for a positive integral n by simple inspection. The fact that we do not know its value at $t = 0$ need not trouble us. If we state that our solution applies only when $t > 0$ we may drop out all terms of this form. The solution at $t = 0$, that is the initial voltage or current, is usually known from physical considerations.

We need then the value of (**2**) when n is fractional, in particular when n is half an integer. This value Heaviside obtained by solving a special case "from both ends" and hence discovering what value must be assigned to give a correct result. The following analysis attempts to derive the value of this expression directly.

The discontinuous function ξ may be expressed by the Fourier line integral

$$\xi = \frac{E}{2\,\pi\,j} \int_{-j\infty}^{j\infty} \frac{\epsilon^{zt}}{z} d\,z \tag{3}$$

the path of integration being the axis of imaginaries; avoiding the origin, which is a first order pole of the integrand, by a small semicircle on the positive side. It may readily be checked that the integral over this path properly describes our function.

Applying p^n to this expression

$$p^n\ \xi = \frac{E}{2\,\pi\,j}\,\frac{d^n}{dt^n} \int_{-j\infty}^{j\infty} \frac{\epsilon^{z}}{z}\,d\,z \tag{4}$$

The variables z and t are entirely independent; hence we may differentiate under the sign of integration, and write

$$p^n\ \xi = \frac{E}{2\,\pi\,j} \int_{-j\infty}^{j\infty} \frac{\epsilon^{zt}}{z^{-n+1}}\,d\,z \tag{5}$$

Consider now the case where $n < 1$. The integrand of expression (**5**) is regular along the above path, and also everywhere on the negative side of the axis of imaginaries; and it is zero at ∞ on this side of the axis. Hence the path of integration may be deformed to a circle taken once positively about the origin.

$$p^n\ \xi = \frac{E}{2\,\pi\,j} \int_{0} \frac{\epsilon^{zt}}{z^{-\ +1}}\,d\,z \tag{6}$$

Let us now consider Cauchy's Integral Formula

$$f^n(Z_0) = \frac{\Gamma(n+1)}{2\pi j}\int_0 \frac{f(z)}{(z-z_0)^{n+1}}\,dz \qquad (7)$$

which gives the nth derivative of a complex function at a point z_0 in terms of a line integral about a boundary surrounding this point. When written as above with the gamma function in place of the factorial, in accordance with

$$\Gamma(n+1) = n!$$

the formula (**7**) serves as an expression also for the fractional derivative.

If now Z_0 is taken at the origin, and if we consider the function

$$f(z) = \epsilon^{zt}$$

(**7**) reduces to

$$p^n(\epsilon^{zt})_{z=0} = \frac{\Gamma(n+1)}{2\pi j}\int_0 \frac{\epsilon^{zt}}{z^{n+1}}\,dz \qquad (8)$$

or

$$\frac{t^n}{\Gamma(n+1)} = \frac{1}{2\pi j}\int_0 \frac{\epsilon^{zt}}{z^{n+1}}\,dz \qquad (9)$$

Comparing now with (**6**) we have immediately

$$p^n\xi = \frac{E\,t^{-n}}{\Gamma(-n+1)} \qquad (10)$$

where $n < 1$

In particular, since

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$$

we may write

$$p^{1/2}\xi = \frac{E}{\sqrt{\pi t}} \qquad (11)$$

the expression noted in the paper, which was assumed by Heaviside.

By applying p successively we may then obtain

$$p^{3/2}\xi = -\frac{E}{2\sqrt{\pi t^3}} \text{ etc.} \qquad (12)$$

The above results, applied to the expansion of the operator in powers of p, justify this method as far as its interpretation is concerned, and allow us to avail ourselves of its directness.

Mr. Carson notes correctly that while the method of expansion in a power series often obtains solutions with remarkable ease, the resulting series are often inconvenient for computation. There are usually two such series, one excellent for very large values of t, and the other for very small values of t. In the intermediate region either series will often lead to prohibitive labor of computation.

Another method of applying the operator which works excellently for certain problems is to expand in partial fractions in $p^{1/2}$ instead of in series.

For the cable without inductance or leakage, but with terminal apparatus, this may often be done to advantage.

The value of

$$\frac{1}{1 + m\,p^{1/2}}\,\xi \tag{13}$$

may be worked out once for all, for real and complex values of m, and tabulated. A solution then consists of first splitting the operator up into partial fractions of this form, and then looking up in these tables.

Effects out on the line may be found similarly by tabulating

$$\epsilon^{-lp^{1/2}}\frac{1}{1 + m\,p^{1/2}}\,\xi \tag{14}$$

the operator

$$\epsilon^{-lp^{1/2}} \tag{15}$$

giving the effect at a distance

$$\frac{l}{\sqrt{R\,C}}$$

out on the line.

This tabulation requires a curve sheet for each assumed value of l.

I have not been able to bring this method of development in partial fractions to a point such as to warrant publication as yet, for my efforts have been otherwise directed during the war. It holds considerable promise. One result was however obtained incidentally by this method which should be of interest in connection with submarine cables. It has been found, if in addition to a terminal series condenser a coil be used between cable and ground, that very interesting results are obtained. The current entering the cable reverses after the initial pulse. The wave form at a certain distance out on the cable is practically without a "tail". The form may of course be controlled by proper choice of the terminal constants.

It seems to me probable that this might be used to advantage in rapid cable telegraphy, adjusting the constants such that the arrival wave would be substantially without tail.

In conclusion I wish to emphasize that the application of this kind of analysis is greatly aided if we keep in mind the analogy with the solution for the alternating-current circuit in the steady state. The function $Z(p)$ of the paper which has been previously called the *threshhold impedance,* may be immediately formed from the ordinary impedance of the circuit by replacing ω with p. This is an excellent guide in formulating the *indicial admittance,* which is derived from the Heaviside expansion theorem, and which forms the starting point of Mr. Carson's admirable paper.

J. H. Morecroft: (communicated after adjournment): The record of this accomplishment in the field of radio telephony adds one more victory to the credit of the commercial research laboratory. It is a record which illustrates how well and quickly an organized corps of research and construction engineers can solve a problem, which is definitely stated, the solution of which is possible.

It has been my opportunity to analyze in some detail, the construction and operation of the set known as the *S C R* 67, one of the sets described in the Craft-Colpitts paper. When the Signal Corps sent men to Columbia University to be trained as radio officers it fell to my lot to explain the principles and operation of the radio telephone outfits which were to be used. When I first opened the cover of the *S C R* 67 and saw the mass of tubes, coils, resistances, condensers, batteries, the compactness of the design, the spring suspension of the tubes, etc. I realized what a tremendous advance in radio telephony had been suddenly accomplished. Upon first seeing one of these sets one naturally anticipates all sorts of trouble in maintenance and adjustment but in this respect we were happily disappointed; the sets functioned very regularly and no special care was required.

In gathering sufficient data to properly explain the operation of the set I did considerable laboratory work in obtaining the principal characteristics; as these technical points are not brought out in the paper, and as there are undoubtedly many electrical engineers who are not acquainted with the functioning of vacuum tubes, it seems worth while to give with the paper a brief discussion, technical, but elementary in character. My remarks will deal only with the power generating tube; the modulator action and the amplifier action are much more evident than is that of the oscillating tube and its circuit.

As explained in the paper the grid potential of the three-electrode tube controls the plate current, this current increasing as the grid goes positive and decreasing as the grid becomes negative; the curves in Fig. 29 of the paper show this action. If the tube is connected to a closed circuit made up of L, R, and

C, in series, and the grid, filament, and plate are connected to the proper points on this circuit, the tube will act as a generator of alternating-current power, the frequency of the alternating current being fixed (nearly) by the well known equation for resonance,

$$f = \frac{1}{2\,\pi\,\sqrt{L\,C}}\,.$$

The simplest possible circuit is shown in Fig 1. The input to the tube comes from the plate battery and is equal to the product of the plate voltage E_p and the plate current I_p. In the tube used with the telephone set the normal plate current is about 50 milliamperes and the plate voltage 300, giving an input of 15 watts. In these low power tube circuits the efficiency is seldom as great as 30 per cent, we might expect then from the tube used with this set an output of about 5 watts; this much may be obtained when the proper adjustments are made.

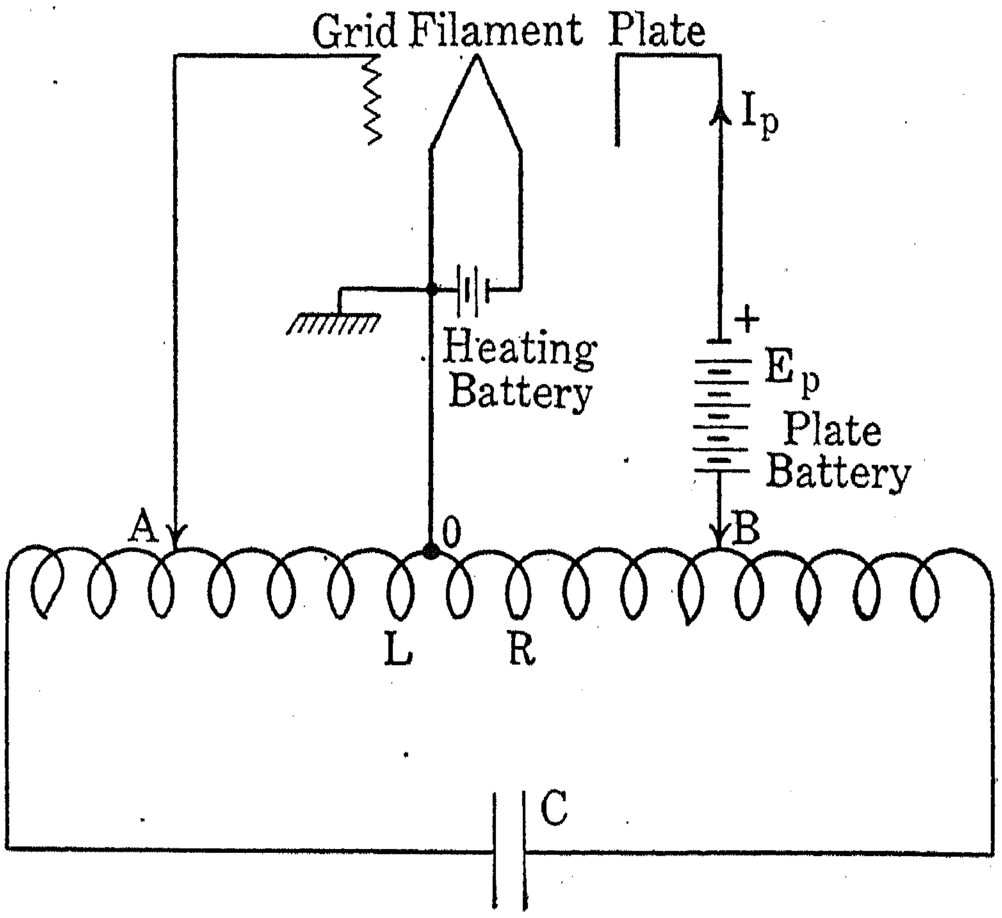


FIG. 1

Many, and apparently different, circuits have been devised for use with oscillating tubes, but they all function to produce the same result. Suppose the oscillating condition has been established in the circuit of Fig. 1. There is an alternating current flowing around the circuit L—C; a pulsating current flowing in the plate circuit, through the tube, and through that part of the inductance between O and B; and a comparatively small current in the grid circuit. This latter current flows only when the grid is positive with respect to the filament and so consists of a series of unidirectional pulses. For the $V\,T\,2$

tubes (used in the $S\ C\ R$ 67) the relative value of these currents might be as follows—the alternating current in the L-C circuit from 0.25 to one ampere, depending upon the resistance of the circuit; plate current of average value 40 milliamperes, fluctuating between 10 and 70 milliamperes; grid current having a maximum value of 5 to 10 miliamperes, its average value being about 3 milliamperes.

If one should connect a tube and circuit as shown in Fig. 1 it would be found that certain conditions must be satisfied before the tube will oscillate. The alternating current flowing in the closed circuit L-C will produce an alternating potential difference along the coil L; when one end of the coil rises in potential the other will fall. This rise and fall of potential are referred to the potential of the filament which, for the sake of discussion, is grounded and so held at zero potential.

The first and fundamental condition to be satisfied is this—the connection must be such that when, due to the alternating current in L, *the plate rises in potential the grid falls in potential* and vice versa. This means that in such a circuit as that in Fig. 1 the grid and plate must be connected on opposite sides of the filament, for if they are connected on the same side they will rise and fall in potential at the same time, instead of having a phase difference of 180 degrees.

Not only must the above condition be satisfied but *the rise and fall must be sufficient in amount.* The rise and fall are fixed in magnitude by two things, the amount of reactance between O and the two points of connection, A and B, and upon the amount of current flowing in the circuit L-C.

Suppose this oscillating circuit has a high-frequency resistance of 4 ohms; this resistance must be measured at the high frequency at which the circuit is going to operate because the high-frequency resistance may (and probably will) be several times as great as the continuous-current resistance. As the tube can furnish about 4 watts of power we may expect one ampere of alternating current, putting $I^2 R = 4$ watts. Then the reactance between O and B must be such as to give, with one ampere of current, a reactance drop the maximum value of which is equal to the plate voltage E_p. The amount of reactance between O and A determines the voltage impressed on the grid and this should be about one half as much (for the $V\ T$ 2) as the drop between O and B. This ratio of grid volts to plate volts depends upon the type of tube used, the finer the mesh of the grid the less is the ratio.

The amount of voltage impressed on the grid as a result of the above consideration, will result in the flow of too large a current in the grid circuit if the average potential of the grid is zero; for this reason it is customary to introduce an extra voltage in the grid circuit to keep its average potential several volts (perhaps 20) negative with respect to the filament. If this is not done there will be excessive loss of power in the grid circuit and there is danger of burning out the tube.

To get maximum power output from a tube the impedance of the oscillating circuit, when measured between the two points to which the filament and plate are connected, must be resistance only, and this resistance must be equal to the resistance of the tube. The resistance of the tube is variable, depending upon the grid voltage, filament current, etc. and is moreover different for an alternating current than for a steady current. The alternating resistance is about one half the continuous-current resistance. This resistance (c. c.) is found by dividing the plate voltage by the plate current; for a plate voltage of 300 and plate current of 0.04 amperes it would be 7500 ohms. The a-c. resistance for this condition would be about 3750 ohms.

Now it would seem that the circuit of Fig. 1, containing *L*, *R* and *C* in series could not have a power factor of unity between the points *B* and *O*, but if this power factor is not unity the tube cannot deliver its maximum power. Here however an interesting property of resonant circuits is encountered. Suppose the circuit of Fig. 2, consisting of *L*, *R*, C_1 and C_2 in series, is connected to an a-c. line at the points *A*, *B*, and suppose further that the impressed frequency is such as to satisfy the relation

$$f = \frac{1}{2\pi\sqrt{LC}}$$

where

$$C = \frac{(C_1 \times C_2)}{C_1 + C_2}$$

This circuit will show parallel resonance; the line power factor will be unity and the impedance between *A* and *B* will be a resistance only. This effective resistance will not be the actual resistance of the circuit but many times greater. In fact it will be equal to the actual resistance multiplied by the ratio $\left(\frac{\omega L}{R}\right)^2$. That is, the lower the actual resistance, the higher is the effective resistance between the points *A* and *B*.

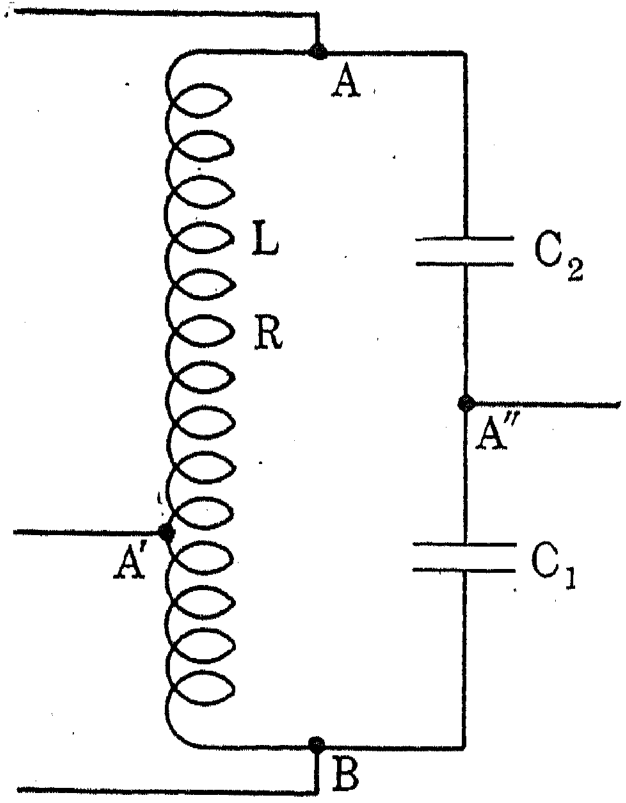


FIG. 2

Now if the power supply is connected to the points *A′*-*B* or *A″*-*B* or *A′*-*A″*, the power factor will still be unity; in fact the power supply may be connected at *any two points* in the circuit and the circuit will offer reisistance only. The value of this resistance will vary of course, as the points of application of the power are changed; it has a maximum value for the points *A*-*B* and for any other points of application it is equal to the value for the points *A*-*B* multiplied by the square of ratio of the reactance in either arm of the parallel path (the reactances of the two branches are equal in magnitude for all points of application) to ωL.

The circuit of Fig. 2 is used in the oscillating tube circuit of *S C R* 67; the manner in which the tube is connected in is shown in Fig. 3. The filament, instead of being connected to the inductance at point *O*, between *A* and *B*, as shown in Fig. 1, is connected at point *O′*, which is the electrical antipode of point *O*. In every closed electric circuit there must be two points which are continually at the same potential and such points are *O* and *O′*; if therefore *O* is a proper point for connecting the filament so is *O′* such a point.

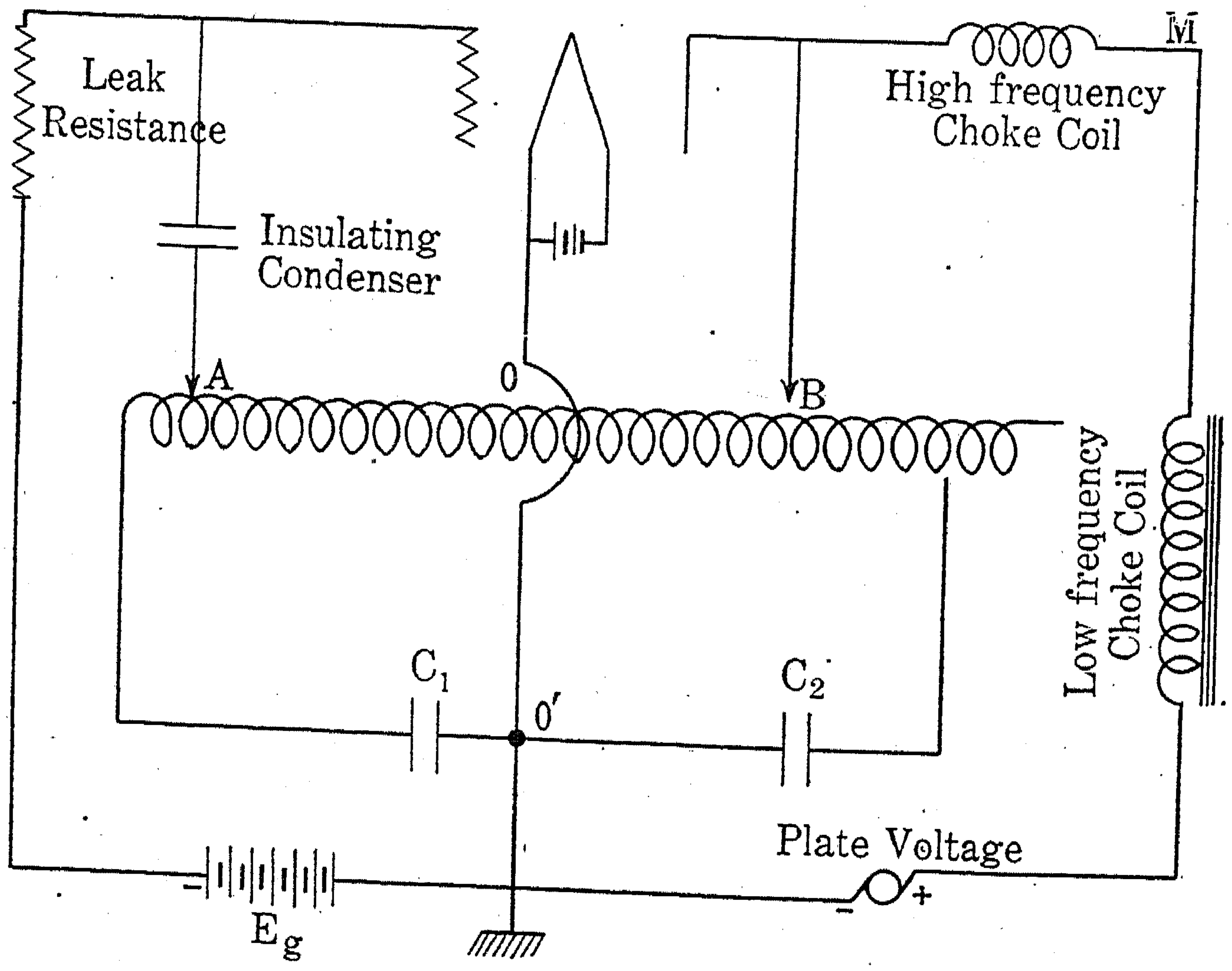


FIG. 3

The tube then oscillates and furnishes power to the circuit *L*, C_1 and C_2; the oscillating circuit is connected to the power supply (the tube) at points *O′* and *B*. The impedance offered between these points will be a resistance and the value of the resistance depends upon the location of the point *B*, which is a movable contact in the actual set. Although the actual resistance of the oscillating circuit is only a few ohms the resistance offered between points *O′* and *B* is perhaps 3000 ohms in normal operation.

The voltage impressed by the oscillating tube between points *O′* and *B* is alternating, with a maximum value about equal to the plate voltage E_p, say 300 volts. The alternating current which the tube supplies to these points varies from − 0.030

amperes to $+0.030$ amperes, its maximum value being somewhat smaller than the normal plate current of the tube. The power supply to the oscillating tube may now be calculated as it would be for any a-c. circuit having a power factor of unity, by getting the product of the effective voltage and current, it is equal to

$$\frac{E_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}},$$

or

$$\frac{300}{\sqrt{2}} \times \frac{0.030}{\sqrt{2}} = 4.5 \text{ watts},$$

which is about what we have said the tube would supply.

The function of the low-frequency choke coil is to maintain through itself an essentially constant current; the plate of the modulator tube is connected at M and so by the rise and fall of the modulator grid volts, takes more or less current from the oscillator tube as explained in the paper. The high-frequency choke coil prevents the partial short-circuiting of the points O'-B by the plate-filament circuit of the modulator tube. Its reactance must be of a proper value to get maximum power from the oscillator tube; in the set it is about 50,000 ohms.

The grid of the oscillating tube is excited by the drop between A and O'; in the *S C R* 67 this drop is varied by changing the capacity of the condenser C_1, the point A being permanently fixed. The grid cannot be directly connected to the point A as this point is positive about 300 volts with respect to the filament, while the grid must actually be held negative with respect to the filament. So a condenser is introduced between the grid and point A, permitting the action of the alternating potential difference but at the same time making it possible to maintain the average grid potential at any desired value by the battery E_g acting through the so called *leak resistance.* This resistance acts as a partial short circuit between the two points O' and A of the oscillating system and so consumes some power; it must be of suitable value for the satisfactory operation of the tube, for the *V T* 2 it is about 10,000 ohms. If too small the resistance consumes too much power and if too large the plate current is caused to decrease, unduly decreasing the power generated by the tube.

In the actual circuit of the *S C R* 67 the condenser C_1 is a group of mica condensers having a total capacity of about 3 millimicrofarads adjustable in about 12 equal steps. The condenser C_2 is really the antenna, but in parallel with the antenna there is a variable condenser occupying a position in the circuit the same as that of C_2 in Fig. 3; by means of this condenser and the taps provided on the coil L the oscillating circuit may be set at any wave length desired within the range of the set.

In Figs. 4, 5, and 6 are shown the more important operating curves of such a set. These curves are qualitative rather than quantitative as they will vary considerably according to the antenna employed.

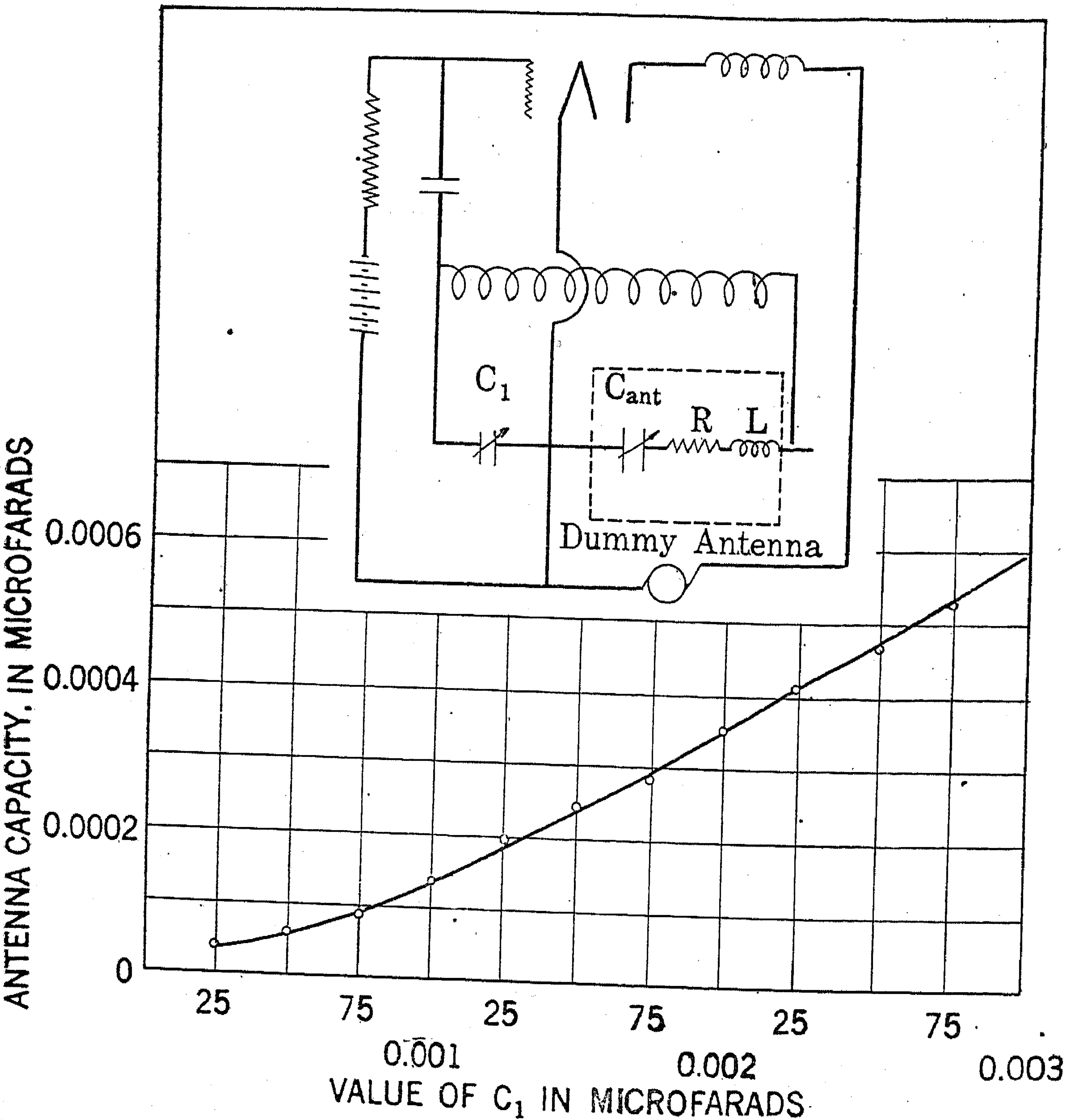


Fig. 4—Curve Showing Necessary Minimum Value of Antenna Capacity to Produce Oscillations, for a Given Setting of C_1

With antenna capacity less than the value given by the curve (for a given setting of C_1) the grid does not get sufficient excitation to produce oscillations. Values depend upon the tube, varying to some extent for different tubes of the same type.

It is evident from the analysis of the action of the circuit, that if the grid is not sufficiently excited the set will not oscillate. As the proportion of the total voltage in the oscillating circuit impressed on the grid is determined by the ratio of the condensers C_1 and C_2, Fig. 3, it seems as though for a given setting of C_1, C_2 must have a certain minimum value to produce oscillations; if less than this critical value the grid will not get enough voltage to sustain oscillations. With the idea of bringing out this operating feature of the set a series of readings were taken to determine the maximum permissible value of C_1 for a given antenna capacity. C_1 was set at a certain value, the antenna capacity being zero. The antenna capacity was then increased until oscillations started and the value of C_2 noted. Then C_1 was set for its next larger adjustment and the new critical value of C_2 found. The results Fig. 4 show that for the conditions obtaining in the test the grid must be supplied with at least one fifth of the total voltage in the oscillating circuit, this ratio being nearly the inverse ratio of grid capacity to antenna capacity for the critical condition.

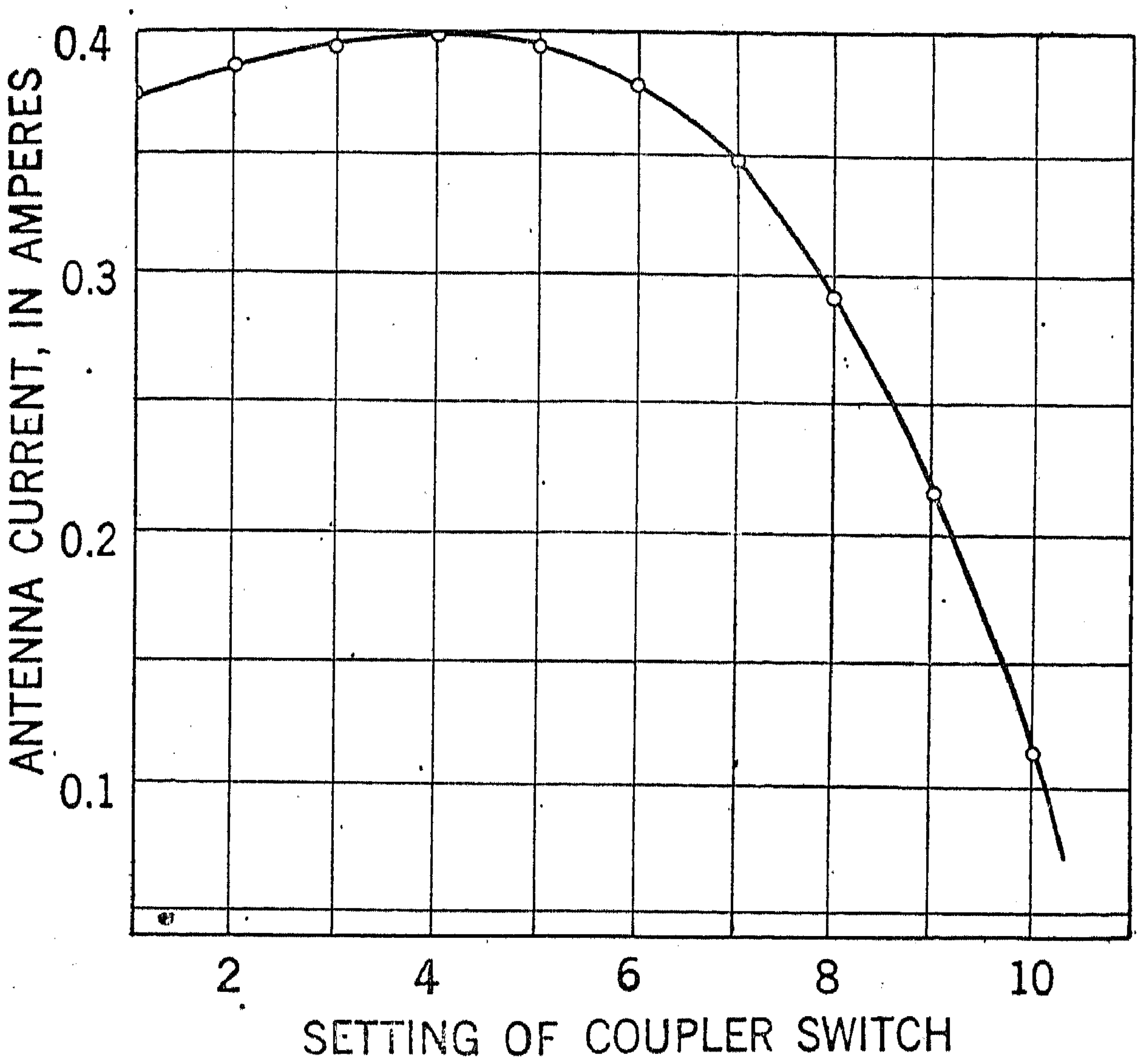


FIG. 5—EFFECT OF VARYING THE COUPLING BETWEEN THE TUBE AND OSCILLATING CIRCUIT. LARGER NUMBERS CORRESPOND TO WEAKER CURRENT.

Wave length = 406 meters. Dummy antenna used had constants: $C = 0.0004\ \mu f$ $R = 8$; $\lambda = 325$ meters; C_1 set at $0.001\ \mu f$.

Fig. 5 shows the effect of changing the position of contact *B*, Fig. 3; this is called the *coupling* adjustment of the set. The dummy antenna used had a resistance of about 8 ohms. The ωL of the antenna circuit for the conditions used was 400 ohms giving the oscillating circuit a maximum effective resistance of 20,000 ohms, much too large to match the impedance of the tube. The exact values of coupling used in the results of Fig. 5 were not known but it is evident that a certain coupling gives a maximum output which was equal to $0.4^2 \times 8 = 1.2$

watts. (The grid was not properly excited to give maximum tube output in this run, as shown by the next figure).

The effect of varying the grid excitation on the output and wave length of the set is shown in Fig. 6; in this run C_1 was the only thing varied. The dummy antenna had an actual resistance of 8 ohms so that the maximum output was

$$0.725^2 \times 8 = 4.2 \text{ watts.}$$

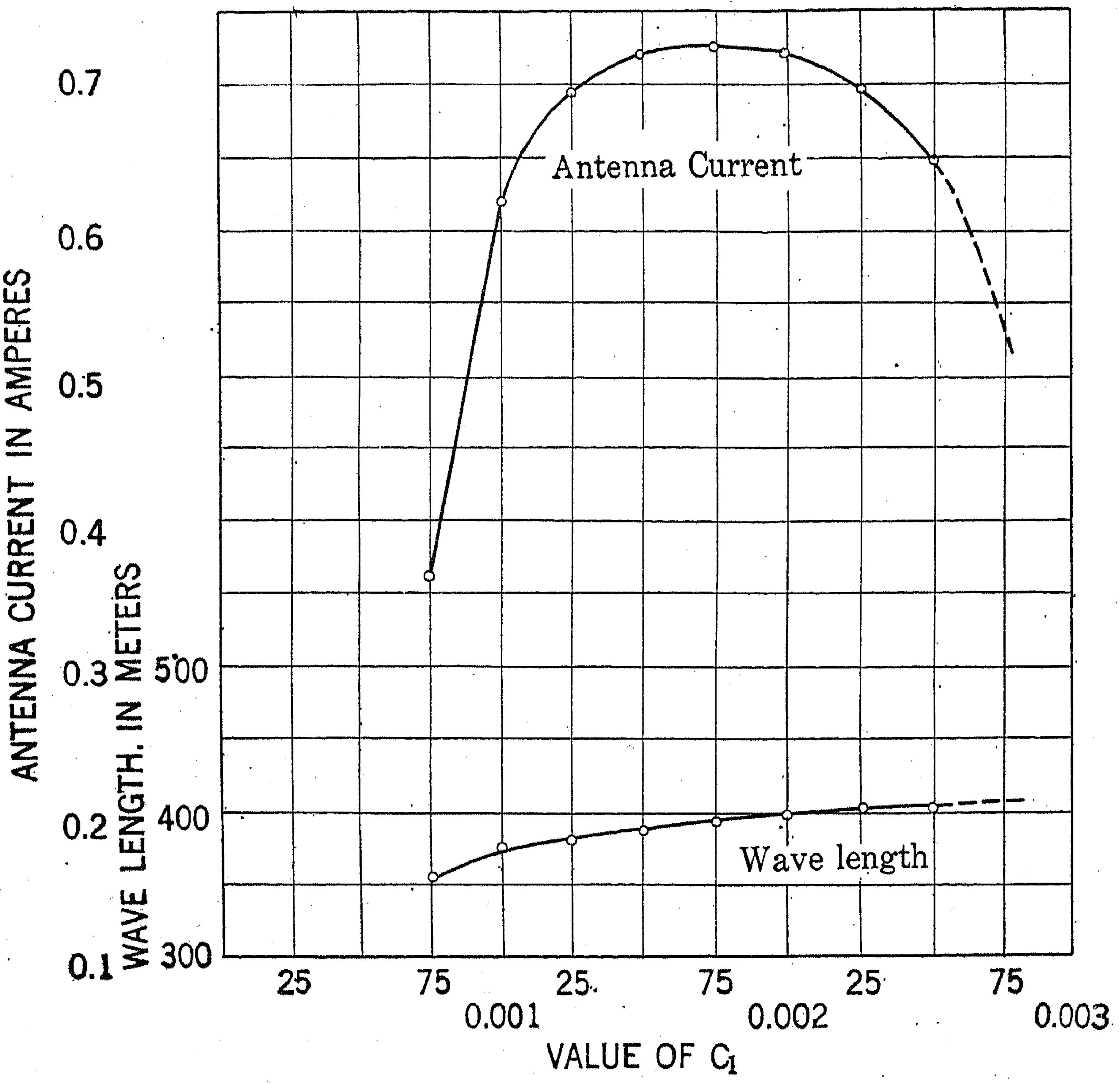


FIG. 6

The wave length increases as C_1 increases, more rapidly for the low than for the higher values as would be expected. When getting a maximum output in this run the continuous current input was 0.040 amperes at 375 volts, giving an efficiency of 28 per cent. In calculating the efficiency the tube was not charged with the power required to heat the filament; if this is taken into consideration the efficiency of the tube generator is only about 15 per cent.

The telephone sets furnished for instruction purposes at

Columbia operated excellently; they seldom gave any trouble and then it was generally some minor defect such as failure of battery, etc. The engineers who designed and constructed these sets have contributed immensely to the radio art.

A. G. Chapman: In checking over the late Mr. Crawford's paper an error was noted in formula (**71**) on the last page of the paper. In the interests of accuracy it should be noted that the values of capacitance given by this formula are too large by a factor of 4. This slight error in the otherwise excellent paper would, no doubt, have been corrected in a final revision by Mr. Crawford.

Harold S. Osborne: Only men who have worked with the problem of telephone transposition systems can appreciate the large amount of careful work which has gone into the production of Mr. Crawford's paper. I do not however concur in the practical conclusions which he has reached regarding the relative desirability of different arrangements of wires and of transpositions for use on commercial lines.

As pointed out by the author, the arrangements which he is suggesting are not new. He does not point out, however, that the square phantom arrangement has, for a long time, been the standard of the British Post Office. It was used for two-wire circuits before the development of the phantom circuit, and as such was tested in England in 1899, by Mr. Gherardi and other American engineers. The arrangements tested were found to be inferior to the flat arrangement of two-wire circuits then in use in this country. The square phantom system was studied again in 1913 by the American Telephone and Telegraph Company, to determine the relative economy and desirability of the two systems when applied to phantomed circuits. The square phantom system was found to be less desirable than the flat phantom system which was adopted as standard in this country.

In 1915 the results obtainable with the square phantom system were again carefully reviewed. In this investigation particular attention was paid to the possibilities which the square phantom system gives of providing for a transposition balance against external interferences. The matter came up in connection with the work of the Joint Committee on Inductive Interference of California, and the proposal that the American Telephone and Telegraph Company should develop a system of transpositions better adapted for coordination of transpositions with parallel 3-phase power circuits than those which were then standard. This study, therefore, took into special consideration the possibilities offered by the square phantom arrangement in obtaining coordinated arrangements of transpositions in parallel telephone and power circuits. This study, like its predecessors, led to the conclusion that the square phantom arrangement was not as desirable as the flat phantom arrangement for use in any of the conditions then prevailing.

A brief outline is given below of the factors of the problem which lead to this conclusion.

1. *Unbalance between telephone circuits.*

Most of the work which has been done in studying the relative tendency for crosstalk between telephone circuits in accordance with the two systems has been based on consideration of mutual inductances only. This is for the reason that the mutual inductances can be computed, whereas, elaborate experimental investigation is necessary to study the mutual capacity unbalances.

If perfect spacing be assumed the advantage in low mutual inductances is somewhat in favor of the square phantom.

The square phantom is much worse than the flat phantom arrangement in liability to large unbalances due to slight irregularities in construction. With the square phantom arrangement the two side circuits of a phantom are much more intimately related than with the flat phantom system, and therefore slight changes often introduce relatively very large mutual inductances. For example, a 2 or 3-degree change in the angle of a crossarm, such as is often caused by warping or other practical construction conditions, introduces a large mutual inductance between the pairs of a square phantom as exists normally between the pairs of a flat phantom. This unbalance is, of course, much more difficult to neutralize than the corresponding unbalance in the flat phantom system, because it enters as an irregularity, varying in an entirely unknown way from one span to the next, whereas in the flat phantom the unbalance is fixed in amount and therefore can largely be neutralized by opposing the effects in adjacent sections.

The electrostatic unbalances are not neutralized to as high a degree as the mutual inductances. This is because the mutual capacity unbalances between the circuits depend not only on the location of the circuits themselves, but also upon the location of other circuits on the pole line. Square phantoms having perfect spacing, therefore, and consequently having zero mutual inductances between the circuits of a given phantom group, generally do have mutual capacity unbalances, and under some conditions, these are large.

As a conclusion, it may be stated that the existing evidence is a little uncertain regarding whether, on the average, the flat phantom arrangement or the square phantom arrangement gives greater ease in reducing unbalances between telephone circuits. It seems probable, however, that because of the large effect in the square phantom system of slight irregularities of construction, the flat phantom system may be somewhat better.

2. *Induction from outside circuits.*

The coefficient of induction into the phantom circuit from outside circuits is very much lower with square phantoms than with flat phantoms. This characteristic is urged by the proponents of the square phantom system as a chief advantage of that system. However, in general, the coefficients of induction to the side circuits are worse with the square phantom system because the wires of the side circuits are more widely spaced for a given minimum separation of wires at crossarms. In some cases only one of the side circuits is worse, and one better. It is difficult to give a single figure which would represent the relative magnitudes of the coefficients of induction with the two systems of telephone transpositions because of the large difference in the comparison for different conditions and the difference in importance of different coefficients of induction. The average of computations on a phantom group for four different conditions with horizontal and triangular arrangement of the power circuit gives an induction between wires from balanced voltages on the power circuit which is somewhat greater with the square than with the flat phantom system. These coefficients are generally the most important in producing noise. The coefficients for the same conditions for induction from balanced currents are materially lower with the square phantom system. For both currents and voltages, however, the coefficients computed for two cases with a vertical arrangement of the power circuit are very much greater with the square phantom than with the corresponding flat phantom arrangement. The coefficients for residual voltage and current computed for two cases average somewhat lower for the square phantom arrangement.

It should be noted that the arrangement of the telephone conductors does not materially influence the magnitude of the voltages induced between telephone conductors in ground and these voltages are in many cases an important factor in determining the amount of interference.

It may be concluded, therefore, that from the standpoint of coefficients of induction from outside circuits, the investigations which have been made show no large difference between the square phantom and the flat phantom arrangements.

3. *Cost.*

With the square phantom arrangement, some of the transpositions are cheaper because they are obtained by a mere rotation of the wires and some must be made in ways similar to the present. The rotation of the wires, however, very materially increases the cost of stringing the wires, as compared with American practises.

In order to obtain as good a comparison as possible between the costs of systems built in accordance with these two plans, there was included in the 1915 investigation a complete design of a square phantom system for a forty-wire pole line intended to meet the same requirements as the "Exposed Line System" which uses flat phantoms. The best system which could be designed with the available information, according to the square phantom arrangement, gave a slightly greater cost than the flat phantom system for the same limits.

4. *Maintenance.*

When applied to 8 or 10-pin crossarms, the square phantom system brings in difficulties in maintenance because the crossarms are separated by only one foot instead of two. This leaves hardly sufficient space for a lineman to reach through to the outer pins. This difficulty is overcome in some cases in British construction by using 4-pin crossarms but limiting the crossarms in this way adds to the construction cost.

5. *Application to Existing Plant.*

The above factors relate to newly constructed lines. When one considers the application of the square phantom system to lines already built, additional difficulties are encountered. These difficulties are of a good deal of practical importance because most of the cases in which the use of the square phantom system would be considered are cases in which inductive interference arises from the building of power lines near existing telephone toll lines.

Modern telephone toll circuits for long distance service are very generally provided with loading coils and very frequently must work with telephone repeaters. For successful operation under these conditions it is necessary that the inductance of all loading coils on a given circuit be very exactly the same and that the capacity of the sections of circuit between adjacent loading coils also be very exactly the same. Furthermore, in order to avoid great difficulties in reducing the crosstalk between the circuits, all circuits must have their loading coils placed at the same points. The phantom circuits have a different capacity from the side circuits and correspondingly have loading coils of a different inductance.

With the square phantom arrangement the capacity of a phantom circuit per unit length is much greater and the capacity of the side circuits per unit length materially smaller than with the flat phantom arrangement. If, therefore, it is proposed to retranspose according to the square phantom arrangement a short section of existing line which is transposed by the flat system, difficulty is at once encountered. To meet the conditions outlined above for loaded lines, side circuit loading coils must be spaced farther apart in the square phantom section than in the flat phantom section and the phantom circuit loading coils must be spaced nearer together. However, this makes it no longer possible to place phantom coils and side circuit coils at the same points and constitutes a very serious difficulty.

This discussion may therefore be summed up by the statement that the possibility of obtaining advantage by the use of the square phantom or some similar arrangement has for a great many years been present in the minds of telephone engineers, that it has been very carefully studied at various times with a view to the possibility of its proving advantageous but that as yet no condition has arisen in which it seems preferable to the flat phantom system which is in use here.

J. B. Carson: In reply to Professor Pupin's interesting remarks I do not believe that any apology is needed for limiting the explicit treatment of the problem to systems which can be described by a set of simultaneous linear differential equations with constant coefficients. This limitation is, as Professor Pupin remarks, not serious and leaves the scope of treatment co-extensive with the problems in alternating-current theory which can be dealt with by the usual symbolic method involving the operator j. As an illustration of the limitation involved, however, Professor Pupin has instanced the induction motor as a problem outside the scope of the methods developed in my paper. Professor Pupin has himself, however, shown how to build up the steady-state theory of the induction motor from the usual periodic solution and by precisely analogous methods its transient states can be investigated by the present theory. In fact the method can be employed to deal with systems in which non-linear differential equations appear, such as the three-element vacuum tube.

I have re-read Professor Pupin's important 1899 and 1900 papers to which he calls attention in his discussion. Without in any sense minimizing their importance and originality, I cannot agree with him that they treated the subject exhaustively if by that Professor Pupin means the transient phenomena and oscillations of such structures. Pupin's treatment of this phase of the problem is limited to writing down the transients as a sum of characteristic damped vibrations, each of which is multiplied by an undetermined constant of integration. Now this is correct as far as it goes but it is in fact merely a formulation of the problem and not a solution. I feel sure that Professor Pupin will agree with me that the real problem lies in determining the integration constants in accordance with the initial configuration of the system and the type of applied forces, and in then throwing the solution into a manageable form which enables us to take account of terminal impedances.

Mr. Bush in his remarks raises some very interesting questions regarding the Heaviside operational methods, to which, however, I do not see that he has given satisfactory or conclusive answers. Consider the symbolic or operational expansion (equation (**1**) of Mr. Bush's discussion) of the problem discussed and solved on pages 405–409 of my paper. Now the problem involved in this operational expansion is two-fold; first to justify the expansion in the operator p itself, and secondly to determine (adopting Mr. Bush's notation) the significance of the symbols $p^n \zeta$. The first phase of the problem which can in no sense be neglected in a rigorous discussion, Mr. Bush ignores entirely and the second he begs by indentifying the symbol p^n with the differential operator d^n/dt^n at the start of the discussion. This is all the more unjustified because he subsequently assigns non-integral values to n which makes the operator d^n/dt^n meaningless. Now it is true that if we start with the solution expressed as the Fourier integral given on page 359 of the paper it is possible to evaluate the symbolic expression $p^n \zeta$ with some reservations involving the legitimacy of differentiating under the integral signs but this leaves the first phase of the problem still untouched. For we have merely shown that *assuming the legitimacy of the expansion in the operator p,* the series resulting is a *formal* solution. The real problem as I see it, still remains: to establish the legitimacy of the expansion in the operator p and then to investigate the significance of the resulting divergent series in inverse powers of t and its relation to the actual convergent solution. It is in the investigation of these important questions, without which the solution must remain in doubt, that I have found the application of integral equations, as developed in my paper, of the greatest value.

Owing to the importance of the foregoing I shall elucidate it in connection with the specific problem stated and solved on pages 405–409 and employed by Mr. Bush in his discussion: namely the evaluation of the voltage $V(t)$ existing across the cable terminals where a "unit e.m.f." is impressed on the cable through a condenser.

The solution from the integral equation of the problem, is, as given at the top of page 407:

$$V(t) = e^{\pi h t} - \sqrt{h} \int_0^t d\lambda \, . \, \frac{e^{\pi h \lambda}}{\sqrt{t - \lambda}}$$

and from the theory of integral equations we know that the solution is unique and absolutely convergent. Following this solution on page 407 it is shown that the integral equation is reducible to a differential equation which is *formally* satisfied by the series solution:—

$$V(t) \sim \frac{1}{\pi \sqrt{h\, t}} \left(1 - (1/2\, \pi\, h\, t) + 1.3\, (1/2\, \pi\, h\, t)^2 \, . \; . \; . \right)$$

This is identical with that gotten by the Heaviside operational method. So far, however, we have merely shown that this is a *formal* solution and we have established no relation, numerical or otherwise between it and the actual solution gotten from the integral equation. It is just here that the essential weakness of the Heaviside operational solution appears; it is quite powerless to establish any such relation, except empirically, or to show that the divergent series is a true asymptotic expansion of the actual solution. The solution derived from the integral equation, however, enables us to answer this question at once. To show this we write the solution as

$$V(t) = \epsilon^{\pi h t} - \sqrt{h}\, \epsilon^{\pi h t} \int_0^t d\lambda\, \frac{\epsilon^{-\pi h \lambda}}{\sqrt{\lambda}}$$

$$= \sqrt{h} \int_t^\infty d\lambda\, \frac{\epsilon^{-\pi h \lambda}}{\sqrt{\lambda}}$$

If we now perform repeated integration by parts in this expression we get:—

$$V(t) = \frac{1}{\pi \sqrt{h\,t}} \left\{ \begin{array}{l} 1 - (1/2\,\pi\,h\,t) + 1.3\,(1/2\,\pi\,h\,t)^2 \\ + (-1)^n\, 1\,.\,3\,.\,5\,\ldots\,(2\,n-1)\,.\,(1/2\,\pi\,h\,t)^n \end{array} \right\} - R_n$$

Where the remainder R_n is

$$(-1)^n \frac{\epsilon^{\pi h t}}{\pi \sqrt{h}}\; \frac{1\,.\,3\,.\,5\,\ldots\,(2\,n-1)}{(2\,h)^n} \int_t^\infty \frac{\epsilon^{-\pi h \lambda}}{\lambda^{n+1}\,\sqrt{\lambda}}$$

Clearly the remainder R_n is less than the last term of the series, consequently the series is a true asymptotic expansion of the solution, and may be used for numerical calculation with an error less than the value of the last term included.

The investigation of the operational solution by rigorous methods is by no means of merely academic interest. For example the method breaks down in at least one specific case which is gotten by replacing the condenser of the present problem by an inductance coil. Here the divergent series obtainable by a straight forward application of the operational method is not a true asymptotic expansion. Heaviside noted this himself and added an extra term which he justified by reasoning which I have been unable to follow, involving as it does an appeal to his somewhat esoteric theory of "generalized differentiation."

The foregoing merely touches on questions of the greatest mathematical interest which I hope to deal with more fully in a future paper.

Another important property of the solution, when expressed as an integral equation, which is not mentioned in the paper, is that it enables us to determine the current and voltage surges where an energized system is suddenly opened. I have gotten solution of problems of this type with a very considerable saving of labor as compared with older methods.

A final word, in the interest of historical accuracy, concerning formula (**11**). At the time my paper was published I believed that this formula originated in the independent work of Mr.

Fry and myself. I have since learned that Vaschy, "Electricite et Magnetisme," Vol. II, page 54, in his discussion of telephony gives a formula for the arrival current which is in the notation of the present paper:—

$$\int_0^t f'(\lambda)\,A\,(t-\lambda)$$

Vaschy does not make a general application of this formula nor does he appear to have appreciated its fundamental importance, and it has gone unnoticed all these years. By a curious coincidence Colonel J. B. Pomey, Chief engineer of the French Telegraph, published in the issue of February 8, 1919 of the *Revue Generale de l'Electricite* a formula superficially the same as that given in note 3 of my paper. Pomey, however, bases his formula on a consideration of the differential equation

$$\left(a_n \frac{d^n}{dt^n} + a_{n-1} \frac{d^{n-1}}{dt^{n-1}} + a_0\right) I = f(t)$$

for which his formula is a correct formal solution. In the problem of electrical networks, however, we are not at liberty to start with such a differential equation but are under necessity of dealing with a set of simultaneous differential equations equal in number to the degrees of freedom of the system. For such a problem it is easy to show that Pomey's formula except for an extremely restricted type of applied force, does not and cannot apply.

Presented at the 7th Mid-winter Convention of the American Institute of Electrical Engineers, New York, February 21, 1919.

Copyright 1919. By A. I. E. E.

IONIZATION OF OCCLUDED GASES IN HIGH-TENSION INSULATION

BY G. B. SHANKLIN AND J. J. MATSON

ABSTRACT OF PAPER

This paper deals with the ionization of occluded gases in solid insulation from the standpoint that these gas spaces are the weakest part of an insulation design and should receive first consideration. The stress at which ionization starts in different types of built-up insulation, such as used in cables and coils, is measured and from these measurements a safe working stress determined. The paper brings out, more than anything else, the importance of reducing the gas spaces to a minimum size, and using materials of the lowest possible permittivity, since the higher the permittivity the greater the stress on the gas spaces

If an insulation design is worked at a stress so high that internal ionization occurs it should be considered as "over-stressed" and its life will certainly be shortened. Just how much it will be shortened depends upon such factors as; type of insulation used, degree of over-stressing, looseness and porosity, etc. Roughly, the life of an over-stressed design is anywhere from six months to six years, judging from numerous tests made. This shows that the damage is very gradual.

INTRODUCTION

THE selection of a safe voltage rating for high-tension equipment, such as cables and coils, or more properly speaking, the selection of a safe thickness of insulation, has not, like other factors in electrical engineering, been based on scientific grounds. The method of selection has been more a question of judgment and experience, than any thing else.

It is generally admitted that high-potential testing and dielectric strength data are very unsatisfactory criterions. They are of value only in testing for flaws and defects. But assuming the insulation in perfect condition physically and as having a dielectric energy loss sufficiently low at maximum operating temperature to assure against accumulative heating or undue rise in temperature, no satisfactory method of predetermining the proper thickness has been available.

In studying the service strength of any design the weakest point has first consideration. Occluded gas is the weakest point of insulation designs, such as considered above. The solid insulation alone, always assuming, of course, that it is a good grade of material and workmanship, will safely withstand much greater stress than when occluded gases are present. When the stress is high enough to ionize these gases, the insulation should be considered "over stressed." This has been generally recognized but the fact that occluded gas is, almost without exception, inherent in organic insulation has received little recognition.

It is the purpose of this paper to present the results of an extensive study of these gas spaces and the effects they introduce. In doing so we do not wish to over emphasize the importance of this one factor. What we do wish to emphasize is that this method of study or test fills a breach in insulation engineering. The method of "judgment and experience" has, as is usually the case, given correct results. It is interesting to note in studying the data given in this paper how closely the maximum allowable working stress agrees with that which is accepted as good practise.

OCCLUDED GAS AND ITS CAUSES

There are so many variables that enter into the formation and characteristics of gas pockets, from the most minute pocket to the largest, that it is impossible to give a complete analysis at the present stage of the investigation. A certain amount of speculation, although an effort will be made to reduce it to a minimum, must be indulged in.

Each detail in the process of manufacture and application of materials introduces some new factor into the formation of gas spaces. Quite often the methods of installation and operation introduce additional factors.

Impregnated Paper Cable, is usually made by first wrapping dry paper tape around the conductor. To simplify the illustration, single-conductor, lead-sheath cable only will be considered.

The dry paper cable is put into a vacuum tank and the greater part of the moisture and air extracted under heat. It is not a perfect vacuum and a certain amount of impure air remains. Melted filling compound is then let into the tank until the cable is immersed. The compound acts as a valve and stops up all the exposed passages through the paper wrappings. Pressure is then applied and the compound forced thoroughly into the cross section. The slight amount of impure air remaining in the cable is driven back into the cross section and entrapped in the spaces or voids between wrappings. The applied pressure compresses these gas spaces to a most minute size.

After the cable has been in the tank long enough to assure complete impregnation the pressure is reduced to atmospheric and the heat cut off. The gas spaces thereby automatically expand to atmospheric pressure and remain at this pressure during cooling. After the cable has cooled down to a temperature at which the compound is somewhat thickened but still thin enough to drip it is lifted out and the excess compound cleaned off. The cable is lead jacketed while still warm and is then immune from external pressure except through the ends. Every one is familiar with the exuding of compound from the exposed ends of cable when heated. This is due to a tendency to equalize the pressure inside and out.

When the cable has cooled to a temperature of about 55 deg. cent. or lower, depending upon the type of filling compound, the compound has thickened to such an extent that pressure is not transmitted to the gas spaces and their size remains constant at lower temperatures, except for the slight enlargement that occurs due to shrinkage of the compound.

At room temperature, then, the pressure in the spaces must be something less than atmospheric. Assuming the temperature at which the compound thickens as 55 deg. cent. the pressure at 25 deg. cent. should be approximately,

$$\frac{760\ (273 + 25)}{273 + 55} = 690 \text{ mm.},$$

or allowing 4 per cent for shrinkage of compound, about 650 mm.

In speculating upon where these spaces are distributed we are inclined to believe that the greater percentage of them are initially down towards the center of the cross section, near the conductor. They are forced down into the cross section by the entrance of the impregnating compound. Either this is the case or the whole cross section contains a maze of gas spaces, for the test results invariably show spaces in the neighborhood of the conductor, where maximum stress occurs.

The aim should be always to keep these spaces reduced to a minimum, for the larger spaces are, the less the stress required to ionize the gas. It is obvious that the paper wrappings should be tight and compact. This prevents the gas from accumulating at one point in the filling compound and forming a bubble of appreciable size between loose layers of paper, or in creases or crinkles.

Varnished Tape and Sheet Materials. The conditions under which these are applied and treated is somewhat different from cable paper. Usually they are wrapped on while exposed to the room air. Varnish or shellac is brushed on between wrappings to stick them together and fill up space. It is reasonable to expect that some air will be entrapped during this application and is originally at atmospheric pressure. When the material is baked or dried out the volatile solvent joins with the air to set up considerable internal pressure and this forces passages through the thickness so that the gases can escape. A cured material is consequently more or less porous and the gas pockets all have small capillary passages to the surrounding atmosphere. They are, then, normally at atmospheric pressure and it is possible to supply fresh air through the outlet pasages as the material cools down. This is called "breathing."

If the tapings or sheets are applied tightly to conductors of round cross section the thickness of the gas spaces should be about in the same order as those experienced in well wrapped paper cables. If, however, the material is applied to conductors of rectangular cross section the thickness of the spaces might vary anywhere between those found in round cross sections and values much greater, depending upon the type of insulation considered, the process of application, conditions of use, etc. There is a tendency for the insulation to bulge away from the rectangular sides, which accounts for the wider variation in thickness of gas space.

IONIZATION CURVES

The loss introduced by ionization of occluded gases is combined with the inherent dielectric loss of the solid material and in obtaining data it is necessary to measure the total loss. Fortunately the ionization loss is an appreciable part of the total and is easily and accurately distinguished.

The dielectric energy loss measurements were made by the "Compensated Dynamometer Wattmeter Method." This was described in detail in an article published previously.[1]

---

[1] "Compensated Dynamometer Wattmeter Method of Measuring Dielectric Energy Loss," G. B. Shanklin, *General Electric Review*, Oct. 1916.

A better conception of the method, its accuracy and the way the data are worked up and analyzed can be obtained from a paper read before the Institute in June, 1917.[2]

During the three years in which we have followed this work all possible precautions have been taken to eliminate external sources of loss, phase-angle errors and other forms of inaccuracy. Numerous checks have been made besides those described in the two publications referred to. Familiarity with the method and analysis of a mass of data make us feel that an accuracy better than 2 per cent is obtained at the lowest dielectric power factors ordinarily met with in solid materials.

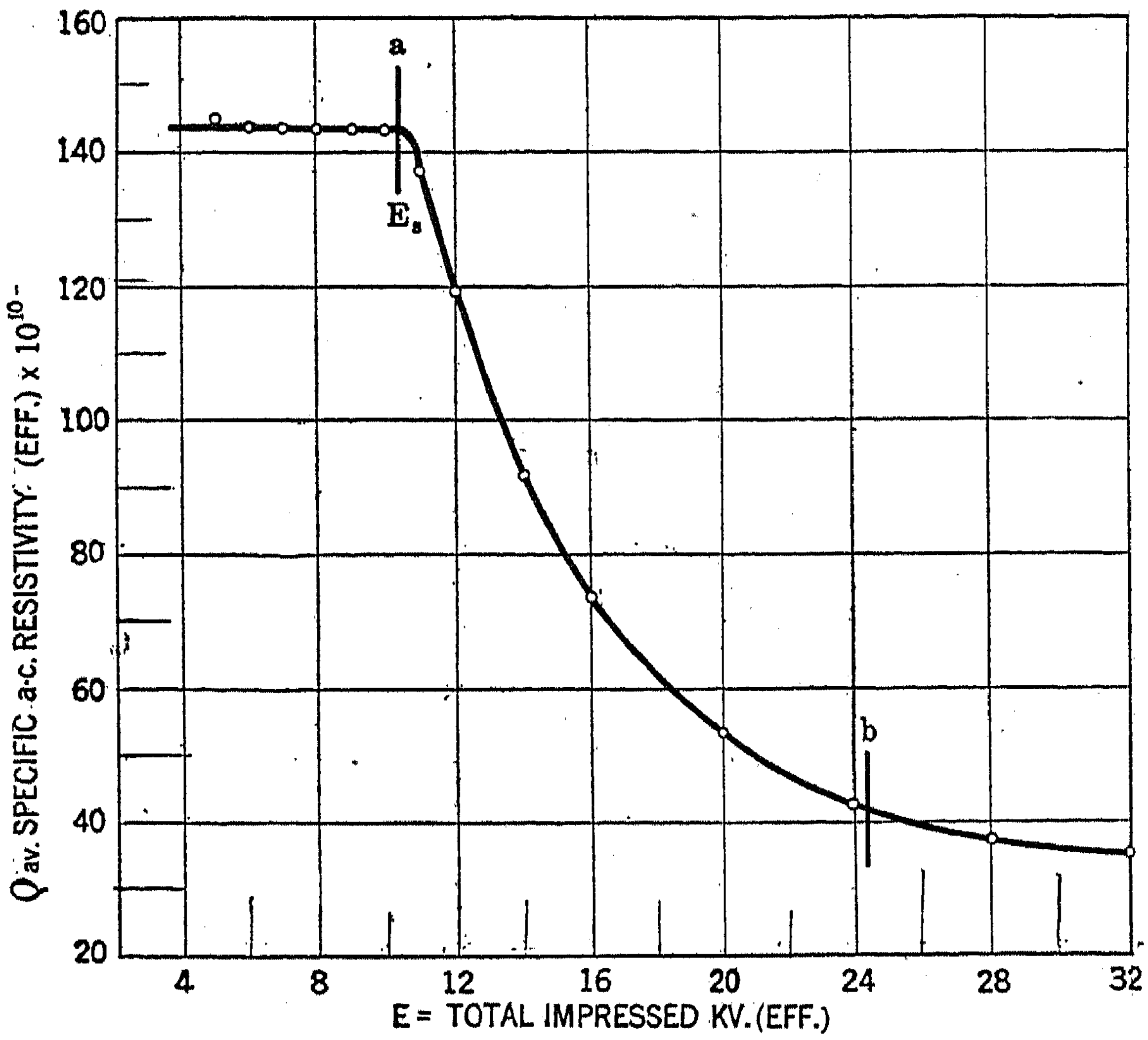


FIG. 1—TYPICAL IONIZATION CURVE

A typical ionization curve is given in Fig. 1. It is calculated from 60-cycle energy-loss data taken at room temperature on a 2/0 single-conductor cable with 9/32 in. thickness of impregnated paper and lead sheath. The watts and current were measured at the impressed voltages indicated. From these readings the effective a-c. restivity per cm^3 ($\rho_{av.}$, average through the thickness) was calculated by formula (**1**) in the Appendix.

Let it be assumed that there are gas spaces sparcely scattered throughout the cross section of cable and that the thicknesses of these spaces vary from zero to that of the largest met with. Up to 10.3 kv. (a) the loss represented by $\rho_{av.}$ is the inherent dielectric loss of the solid material and varies practically as the square of the voltage. The largest gas spaces next to the conductor are the first to be ionized. The voltage gradient at the conductor (in the solid material, not in the gas space) will be

---

[2] "Insulation Characteristics of High-Voltage Cables," Clark and Shanklin, A. I. E. E. TRANS., Vol. XXXVI, 1917, p. 447.

$$g_{max.} = \frac{E}{a \log_\epsilon b/a} = 2.218\ E,$$

and since $E_s = 10.3$ kv., $g_{max.} = 22.8$ kv. per cm.

As the voltage is gradually raised the gradient at the conductor increases and the smaller spaces at this point will be ionized. Likewise, the gradient 22.8 kv. per cm. is traveling out through the thickness ionizing the larger spaces until it finally reaches the sheath at an impressed voltage $E = 24.3$ kv. (b). Complete saturation is not yet reached but the curve is flattening out, showing that the majority of the gas spaces have been ionized. It is impossible to reach complete saturation before the cable breaks down as the extremely small gas spaces, those of microscopic dimensions, require excessive stress to ionize them.

Another factor that determines the slope of these curves is the increase in gas pressure after ionization once starts. This increase in pressure, or "ionization pressure" as it is called, is dealt with in books on gases by J. J. Thompson[3] and J. S. Townsend.[4] A paper by E. H. Warner[5] also covers the subject.

Since the gas pressure at the instant ionization starts is that with which we are dealing, the "ionization pressure" does not bear on our work, but it is mentioned merely as a matter of interest.

The relative permittivity ($K_{av.}$, average through thickness) is calculated from the energy-loss data by formula (**2**) in the Appendix. For a full description of these characteristics reference should be made to the Institute paper previously referred to.

---

[3] "Conduction of Electricity through Gases," by J. J. Thompson, Cambridge Press, 1906.

[4] "Electricity in Gases" by J. S. Townsend, Oxford Press, 1915.

[5] "Ionization Pressure, by E. H. Warner, *Phys. Review*, Vol. 8, 1916, p. 285.

$K_{av.}$ is not the true relative permittivity. In calculating $K_{av.}$ it is assumed that the dielectric circuit is a parallel circuit, that is, the leakage path (a-c.) and capacity are assumed in parallel. Actually the circuit is a maze of series-parallel paths. The series component determines the change in $K_{av.}$ with frequency, depending upon the type of material considered. For instance, in varnished cambric or impregnated paper the difference in $K_{av.}$ at 25 and 60 cycles is small, especially at higher temperatures, because the series component is small. In pasted mica the difference is greater due to the "barrier effect" of the mica flakes.

$K_{av.}$ is an average effective value through the whole thickness and is consequently the value needed in this work. It corresponds to the effective a-c. resistance of a conductor in which skin effect is appreciable. The true permittivity corresponds to the d-c. resistance of the conductor.

Knowing the effective permittivity of the solid insulation and the voltage at which ionization begins, curves can be calculated showing the stress across gas spaces of any assumed thickness. Also, knowing approximately the pressure of the gas and its dielectric strength, cross curves can be plotted and a comparison will indicate the maximum thickness of gas space. This method of dealing with the data will be followed throughout the paper. It would be impossible to present all of the energy-loss data or even the ionization curves alone, as hundreds of them were taken.

In a number of cases measurements were taken at both 25 and 60 cycles. The voltages at which ionization started agreed almost exactly. In other words, the ionization starting point is practically independent of frequency over the usual commercial range.

During the initial stages of this work considerable difficulty was experienced in obtaining uniformly smooth and consistent curves, such as that shown in Fig. 1. In particular, the points taken at the lower voltages would not line up on a smooth curve but plotted in a haphazard manner. We at first came to the conclusion that the ionization was inherently unstable and followed no simple law, but further investigation showed this trouble to be due to two causes:

1. The errors in voltage reading are squared in the calculation of $\rho_{av.}$ and come in once more in the abscissas of the curves. The greatest possible accuracy in voltage reading over the whole range is therefore necessary.

2. The wave shape varied slightly over the voltage range. This not only changed the values of $\rho_{av.}$ but also gave an ionization starting point at variance with that of a sine wave. This error was corrected by connecting a relatively large capacity in parallel with the test sample. A pure sine wave over the whole voltage range resulted.

The effects of a badly distorted wave are shown in Fig. 2 the data being taken at 60 cycles, room temperature, on a sample of mica-paper insulation 0.16 in. thick, wrapped on a conductor of rectangular cross section. Baking at high temperature had caused the insulation to loosen up and form numerous gas spaces. Curve (*a*) shows measurements with a distorted wave and curve (*b*) with a sine wave. Oscillographic records of the two waves are shown in Fig. 3. The higher harmonics in (*a*) were caused by an iron core reactance in the low voltage side. Wave shape (*b*) was taken after 0.05 microfarads capacity was connected in parallel with the test sample.

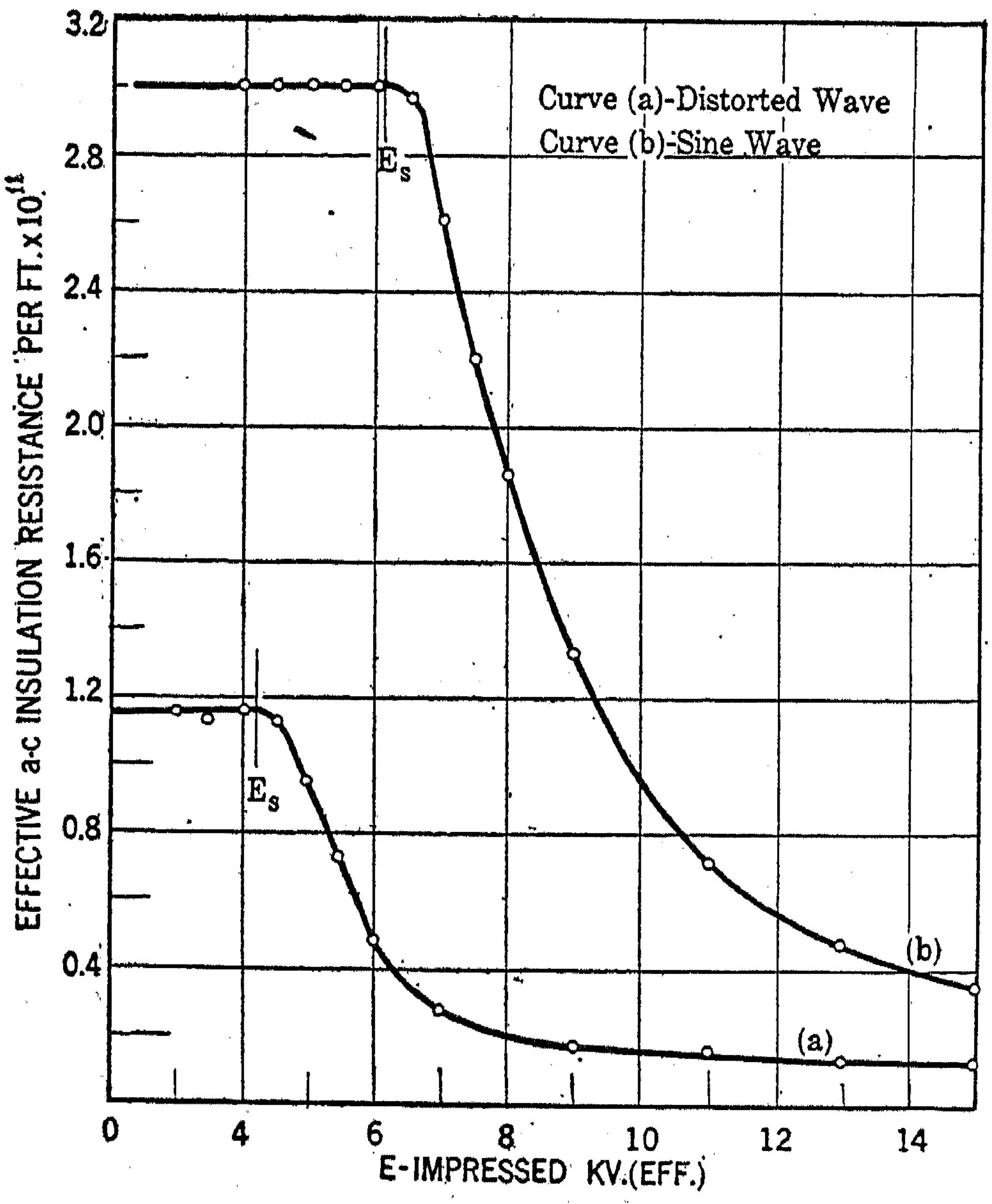
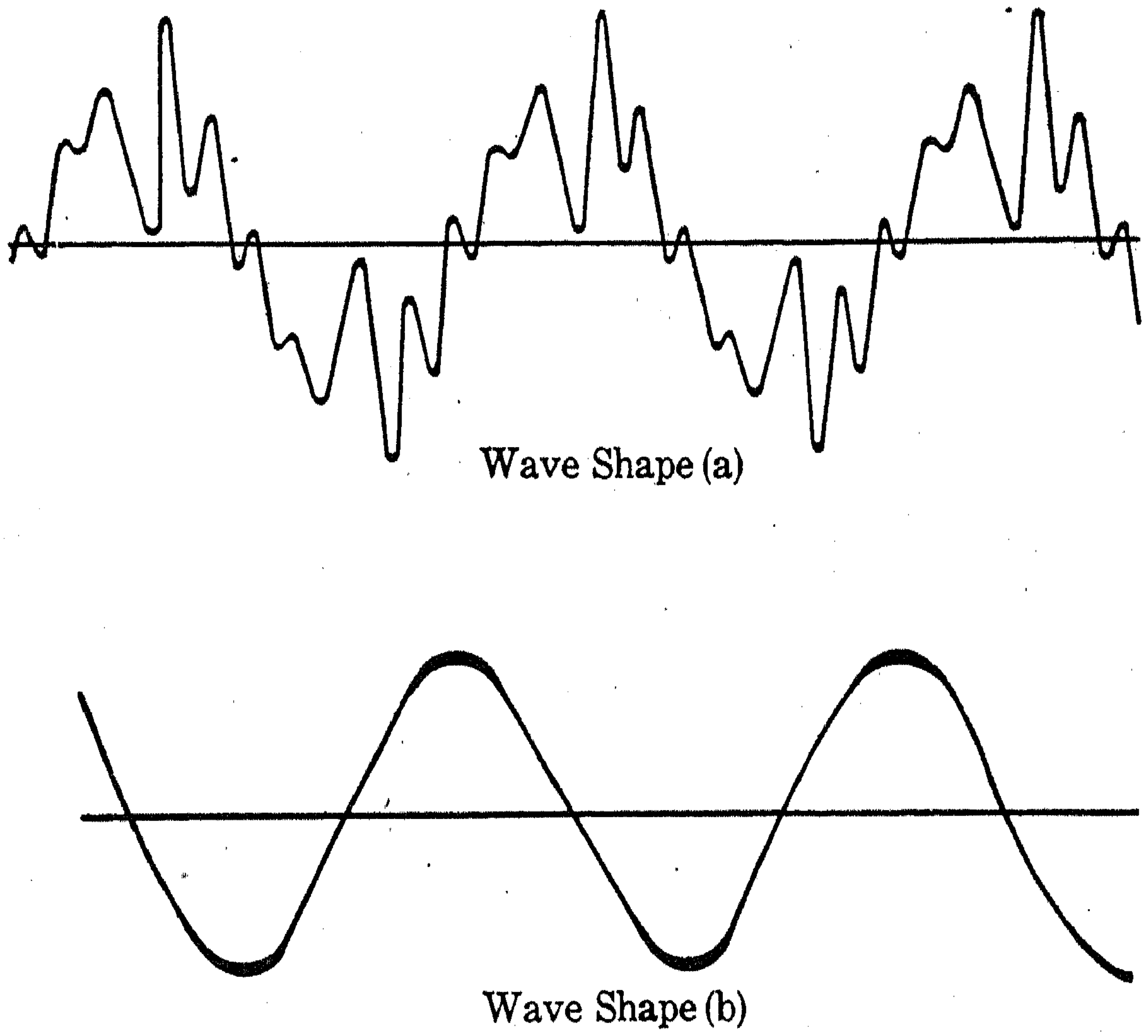


FIG. 2—IONIZATION CURVES SHOWING EFFECT OF DISTORTED WAVE SHAPE

Ionization starts at 6.1 kv. effective with the sine wave, while with wave (*a*) it starts at 4.2 kv. The starting point with wave (*b*) is $6.1 \times \sqrt{2} = 8.62$ kv. maximum, with wave (*a*) it is $4.2 \times 2.03 = 8.53$ kv. maximum. These two values correspond closely.

STRENGTH OF AIR

Before the thickness of gas space in a test sample, can be determined the strength of gas under exactly the conditions, met with must be known. From the nature of these conditions the gas considered must be a mixture of air and the volatile organic contents of the insulation, the amount of impurity depending upon the vapor tension of the different components, the ability to supply fresh air to the gas spaces, etc.


Wave Shape (a)

Wave Shape (b)

FIG. 3—OSILLOGRAPH RECORDS OF WAVE SHAPES REFERRED TO IN FIG. 2

In those cases where components of low vapor tension, such as benzol, are present the nature of the conditions is such that passages are forced from the gas pockets to the outside air and the impurities mostly escape.

Impregnating compounds are heat or vacuum treated before using, and their components have a comparatively high vapor tension. When impregnating compound is admitted to the filling tank the vacuum decreases but little faster than the decrease in gas volume, which shows that the amount of impurity liberated is not great.

We can say roughly, that, the occluded gas in insulation is a mixture of air and from zero to about 50 per cent impurity, the impurity as a general rule being an organic vapor. The best that can be done is to consider the strength of air alone. The amount of impurity will vary in different cases and complete ionization data are available only for air.

On pages 466 to 471 of J. J. Thompson's book he discusses and analyzes data on various gases and vapors. The strength of CO_2 and water vapor is about equal to that of air over the range of pressure and gap spacing needed in our work. The strength of benzol vapor varies from about 1.0 to 2.5 times that of air.

TABLE I
DIELECTRIC STRENGTH OF AIR BETWEEN PARALLEL PLATES

P = pressure in mm.
t = air gap in mm.

$P \times t$	Volts Eff. A-C.	$P \times t$	Volts Eff. A-C.	$P \times t$	Volts Eff. A-C.
1.05	1371.5	9.0	260.0	160.	1025.
1.10	1226.5	10.0	270.0	200	1195
1.35	813.0	15.0	308.0	300	1602
1.50	686.0	20.0	336.5	400	1980
2.00	452.5	25.0	363.0	500	2350
2.50	346.5	35.0	417.5	600	2705
3.00	295.5	45.0	471.5	800	3402
4.00	254.5	60.0	554.0	1000	4075
5.00	249.0	80.0	664.0	1200	4720
6.00	249.5	100.0	756.0	1500	5655
7.00	250.0	120.0	848.0	1700	6260
8.00	252.0	140.0	937.0		

In discussing Bouty's tests on different gases between glass plates, Thompson concludes that the nature of the electrodes (whether metal or dielectric material) has no effect unless the spacing is comparable with the mean free path of the gas molecules. Tests in our laboratory at 60 cycles, atmospheric pressure, showed no difference in the strength of air between brass electrodes and glass plate electrodes down to the minimum spacing (0.05 cm.) that could be measured accurately.

Table No. 1 gives the strength of air in terms of Pachen's Law;

$$Pt = \text{a constant voltage.}$$

where P = mm. pressure, and
t = mm. gap spacing.

The data in this table represent an average of the values obtained by a number of investigators with both direct and alternating voltages. Most of the data were taken from the books by J. J. Thompson and J. S. Townsend.. In some cases original sources were referred to.[6]

Pashen's Law as given above is based on the assumption that $P \times t =$ a constant mass of gas per unit area of gap. Also, the figures in Table I are based on an air density equivalent to a temperature of 25 deg. cent.

It is obvious that when increasing the temperature of a test sample above 25 deg. cent. this condition is not met with. A correction can be made, however, if the gas pressure and temperature are known. Harris[7] has shown that a change in air density produced the same effect on dielectric strength whether due to temperature or pressure.

In paper cables, for instance, the gas pressure is assumed as 650 mm. at 25 deg. cent. and 760 mm. above 55 deg. cent., as previously discussed. This can be corrected for by assuming "equivalent pressures" proportional to the absolute temperatures, instead of the actual pressure. The "equivalent pressures" are:

650 mm. at 25 deg. cent.
760 mm. at 55 deg. cent.

$$760 \times \frac{328}{373} = 668 \text{ mm. at 100 deg. cent.}$$

In wrapped insulation that has been cured the gas pressure is assumed as 760 mm. over the whole temperature range. The "equivalent pressures" therefore are:

760 mm. at 25 deg. cent.

$$760 \times \frac{298}{373} = 607 \text{ mm. at 100 deg. cent.}$$

$$760 \times \frac{298}{473} = 479 \text{ mm. at 200 deg. cent.}$$

---

[6] "Laws of Elec. Discharge in Gases," by W. R. Carr, Phil. Trans. of Royal Society A, Vol. 201, 1903.

"Comparison of A-C. and D-C. Spark Potentials, by J. C. Jansen, *Phys. Review*, Vol. 8, 1916.

"Dielectric Phenomena in High-Voltage Engineering," by F. W. Peek, McGraw-Hill Co., 1915.

[7] W. S. Harris, Phil. Trans. Royal Society, 1834, p. 230.

TABLE II.
SINGLE-CONDUCTOR PAPER CABLE:
FILLED WITH MINERAL HYDRO-CARBON BASE COMPOUND. LEAD SHEATH. NEVER IN SERVICE.

Sample No.	Type of conductor	Dia. cond. inches	Thick. insul. inches	G_{max} kv. per cm. at cond.	Temp. deg. cent.	Freq.	K_{av}	E_s Start of ionization kv.	$G_{max.}$ at start kv. per cm.	Remarks
No. 1	stranded	0.412	9/32	2.218E	25	60	3.28	10.2	22.65	Wrappings compact and well filled. Inherent loss higher than average.
					50	60	3.30	10.9	24.20	
					75	60	3.45	11.0	24.40	
No. 2	stranded	0.412	9/32	2.218E	21.5	60	2.93	10.1	22.40	Wrappings compact but not very well filled. Inherent loss less than average.
					51.5	60	2.96	11.2	24.85	
					73.0	60	2.90	12.0	26.60	
					100.0	60	2.93	13.2	29.30	
No. 3	stranded	1.346	14/32	1.17E	25.0	60	2.85	20.0	23.40	Wrappings compact and well filled. Inherent loss less than average.
					47.5	60	2.87	22.0	25.75	
					72.5	60	2.92	24.0	28.07	
No. 4	stranded	1.186	26/32	0.770E	23.	60	3.17	30.0	23.10	Wrappings compact and fairly well filled. Average inherent loss.
					60	60	3.27	31.5	24.25	
					100	60	3.67	33.5	25.80	
					60	25	3.55	31.5	24.25	
					100	25	4.20	33.5	25.80	
No. 5	smooth	1.311	24/32	0.7875E	26	60	3.10	28.5	22.45	Conductor surface smoothed with 1/16 in. coating of lead. Wrappings compact and fairly well filled. Average inherent loss.
					60	60	3.23	32.0	25.20	
					100	60	3.70	32.75	25.80	
					28	25	3.30	29.0	22.85	
					60	25	3.50	32.5	25.60	
					100	25	4.20	33.0	26.00	
					SAMPLE WITH LARGE GAS SPACES.					
No. 6	stranded	1.186	22/32	863.E	23.5	60	2.91	15.0	12.95	Wrappings loose and badly wrinkled. Only partly filled. Inherent loss less than average.
					59.0	60	3.05	16.0	13.81	
					97.0	60	3.20	17.0	14.68	
					23.5	25	3.11	15.5	13.38	
					59.5	25	3.24	16.0	13.81	
					98.0	25.	3.68	16.5	14.25	

When determining the strength of air from Table I the "equivalent pressures" should be used instead of the actual pressures.

IONIZATION DATA

Single-Conductor Paper Cable: Filled with Mineral Hydrocarbon Base Compound. Lead Sheath. In Table No. 2 data are given for samples of single-conductor paper cable filled with mineral hydro-carbon base compound. The various cross sections cover a wide range. The voltage at which ionization starts (E_s) was taken from measured curves similar to that in Fig. 1. In every case it is the minimum value measured on two or more separate samples.

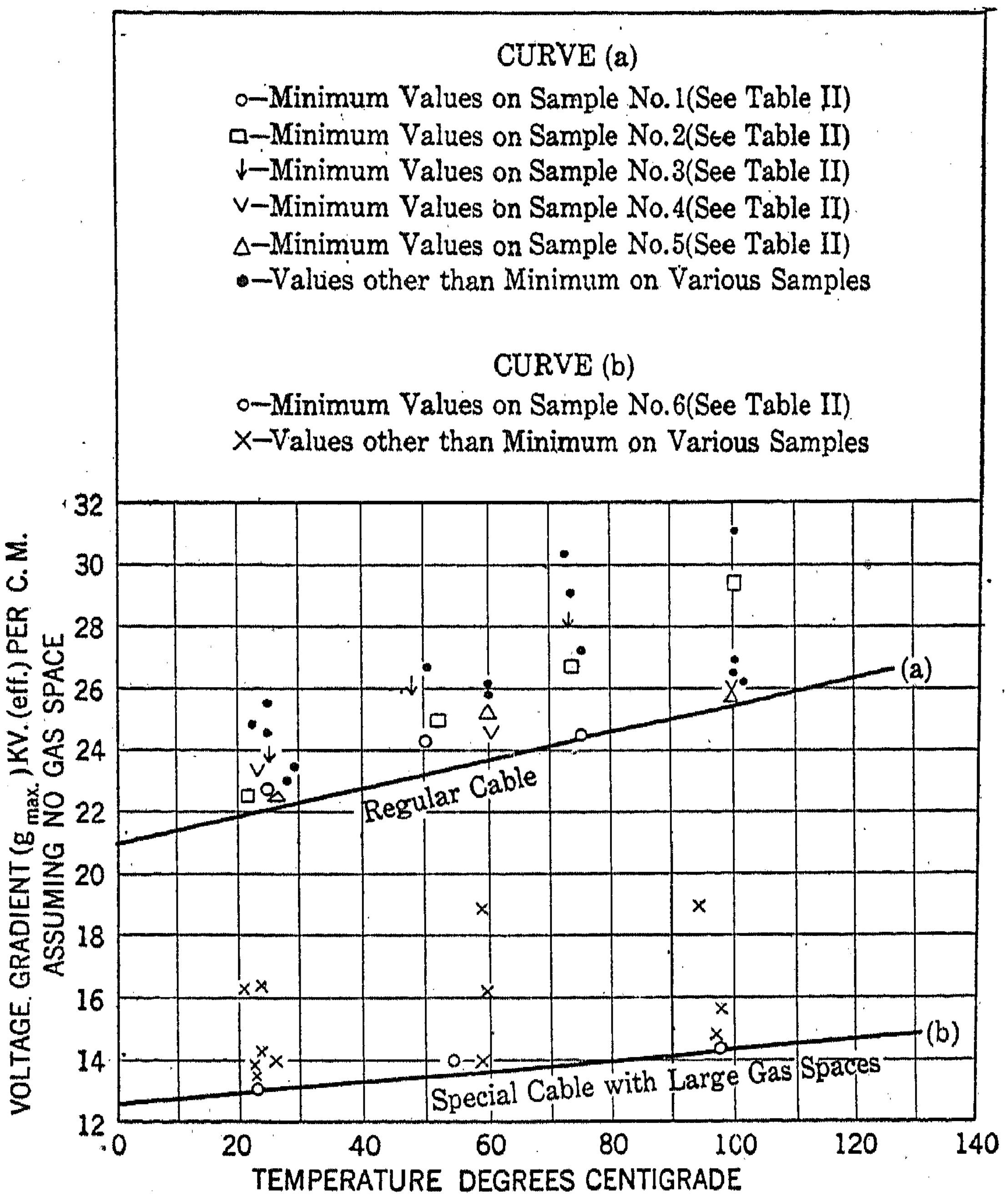


FIG. 4—MAXIMUM VOLTAGE GRADIENT BETWEEN CONDUCTOR AND SHEATH (ASSUMING NO GAS SPACE) AT START OF IONIZATION

Single-conductor paper cable—filled with mineral hydro-carbon base compound— new cable; never in service

In Fig. 4 the voltage gradient (effective kv. per cm.) in the solid insulation at the conductor, that is, assuming no gas spaces present, is given for the different cables at the instant ionization starts. The assumption of no gas spaces is allowable, for even when present they are so small as not greatly to effect the distribution of stress in the solid material. Also, it would be expected that the larger number of the stress lines between conductor and sheath pass only through solid material without encountering gas spaces.

The gradient (g_{max}) agrees surprisingly well for the different cross sections. All of the points above curve (a) in Fig. 4 are representative of the values met with in ordinary commercial cable of this type. In a number of cases we found values of g_{max} higher than those given in Table II. These are given in Fig. 4 but are not included in the table, as the minimum values are of greater importance. Occasionally there would apparently be no ionization at all above 70 deg. cent.

Extending the minimum curve (a) in Fig. 4 back to 0 deg. cent., a temperature lower than would be met with in operation, gives a value of 21. kv. per cm. Allowing a further margin of safety it would appear that 19 or 20 kv. per cm. should be selected as the maximum allowable working stress in a cable of this type, assuming, of course, that the cable is compactly made up and in good condition and further that the working voltage has approximately a sine wave.

Ionization starts in sample No. 5 at the same value of g_{max} as in the other samples. This sample had a smooth surfaced conductor while the others had stranded conductors. We have made a close study of smooth and stranded conductors and have never found any difference between their effects. Either the gas spaces occur slightly out through the thickness of insulation from the conductor, or stranded conductors give a uniform radial stress without appreciable distortion next to the conductor.

It now remains to determine approximately the maximum thickness of gas space met with in this type of cable. Taking sample No. 5 as a representative sample the stress across gas spaces of assumed thicknesses was calculated at 26 deg. and 100 deg. cent. using formula (3) in the Appendix. The stress (g_a) across the gas space is calculated as the average by making $x = a + 1/2\ t_a$ when the gas space is assumed next to the conductor, or $x = b_1 + 1/2\ t_a$ when the space is assumed out in the thickness of insulation, which is allowable because there is only one or two per cent difference between the maximum and minimum stress across the gas space. In other words, the lines of stress across the gas space are practically parallel. This is due to its extreme thinness when compared with radius a.

The calculated curves discussed above are given in Fig. 5. The gas space (t_a) is first assumed as next to the conductor and then 5 and 15 per cent out in the thickness of insulation from the conductor. The difference in the permittivities of gas and solid material makes the stress much higher in the gas space, as can be seen by comparing Figs. 4 and 5.

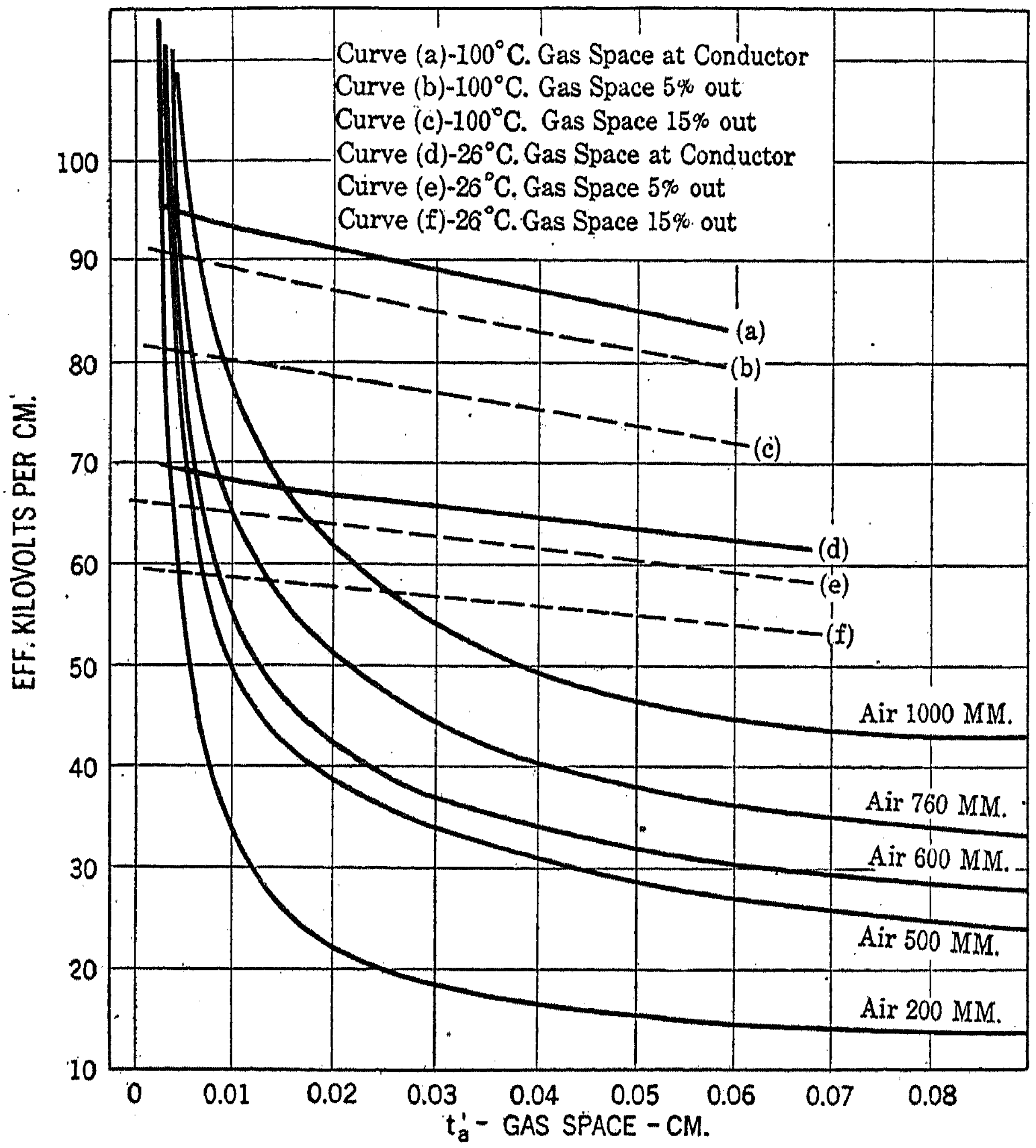


FIG. 5—CROSS CURVES FOR DETERMINING THICKNESS OF GAS SPACE—SAMPLE NO. 5 (SEE TABLE II)

The strength of air, taken from Table I, is cross plotted in Fig. 5 at various assumed pressures. It is questionable whether the strength of pure air is closely representative of the impure mixture in the gas spaces. However, it serves the purpose of approximation and comparison of the different materials considered. It is interesting to note that according to Thompson the strength of the impure mixture must be higher, not lower, than that of pure air.

Assuming a gas pressure of 650 mm. at 25 deg. cent. and 760 mm. at 100 deg. cent., as previously considered, the corresponding "equivalent pressures" are;

650 mm. at 25 deg. cent.
668 mm. at 100 deg. cent.

With the gas space assumed next to conductor the values of t_a, estimated from Fig. 5 at points where the curves cross are:

0.0068 cm. at 25 deg. cent.
0.0036 cm. at 100 deg. cent.

This decrease of t_a with increase in temperature apparently does not check up with the assumption that the gas pressure is only 760 mm. at 100 deg. cent. for the gas is totally enclosed and must expand or the pressure must increase.

This discrepancy might be accounted for in the following way; t_a measures between adjacent bare surfaces of the slightly loose paper wrappings, or between the under surface of the first wrapping and the conductor. In the general re-adjustment which occurs when the solid material, liquid filler and gas expand the paper wrappings are pressed closer together and the gas allowed to expand in a lateral direction between the paper surfaces, giving greater area but less thickness. This could also account for the disappearance of ionization at higher temperatures that is sometimes met with. The thickness of gas space (t_a) may in these cases be so slight that the stress across it is insufficient to ionize the gas.

Similar curves for sample No. 2 of much smaller cross section are given in Fig. 6. The values of t_a measured from the curves in Figs. 5 and 6 are only slightly different and this is due to the difference in permittivities and starting gradient (g_{max}). The values of t_a are independent of cross section as long as t_a is small compared with the thickness of insulation.

Cable With Large Gas Spaces. Sample No. 6 in Table II was a special length made up with comparatively large gas spaces for test. The first 5 per cent of the paper wrappings were applied loose and crinkled, the remaining wrappings being uniformly firm. The length was then vacuum treated in the usual way but was not left in the compound tank long enough to thoroughly fill it. Compound penetrated the whole cross section but not thoroughly, as was shown by examination after test, the gas spaces in the loose crinkled section being plainly visible when the wrappings were carefully removed.

Curve (*b*) in Fig. 4, extended back to 0 deg. cent. gives a voltage gradient of 12.5 kv. per cm., a dangerously low value. Curves for determining the maximum thickness of gas space are given in Fig. 7. The "equivalent pressures" are:

$$650 \times \frac{296.5}{298} = 647 \text{ mm. at } 23.5 \text{ deg. cent.}$$

$$760 \times \frac{328}{370} = 674 \text{ mm. at } 97 \text{ deg. cent.}$$

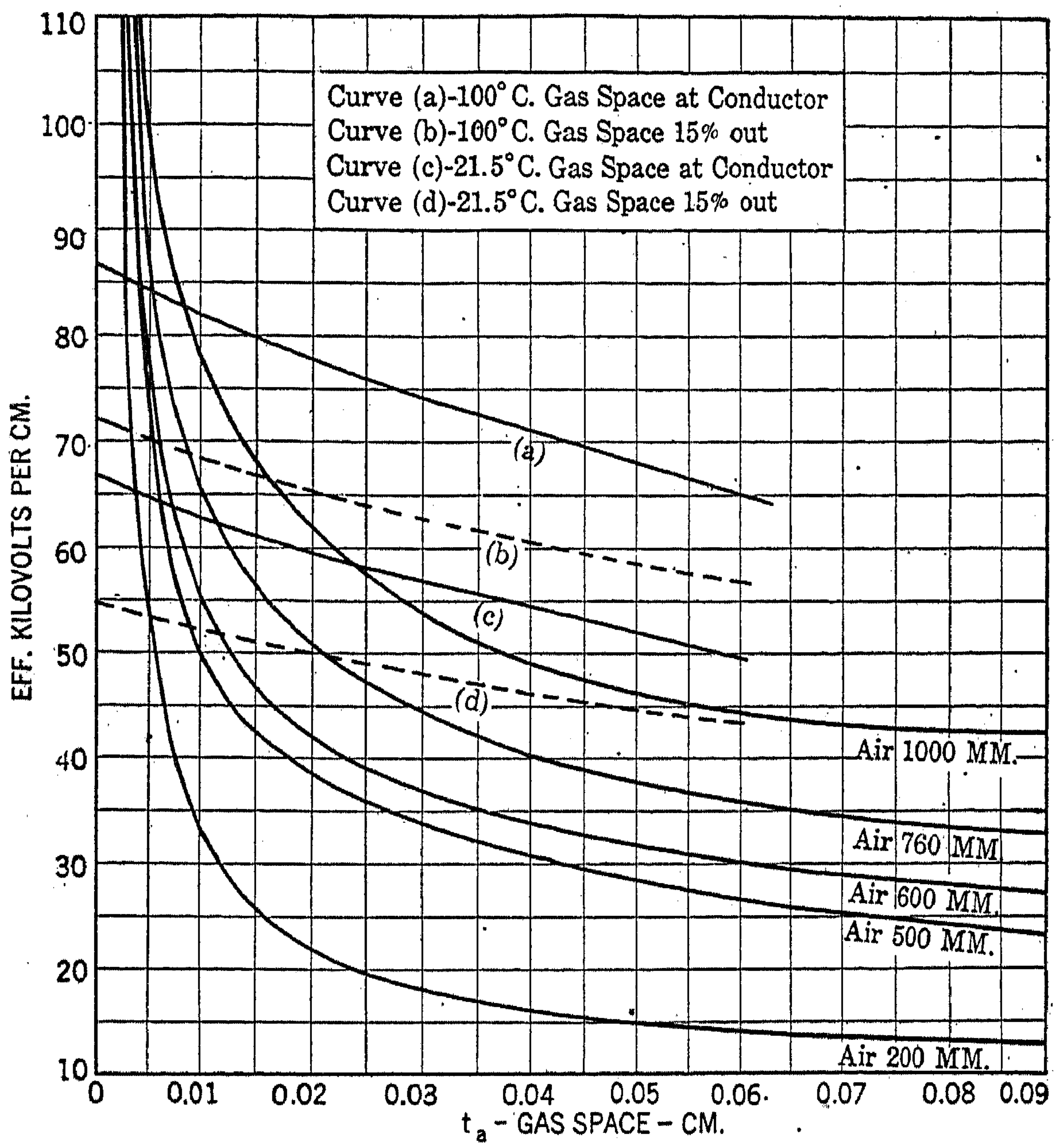


FIG. 6—CROSS CURVES FOR SAMPLE NO. 2 (SEE TABLE II)

With gas spaces assumed next to conductor the values of t_a are:

0.047 cm. at 23.5 deg. cent.
0.023 cm. at 97 deg. cent.

The spaces in this sample are therefore about seven times larger than those in samples Nos. 2 and 5 which are representative of average good cable of this type. It is obvious that large gas spaces are dangerous. All engineers have recognized this. The data given above merely verify a generally accepted view.

It will be noted that the 23.5 deg. cent. curve in Fig. 7 falls below the curve of the dielectric strength of air at 760 mm. This is very good proof that the pressure in the gas spaces is something less than atmospheric at room temperature.

Three-Conductor Paper Cable, Filled with Mineral Hydrocarbon Base Compound. We have had very poor success in attempting to investigate internal ionization and other characteristics of three-conductor cables with actual three-phase voltage. The degree of accuracy in the measurements has not been sufficient to obtain uniform and consistent ionization curves. Our work in this direction agrees with that of F. M. Farmer,[8] in that, as far as can be determined, there is no difference in the actual three-phase losses and those calculated from single-phase measurements. The investigation is being continued and we are hoping for better results later.

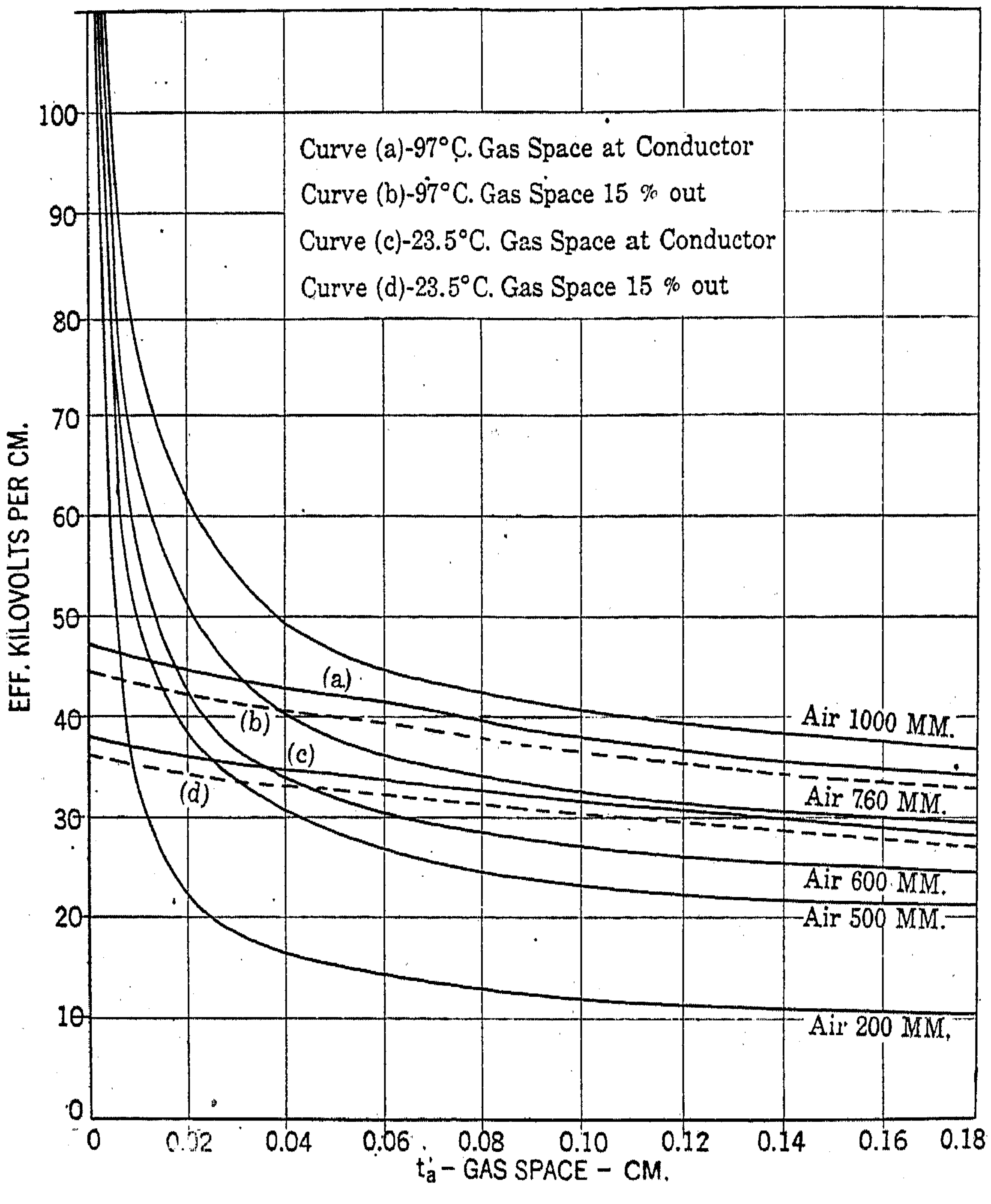


Fig. 7—Cross Curves for Sample No. 6 (See Table II)

---

[8] Measurements of Power Loss in Voltage of Three-Conductor High-Tension Cables, by F. M. Farmer, Trans. A. I. E. E., 1918, p. 221.

TABLE III.
THREE-CONDUCTOR PAPER CABLE:
FILLED WITH MINERAL HYDRO-CARBON BASE COMPOUND. LEAD SHEATH. STRANDED CONDUCTORS.

Sample No.	Dia. cond. inches	Thick. insul. inches	*Gmax.* kv. per cm. at cond.	Temp. deg. cent.	Freq.	*Kav*	*Es* start of ionization	*Gmax.* at start kv. per cm.	Remarks
				New Samples—Never in Service					
No. 7	0.412	$\frac{9}{32} \times \frac{7}{32}$	1.45*E*	25	60	3.26	15.5	22.48	Three conductors in.
				25	60	3.26	15.5	22.48	Two of conductors withdrawn.
No. 8	0.412	$\frac{9}{32} \times \frac{7}{32}$	1.45*E*	22.5	60	3.24	15.0	21.75	Wrappings compact but not very well filled. Loss less than average.
				60	60	3.33	16.0	23.20	
No. 9	0.412	$\frac{9}{32} \times \frac{7}{32}$	1.45*E*	48.	60	3.53	16.0	23.20	Average well filled. Loss about average.
				74.	60	3.60	16.4	23.79	
				94.	60	3.61	16.8	24.37	
No. 10	0.728	$\frac{7}{32} \times \frac{7}{32}$	1.396*E*	23	60	3.69	17.6	24.57	Well filled. Loss about average.
No. 11	Semi Sector	$\frac{9}{32} \times \frac{5}{32}$	1.121*E*	21	60		20.1	22.53	Assumed as concentric single cond. a = 2.104 cm.
				98	60		22.0	24.70	
				Old Samples—Had Been in Service.					
No. 12	0.373	$\frac{9}{32} \times \frac{7}{32}$	1.51*E*	24	60	3.23	20.0	30.20	Well filled. Loss a little higher than average.
				51	60	3.40	22.0	33.20	
				77	60	3.44	22.6	34.15	
				101	60	3.53	23.4	35.30	
No. 13	0.412	$\frac{9}{32} \times \frac{7}{32}$	1.45*E*	26	60	3.35	20.0	29.00	Well filled. Loss a little higher than average.
				64	60	3.45	22.0	31.90	
				100	60	3.52	24.0	34.80	

The start of ionization between any two conductors cannot be studied with single-phase voltage, but only that between conductor and sheath. This is unfortunate, for in actual three-phase operation the stress is greatest between conductors, whereas with, single-phase voltage the stress is greatest between conductor and sheath, and ionization naturally starts here first, blanketing the results between conductors. A study of the stress between conductor and sheath is instructive, however, and will be given.

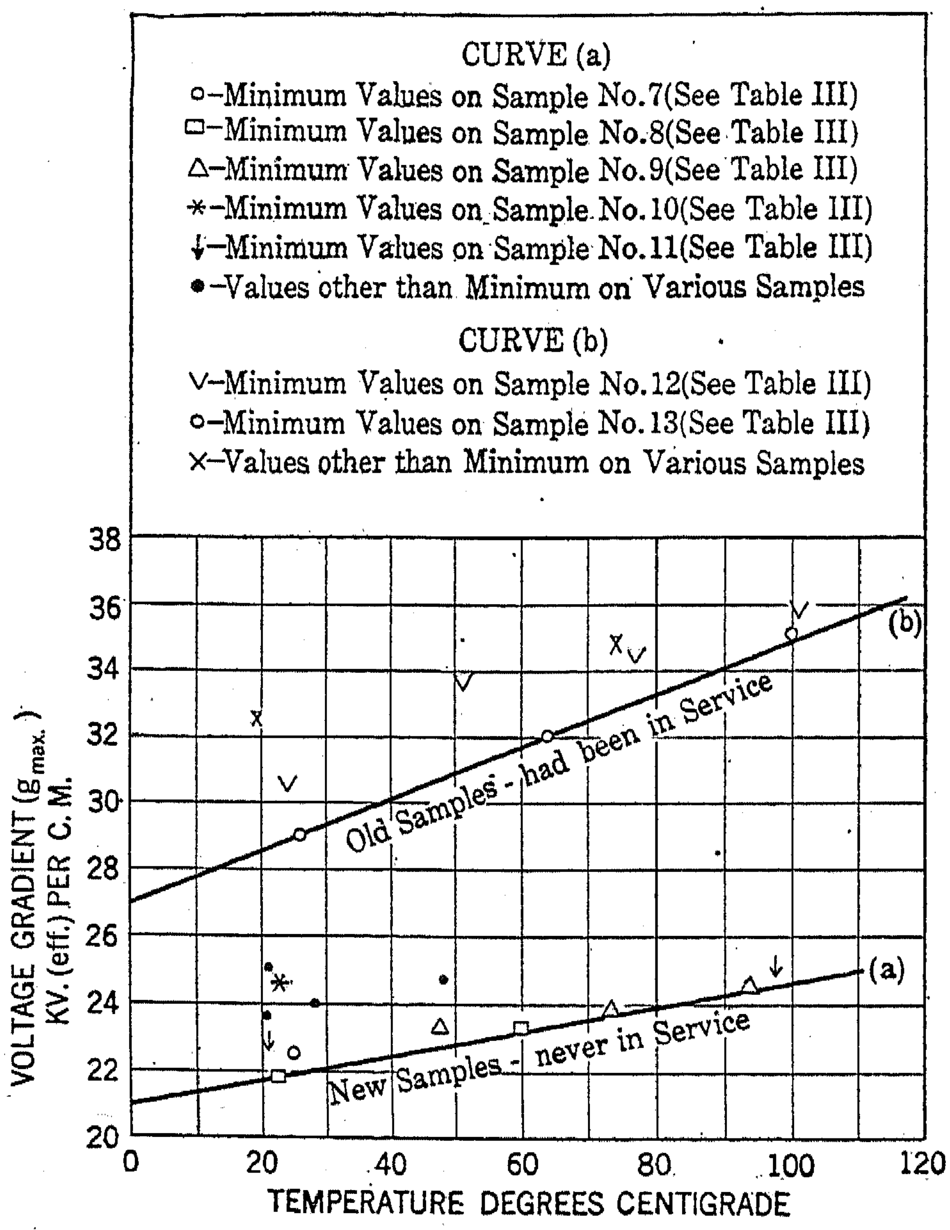


FIG. 8—MAXIMUM GRADIENT BETWEEN CONDUCTOR AND SHEATH (ASSUMING NO SPACE) AT START OF IONIZATION

Three-conductor paper cable filled with mineral hydro-carbon base compound

Calculation of the maximum stress between conductor and sheath (assuming no gas space present) is made by formula (5) in the Appendix. In this formula, it is assumed that the proximity of the other two conductors has no effect on g_{max}. The correctness of this assumption is verified by a comparison of the values of g_{max} for new cable in Table III and Fig. 8 with those of single-conductor cable in Fig. 4. A special test on sample No. 7 in Table III gives, perhaps, better verification.

Sample No. 7 was first tested by applying high voltage to all three conductors and grounding the sheath. The voltage at which ionization started (E_s) was found to be 15.5 kv. Two of the conductors were then carefully withdrawn from the sample, strand at a time, and the test repeated. E_s agreed exactly with that found in the first test.

The minimum values of g_{max}. for new samples of three-conductor cable, in Fig. 8 (curve *a*), agree closely with the corresponding values for single-conductor cable, Fig. 4 (curve *a*), and it must be concluded that the conditions, such as thickness of gas space, gas pressure, etc. are the same in both types of cables. This would be expected, since they are made of the same materials and by the same process.

Sample No. 11 has semi-sector shaped conductors. The outer circumference of these conductors are arcs, concentric with the lead sheath. As far as the maximum stress between conductor and sheath is concerned this type can be considered as a single-conductor cable and the max. stress calculated in the usual way;

$$g_{max} = \frac{E}{a \log_\epsilon b/a}$$

where, a = the radius to the outer surface of conductor, and b = inner radius of sheath.

Tests on old samples that had been in service, samples No. 12 and 13, gave very interesting results. Curve (*b*) in Fig. 8 shows these to have much higher values of g_{max} when ionization starts. This same permanent increase in E_s has been noted several times in new samples after they had been heated above the melting point of the mineral compound filler. We can only account for it by suggesting some sort of "settling" or re-adjustment of the melted compound and gas spaces in such a way as to make the spaces permanently smaller, at least those nearest the conductor. This probable explanation has previously been offered for the increase of g_{max}. with temperature, although in that case the increase considered was only temporary. The two effects are probably closely related, as will be brought out later.

The study of old samples has been limited and we do not know whether this settling effect is inherent or not. It would hardly be safe to rely upon improvement with aging in every case, and the minimum curve (*a*) in Figs. 4 and 8 should stand as a safer guide. However it is re-assuring to know that the change in ionization with aging is in the right direction.

TABLE IV.

THREE-CONDUCTOR PAPER CABLE:

FILLED WITH VEGETABLE HYDRO-CARBON BASE COMPOUND. LEAD SHEATH. STRANDED CONDUCTORS.

Sample No.	Dia. cond. inches	Thick. insul. inches	G_{max}. kv. per cm. at cond.	Temp. deg. cent.	Freq.	K_{av}	E_s start of ionization	G_{max}. at start kv. per cm.	Remarks.
				New Samples—Never in Service.					
No. 14	0.528	$\frac{7}{32} \times \frac{7}{32}$	1.624E	21	60	3.88	13.5	21.93	Well filled. Losses high.
				58	60	4.43	12.5	20.30	
				100	60	5.40	11.0	17.87	
No. 15	Semi sector	$\frac{6}{32} \times \frac{6}{32}$	1.243E	25.5	60		13.0	16.16	Assumed as concentric single cond. a = 2.44 cm. Losses high.
				110.5	60		14.5	18.02	
				Old Samples—Had Been in Service.					
No. 16	0.528	$\frac{7}{32} \times \frac{7}{32}$	1.624E	29	60	5.03	10.0	16.24	Charred in center. Losses very high.
				55	60	5.52	10.2	16.56	
				75	60	6.10	10.4	16.90	
				111	60	9.37	11.6	18.85	
No. 17	0.528	$\frac{7}{32} \times \frac{7}{32}$	1.624E	25	60	4.25	8.8	14.30	Undamaged. Poorly filled. Losses high.
				75	60	5.29	10.0	16.24	
				100	60	7.19	10.1	16.40	
No. 18	0.528	$\frac{7}{32} \times \frac{7}{32}$	1.624E	27	60	4.74	9.5	15.45	Undamaged. Well filled. Losses very high.
				100	60	8.36	10.0	16.20	
No. 19	Semi sector	$\frac{6}{32} \times \frac{6}{32}$	1.243E	25.5	60		15.0	18.65	Assumed as single cond. a = 2.44 cm. Losses high.
				106	60		16.0	19.90	

The relative permittivity (K_{av}) of the insulation between conductor and sheath, as given in Table III, is calculated with a formula for three-conductor cables developed by Alex. Russell. It is given as formula (**6**) in the Appendix. The calculated K_{av}. agrees favorably with that for single-conductor cable.

Three-Conductor Paper Cable; Filled with Vegetable Hydrocarbon Base Compound. Before discussing the tests on paper cable filled with vegetable hydrocarbon base compound we wish to point out that our tests were limited to only two lots of this kind and, in particular, that a study of these was made because they had given trouble in operation.

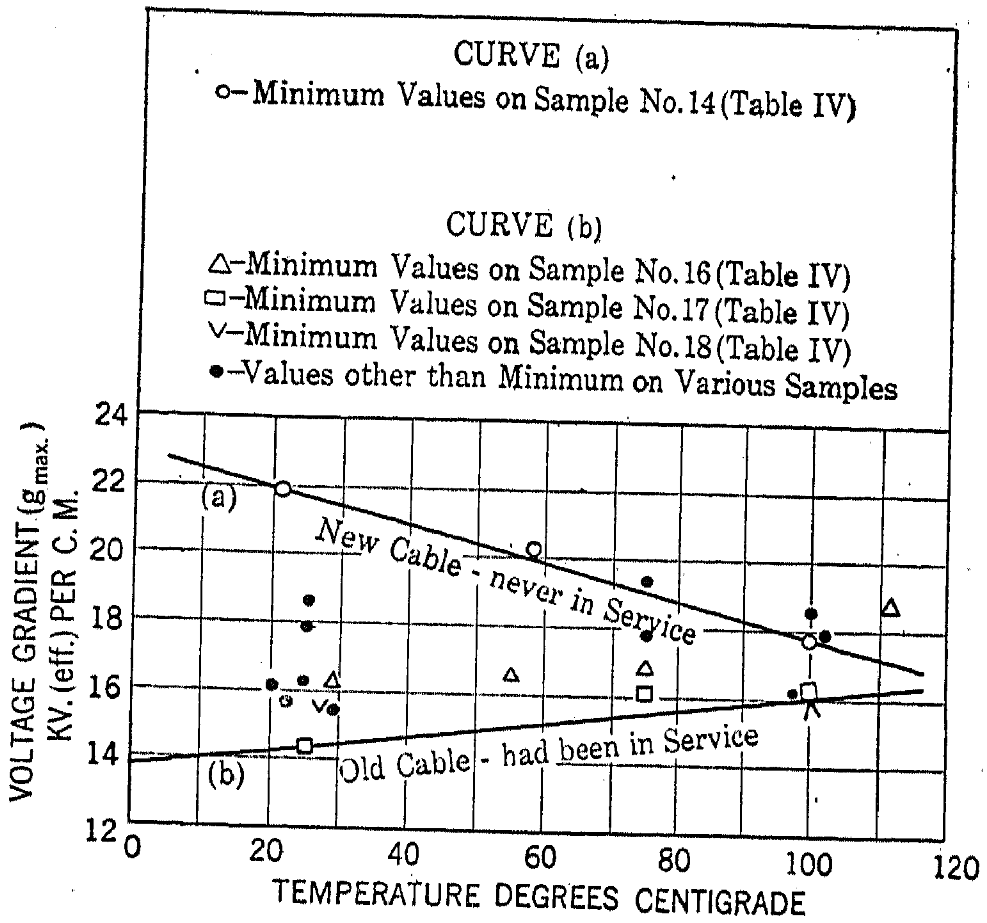


Fig. 9—Maximum Gradient Between Conductor and Sheath (assuming no gas space) at Start of Ionization

Three-conductor paper cable—filled with vegetable hydro-carbon base compound

Ionization data are given in Table IV. Samples Nos. 14, 16, 17, and 18 are from the same batch of cable described in the June, 1917, Institute paper previously referred to. Sample No. 14 had never been in service. The others were cut from the lines near points of failure after five years operation. The start of ionization is shown in Fig. 9. It is very low compared with that in Fig. 8, which shows that this lot of cable could not be safely operated at so high a voltage as the other type.

The dielectric losses and temperature coefficient were extremely high, so high in fact, that the temperature rise in operation was appreciable even without load on the cables. This factor alone would limit the voltage rating. With internal ionization and the deterioration it must cause probably included, there is no wonder that the cable gave trouble.

The stress between conductors at which ionization starts could not be determined with single-phase voltage. We cannot say definitely, then, that there was ionization during operation, but judging from the low values of g_{max} between conductor and sheath in Fig. 9 it seems very probable that there was. The paper wrappings between conductors were badly charred in spots and showed general signs of deterioration. The greater part of this deterioration may have been caused by tangential stresses and leakage over the surfaces of the wrappings as discussed in the previous paper, but ionization must also have had something to do with it. Hochstadter[9] describes ionization in this section of a cable as "glow discharge."

Fig. 9 not only shows a very low voltage gradient for both new and old samples but in addition it shows:

1. that $g_{max.}$ decreases with increase in temperature in the new sample. (curve *a*).
2. that $g_{max.}$ decreases with aging (curve *b*).

It is interesting to compare Figs. 8 and 9. The effects are reversed. The tests so far made on different types of cable insulation have shown that whenever there is an increase or decrease in $g_{max.}$ with temperature in new samples there is a corresponding increase or decrease with aging. This would tend to show that the two effects, whatever their cause are closely related.

Formula (3) in the Appendix does not apply to three-conductor cable and there is no way of approximating the thickness of gas space t_a. It is apparent, however, that t_a is greater than that found for paper cable filled with mineral hydrocarbon compound, although the higher value of permittivity is also a determining factor in making $g_{max.}$ lower.

Samples No. 15 and 19 in Table IV are from a different lot of cable than those dealt with above. The curves for these samples are in Fig. 10. Their characteristics are not the same as those of the first lot, more nearly resembling those in Fig. 8 although the values of $g_{max.}$ are considerably lower.

---

[9] "Three-Core Cable," by M. Hochstadter, *Electrotechische Zeitschrift*, Heft 47, 1915.

This lot of cable did not give as much trouble in operation as the first, even though the dielectric losses were just as high. Allowing for possible differences in the operating conditions it would still seem that the lesser trouble experienced with this lot was due to its more favorable ionization characteristics, that is, improvement with aging.

Varnished Cambric Tape; Wrapped Half-lap on Conductors of Round Cross Section. The ionization data for 0.010 in. black varnished cambric tape, wrapped half-lap on round conductors is given in Table V.

Two samples of lead-sheathed single-conductor cable (samples No. 20 and 21) are also given in this table. Sample No. 20 is varnished cambric cable and sample No. 21 varnished paper. They both were well filled with cable compound, this being accomplished by running the tape through the compound.

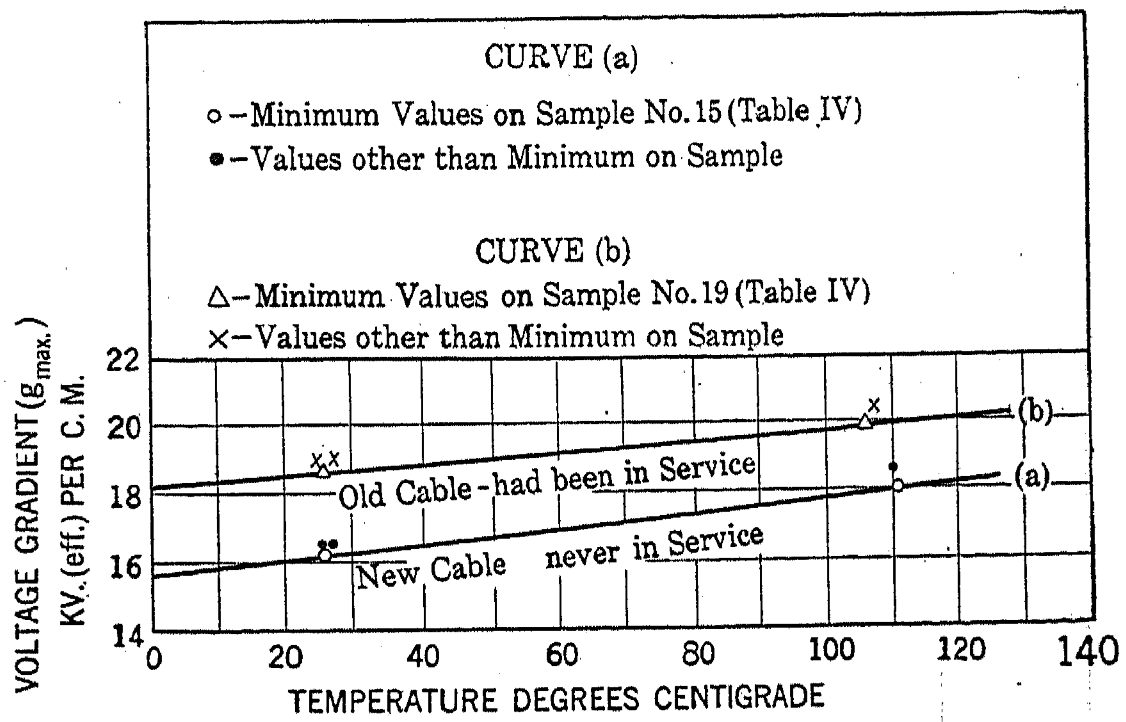


FIG. 10—MAXIMUM GRADIENT BETWEEN CONDUCTOR AND SHEATH (ASSUMING NO GAS SPACE) AT START OF IONIZATION

Three-conductor paper cable—filled with vegetable hydro-carbon base compound

The other samples under the heading, "Filled as Tapings were Applied." were wrapped on brass tubes, wet varnish being brushed on as the tapings were applied. These were first tested without heat treatment, and then after baking at 100 deg. cent., as designated in the table. Metal foil tape was wrapped over the surface and bound down tightly with adhesive tape.

The minimum curve of $g_{max.}$ *vs.* temperature for the filled samples is in Fig. 11 and the cross curves of stress in gas space

TABLE V.
0.010″ VARNISHED CAMBRIC TAPE:
WRAPPED HALF-LAP ON CONDUCTORS OF
ROUND CROSS SECTION.

Sample No.	Type of conductor	Dia. cond. inches	Thick. insul. inches.	*gmax.* kv cm. at cond.	Temp. deg. cent.	Freq.	K_{av}	E_S start of ionization	*gmax.* at start kv. cm.	Remarks.
					FILLED AS TAPINGS WERE APPLIED.					
No. 20	stranded	0.412	9/32	$2.218E$	25	60	4.07	8.5	18.85	Black varnished cambric cable. Lead sheath.
					50	60	4.55	8.4	18.64	
					75	60	5.20	none		
No. 21	stranded	0.412	9/32	$2.218E$	35	60	3.53	8.3	18.41	Black varnished paper cable. Lead sheath.
					75	60	4.15	8.5	18.85	
					100	60	4.76	10.1	22.40	
					125	60	5.08	10.6	23.50	
No. 22	smooth	0.538	.331	$1.82E$	23.5	60	4.20	10.2	18.56	Black varnished cambric wrapped on brass tubes. Tin foil wrapped tightly outside.
					58	60	4.80	10.1	18.39	
					104	60	5.95	11.2	20.40	
					(a) 20.5	60	4.01	10.0	18.20	
No. 23	smooth	0.528	.0975	$4.74E$	23.5	60	4.05	4.0	18.95	Same as Sample No. 22.
					58	60	4.55	4.2	19.90	
					103	60	6.20	4.5	21.30	
					(a) 21	60	3.80	3.8	18.02	
No. 24	smooth	1.527	.277	$1.665E$	22.5	60	4.46	11.5	19.16	Same as Sample No. 22.
					57.5	60	4.72	11.8	19.66	
					101	60	5.60	12.9	21.50	
					(a) 20	60	4.17	11.1	18.50	
No. 25	smooth	1.533	.0845	$4.92E$	23.5	60	4.00	3.8	18.70	Same as Sample No. 22.
					56.5	60	4.51	4.0	19.68	
					103.	60	6.40	4.3	21.15	
					(a) 21	60	3.70	3.7	18.21	

TABLE V.—(*Continued*)
0.010″ VARNISHED CAMBRIC TAPE
WRAPPED HALF-LAP ON CONDUCTORS OF
ROUN CROSS SECTION.

Sample No.	Type of conductor	Dia. cond. inches.	Thick. insul. inches.	g_{max} kv. cm. at cond.	Temp. deg. cent.	Freq.	*Kav*	*Es* Start of ionization	g_{max} at start kv. cm.	Remarks.
					(b) No Filling Taping Applied Dry.					
No. 26	smooth	0 .530	0.294	1.988*E*	23.5	60	4.16	9.0	17.90	Black varnished cambric wrapped on brass tubes. Tin foil wrapped tightly outside.
					62	60	4.68	10.0	19.88	
					102.5	60	5.15	11.1	22.07	
					(c) 23.5	60	4.08	8.8	17.50	
					23.5	25	4.08	9.1	18.10	
					62.	25	4.71	10.5	20.87	
					103.	25	5.43	11.1	22.07	
					(c) 23.	25	4.08	8.8	17.50	
No. 27	smooth	0.534	0.133	3.63*E*	19.5	60	4.87	4.6	16.70	Same as Sample No. 26.
					61.5	60	5.62	5.2	18.90	
					101.	60	5.96	5.5	19.95	
					(c)23.	60	4.79	4.6	16.70	
					23.5	25	5.00	4.6	16.70	
					62.5	25	5.96	5.2	18.90	
					102.	25	6.21	5.6	20.33	
					(c) 22.5	25	4.82	4.7	17.05	
No. 28	smooth	1.529	0.292	1.592*E*	23.5	60	3.90	11.0	17.50	Same as Sample No. 26.
					59.5	60	4.35	12.2	19.40	
					101.5	60	4.76	13.0	20.70	
					(c) 23.5	60	3.94	11.0	17.50	
					19.5	25	4.10	11.4	18.15	
					61.5	25	4.50	12.2	19.40	
					102.5	25	4.95	12.8	20.40	
					(c) 23.	25	3.98	11.0	17.50	
No. 29	smooth	1.510	0.0925	4.525*E*	21	60	3.75	4.0	18.10	Same as Sample No. 26
					57	60	4.35	4.2	19.00	
					102.5	60	4.86	4.7	21.27	
					(c) 24	60	3.88	3.9	17.64	
					24	25	4.00	4.2	19.00	
					62	25	4.99	4.2	19.00	
					102.5	25	5.22	4.7	21.27	
					(c) 23.	25	3.90	4.0	18.10	

(a) Note: These measurements were taken after samples had been baked in oven 24 hrs. at 50 deg. cent. and 5 hrs. at 100 deg. cent.
(b) All of the samples under this heading were baked 72 hrs. at 50 deg. cent. and 24 hrs. at 100 deg. cent. before testing.
(c) Baked an additional 24 hrs. at 50 deg. cent. and 5 hrs. at 100 deg. cent.

and strength of air in Figs. 12 and 13. The stress in the gas spaces (g_a) was calculated by determining values of E_s from the minimum curve in Fig. 11 and using an average low value of K_{av} from Table V.

The thickness of gas space (t_a) measured from the cross curves in Figs. 12 and 13 is therefore the maximum expected in this type of insulation.

Assuming the gas pressure as 760 mm. at both 25 deg. cent. and 100 deg. cent. the "equivalent pressures" are:

760 mm. at 25 deg. cent.
607 mm. at 100 deg. cent.

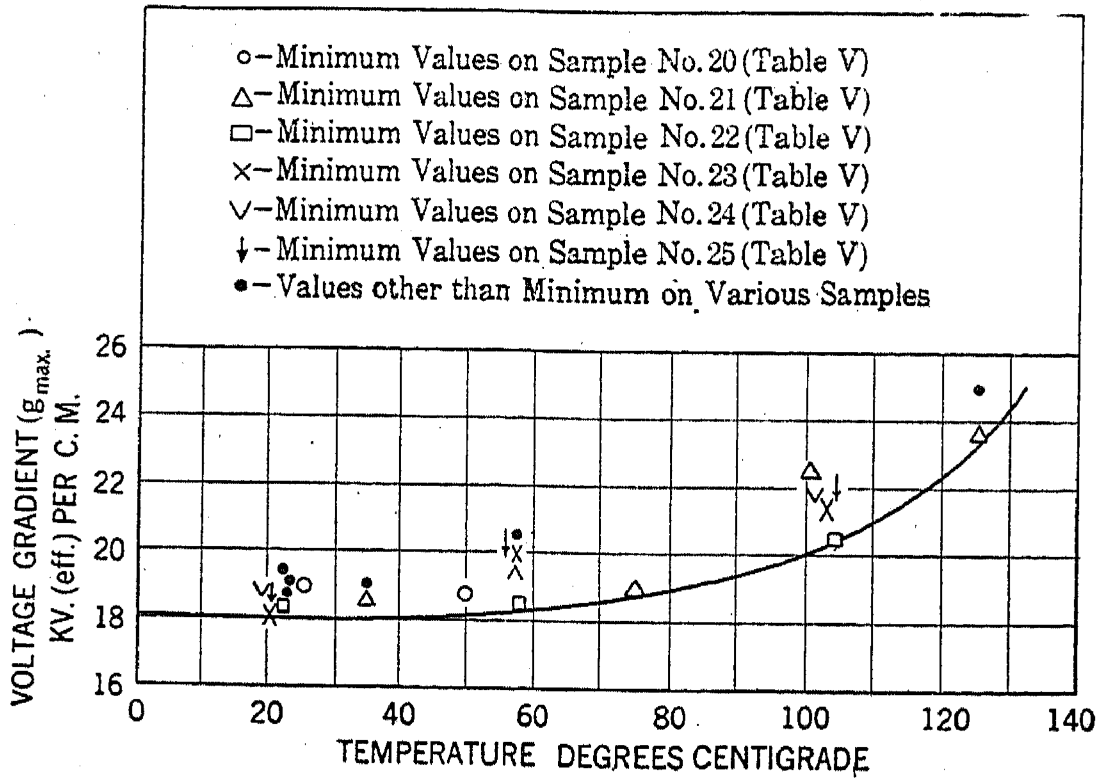


FIG. 11—MAXIMUM GRADIENT BETWEEN CONDUCTOR AND OUTER METALLIC WRAPPING (ASSUMING NO GAS SPACE) AT START OF IONIZATION

Varnished cambric tape—wrapped half lap on conductors of round cross section—filled as tapings were applied

With the gas space assumed next to conductor the values of t_a are:

0.008 cm. at 25 deg. cent.
0.003 cm. at 100 deg. cent.

The thickness of gas space given above is within the same order as that for paper cables filled with mineral hydrocarbon compound. The values of $g_{max.}$ however, are lower because of the higher permittivity of varnished cambric.

Ionization data are given under a separate heading in Table V for samples of 0.010-in. black varnished cambric tape wrapped on brass tubes without a filler, that is, no wet varnish brushings were applied. These samples were thoroughly dried out in an oven just before testing to make sure that no excess moisture was present. The minimum curve of $g_{max.}$ is in Fig. 14. It is surprising to note that it is only slightly lower than the corresponding curve in Fig. 11. Larger gas spaces at the edges of the overlapped wrappings would naturally be expected.

The values of t_a corresponding to those given for the filled samples are:

0.011 cm. at 25 deg. cent.
0.0035 cm. at 100 deg. cent.

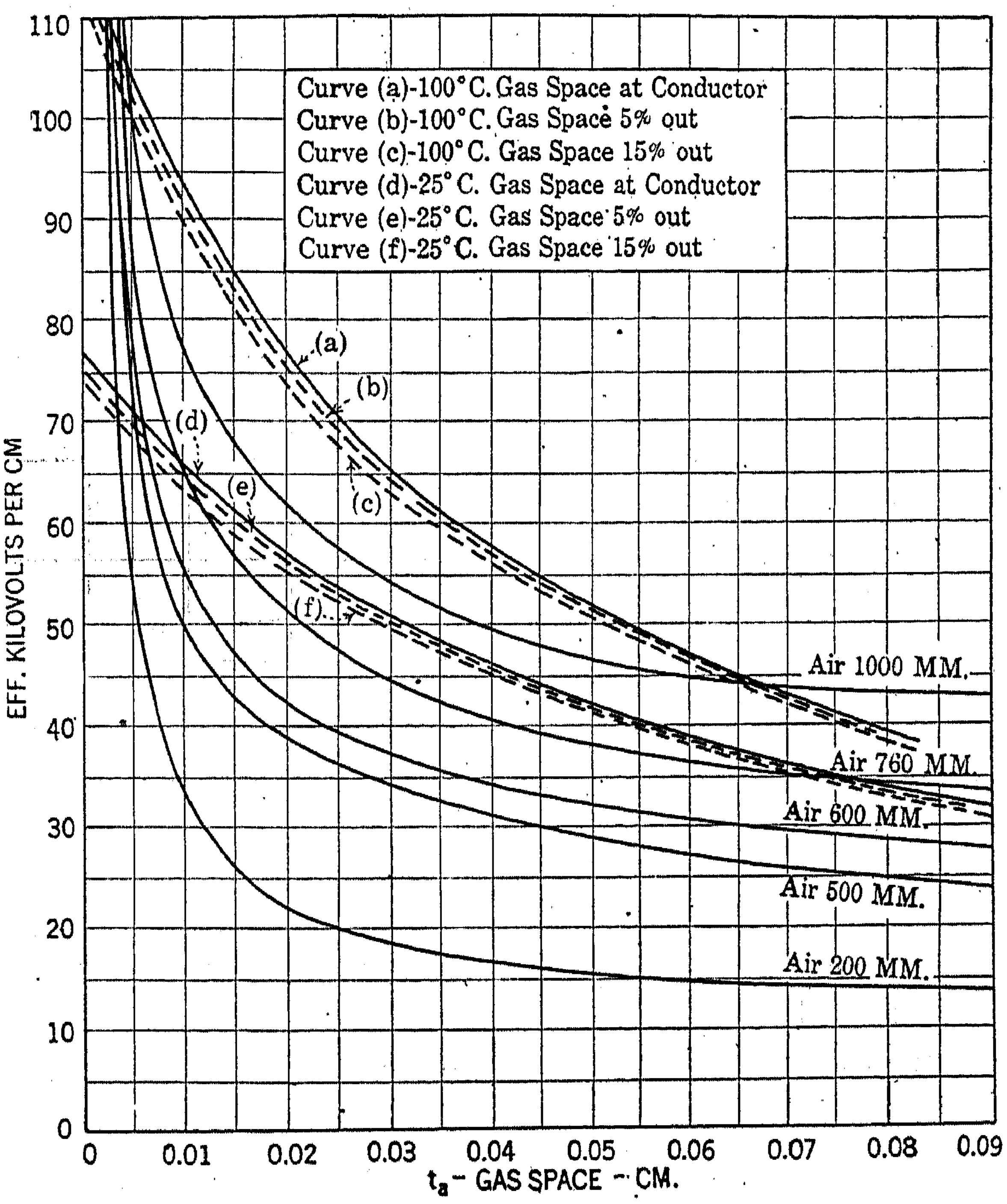


Fig. 12—Cross Curves for Sample No. 25 (See Table V)

These values were taken from curves similar to the curves in Figs 12 and 13.

The actual maximum thickness of the space at the edge of the wrappings must be greater than estimated above, as the wrappings were each 0.010 in. (0.0254 cm.) thick. We believe the discrepancy can be accounted for in the following way:

In calculating g_a it is assumed that the dielectric lines of flux are uniformly straight radial lines. In passing through the thin gas spaces of comparatively large area, usually met with between the surfaces of wrappings, they doubtless are, at least towards the center of the space where t_a is a maximum and ionization first occurs. But, the space at the edge of the wrappings is nothing but a narrow thread-like spiral and the stress lines are partly shunted around it through the solid material of higher permittivity. Consequently, either the ordinary spaces between surfaces are ionized first, in which case the calculations are correct, or the narrow edge spaces are actually ionized at a lower value of g_a than that wrongly estimated.

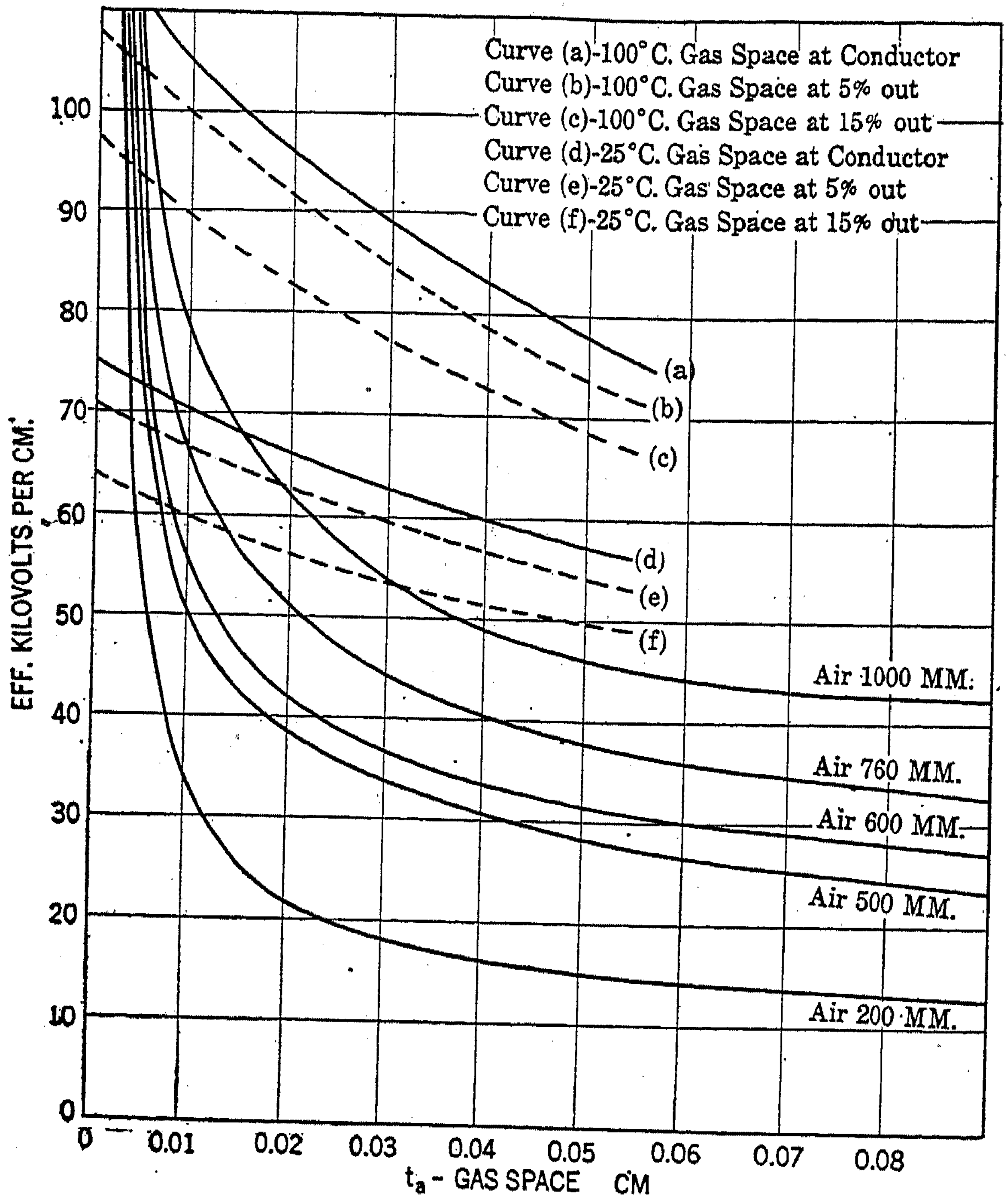


FIG. 13—CROSS CURVES FOR SAMPLE NO. 22 (TABLE V)

This same line of reasoning would apply to unfilled wrinkles or creases in wrappings, and also to the small capillary passages which form as the filler is dried out.

Flexible Mica-Paper Tape; Wrapped Half-lap on Conductors of Round Cross Section. Pasted flexible mica-paper has characteristics somewhat different from the usual run of flexible organic materials. This difference is due to the interruptions, or barrier effect, of the laminated inorganic mica flakes. Pasted mica-paper can be better understood if it is considered as having the combined characteristics of mica flakes and varnished sheet material such as paper or cambric.

Pure mica is more permanently stable than any other insulating material. Its electrical characteristics, such as dielectric strength, permittivity and energy loss are almost independent of length of voltage application and temperature between 0 deg. and 300 deg. cent.

The organic paper backing and sticker introduce between the flakes of mica those characteristics usually met with in organic solid materials, that is, comparatively high temperature coefficient, and deterioration and porosity with aging.

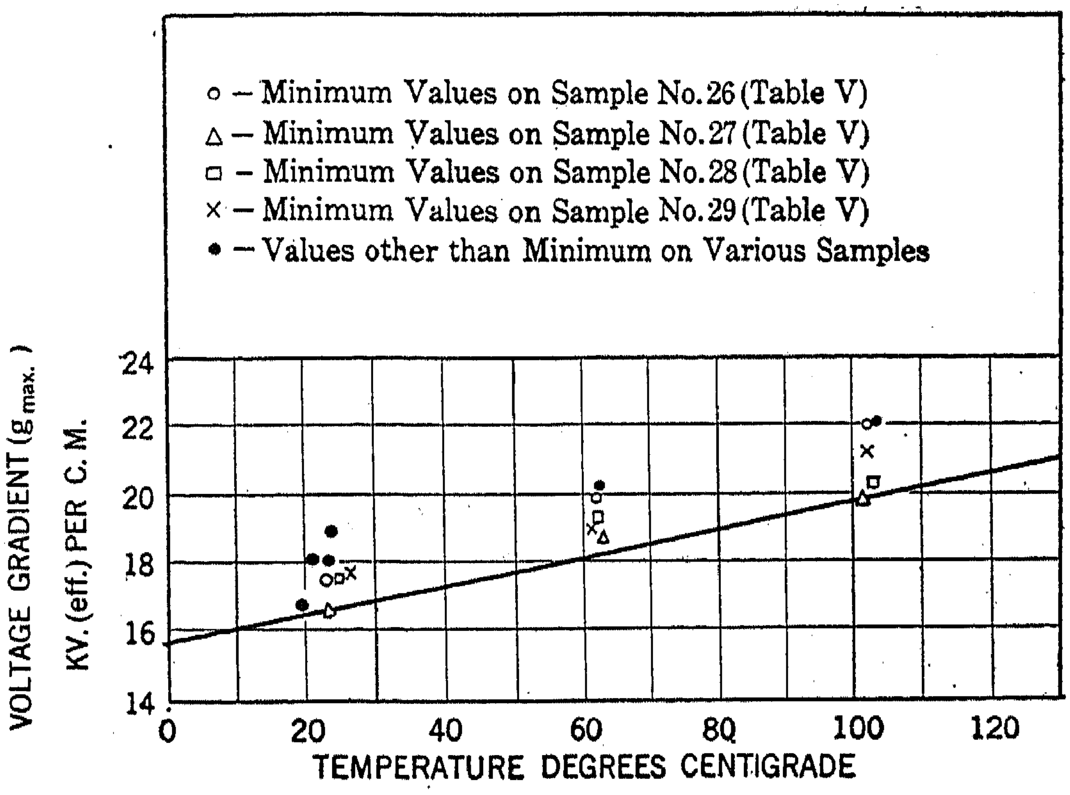


FIG. 14—MAXIMUM GRADIENT AT START OF IONIZATION (ASSUMING NO GAS SPACE)

Unfilled varnished cambric tape wrapped half-lap on conductors of round cross section

The a-c. leakage path (also porosity and deterioration when present) follows the path of the organic components, zig-zagging in and out around the mica flakes and wrappings searching out the shortest and easiest path through the thickness.

When mica-paper is wrapped on conductors of rectangular cross section extreme care must be taken to prevent the insulation from loosening up and bulging away from the sides. The natural spring of mica flakes and softening of the sticker under heat treatment tend to cause this bulging. On round cross sections there is no tendency to bulge and, hence, one of

TABLE VI.
FLEXIBLE MICA PAPER TAPE:
WRAPPED HALF-LAP WITH STICKING VARNISH ON CONDUCTORS OF ROUND CROSS SECTION. STICKER DRIED BY HEAT CURING.

Sample No.	Dia. Cond. inches	Thick. insul. inches	g_{max} kv. per cm. at cond.	Temp. deg. cent.	Freq.	K_{av}	E_s start of ionization	g_{max} at start kv. per cm.	Remarks.
				Fresh Samples—Cured but not Aged.					
No. 30	0.536	0.07075	6.27*E*	26.	60	2.98	4.2	26.30	Metal foil tape wrapped over outer surface and tightly bound down with adhesive tape.
				65.	60	4.11	4.0	25.10	
				105	60	5.46	4.1	25.70	
				25	25	3.10	4.2	26.30	
				67	25	5.20	4.0	25.10	
				102	25	6.40	4.0	25.10	
No. 31	0.535	0.2512	2.22*E*	26	60	3.05	11.7	26.00	
				68	60	4.16	11.4	25.30	
				105	60	5.70	11.4	25.30	
				26	25	3.28	11.4	25.30	
				67	25	5.16	11.6	25.75	
				104	25	6.51	11.8	26.20	
No. 32	1.51	0.073	5.66*E*	26	60	3.15	4.7	26.60	
				64	60	4.09	4.25	24.05	
				105	60	5.99	4.3	24.35	
				25	25	3.36	4.7	26.60	
				68	25	5.22	4.3	24.35	
				103	25	6.77	4.5	25.50	
No. 33	1.51	0.118	3.58*E*	22	60	3.22	7.0	25.06	
				61	60	4.31	6.9	24.71	
No. 34	1.53	0.211	2.155*E*	25	60	3.02	12.2	26.30	
				106	60	5.38	12.0	25.90	
				28	25	3.25	13.0	28.00	
				65	25	5.26	12.0	25.90	
				103	25	6.65	12.0	25.90	
				Old Samples—Aged in Oven at High Temp.					
No. 35	1.51	0.120	3.53*E*	24	60	2.95	6.0	21.18	Baked 24 hr at 50 deg. cent.
				62	60	3.20	6.1	21.53	" 24 hr at 75 deg. cent.
				103	60	3.99	6.0	21.18	" and 84 hr. at 100 deg. cent.
No. 36	1.51	0.116	3.66*E*	25	60	2.87	5.7	20.87	Same baking as sample No. 35 and in addition 84 hr. at 150 deg. cent.
				101	60	3.10	5.6	20.50	
				152	60	3.51	5.4	19.77	
No. 37	1.51	0.118	3.58*E*	28	60	2.65	5.8	20.75	Same baking as sample No. 36 and in addition 290 hr. at 200 deg. cent.
				106	60	2.85	5.6	20.05	
				200	60	3.12	5.5	19.70	

the most troublesome characteristics of pasted mica is eliminated when used in this way.

Ionization data for representative samples of mica paper insulation are given in Table VI and the minimum curves of g_{max} in Fig. 15. The wrappings were compactly applied half-lap with sticking varnish on round brass tubes, and heat cured to drive out the solvent.

The minimum curve of g_{max}. for fresh material in Fig. 15 is higher than that for any other material so far considered.

Samples No. 35, 36 and 37 in Table VI were aged in an oven respectively at 100 deg. cent., 150 deg. cent. and 200 deg. cent. They are minimum representatives of a number of samples aged for different periods of time at the above temperatures.

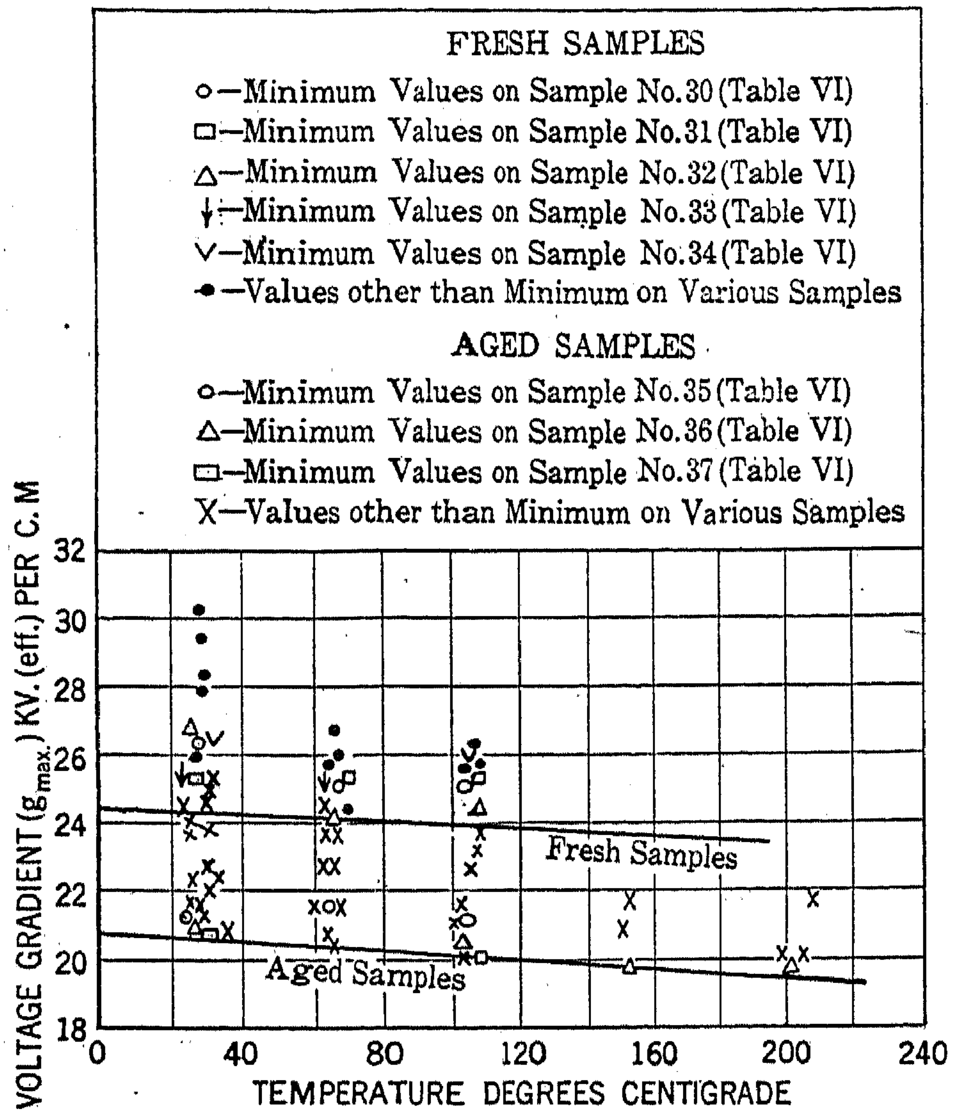


FIG. 15—MAXIMUM GRADIENT AT START OF IONIZATION (ASSUMING NO GAS SPACE)

Flexible mica-paper-wrapped half-lap on conductors of round cross section.

Baking at high temperature volatilizes a considerable part of the organic components and renders the insulation brittle and extremely porous. The degree of porosity was shown clearly in two ways:

1. The additional dielectric loss caused by ionization was much greater than the corresponding loss in fresh samples. This was shown by comparison of ionization curves similar to that in Fig. 1.

2. When aged samples were exposed to moisture the absorption by the capillary passages of the porous organic components increased the dielectric loss enormously. In fact it was almost impossible to reach voltage E_s without breakdown when the samples were well saturated.

The minimum curve of g_{max} for aged samples in Fig. 15 indicates that aging enlarges the gas spaces, as would be expected.

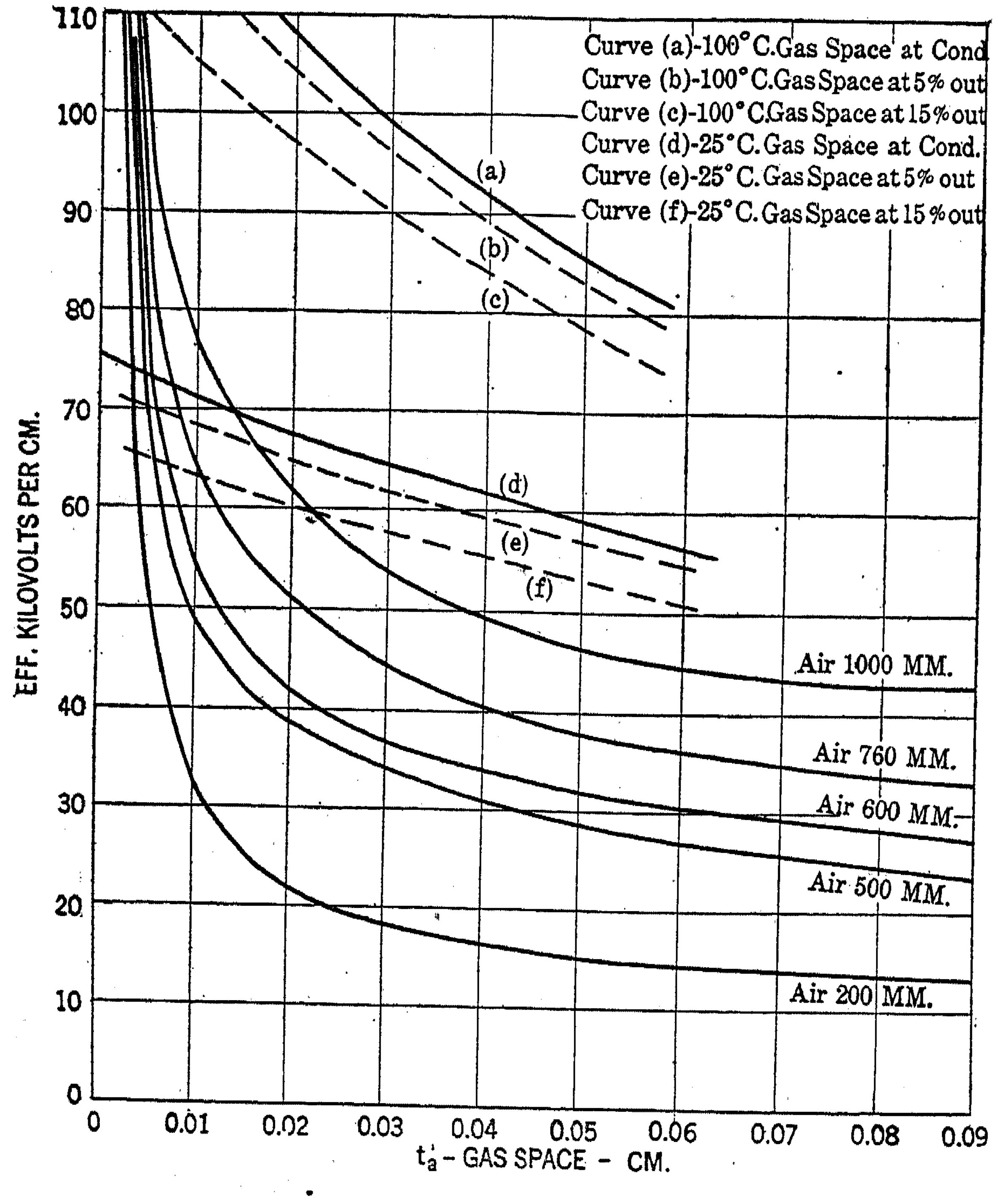


Fig. 16—Cross Curves for Sample No. 31 (See Table VI)

The cross curves for determining the maximum thickness of gas space for fresh samples are in Fig. 16. The curves of g_a were calculated by taking values of E_s from the minimum curve in Fig. 15 and minimum average values of K_{av}. The maximum ratio of b/a (sample No. 31) is considered.

Assuming the gas pressure as 760 mm. both at 25 deg. cent. and 100 deg. cent. the "equivalent pressures" are:

760 mm. at 25 deg. cent.
607 mm. at 100 deg. cent.

From the cross curves the values of t_a assumed next to conductor are:

0.0075 cm. at 25 deg. cent.
0.0025 cm. at 100 deg. cent.

The cross curves for sample No. 37, a representative aged sample, are shown in Fig. 17. Assuming a constant gas pressure of 760 mm. the "equivalent pressures" are:

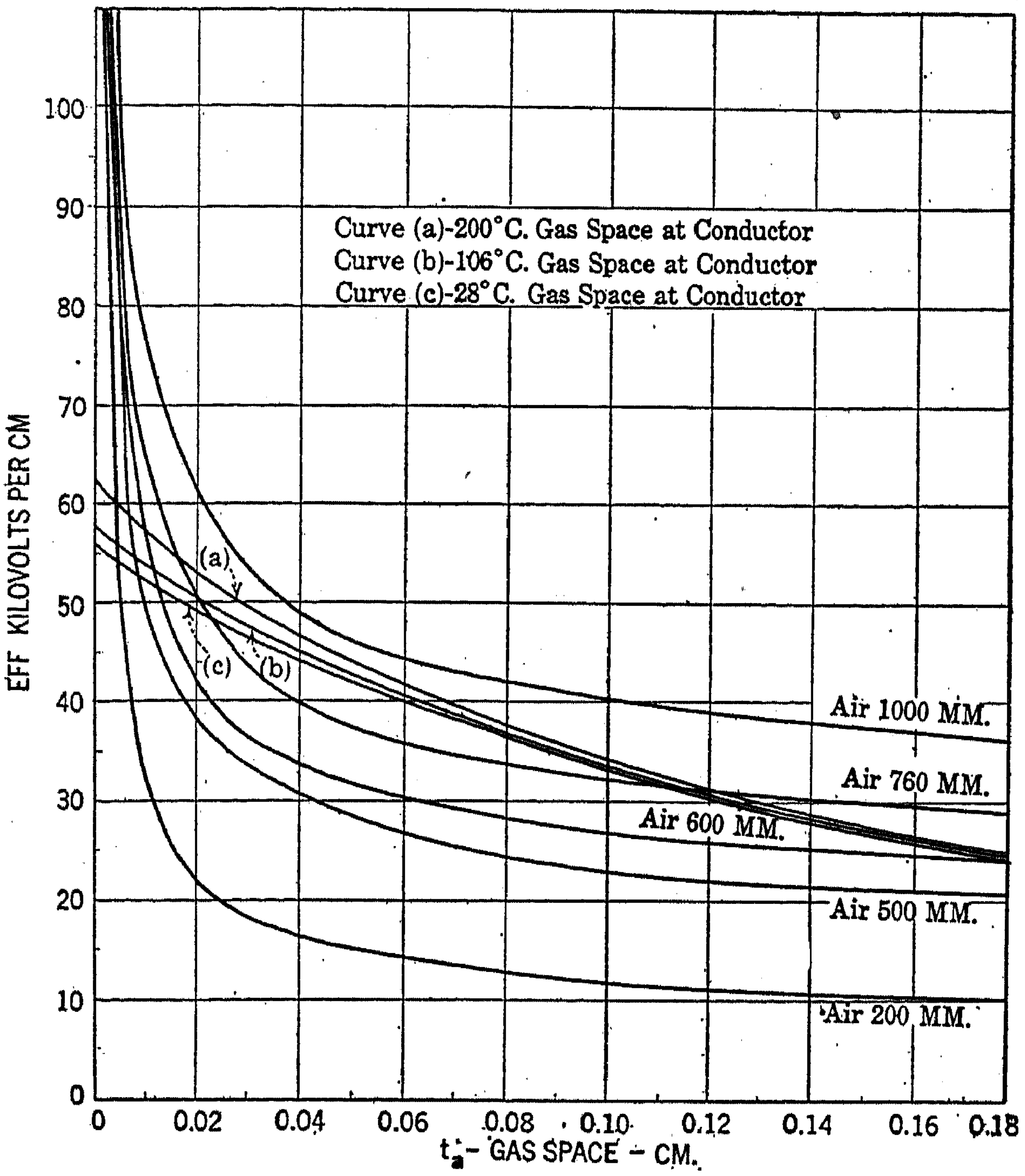


Fig. 17—Cross Curves for Sample No. 37 (See Table VI)

$$760 \times \frac{298}{304} = 752 \text{ mm. at 28 deg. cent.}$$

$$760 \times \frac{298}{379} = 598 \text{ mm. at 106 deg. cent.}$$

$$760 \times \frac{298}{473} = 479 \text{ mm. at 200 deg. cent.}$$

From the cross curves the values of t_a assumed next to conductor are.

0.023 cm. at 28 deg. cent.
0.012 cm. at 106 deg. cent.
0.0065 cm. at 200 deg. cent.

Aging has enlarged the gas spaces appreciably.

INSULATION WRAPPED ON CONDUCTORS OF RECTANGULAR CROSS SECTION

The results given thus far in the paper have been fairly consistent because samples of round cross section only have been considered. The way the values of $g_{max.}$ in Figs. 4, 8, 9, 10, 11, 14 and 15 have bunched has made it possible to assign definite minimum curves of $g_{max.}$ for the different materials, and from these determine a safe working stress. This has been possible because of the fact that the thickness of gas space (t_a) has been so small that $g_{max.}$ has been practically independent of both conductor diameter and thickness of insulation. It is dependent only upon the kind of material used, method of application, treatment and aging.

An attempt to give minimum curves of the stress at which ionization starts (g_s, assuming no gas space) for samples of rectangular cross section leads only to misrepresentation and confusion. The tendency for the insulation to bulge away from the straight sides of the conductors, and therefore the maximum thickness of gas space, is not only more vitally dependent upon those factors associated with samples of round cross section, but also upon the dimensions of the conductor and thickness of insulation. Then too, we are approaching values of t_a that are appreciable when compared with the thickness of insulation and for this reason g_s varies with thickness.

The most satisfactory way to deal with rectangular cross sections, such as armature coils, is to measure the voltage at which ionization starts in each individual case. The value of E_s will indicate whether the particular type of coil can be safely operated at its rated voltage or not. Also, the thickness of gas space determined from these measurements will indicate the degree of puffing or bulging by comparison with a similar sample of round cross section, which can be considered as a standard representing the best possible attainment, that is, as having the least values of t_a for the given conditions.

Before dealing with actual tests on coils consideration will again be given to round cross sections to illustrate, by indirection, a peculiar characteristic which has been met with only in rectangular cross sections.

Referring to Fig. 12, as representative of round cross sections, the 25 deg. cent. curve of stress in gas space at conductor crosses the 760 mm. strength of air curve at both 0.009 cm. and 0.075 cm. It is evident that for any thickness smaller than 0.009 cm. or larger than 0.075 cm. there will be no ionization at the impressed voltage of 3.64 kv.

Now, the actual maximum thickness of gas space is selected as 0.009 cm. instead of 0.075 cm. for the following reasons:

a. For any thickness between the above limits, ionization would have started at a lower value of E_s.

b. If a gas space between surfaces is considered as having 0.075 cm. as its maximum thickness, then from the nature of its shape the thickness must vary from zero at its edges to 0.075 cm. in the center, consequently somewhere through the gas space there will be a critical thickness, which we will call t_c, at which ionization will start at a minimum E_s lower than 3.64 kv. It would have the effect of lowering the 25 deg. cent. stress curve so that it would be just tangent to the 760-mm. curve at t_c, (in this case t_c is approximately 0.035 cm.)

This critical thickness has not been experienced in round cross sections because the maximum thickness of gas space has always been something less than this. It is, however, experienced quite often in coils and leads to an interesting and important conclusion concerning them, in that, any degree of puffing or bulging beyond t_c does not lower the voltage E_s at which ionization starts.

Ionization data for a few representative coils are given in Table VII. Sample No. 38 is one of a number made up and tested under conditions whereby the gas spaces would be reduced to the smallest possible size. The insulation was applied as compactly as possible and it was tightly clamped in moulds during the treating process and throughout test. Metal foil was wrapped over the outer surface and tightly bound down, serving as one electrode.

The voltage stress (g_s) at which ionization starts is found merely by dividing E_s by the thickness of insulation. This is a closer approximation than would at first appear, as the bulging is always a maximum at the center of the widest straight side where the lines of stress are practically parallel. Divergent

TABLE VII.
FLEXIBLE MICA-PAPER TAPE:
WRAPPED HALF-LAP WITH STICKING VARNISH ON CONDUCTORS OF RECTANGULAR CROSS SECTION.
CURED DURING APPLICATION.

Sample No.	Cond. inches	Thick. insul. inches	g kv. per cm.	Temp. deg. cent.	Freq.	Kav	Es start of ionization	gs at start kv. per cm.	Remarks.
				TESTED IN MECHANICAL CLAMPS.					
No. 38	1/2 × 3	0.0960	4.10E	24.7	60	4.00	5.6	22.97	Insulation compactly applied and well cured while clamped. No aging.
				100	60	5.20	5.5	22.55	
				151	60	6.20	5.6	22.97	
		0.0930	4.23E	25	60	3.10	5.5	23.27	Aged in an oven while clamped for 170 hr. at 200 deg. cent.
				100	60	3.30	5.3	22.42	
				198	60	3.60	5.6	23.70	
				COMPACT COILS TESTED WITHOUT CLAMPS					
No. 39	1/2 × 3	0.112	3.51E	25	60	3.80	5.1	17.91	Insulation applied and cured compactly. Good grade. No aging.
				60	60	4.40	5.1	17.91	
				102	60	4.90	5.1	17.91	
No. 40	1/2 × 3	0.322	1.22E	26	60	3.80	12.4	15.13	Same as Sample No. 39.
				60	60	4.40	12.3	15.01	
				101	60	4.90	12.2	14.88	
				TESTED WITHOUT CLAMPS—NOT VERY COMPACT.					
No. 41	1/2 × 3	0.110	3.58E	25	60	3.87	4.5	16.11	Insulation bulged but not badly.
				61	60	4.40	4.5	16.11	
				100	60	4.90	4.4	15.75	
No. 42	1/2 × 3	0.322	1.22E	25	60	3.80	10.0	12.20	Same as Sample No. 41.
				60	60	4.40	10.0	12.20	
				102	60	4.90	10.0	12.20	
				COIL BADLY PUFFED—NO CLAMPS.					
No. 43	1/2 × 3	0.328	1.196E	25	60	3.85	10.0	11.96	Insulation caused to bulge badly by sudden application of high temperature.
				61	60	4.30	8.4	10.05	
				100	60	4.75	7.2	8.61	

stress lines occur only at the corners and the insulation is very compact here.

The values of g_s are comparable with corresponding values of $g_{max.}$ for mica samples of round cross sections in Table VI, showing how compact this coil was. The clamps prevented bulging when the coil was aged at 200 deg. cent. and consequently there was not much change in g_s.

The working stress of coils varies over a wider range than for other insulation designs. It is usually somewhere between 20 and 37 volts per mil (7.87 and 14. 57 kv. per cm.) depending upon such factors as mechanical stress and abrasion, working temperature, etc. Ionization started in sample No. 38 at 57 volts per mil (22.5 kv. per cm.) If it were possible to actually operate this coil under the same favorable conditions it is evident that it could be safely stressed much higher than the very highest stress used in ordinary operation.

The stress in gas spaces of assumed thicknesses cannot be calculated unless the effective permittivity (K_{av}) is known. K_{av} cannot be determined from measurements on rectangular cross sections because the dielectric field is not symmetrical. The best that can be done is to approximate g_a by selecting values of K_{av} by judgment, using those values found in round cross sections of similar material as a guide. The results are fairly good but K_{av} varies too much in different batches, treating processes, etc., to make this method more than a fair approximation. Whenever the maximum thickness of gas space is equal to or greater than the critical thickness (t_c), K_{av} can be found directly in the following way:

For a first trial a value of K_{av} is selected as above to correspond as closely as possible to the true value. The curve g_a vs. t_a is calculated and compared with the strength of air curve of the right "equivalent pressure." If K_{av} is properly chosen it will be almost tangent to the strength of air curve. Another curve is drawn in parallel to the first and exactly tangent. The point of tangency will indicate the value of t_c. Knowing this, the true value of K_{av} and the correct curve of g_a vs. t_a can be calculated.

The cross curves for sample No. 38 are in Fig. 18. The g_a vs. t_a curves all cross their corresponding "equivalent pressure" curves, showing that the gas space is less than t_c. In selecting the values of K_{av} weight was given to the following factors:

(a) K_{av} should be higher than for the round cross sections in Table VI as the insulation was applied differently in the two cases. For the round cross sections all of the insulation was wrapped on with wet sticker between wrappings. The whole was then cured at high temperatures until the varnish was well dried out. The escaping gases consequently caused porosity, and added to this, there must have been many very small spaces formed between surfaces from shrinkage of the sticker as it dried out. The percentage of solid material was reduced and thereby the effective permittivity. For the rectangular cross sections, the layers were carefully cured and moulded as applied, largely closing up capillary passages and other small spaces generally distributed through the thickness and increasing the effective permittivity.

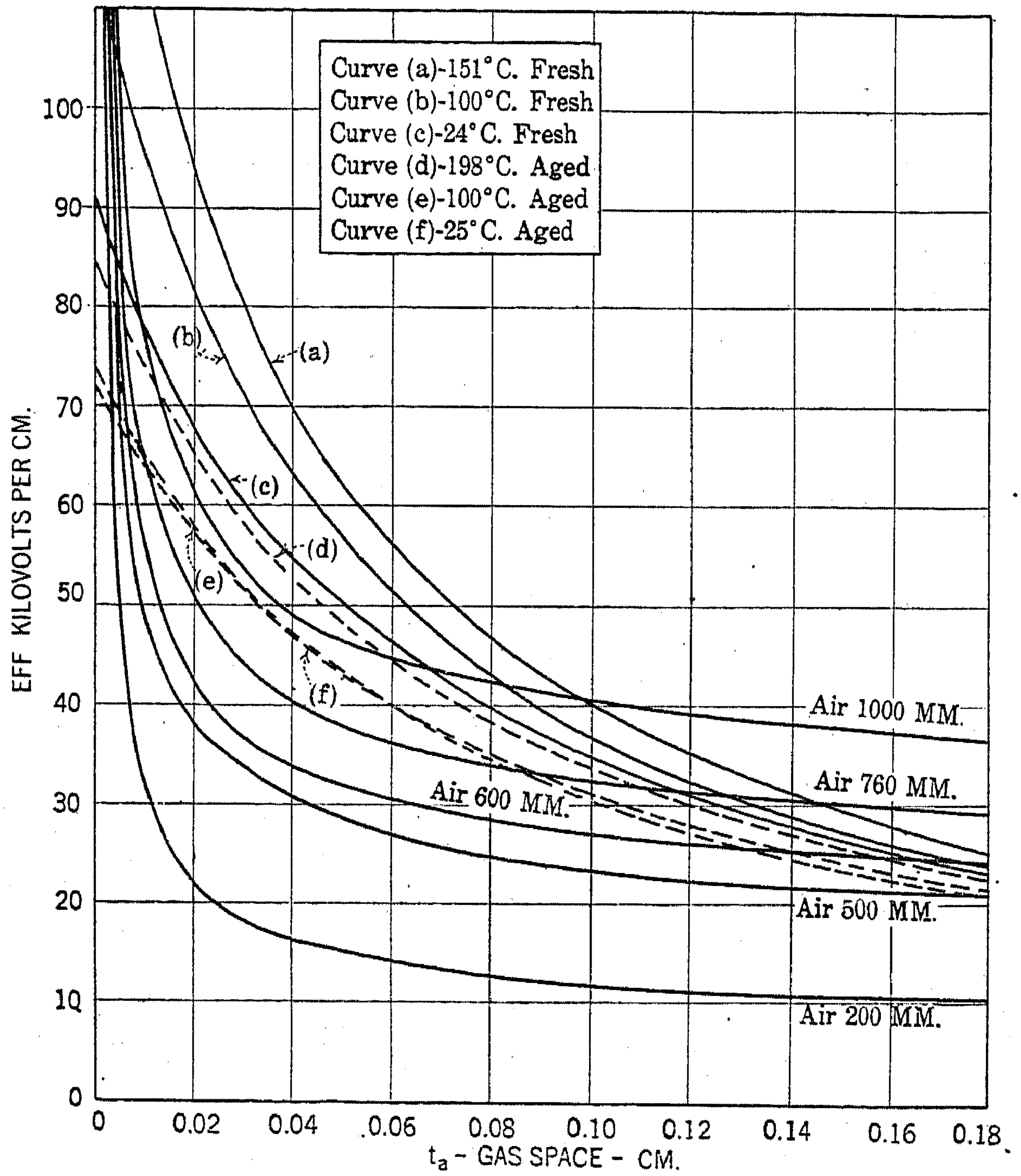


FIG. 18—CROSS CURVES FOR SAMPLE NO. 38 (TABLE VII)

(b) Energy loss measurements give the total capacity of the sample and show how this capacity varies with temperature. The actual permittivity should, of course, vary exactly the same. There remains, then; only to select an arbitrary value of K_{av} for one temperature.

(c) In samples of the same kind, the insulation can be caused to bulge, by sudden application of high temperature until t_a is equal to or greater than the critical thickness at room temperature. Tangent curves as previously discussed will give K_{av} direct. This K_{av} should, however, be a little less than that for sample No. 38, (see sample Nos. 41 and 42) which was tightly clamped and more compact.

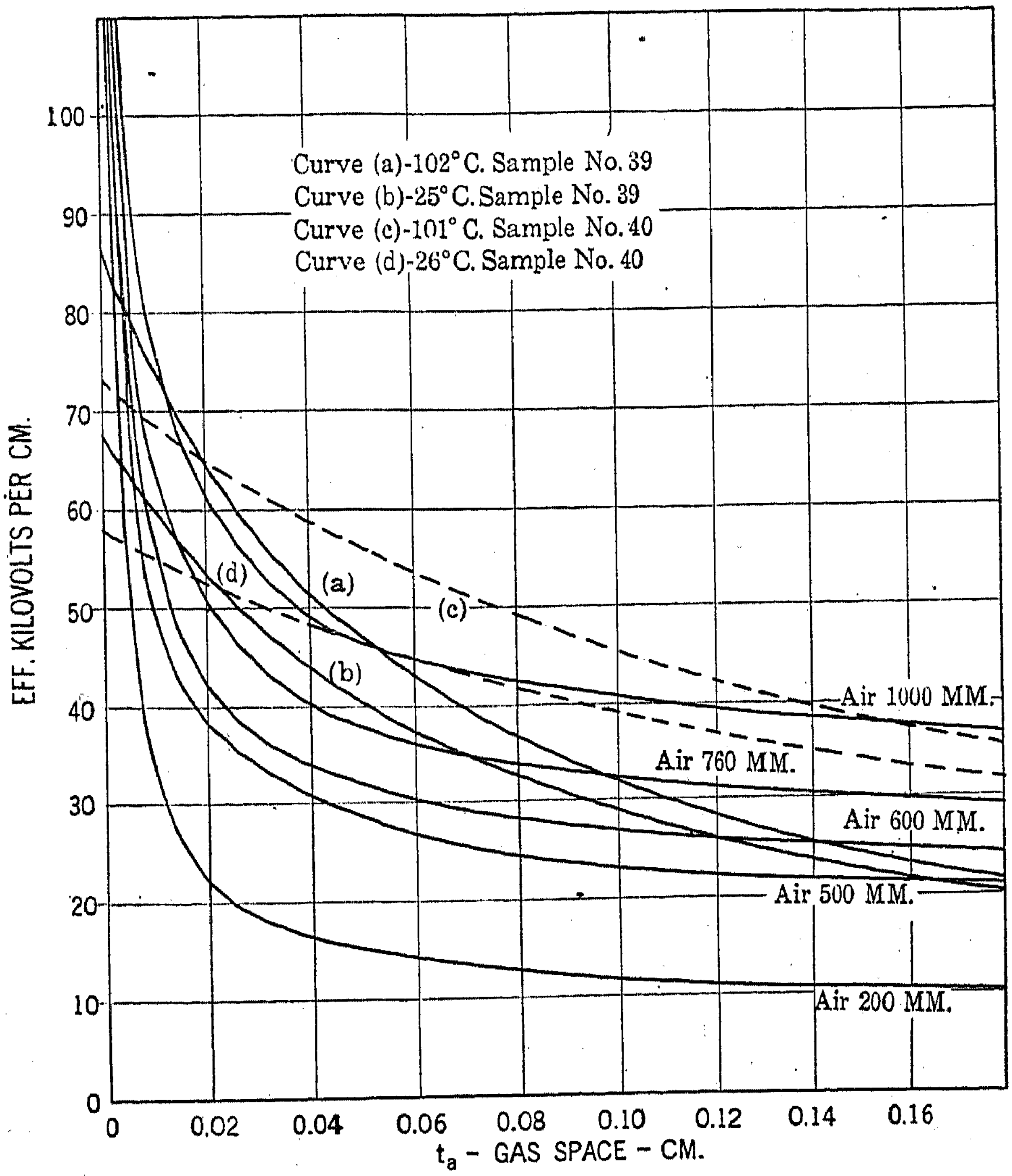


FIG. 19—CROSS CURVES FOR SAMPLES NOS. 39 AND 40 (SEE TABLE VII)

Using "equivalent pressures" in the usual way, the values of t_a in Fig. 18 are:

FRESH	AGED
0.005 cm. at 24 deg. cent.	0.010 cm. at 25 deg. cent.
0.003 cm. at 100 deg. cent.	0.007 cm. at 100 deg. cent.
0.0025 cm. at 150 deg. cent.	0.0045 cm. at 198 deg. cent.

These thicknesses are even smaller than those in samples of round cross section. We believe this is accounted for partly by the tight clamping and partly by the more compact nature of the material, that is, presence of more organic sticker.

Samples No. 39 and 40 are exactly similar to No. 38 but were not tested in clamps. They are representative of good grade compactly made coils in which every reasonable care has been taken to reduce the gas spaces to a minimum. No. 39 has 0.2845 cm. (0.112 in.) thickness of insulation and ionization starts at a stress of 17.91 kv. per cm. (45.5 volts per mil) No. 40 has 0.818 cm.(0.322 in.) thickness and ionization starts at only 15.0 kv. per cm. (38.2 volts per mil). In other words, the thicker the insulation the less the stress it will stand. This characteristic is inherent in rectangular cross sections and is due to the gas spaces being appreciable compared with thickness of insulation. It is evident that there will be no ionization in sample No. 39 at the very highest stress ordinarily used in operation, while No. 40 would be limited to a working stress of about 14.17 kv. per cm. (36 volts per mil), which is a little

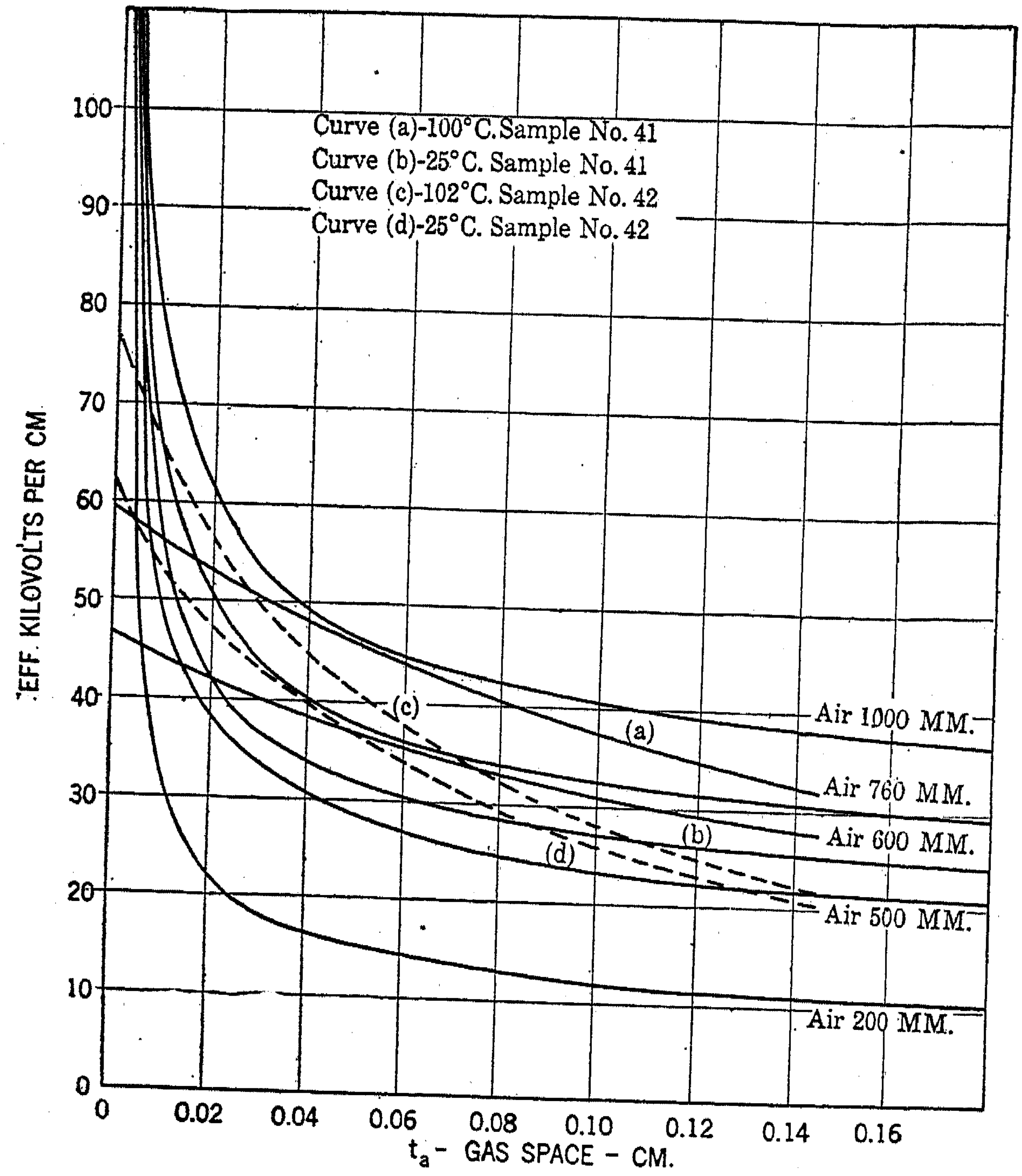


FIG. 20—CROSS CURVES FOR SAMPLES NOS. 41 AND 42 (TABLE VII)

less than the maximum limit but actually higher than would be required in operation for so great a thickness. A working stress of 36 volts per mil is based on the assumption that the operating conditions, such as maximum temperatures, etc., will be such that the gas spaces will not be enlarged with aging. The cross curves for No. 39 and 40 are in Fig. 19. The values of t_a are:

SAMPLE NO. 39	SAMPLE NO. 40.
0.015 cm. at 25 deg. cent.	0.018 cm. at 26 deg. cent.
0.005 cm. at 102 deg. cent.	0.0065 cm. at 100 deg. cent.

Samples No. 41 and 42 in Table VII are representative of poor grade coils in which the insulation has bulged out, not extremely so, but enough to be visible. Ionization starts at 16 kv. per cm. (40.7 volts per mil), in No. 41 and 12.2 kv. per cm. (31 volts per mil) in No. 42. This points to the conclusion that the higher the voltage rating of a coil, in other words, the greater the thickness of insulation, the more carefully bulging must be guarded against.

The cross curves in Fig. 20 indicate that at room temperature the maximum gas space is equal to or greater than the critical in both No. 41 and 42, for the g_a vs. t_a curves are tangent to the 760 mm. curves. At 60 and 100 deg. cent. the stress curves cross their "equivalent pressure" curves, which tends to show that at room temperature the maximum and critical gas spaces must have been almost the same, otherwise expansion of the insulation could not have closed the spaces up so effectively at higher temperatures. The thickness of gas spaces in Fig. 20 are:

SAMPLE NO. 41.	SAMPLE NO. 42.
0.035 cm. at 25 deg. cent. (critical)	.06 cm. at 25 deg. cent. (critical)
0.007 cm. at 100 deg cent.	.011 cm. at 102 deg. cent.

This apparent reduction in thickness of gas space with increase in temperature is based entirely on the assumption adhered to throughout the paper of atmospheric pressure at the higher temperatures as well as at room temperature. If the pressure is assumed to increase with temperature it is possible to even get an increase in gas space with temperature. We are confident that the assumption of atmospheric pressure at room temperature is correct but are free to confess that we have sometimes doubted the assumed pressure at higher temperatures because of the unreasonable decrease in gas space. Every bit of evidence we have, with the exception of this one contradiction, has forced us to the conclusion of atmospheric pressure at all temperatures. When the temperature is raised too rapidly there is certainly an increase in gas space, due to internal gas pressure, as bulging is plainly visible. In all of our tests, however, the temperature is brought up very slowly and everything allowed to stabilize before readings are taken.

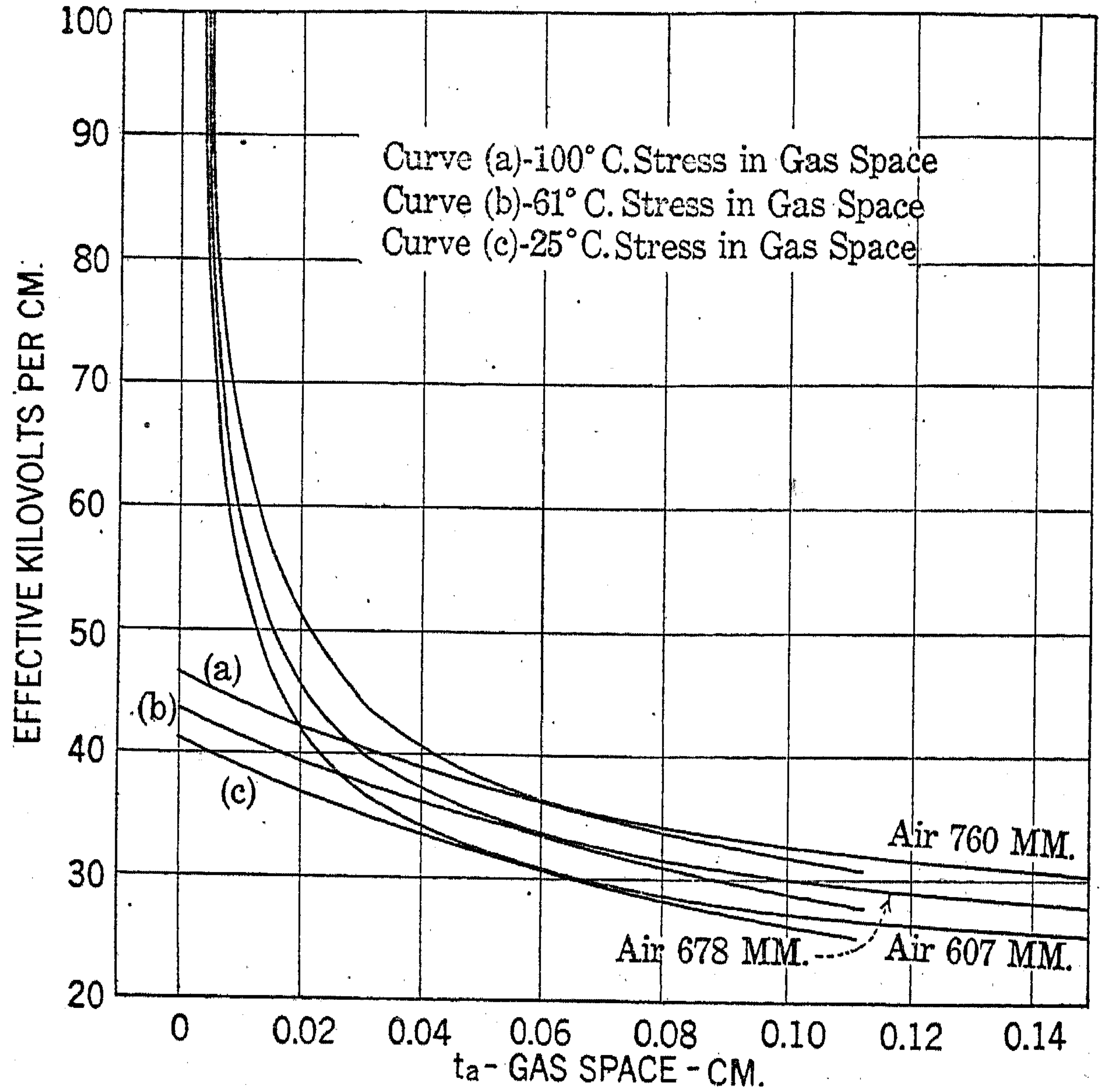


FIG. 21—TANGENT CURVES FOR SAMPLE NO. 43 (TABLE VII) SHOWING CRITICAL THICKNESS OF GAS SPACE

For this reason, it is possible that the pressure is held at atmospheric by the small capillary orifices.

The data on sample No. 43, a badly puffed coil, prove beyond any doubt that the gas pressure was practically atmospheric for all temperatures. Whether the internal gas pressure of a coil in this condition is representative of the more compact coils is problematical. The g_a vs. t_a curves in Fig. 21 are all tangent to their respective "equivalent pressure" curves and the permittivity K_{av}, found in this way is so nearly that which would be expected that there is little doubt of the assumed pressures being correct. Apparently the maximum thickness f gas space is considerably greater than the critical, at least t room temperature. The values of t_c are:

0.063 cm. at 25 deg. cent.
0.062 cm. at 61 deg. cent.
0.061 cm. at 100 deg. cent.

No. 43 had an insulation thickness of 0.833 cm. (0.328 in.) t 100 deg. cent. ionization starts at a minimum stress of 8.61 v. per cm. (21.8 volts per mil). With lesser thickness of in-ulation, ionization would not have started at such low stress, ut it is apparent that in any badly bulged coil there is danger f ionization under normal operating conditions.

EFFECT OF INTERNAL IONIZATION

The damage caused by internal ionization in samples of ectangular cross section having various types and thicknesses f insulation has been studied for the past four years. The amples are over stressed for long periods of time, some con-inuously, others intermittently to produce "breathing" effect. The result can be summarized by saying that without ex-eption a chemical analysis[10] of the deteriorated material eposited on the walls of the gas spaces showed it to be a nitro-roduct. Exactly the same effect is produced by the attack of itric acid. This can only mean that some source of air supply vas available and the conclusion is that air was supplied through he capillary passages.

In general the degree of damage depended upon the ability o supply fresh air. The damage in loose porous samples was nuch greater than in compact ones. Likewise it was greater n those compact samples operated intermittently, where 'breathing" supplied additional air. Another feature was hat the voltage stress at which ionization started seemed to e independent of the degree of damage. There was no de-rease in the starting voltage as the damage progressed.

We have never tested impregnated paper cable in this way. here is no source of air supply in lead sheathed cable and such test would be interesting.

ACKNOWLEDGMENT

In concluding this paper, we wish to express our appreciation nd thanks to Dr. C. P. Steinmetz and Messrs. W. S. Clark,

---

[10] Thanks are due to Dr. C. G. Van Brunt who analyzed these ma-erials.

H. G. Reist, E. Knowlton, D. A. Ballard, H. L. Schermerhorn, and W. C. Hayman for their cooperation and helpful advice.

APPENDIX

For a concentric cross section, such as a single-conductor sheathed cable, the effective a-c. resistivity per cm³., (average through thickness of insulation) can be calculated by the following formula:

$$\rho_{av.} = \frac{2\,\pi\,l\,E^2}{W\,\log_\epsilon \frac{b}{a}} \qquad (\mathbf{1})^{11}$$

where, $\rho_{av.}$ = average effective a-c. resistivity per cm.³
l = length of sample considered in cm.
a = radius of conductor in cm.
b = radius of inner side of metal jacket in cm.
E = impressed volts (effective)
W = watts dielectric loss in length, l.

For the same conditions above the relative permittivity (average through thickness,) is:

$$K_{av.} = \frac{I \sin\theta \cdot \log_\epsilon \frac{b}{a}}{(2\,\pi)^2\, l\, f\, E\, K_a} \qquad (\mathbf{2})$$

where, I = amperes, dielectric current for length, l.
θ = dielectric power factor angle.
f = frequency.
K_a = capacity per cm³ of air = 0.08842×10^{12} farads.

With the following nomenclature and that already given the formula for concentric cross sections having dielectric layers of different permittivities is derived below:

g = voltage gradient $\left(\frac{d\,e}{d\,x}\right)$ at radius x.

C = capacity in farads = $\frac{I \sin\theta}{2\,\pi\,f\,E}$

$b_1, b_2 \ldots\ldots b_n$ = radius in cm. to outer surface of layers.
K = relative permittivity at radius x.
$K_1, K_2 \ldots\ldots K_n$ = relative permittivity of layers.

---

[11] For derivation of this formula see, "Insulation Characteristics of High-Voltage Cables," Clark and Shanklin, loc. cit.

$$\frac{1}{C} = S = S_1 + S_2 \,.\,.\,.\,.\,.\,.\,.\,.\,. + S_n$$

$$S = \frac{1}{2\,\pi\,l\,K_a} \cdot \left(\frac{\log_\epsilon \dfrac{b_1}{a}}{K_1} + \frac{\log_\epsilon \dfrac{b_2}{b_1}}{K_2} \,.\,.\,.\,.\,.\,.\,. + \frac{\log_\epsilon \dfrac{b_n}{b_{n-1}}}{K_n} \right)$$

$$d\,s = \frac{1}{K} \cdot \frac{1}{2\,\pi\,l\,K_a} \cdot \frac{d\,x}{x}$$

$$d\,e = \frac{I \sin\theta\, d\,s}{2\,\pi f} = \frac{E}{S} \cdot \frac{1}{K} \cdot \frac{1}{2\,\pi\,l\,K_a} \cdot \frac{d\,x}{x}$$

$$g = \frac{d\,e}{d\,x} = \frac{E}{S} \cdot \frac{1}{K} \cdot \frac{1}{2\,\pi\,l\,K_a} \cdot \frac{1}{x}$$

$$g = \frac{1}{x\,K} \left(\frac{E}{\dfrac{\log_\epsilon \dfrac{b_1}{a}}{K_1} + \dfrac{\log_\epsilon \dfrac{b_2}{b_1}}{K_2} \,.\,.\,.\,.\,.\,.\, + \dfrac{\log_\epsilon \dfrac{b_n}{b_{n-1}}}{K_n}} \right) \qquad (3)$$

When $a = \alpha$ the dielectric lines are parallel between plane surfaces.

$$t_1 = b_1 - a \qquad t_2 = b_2 - b_1 \qquad t_n = b_n - b_{n-1}$$

$$g = \frac{E}{K\left(\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} \;.\,.\,.\, + \dfrac{t_n}{K_n}\right)} \qquad (4)$$

In calculating the maximum voltage gradient between conductor and sheath in a three-conductor cable it is assumed that g_{max} is not influenced by the proximity of the other two conductors. The problem then simplifies to the determination of g_{max} for two eccentric conductors.

a = radius in cm. of conductor.
b = radius in cm. to inside surface of sheath.
d = distance in cm. between axis of a and b.
g_{max} = voltage gradient (volts/cm.) at surface of cond.

From Russell's "Alt. Currents", Vol. 1, pp. 169-170.

$$g_{max} = \frac{E}{a\ \omega} \left[\frac{b^2 - (a - d)^2}{b^2 - (a + d)^2} \right]^{1/2} \qquad (5)$$

where,

$$\omega = 2 \log_\epsilon [\sinh (\omega/2) + \cosh (\omega/2)]$$

and,

$$\sinh (\omega/2) = \left[\frac{(S - a)\ (b - S)}{a\ b} \right]^{1/2}$$

$$\cosh (\omega/2) = \left[\frac{S\ (S - d)}{a\ b} \right]^{1/2}$$

where,

$$S = \frac{a + b + d}{2}$$

The relative permittivity (K_{av}) of the insulation between conductor and sheath of a three-conductor cable is determined as follows:

Let,

a = radius in cm. of conductor.
b = radius in cm. to inside surface of sheath.
d = distance in cm. between axis of a and b.
c_1 = 1/3 the total measured capacity between the three conductors in parallel and sheath, in farads per ft. length of cable.

From Alex. Russell's, "Alt. Currents" Vol. 1, p. 193.

$$K_{av} = 5.904 \times 10^{10}\ c_1 \log_\epsilon \frac{b^6 - d^6}{3\ (b^3\ d^2\ a)} \qquad (6)$$

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 21, 1919.

Copyright 1919. By A. I. E. E.

THE DIELECTRIC STRENGTH OF AIR FILMS ENTRAPPED IN SOLID INSULATION AND A PRACTICAL APPLICATION OF THE PROBLEM FOR ALTERNATOR COILS AND CABLES

BY F. DUBSKY

ABSTRACT OF PAPER

It is of theoretical importance, as well as of practical importance in design, to be able to predetermine the dielectric strength of air films or bubbles entrapped in solid insulation.

If such films are over-stressed the solid insulation may be damaged by chemical action, local heating, mechanical action etc., due to corona.

An experimental investigation was made of the strength of air films of various thicknesses between glass plates. In the arrangement used, the breakdown of the air or the starting point of corona could be readily observed. Tests were also made of the dielectric strength of air films between other solid insulations.

It was found that the dielectric strength of air films between insulations was practically the same as the dielectric strength of air films between conductors.

The second part of the paper is devoted to the practical application of the data to the design of armature coils and cables, and several specific examples are given.

Part I

THE DIELECTRIC STRENGTH OF AIR FILMS

THIN air films or bubbles are often entrapped in the solid insulation used in coils, cables and other electrical apparatus. Air is weaker than solid insulations, and when placed in series with them in an electric field, as is usually the case for entrapped air, it takes a much greater unit stress. This follows because the same flux passes through the different dielectrics and the air has the lowest permittivity or offers the highest resistance to the flux.

It becomes of great practical importance to be able to predict the behavior of these films and the voltage at which their break-down occurs. The break-down of an air bubble or film usually will not cause immediate break-down of the solid insulation; it may result, however, in gradual deterioration for the following reasons:

1. Heating due to the losses in corona.
2. Chemical action due to the formation of ozone and nitric acid.
3. Mechanical action due to a rapid movement of the corona streamers.
4. The formation of conducting paths by chemical action and carbonization.

The dielectric strength of air in the form of very thin spacings or films *between conductors* has been studied by a number of investigators†; it has recently been treated very thoroughly by F. W. Peek, Jr. and the resulting laws and data are published in several papers and in his book‡.

It is the object of the present investigation to determine experimentally if the laws and data referred to apply when the air films are not between metallic conductors, but exist as pockets in the solid insulation. For this reason, in the tests which will be described later, the break down strength of air was measured on films between glass plates and between insulated conductors.

The data on air films between metal surfaces referred to above will first be discussed.

Air Films Between Metal Surfaces. The dielectric strength of air is the voltage gradient or the volts per unit distance which must be reached before the spark-over of an air space in a uniform electric field can occur. For the thin films, or small spacings under consideration here, the field may be considered as uniform.

The dielectric strength of air at large spacings is proportional to the air density, which varies directly with the pressure and inversely as the absolute temperature. If the air density at a temperature of 25 deg. cent. and a barometric pressure of 76 cm. is taken as unity, the relative air density δ at other temperatures and pressures may be expressed in terms of it, thus:

$$\delta = \frac{3.92\, b}{273 + t} \qquad (1)$$

---

†Harris J. Ryan, TRANS. A. I. E. E., 1911, Vol. XXX, Part I, page 8 Fig. 6, giving the Baille-Paschen-Schuster results.

‡F. W. Peek, Jr., TRANS. A. I. E. E., 1913, Vol. XXXII, Part II, pages 1335 to 1355, *Law of Corona and Dielectric Strength of Air.*

F. W. Peek, Jr., In his book "Dielectric Phenomena in High-Voltage Engineering", Chap. III on Visual Corona, Chap. X on Design of Apparatus where Solid, Liquid and Gaseous Insulations enter in Combinations.

where b = barometric pressure in centimeters,
t = temperature in degrees centigrade.
If G_0 = dielectric strength of air in effective volts per mil in a uniform electric field at 25 deg. cent. and 76 cm. barometric pressure, and
G = dielectric strength of air in effective volts per mil in a uniform electric field at temperature of t deg. cent. and barometric pressure of b cm.

$$G = G_0 \, \delta \tag{2}$$

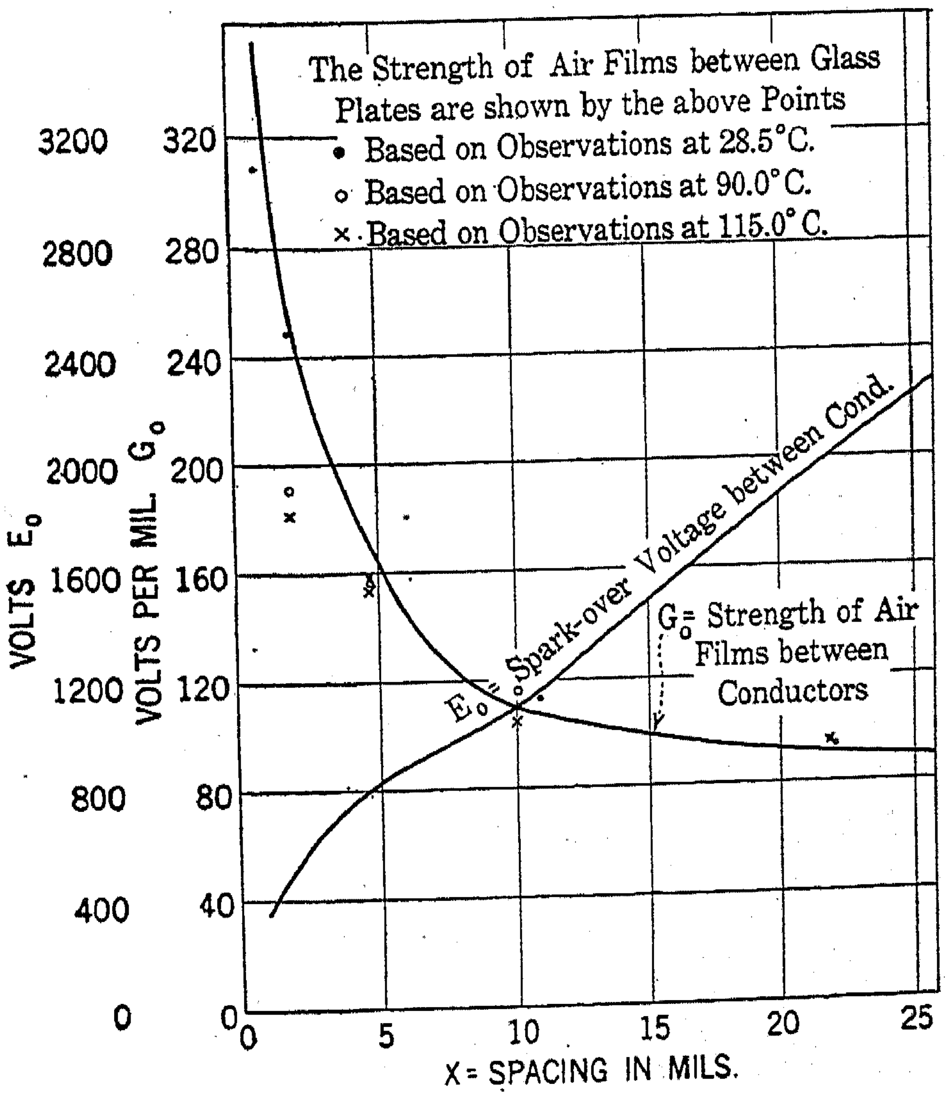


FIG. 1A—DIELECTRIC STRENGTH AND SPARK-OVER VOLTAGE OF AIR FILMS AT t = 25 DEG. CENT. AND b = 76 CM.

Further, if X = air space in mils, since the field is uniform,

$$E_0 = X \, G_0 \tag{3}$$
$$E = X \, G \tag{4}$$

where E_0 and E are respectively the effective sine-wave spark-over voltages of the air space.

The relation $G = G_0 \, \delta$ does not hold for spark-over between parallel plates of small spacings at low air densities. It is used here for simplicity. It is of sufficient accuracy for the range of spacing and air density used in this paper. If greater accuracy is desired the following relation should be used:

According to Paschen's law the spark-over voltage is a function of the product of gas pressure and spark length; the spark-over voltage remains constant when both the gas pressure and the spark gap are altered in such a way that their product does not change.

From the investigations referred to in "Conduction of Electricity through Gases" by J. J. Thompson, *Cambridge Press*, 1906, pages 430 to 528, it follows that the spark-over voltage in gases diminishes as the product $X\,\delta$ of spark length X and relative air density δ decreases, until at a certain critical value of $X\,\delta$ the spark-over voltage reaches a minimum; with still further decrease of $X\,\delta$, below its critical value, the spark-over voltage first increases rapidly to some maximum and then drops to zero.

For air the critical value of $X\,\delta = 0.25$ and the corresponding minimum spark-over voltage $E = 249$ volts effective.

The dielectric strength of air for a very small spacing at various low relative air densities δ, actually decreases at a lesser rate than that given by equation (**2**). The following two formulas (**2a**) and (**4a**) give, respectively, with sufficient accuracy, the dielectric strength and the spark-over voltage for air:

$$G = \frac{225}{X} + n\,\delta \qquad (\mathbf{2a})$$

$$E = X\,G = 225 + n\,X\,\delta \qquad (\mathbf{4a})$$

Where $n = 106$ for $X\,\delta = 0.25$ to 4
$n = 100$ " $X\,\delta = 6$
$n = 96$ " $X\,\delta = 8$

$n = 93$ " $X\,\delta = 10$
$n = 89$ " $X\,\delta = 15$
$n = 82$ " $X\,\delta = 25$

In Figs. 1A and 1B are plotted two curves showing the strength G_0 and the spark-over voltage E_0 for different thicknesses of air films between metal surfaces at 25 deg. cent. and 76 cm. barometer. These data were obtained from the investigation referred to above*; the spacing is expressed in terms of mils to be more conveniently applied.

It would be expected that this strength of air would hold also whenever thin air spacings occur in combination with layers of various solid insulations. In order to apply this data, however, it is first necessary to calculate the stress or gradient on the entrapped air. That this may be simply done is shown below.

---

*F. W. Peek, Jr., TRANS. A. I. E. E., Vol. XXX, Part II, 1913, p. 1348.

Calculation of the Stress or Gradient in Air Films in Solid Insulation. Taking a general case when n dielectrics of thicknesses $X_1, X_2 \ldots X_i \ldots X_n$ mils and of relative permittivities $k_1, k_2 \ldots k_i \ldots k_n$ respectively, are arranged in a uniform electric field in series so that the same dielectric flux density passes through each material, the expression for the gradient in any dielectric of the combination of the n insulations is

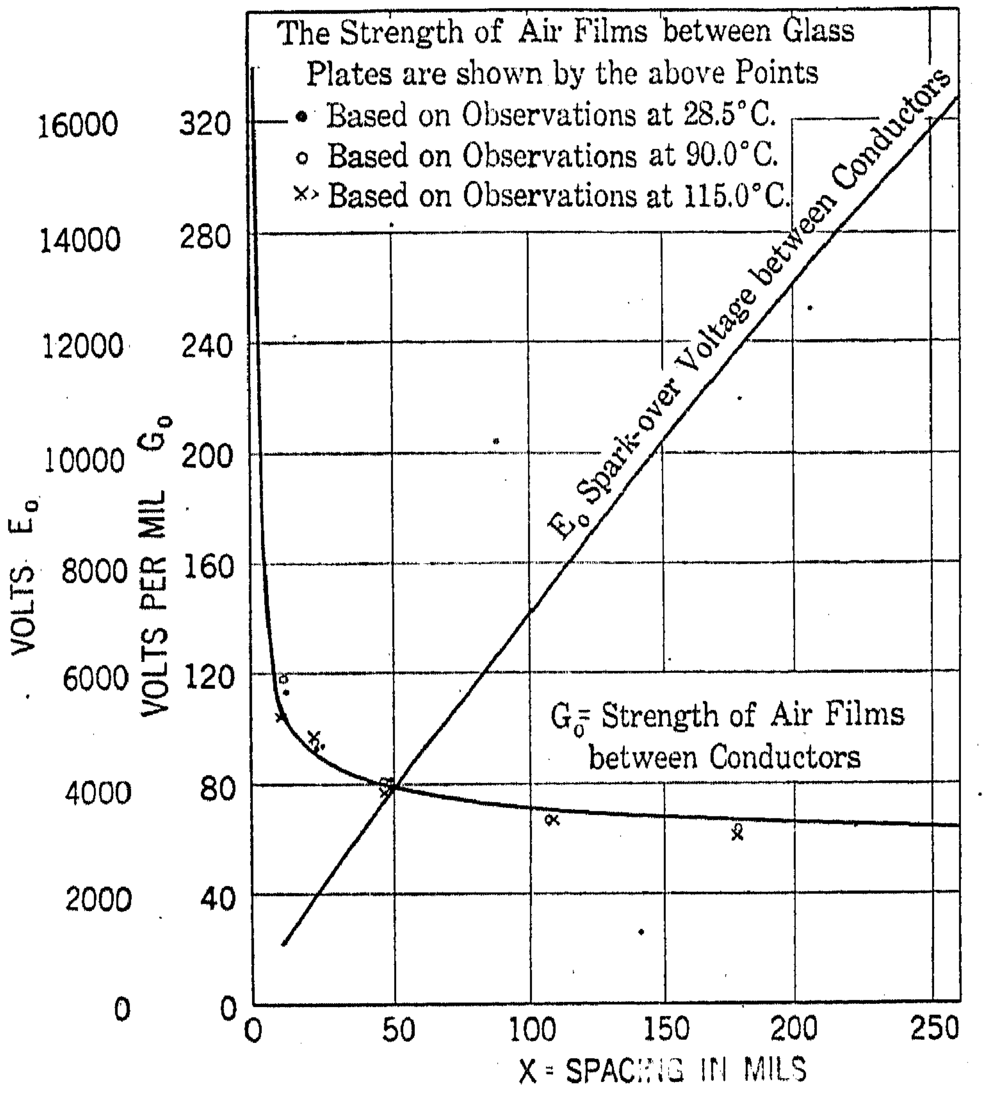


FIG. 1B—DIELECTRIC STRENGTH AND SPARK-OVER VOLTAGE OF AIR AIR FILMS AT t = 25 DEG. CENT. AND b = 76 CM.

$$g_i = \frac{E}{k_i\left(\frac{X_1}{k_1} + \frac{X_2}{k_2} + \ldots + \frac{X_i}{k_i} + \ldots + \frac{X_n}{k_n}\right)} \quad (5)$$

where E = voltage across the terminals in volts.

If X_1, g_1 and k_1 are respectively the thickness, gradient and permittivity of the air film, and hence $k_1 = 1$, it follows that

$$g_i = \frac{E}{k_i\left(X_1 + \frac{X_2}{k_2} + \ldots + \frac{X_i}{k_i} + \ldots + \frac{X_n}{k_n}\right)} \qquad (6)$$

and
$$g_1 = \frac{E}{\left(X_1 + \frac{X_2}{k_2} + \ldots + \frac{X_i}{k_i} + \ldots + \frac{X_n}{k_n}\right)} \qquad (7)$$

The latter equation is used to determine the dielectric stress in air whenever the air enters into combination with solid insulation. The stress on the relatively weak air will always be much greater than the stress on the solid insulation. When the dielectric stress reaches the critical value, the air film breaks down with the appearance of visual corona, characterized by a pale violet glow, a hissing noise and the odor of ozone.

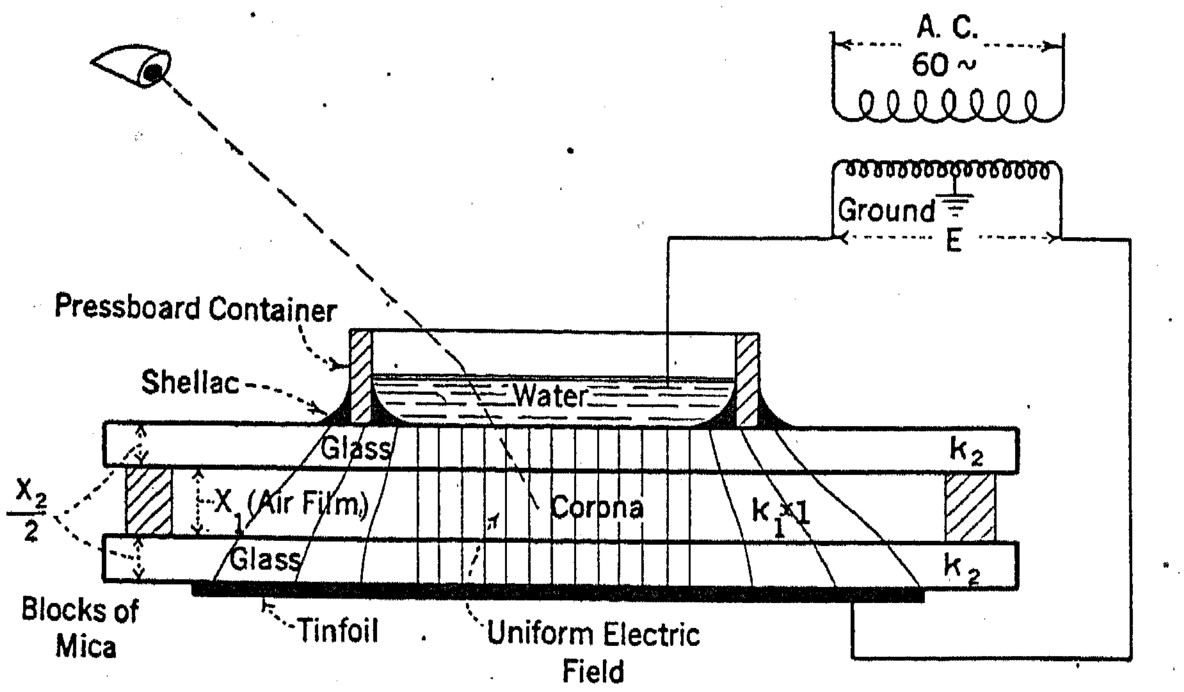


FIG. 2—CORONA TESTS ON AIR FILMS BETWEEN TWO GLASS PLATES

Description of Tests to Determine the Strength of Air Films between Solid Insulations. The corona tests were made with varying air space between two pieces of white French plate glass each 8 in. by 15 in. (see Fig. 2). Small blocks of mica were used to secure the desired space between the glass plates. The tests were made at practically sine-wave voltage, at a frequency of 60 cycles per second.

The lower terminal was a piece of tinfoil 6 in. by 12 in. pressed into intimate contact with the glass. The upper terminal consisted of water held in place by a container of pressboard fastened to the glass with shellac. Water was used as the upper electrode so that the start of corona could be readily observed through the water and the glass plate. The air film did not make contact with either electrode, but did with the glass plates.

The tests were made first at room temperature and then at higher temperatures by the use of an oven. In the latter case the same arrangement was used with the exception of the top terminal, to which calcium chloride was added in order to raise the boiling point of the water.

The voltage was very gradually increased and the start of corona was observed in a dark room by looking through the water terminal and the upper glass plate. The break-down point, as indicated by the pale violet glow in the air film, could be very accurately determined.

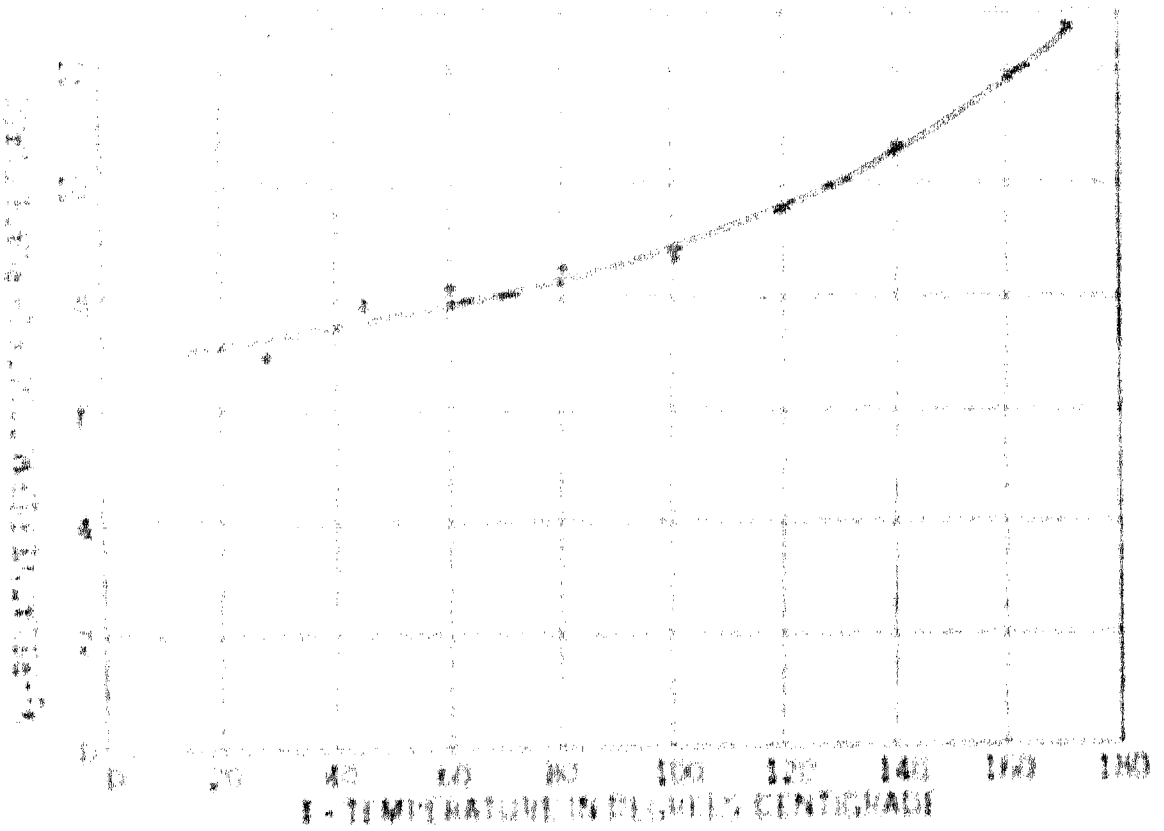


Fig. 3.—Variation of Permittivity of Plate Glass with Temperature.

This interesting experiment was suggested by Mr. F. W. Peek, Jr.

The observed data and the values calculated therefrom are given in Table I where the following designations are used:

X_1 = air space in mils,
X_2 = average thickness of both glass plates,
E = effective sine wave volts between terminals,
b = barometric pressure in cm.,
t = average temperature in deg. cent. of glass plates and air space,

k_2 = relative permittivity of the plate glass. In Fig. 3 is plotted a curve giving the relation between the relative permittivity of the plate glass (used in the above tests) and the temperature from 25 to 170 deg. cent. The data for this curve were obtained from capacitance measurements, on one of the 8 in. by 15 in. glass plates. A sheet of tinfoil 6.625 in. by 13.625 in. was pasted on each side of the glass plate with dry shellac and smoothed out with a hot iron. The maximum, minimum and average values of thickness in 20 measurements were 0.194 in., 0.160 in. and 0.176 in. Capacity measurements were made with a Hoxie capaciometer.

$$g_2 = \frac{E}{k_2\left(X_1 + \frac{X_2}{k_2}\right)}$$ = gradient in the glass at the start of corona, expressed in effective volts per mil at observed temperature and barometric pressure.

$$g_1 = \frac{E}{X_1 + \frac{X_2}{k_2}}$$ = critical gradient in the air between the glass plates at the start of corona, expressed in effective volts per mil at observed temperature and barometric pressure.

$$\delta = \frac{3.92\, b}{273 + t}$$ = relative air density at observed temperature t in deg. cent. and barometric pressure b in cm.

$g_{1,0} = g_1/\delta$ = critical gradient in air between glass plates at start of corona, expressed in effective volts per mil reduced to 25 deg. cent. temperature and 76 cm. barometric pressure = the dielectric strength of air between glass plates, expressed in effective volts per mil at 25 deg. cent. temperature and 76 cm. barometric pressure.

In the last column of Table I is given

G_0 = dielectric strength of air *between conductors* in effective volts per mil at 25 deg. cent. temperature and 76 cm. barometric pressure which values were taken for comparison from Figs. 1A and 1B for the air spacings X_1 referred to in Table I.

Finally it should be stated that the values $g_{1,0}$ obtained in Table I are plotted in Figs. 1A and 1B along the curve giving G_0. The three symbols represent points based on three different temperatures of observation, (all reduced to $t = 25$ deg. cent. and $b = 76$ cm.).

The points, (especially those based on observations at 28.5 deg. cent.), practically coincide with the curves if the relative accuracy due to small spacings between glass plates is taken in consideration.

The results from corona tests on air space between two insulated conductors are given in the lower part of Table I in which case X_2 designates the both sides thickness of the conductor insulation built up of tapings.

TABLE I

CORONA TESTS ON AIR FILMS BETWEEN TWO GLASS PLATES

X_1	E	b	t	X_2	k_2	$\frac{X_2}{k_2}$	$X_1 + \frac{X_2}{k_2}$	g_2	g_1	δ	$g_{1,0}$	G_0
1	14,900	73.6	28.5	357	7.2	49.6	50.6	41	294	.96	307	353
2.1	12,300	"	"	"	"	"	51.7	33	238	"	247	250
4.8	8,200	"	"	"	"	"	54.4	21	151	"	157	170
11.0	6,600	"	"	"	"	"	60.6	15.1	109	"	113	106
22.2	6,400	"	"	"	"	"	71.8	12.4	89	"	93	90
48.0	7,600	"	"	"	"	"	97.6	10.8	78	"	81	79
108	10,100	"	"	"	"	"	157.6	8.9	64	"	67	70
179	13,500	"	"	"	"	"	228.6	8.2	59	"	61.5	66
2.0	6,500	72.3	90	357	8.5	42	44.0	17.4	148	.78	190	256
4.8	5,750	71.5	86	"	8.4	42.5	47.3	14.5	122	.78	156	170
10.2	4,810	71.5	87	"	8.4	42.5	52.7	10.9	91	.78	117	107
22.0	4,750	71.5	89	"	8.5	42	64.0	8.7	74	.775	95	91
47.3	5,625	72.2	88	"	8.45	42.3	89.6	7.4	63	.785	80	79.5
107	7,750	72.5	89.5	"	8.5	42	149	6.1	52	.785	66	70.5
179	11,000	72.5	88	"	8.45	42.3	221.3	5.9	50	.785	64	66
2.0	5,370	72.9	114	357	9.3	38.4	40.4	14.3	133	.738	181	256
4.8	4,940	"	114	"	9.3	38.4	43.2	12.3	115	.738	155	170
10.2	3,630	"	120	"	9.5	37.6	47.8	8.0	76	.728	105	107
22	4,125	72.5	116	354	9.4	37.7	59.7	7.3	69	.730	95	91
47.3	4,680	"	117	"	9.4	37.7	85.0	5.9	55	.728	76	79.5
108	7,125	"	114	"	9.3	38.1	146.1	5.3	49	.733	67	70
179	10,000	"	115	"	9.3	38.1	217.1	5.0	46	.732	63	66
Corona Tests on Air Films between Insulated Conductors.												
250	19,000	72.8	25	254	3.2	79	329	18.1	58	.95	61	63
250	15,800	72.8	107	254	3.5	73	323	13.9	48.7	.75	65	63

Conclusions from above Corona Tests. A comparison of the dielectric strength of *air films between conductors,* and *air films entrapped in solid insulation* shows as good agreement as could be expected; in a uniform electric field the phenomenon of start of corona in thin air films between layers of solid insulation apparently follows the same law as between conductors. It is important to show how practically to apply this data. This is done in Part II.

Part II

EFFECT OF AIR FILMS, AIR BUBBLES OR POCKETS, ENTRAPPED IN SOLID INSULATION UPON THE DESIGN OF ARMATURE COILS OF AIR-COOLED HIGH-VOLTAGE ALTERNATING-CURRENT MACHINES AND UPON THE DESIGN OF CABLES.

General Description of Coil Arrangement and Insulation in Alternators, Motors and Induction Regulators. The armature winding of air-cooled high-voltage alternating-current machines is usually of the barrel type. Three-phase windings are preferably connected in Y with grounded or ungrounded neutral. The form-wound coils are embedded in open slots; there are two coil-sections per slot.

There may be one or more turns per coil as shown in Fig. 4. Occasionally, when the number of turns per coil is large, the coil is wound with several layers and with several turns per layer.

The turns are often wound with several copper strands in multiple. Single-cotton covering, compound treatment, or both are used to insulate the strands so as to eliminate eddy current loss between strands.

The first coil of each phase, being connected to the line terminal, has full line potential to ground (volts between lines $/\sqrt{3}$); the potential of other coils to ground is less and decreases down to zero at the common neutral.

The potential difference between two coils and between top and botton leg of coils depends upon the location of the coils under consideration, and varies between a maximum value equal to the voltage between two line terminals and a minimum value equal to the induced voltage of two coils.

Usually all coils are wound and insulated alike. The insulation of each coil to ground must be designed for the line potential to ground, E, and the total insulation from copper to copper between two coils must stand a potential difference $\sqrt{3}\,E$.

Since there are two thicknesses of insulation between coils it is possible to reduce the thickness of the coil insulation in the end portion of coils to about 87 per cent of the thickness of the coil insulation in the slot portion. The mechanical reenforcement rings supporting the end portion of the winding must then be sufficiently insulated and spaced from the insulated coils so as to give from copper to ground at least the same dielectric strength in the end portion as in the slot portion of the winding. The end portions of the insulated coils are properly spaced for ventilation of the winding, and this spacing helps to reduce the gradients in the coil insulation at these points.

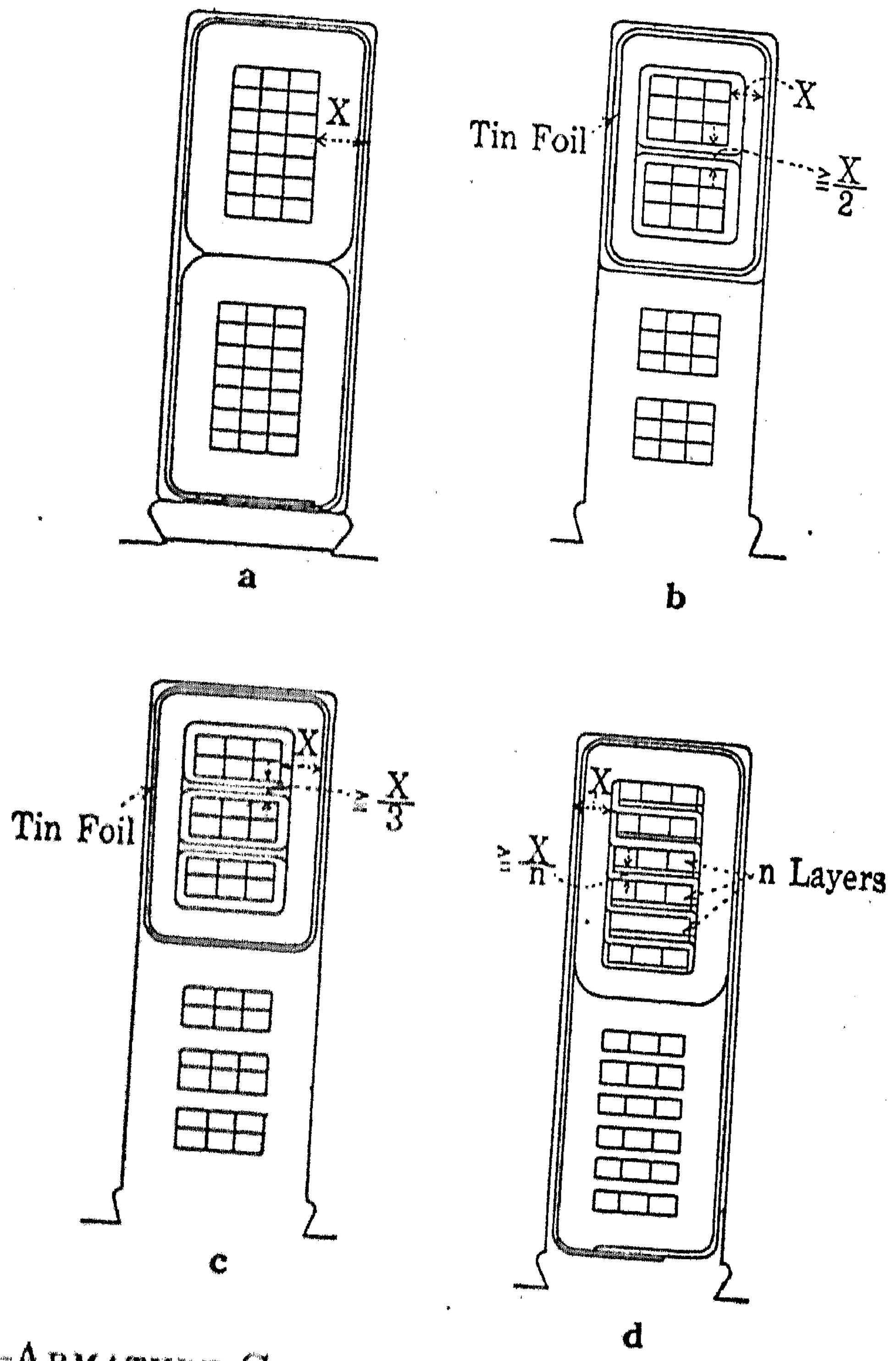


FIG. 4—ARMATURE COILS OF BARREL TYPE WINDING

The coil insulation, being of laminated form, is built up of suitable materials, such as paper and mica, varnished cambric, oiled linen, or micanite, cemented together with some binding sticker and built up to the proper thickness to meet the operating voltage.

Method of Calculating the Stress on the Entrapped Air in Coils with Practical Example. As a simple example, take a coil insulated with a uniform, homogeneous insulation of a thickness X and permittivity k. It is sufficiently accurate to assume the conductor of the coil as a flat plate and the iron core or tinfoil as the other plate. The thickness of the entrapped air bubbles will then be so extremely small compared to X that the capacity or total flux will not be appreciably changed thereby.

If the voltage is E,

$$g_i = \frac{E}{X} = \text{gradient in the solid insulation,}$$

$$g_1 = k\frac{E}{X} = \text{gradient in the air bubbles.}$$

A coil, however, is not usually insulated in the above manner, but by a number of wrappings of a composite tape made up of laminae of various solid insulations, a "sticker", and possibly *very thin films of air,* usually much less than one mil thick, between the laminae. These very thin films need not be considered from the standpoint of dielectric strength, since they are very strong as can be ascertained by examining the curve giving the value of G_0 in Fig. 1A. One mil thickness of air has the strength of 354 volts per mil at $t = 25$ deg. cent. and $b = 76$ cm., a value which is approximately equal to the strength of the varnished cambric or paper and mica taping. There are other more or less avoidable local air bubbles, or pockets, of larger thickness, which are due to imperfect butt joints of the tape, due to occasional wrinkles produced by unequal tension during the taping, and due to bulging of the coil insulation, whose strength requires attention. The very thin air films between laminae must be considered, however, in calculating the resultant permittivity, K, since they reduce the dielectric flux density in the coil insulation.

Calculation of the stresses in any of the insulation materials of the combination may then be made from equation (5),

$$g_i = \frac{E}{k_i\left(\frac{X_1}{k_1} + \frac{X_2}{k_2} + \ldots + \frac{X_i}{k_i} + \ldots + \frac{X_n}{k_n}\right)} \quad (5)$$

For convenience let us substitute for the expression in the parenthesis in the above denominator

$$\left(\frac{X_1}{k_1} + \frac{X_2}{k_2} + \ldots + \frac{X_i}{k_i} + \ldots + \frac{X_n}{k_n}\right)$$

$$= \sum_1^n \frac{X_i}{k_i} = \frac{X}{K} \tag{8}$$

where X is the total thickness of insulation

$$X = X_1 + X_2 + \ldots + X_i + \ldots + X_n = \sum_1^n X_i \tag{9}$$

and K is the resultant permittivity

$$K = \frac{X}{\sum_1^n \frac{X_i}{k_i}} \tag{10}$$

The method of calculating the resultant permittivity, K is best shown by an example. Let a taping be made up as follows:

$X_1 = 1$ $X_2 = 4$ $X_3 = 6$ $X_4 = 1.5$ mils

$k_1 = 1$ $k_2 = 2.6$ $k_3 = 5.5$ $k_4 = 4.5$

$X = X_1 + X_2 + X_3 + X_4 = 1 + 4 + 6 + 1.5 = 12.5$

$$\sum_1^n \frac{X_i}{k_i} = \frac{X_1}{k_1} + \frac{X_2}{k_2} + \frac{X_3}{k_3} + \frac{X_4}{k_4}$$

$$= \frac{1}{1} + \frac{4}{2.6} + \frac{6}{5.5} + \frac{1.5}{4.5} = 3.96$$

$$K = \frac{X}{\sum_1^n \frac{X_i}{k_i}} = \frac{12.5}{3.96} = 3.15.$$

The resultant permittivity is calculated in Table II for several insulation units. It will be seen that the resultant permittivity, K, of the coil insulation with reference to a temperature of 25 deg. cent. is about 3 to 3.5 for the various tapings and about 4 for micanite.

The resultant permittivities, K, in Table II apply for a

TABLE II

RESULTANT PERMITTIVITY $K = \frac{X}{\sum_{1}^{n} \frac{X_i}{k_i}}$ OF COIL INSULATION-UNITS BASED ON PERMITTIVITIES k_i OF DIELECTRICS AT A TEMPERATURE OF 25 DEG. CENT.

Insulation unit	Composed of materials	X_i Thickness in mils	k_i Permittivity at 25 deg. cent.	$\frac{X_i}{k_i}$ at 25 deg. cent.	$\frac{X}{\sum_{1}^{n} \frac{X_i}{k_i}} = K$ at 25 deg. cent.
One-half lap taping with (.005)″ = .001 + .003 + .001 Paper-Mica-Paper Tape	Paper	4	2.6	1.54	$\frac{12.5}{3.96}$ =
	Pasted Mica	6	5.5	1.09	
	Sticker	1.5	4.5	.33	= 3.15
	Air	1	1.0	1.0	
	Total	12.5		3.96	
One-half lap taping with (.012)″ Black Varnished Cambric Tape	B. V. C.	24	3.8	6.30	$\frac{27}{7.66}$ =
	Varnish	2	5.6	.36	
	Air	1	1.0	1.00	
	Total	27		7.66	= 3.52
One-half lap taping with (.008)″ Black Varnished Cambric Tape	B. V. C.	16	3.5	4.57	$\frac{19}{5.93}$ =
	Varnish	2	5.6	.36	
	Air	1	1.0	1.00	
	Total	19		5.93	= 3.2
One-half lap taping with (.005)″ Cotton Tape + 2 Coats of Varnish	Cotton Tape	10	2.6	3.84	$\frac{15}{5.55}$ =
	Varnish	4	5.6	.71	
	Air	1	1.0	1.00	
	Total	15		5.55	= 2.7
One butted taping with (.005)″ Cotton Tape + 2 Coats of Varnish	Cotton Tape	5	2.6	1.92	$\frac{9.5}{3.13}$ =
	Varnish	4	5.6	.71	
	Air	.5	1.0	.50	
	Total	9.5		3.13	= 3.03
Micanite			4.0		
Horn Fibre, varnished			2.8		
Press Board, varnished			3.7		

temperature of 25 deg. cent.; at higher temperatures these values become greater, because the permittivities of the solid dielectrics increase with temperature.

From equations (**8**) and (**5**) we obtain now

$$g_i = \frac{E}{k_i \frac{X}{K}} = \frac{K}{k_i} \frac{E}{X} \qquad \textbf{(11)}$$

and for air $k_i = k_1 = 1$ and $g_i = g_1$

$$g_1 = K \frac{E}{X} \qquad \textbf{(12)}$$

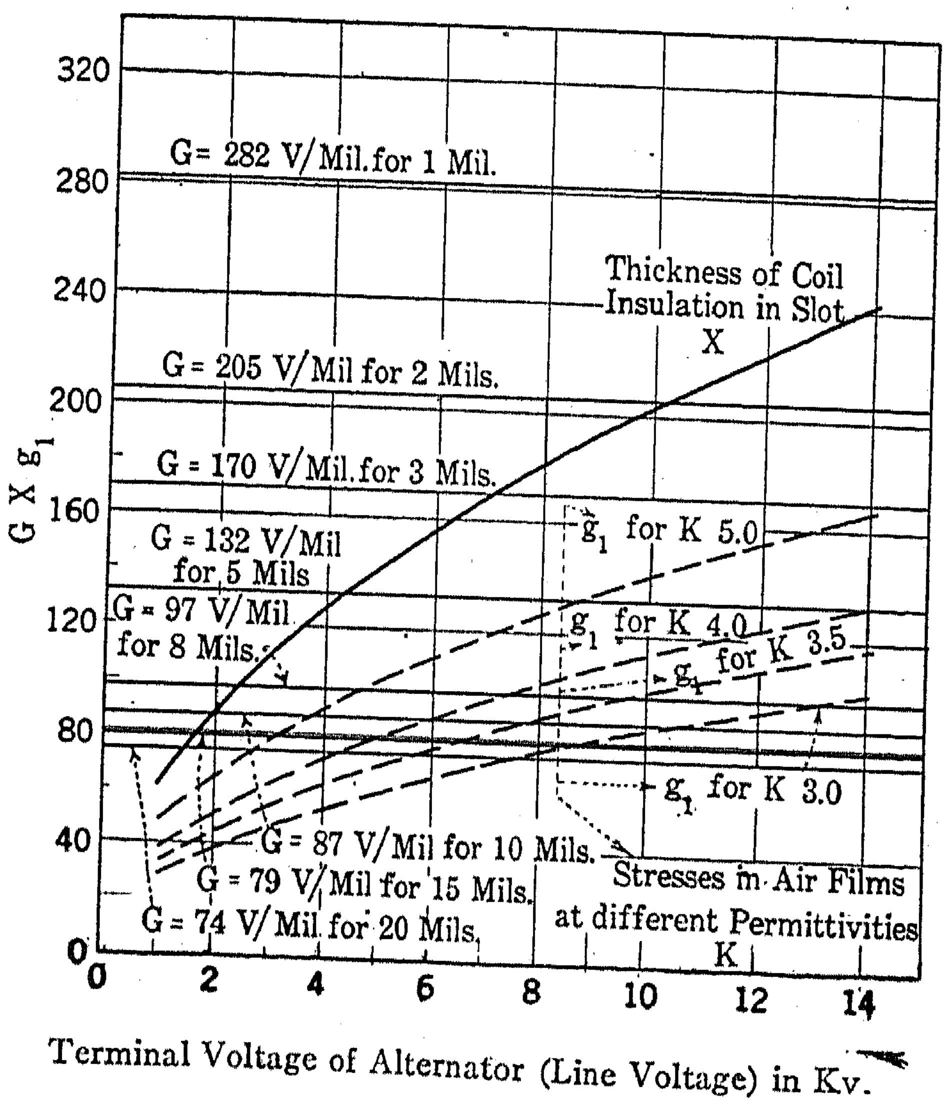


FIG. 5—INVESTIGATION OF START OF CORONA IN COIL INSULATION SHOWN BY A TYPICAL CURVE X

X = Thickness in mils of coil insulation in slot portion (from copper to ground).
g_1 = Dielectric stress in effective volts per mil of entrapped air pockets
G = Dielectric strength of air films at 100 deg. cent. expressed in effective volts per mil

where g_1 = dielectric stress or gradient in air bubbles or pockets, expressed in effective volts per mil;
K = resultant permittivity of the coil insulation;
E = effective sine wave voltage between line terminal and core or grounded tinfoil, expressed in volts = line voltage $/\sqrt{3}$;
X = thickness of coil insulation in slot portion measured from copper to core or grounded tinfoil and expressed in mils;

In case of composite coil insulation which is made up of several layers of different kind of insulation—units, as layer *A* next to the coil, then *B*, then *C*, having layer thicknesses X_A, X_B, and X_C and resultant permittivities K_A, K_B and K_C respectively, the resultant of the resultant permittivities is

$$K = \frac{X}{\frac{X_A}{K_A} + \frac{X_B}{K_B} + \frac{X_C}{K_C}} \tag{13}$$

TABLE III

DIELECTRIC STRESSES IN AIR FILMS IN COIL INSULATION SHOWN BY CURVE *X* IN FIG. 5.

Line Voltage	Potential to ground	Thickness of coil insulation in mils		Calculated gradients in volts per mil in air films of coil insulation $g_1 = K\frac{E}{X}$ for			
$\sqrt{3}E$	E	X	$\frac{E}{X}$	$K = 3$	$K = 3.5$	$K = 4$	$K = 5$
1,000	577	60	9.6	29.0	34	38	48
2,000	1,154	88	13.1	39	46	53	66
3,000	1,731	110	15.8	47	55	63	79
4,000	2,310	128	18.0	54	63	72	90
5,000	2,885	143	20.2	61	71	81	101
6,000	3,460	157	22.0	66	77	88	110
7,000	4,045	170	23.8	71	83	96	119
8,000	4,620	182	25.4	76	89	102	127
9,000	5,200	193	27.0	81	95	108	135
10,000	5,770	204	28.3	85	99	113	142
11,000	6,350	215	29.6	89	104	119	148
12,000	6,930	225	30.8	93	108	123	154
13,000	7,500	233	32.2	97	113	129	161
14,000	8,090	241	33.6	101	118	135	168

where K_A, K_B and K_C are obtained from equation (**10**) or from Table II for the corresponding insulation-unit in the layer *A*, *B* and *C* respectively.

In Fig. 5 is given a typical curve, *X*, showing the thickness of coil insulation in the slot portion of alternator windings—in relation to the terminal voltage (line voltage).*

In Table III this curve is analyzed with respect to the dielectric stresses in air pockets in the coil insulation, by applying equation (**12**) and using the resultant permittivities $K = 3$, 3.5, 4 and 5 respectively.

---

*See Standard Handbook for Electrical Engineers, Fourth Edition, Section 7, Alternating-Current Generators and Motors by Comfort A. Adams and Henry M. Hobart, page 484, Fig 65.

The stresses g_1 in air bubbles for different resultant permittivities, K, are shown in the relation to the terminal voltage by the broken curves in Fig. 5.

In Fig. 6 is reproduced the curve of dielectric strength G_0 of air at $t = 25$ deg. cent. and $b = 76$ cm. in relation to spacing, (see also Fig. 1A), and therefrom is derived—by applying

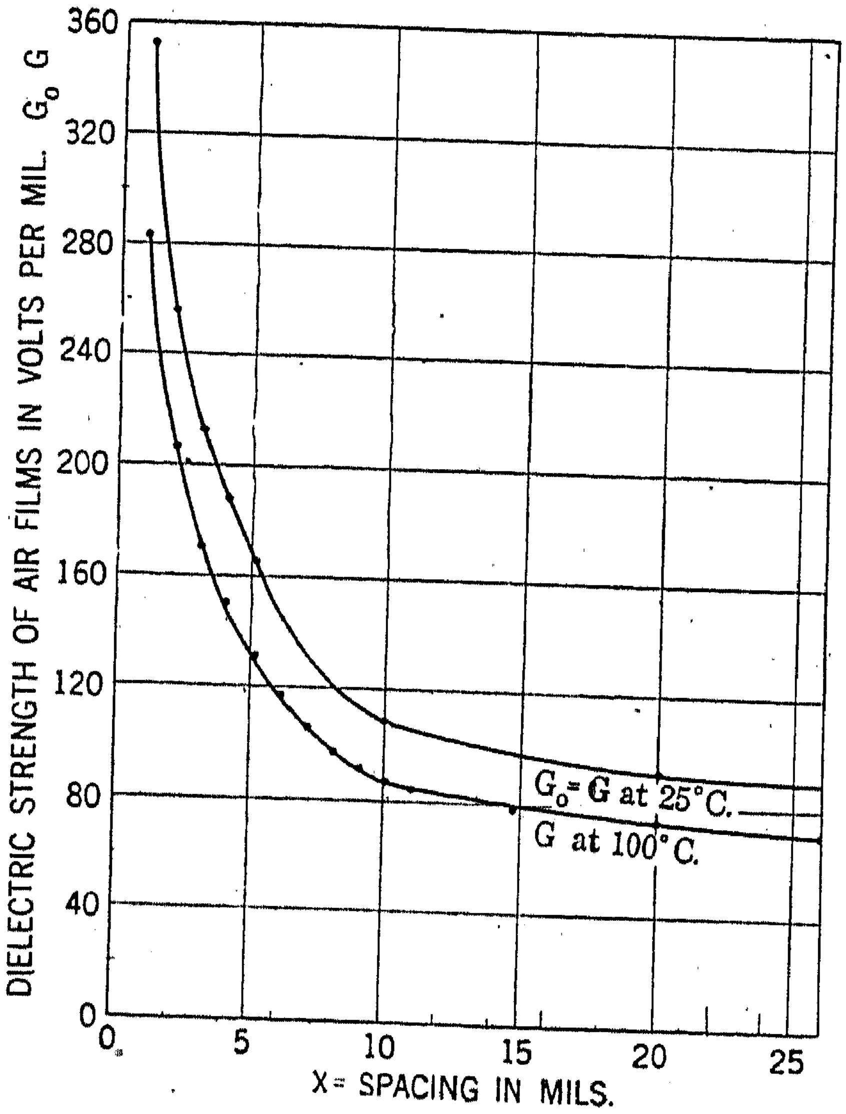


FIG. 6—DIELECTRIC STRENGTH OF AIR FILMS AT $b = 76$ CM. AND $t = 25$ AND 100 DEG. CENT. RESPECTIVELY

equations (**1**) and (**2**)—the curve of dielectric strength G of air films at $t = 100$ deg. cent. and $b = 76$ cm.

The following values of dielectric strength, G, of air at $t = 100$ deg. cent. and $b = 76$ cm. for spacings;
$X_1 =$ 1, 2, 3, 5, 8, 10, 15, 20 mils; namely $G =$ 282, 205, 170, 132, 97, 87, 79, 74 volts per mil are shown by lines parallel to the abscissa axis in Fig. 5 and *their intersections with the broken curves g_1 indicate the starting points of the corona* in the entrapped air bubbles or pockets.

As stated above we have to consider the strength of entrapped air bubbles or pockets of larger thickness only, namely of such air bubbles or pockets which are due to imperfect butt joints of the tape, due to occasional wrinkles produced by unequal tension during the taping, and due to bulging of the coil insulation.

Since air has the lowest value of both permittivity and dielectric strength among the insulating materials which enter into the combination of the coil insulation, we should keep the dielectric stress of those air bubbles or pockets below the critical values which are plotted in Fig. 6 in relation to the air spacing. Both temperature and barometric pressure (altitude) of the location affect the strength of air pockets entrapped in coils, and should be taken into consideration. Tightly wound coils with very thin air pockets withstand higher dielectric stresses than loosely insulated coils.

Let us consider a practical example to illustrate the use of Fig. 5.

Take a 13,200-volt generator. Assume that $K = 3.5$ at 100 deg. cent. and that the method of manufacture is such that there is a probability of 10 mils air pockets in the coil insulation. From curve X the thickness of coil insulation for a 13,200-volt generator is 235 mils from copper to core or grounded tinfoil. The stress on the entrapped air for $K = 3.5$, from the broken curve g_1 for $K = 3.5$, is $g_1 = 113$ volts per mil.

The dielectric strength of 10 mils air pocket at 100 deg. cent., as indicated by the horizontal line $G = 87$ volts per mil for 10 mils, air film, is not strong enough to withstand a stress of $g_1 = 113$ volts per mil, with the result that there will be corona in the 10 mils air pockets.

If the method of manufacture can be improved so that the air bubbles in the coil insulation are all reduced to about 5 mils or less, there will be no internal corona, since the strength of a 5 mils pocket is then 132 volts per mil, as indicated by the horizontal line $G = 132$ volts per mil for 5 mils air film.

The above can, of course, also be calculated for any coil insulation directly from equation (**12**) and the conclusion reached by inspection of Fig. 6.

It should be borne in mind that we have considered so far the dielectric stresses in air in a uniform electric field, which is nearly the case for the sides of the coils adjacent to the core. At the edges produced in the slots by the air ventilating ducts in stator iron and at the edges of the coils, layers and turns, there are some concentrations of the dielectric field which produce somewhat higher local stresses on the coil insulation and the occluded air pockets than at the location of the uniform field.

The edges of stator ducts and stator teeth may produce external corona and cause pitting of the coil insulation at the coil surface. This danger can be guarded against by providing a grounded tinfoil armor extending 1½ to 2 in. at each end beyond the slot portion of the coil. The coil insulation must be properly reenforced under and above the edges at the ends of the tinfoil armor.

The edges of layers and turns are locations of increased stress for the internal coil insulation between layers and turns. The problem is comparatively simple for coils which have but few turns per coil with only one turn per layer. Here the turns may be heavily taped, as shown in Figs, 4B and 4C without much loss of space, because the turn insulation constitutes a part of the coil insulation. It is recommended that coils with n turns per coil receive an insulation between turns equal to at least $1/n$ of the thickness of the coil insulation to ground. Such turn insulation will stand up indefinitely under normal operating conditions and will also withstand occasional line surges when they penetrate the winding. It is in most cases the insufficient turn insulation that, after comparatively short operation, ages and deteriorates due to action of internal corona, and causes break down usually in the line coil.

Coils wound with several layers and several turns per layer are more delicate and must be insulated yet more liberally to withstand safely a long time service. Such coils are usually filled with compound before the insulation is applied; the purpose of this treatment is to eliminate so far as possible all the air between turns and layers inside the coil.

In conclusion it is evident that the dielectric stress on the air bubbles in solid insulation of armature coils should be calculated and kept below the dielectric strength by using suitable thickness of insulation for the required operating voltage and by applying the insulation as tightly as possible. The internal insulation of coils between layers and turns should be adequate and selected in proper relation to the coil insulation to withstand the increased dielectric stresses at the edges of layers and turns and to withstand the line surges.

General Description of Insulation in Cables and Leads. The insulation of cables and leads is also of laminated form, being built up of paper or varnished cambric tape, and occludes also thin air bubbles or pockets. The latter are, of course, much thinner than those in the coil insulation, because the taping of the cylindrical conductor can be done with uniform tension; and in the case of cables the taping is applied with a machine.

The dielectric problems with which we will deal here are, therefore, similar to those encountered in the coil insulation, except that the electric field in the cable insulation is not uniform, but radial, as shown in Fig. 7.

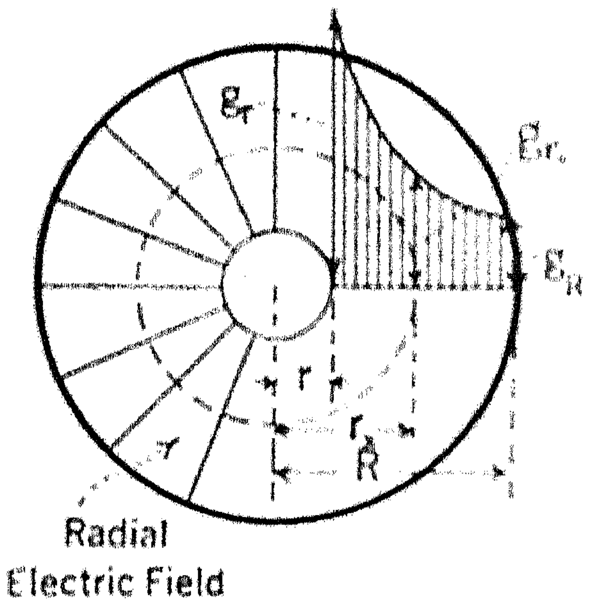


FIG. 7—SINGLE-CONDUCTOR CABLE WITH LEAD SHEATH

Let

r = radius of wire in mils,

r_x = any radius of the insulation around the conductor in mils,

R = inside radius of the lead sheath in mils,

E = effective sine-wave voltage between the conductor and the grounded lead sheath,

g_r = gradient in effective volts per mil on the cable insulation at radius r,

g_{r_x} = gradient in effective volts per mil on the cable insulation at radius r_x.

g_R = gradient in effective volts per mil on the cable insulation at radius R.

The following equations hold for the gradients at different points in the radial electric field of a homogeneous insulation:

$$g_r = \frac{E}{r} \; \frac{1}{\log_e \dfrac{R}{r}} \qquad (14)$$

$$g_{r_x} = \frac{E}{r_x} \; \frac{1}{\log_e \dfrac{R}{r}} \qquad (15)$$

$$g_R = \frac{E}{R} \; \frac{1}{\log_e \dfrac{R}{r}}\,. \qquad (16)$$

If K is the resultant permittivity of the cable insulation, we may determine the dielectric stresses $g_{1,r}$, g_{1,r_x}, $g_{1,\mathrm{R}}$ on occluded air pockets at different points in the radial electric field as follows:

$$g_{1,r} = K g_r = K \frac{E}{r} \frac{1}{\log_e \frac{R}{r}} \tag{17}$$

$$g_{1,r_x} = K g_{r_x} = K \frac{E}{r_x} \frac{1}{\log_e \frac{R}{r}} \tag{18}$$

$$g_{1,\mathrm{R}} = K g_{\mathrm{R}} = K \frac{E}{R} \frac{1}{\log_e \frac{R}{r}} \tag{19}$$

The gradient g_i, r_x, in any dielectric of permittivity k_i at any radius r_x, can be calculated from the following equation:

$$g_i, r_x = \frac{K}{k_i,} \frac{E}{r_x} \frac{1}{\log_e \frac{R}{r}} \tag{20}$$

In case that the cable insulation is made up of one insulation-unit the K for equations (**17**), (**18**), (**19**) and (**20**) can be obtained with sufficient approximation from equation (**10**) or from Table II for the corresponding insulation-unit.

In case that the cable insulation is graded by using several layers of different kind of insulation-units, as layer A next to the conductor of radius r, then B, then C, having outside radii R_{A}, R_{B} and R_{C} and resultant permittivities K_{A}, K_{B} and K_{C} respectively, the resultant of the resultant permittivities, K, for equations (**17**), (**18**), (**19**) and (**20**) is

$$K = \frac{\log_e \frac{R_{\mathrm{C}}}{r}}{\frac{1}{K_{\mathrm{A}}} \log_e \frac{R_{\mathrm{A}}}{r} + \frac{1}{K_{\mathrm{B}}} \log_e \frac{R_{\mathrm{B}}}{R_{\mathrm{A}}} + \frac{1}{K_{\mathrm{C}}} \log_e \frac{R_{\mathrm{C}}}{R_{\mathrm{B}}}} \tag{21}$$

where K_{A}, K_{B} and K_{C} are obtained from equation (**10**) or from Table II for the corresponding insulation-unit in the layer A, B and C respectively.

The gradient $g_{1,r}$ on the air pockets at the conductor radius r has the maximum value because it occurs in the densest electric field.

From equation (**17**) it will be seen that the occluded air is subjected to a higher dielectric stress than the solid insulation. Because the dielectric strength of the air bubbles is lower than the dielectric strength of the solid insulating materials which enter into the combination of the cable insulation, we should, to avoid corona, keep the dielectric stress, $g_{1,r}$ on the air pockets, below the critical values which are plotted in Fig. 6 in relation to air spacing.

Conclusions from the Applications in Part II. To avoid corona in the insulation of armature coils, cables and other high-tension apparatus operating in air, it is necessary that under normal operating conditions the dielectric stress on air is below the dielectric strength of such air spaces, air bubbles or pockets which cannot be avoided in the process of manufacture.

A solid insulation which is not subjected to corona will show not an abnormal deterioration with time but will preserve practically its full dielectric strength and enable it to withstand exceptional high-voltage stresses of very short duration, caused by surges and lightning.

This discussion does not cover, of course, the phenomenon generally known as "aging", that is the fatigue of solid insulations with time due to slow oxidation in the atmospheric air, or from effects of heating and dielectric stress below corona starting point.

It is not within the scope of this paper to deal with the difference in action between air films or thin pockets which have a practically uniform dielectric field and air bubbles or air spaces of such a shape as to distort the field. The latter may introduce additional phenomena as lateral stresses at the surface of division of air and taping.

DISCUSSION ON "IONIZATION OF OCCLUDED GASES IN HIGH-TENSION INSULATION" (SHANKLIN AND MATSON) AND "THE DIELECTRIC STRENGTH OF AIR FILMS ENTRAPPED IN SOLID INSULATION AND A PRACTICAL APPLICATION OF THE PROBLEM FOR ALTERNATOR COILS AND CABLES" (DUBSKY), NEW YORK, N. Y., FEBRUARY 21, 1919.

Henry G. Reist: Injury from corona is an old trouble with us that is continually taking new forms. I remember many years ago the engineers in England and the continent, had very serious trouble from internal corona, about which we knew nothing in this country. It has been the practise of American manufacturers to use form-wound compound filled coils ever since large high-voltage generators were built. In Europe, on the contrary, the conductors were drawn through tubes of paper or micanite, and no filling was used, leaving a very considerable amount of air around the conductors. It was around these conductors that the internal corona was experienced, and the insulation on the individual turns of the conductors was soon destroyed and the machines were short-circuited.

As far as my experience goes, the first trouble experienced with corona was outside of the insulation on the coils. This appeared on machines usually in certain places, more particularly perhaps when they were located in high altitudes. But now we are coming to a new kind of corona trouble, that is, corona within the walls of the external insulation. Due to filling the coils, there is very little corona next to the conductors.

Corona in the external insulation, seems to have been increasing in recent years, probably due to the fact that we are making larger coils than formerly, the present coils being two, three or more inches in depth, with straight sides, on which it is difficult to apply the insulation so that it will lie flat on the surface. There is always a tendency for the insulation to bulge, due to the springing of the material. This tendency is probably greater with mica than when paper or cloth is used, because of the natural elasticity of the mica. If the section of the coils were round, there would be no difficulty in applying the insulation closely.

Insulated coils are usually moulded, and when new the external insulation is very compact, but most of the binding materials soften under heat and will release the insulation from the coil so that air pockets may be formed. These are largely prevented in the slots by the walls of the slot not allowing the insulation to go very far. At the ends it is not so easy to avoid this and prevent pockets from forming.

If we could keep the thickness of the spaces down to 0.003 in. or 0.004 in., we would apparently be free from troubles, as pointed out by the writers of these papers, but it is not always possible to do this.

As a practical remedy, we may use materials that are least affected by the corona, such as, mica and certain varnishes, some of which are much more resistant than others. In this way, paper and cloth may be protected from the acids which are formed by the corona.

L. W. Chubb: The formation of corona in the air pockets has, of course, been known for years, and the endeavor of the manufacturer has always been to reduce this trouble by either confining the insulating material to low specific capacitance, by mechanical means, by filling the coils so that they will not breathe and by using insulation that will not be injured.

Years ago, in considering the problem, it was felt that materials having low specific inductive capacitance and the necessary dielectric strength would be rather hard to find, especially those that would stand the higher temperature, so that the problem was attacked by improving the mechanical methods of insulating coils. The mica is very desirable because it can stand the high temperature. It is, of course, attacked by nitric acid, and there are cases where pitting has been produced.

The way that the problem was solved to a great extent was to put the mica on hot, and by a machine which has ironing action and puts the wrapper in circumferential tension. This operation eliminates all air pockets and by the heavy pressure and ironing action removes all bond except that necessary to fill the irregularities of the voids.

Mr. Reist has told about the great trouble with the flat sided-coil. This is usually very much greater with mica. The machine wrapped coil, however, is so solid and can be made so true to dimensions that when, properly gaged and pressed in, will be supported by the sides of the slot, the tension with which it was wound will keep it from bulging at the corners and edges, where there is no support and a chance to bulge.

Mr. Reist spoke of the end of the coil not being supported. That is true but the end of the coil has not as much potential gradient, because of no grounded material outside.

In Mr. Dubsky's paper (second paragraph), a list of troubles is given. The first one is: "Heating due to the losses in corona". This is not so great at commercial frequencies. At high frequencies it is quite important, and protection must be obtained so that local heating will not be produced with high-frequency troubles. The next item in the list of troubles is: "Chemical action due to the formation of ozone and nitric acid". Nitric acid is the more important. Its effects can be greatly reduced by the filling of coils. If a coil is not allowed to breathe, the troubles, even though occluded air is broken down, will not be very serious, and the corrosion will be very slight. The next trouble is indicated as being due to, "Mechanical action due to a rapid movement of the corona streamers". I do not understand just what is meant by this and would like the author to explain it. The fourth reason given is: "The formation of conducting paths by chemical action and carbonization." It seems to me, that fourth item is an advantage rather than a disadvantage. If we have an air film the surfaces of which becomes conducting, the space between will be subjected to no potential gradient. Such conducting surfaces will be an equipotential surface and remove all stress from the entrapped air.

The method of using the water for the upper electrode for observing corona was quite ingenious. It seems as though Dr. Whitehead's schemes of recording the breakdown of the air could be used to get very much closer indications of the start of corona.

John B. Whitehead: Both of the foregoing speakers have emphasized the importance of the subject of these papers, but it is noticeable that neither of them has made any comment on the material presented. While agreeing entirely as to the importance of the subject, I, too, find it very difficult to make specific comment. The explanation lies, in my opinion in the unfortunate manner in which the material is presented.

Both papers deal with the presence of air spaces in insulation and each discusses the subject as though these air spaces existed in the form of thin films. Neither paper presents satisfactory evidence of the existence of such air films and if, as seems probable, air is entrapped in insulation, it is quite certain that it does not exist in the shape of uniform films. On the contrary, it is probably in the form of bubbles of highly irregular shape and size. It appears to me therefore unwarrantable to reason as to the permissible electric intensity in insulation from the premises assumed by the authors.

The late Professor Rowland used to conduct a journal meeting, in which the graduate students reported on current articles in physics. The painstaking student would often undertake an exhaustive description of all the subject matter of a paper. Rowland was anxious only for the results presented and would exclaim with impatience "Stop all that, tell me what he did and not what he thinks." While the anecdote may over stress the point to be made, I think it may fairly be said that the authors of the present papers have extended their speculations, as based on the assumptions of uniform thin films, to a point where the actual results they have to present are considerably confused. There is an increasing tendency, in the presentation of papers to the Institute to reduce to an extreme minimum, sometimes only to the drawing of a curve, the statement of the conditions of observation and accuracy of experimental results. The criticism applies not only to the papers before us, but to numerous others. What is wanted in a paper describing experimental observations is a simple statement of the problem, the conditions of observation, a reasonable account of the observations themselves, showing their accuracy, and a description of conclusions that may fairly be drawn from the experiments.

Returning to the subject of the papers, there are two obvious methods of investigation which would appear to offer promise of useful result: first, insulate a number of armature conductors in as many different ways as can be thought of for the exclusion of air, then measure the losses in different types of insulation by the very interesting methods described in the Shanklin and Matson paper; second, if it can be found that gas and air are actually present in the insulation in the form of films, prepare insulated conductors in which films of known thickness are actually included. Measurements may then be made on this type of conductor also.

In the face of the painstaking and earnest work that has been put on these papers, it is perhaps ungracious to appear to criticize them. The spirit, however, is intended to be constructive and to make an earnest plea for a more scientific method in our experimental work and in the form of its presentation.

C. A. Adams: I think all of those present who have been concerned with research work in any thorough and careful way will appreciate the force of what Major Whitehead has said. In every experimental research we start out with some problem to solve, certain definite results we wish to obtain, and with some recognition of the variables involved. If we analyse the results carefully as the work progresses we will usually find some inconsistency between the results and our theory, which takes account of only those variables previously considered as significant. We are thus forced to take account of other variables, possibly of some not previously even recognized. In fact the chief value of a piece of research is dependent very largely upon the insistence on knowing the reason why; the digging down deeper and deeper, after each step is made, into the theory of the problem, introducing one by one additional factors until we have been able to explain completely all the observations that we have made.

I do not like to take up your time to discuss at any length one of my hobbies, one to which I have been devoting a large amount of time for the past year or two; but I cannot resist the opportunity to emphasize briefly the importance of co-operative research.

In almost any industrial field you will find several researches being carried on in several places by different men, all overlapping more or less, and frequently none of them comprehensive or thorough. It often happens in research work that a suggestion made by another mind, with a slightly different point of view, may throw important light on the subject which at a particular juncture will turn the direction of the research at right angles, or more, and finally result in something which is really worth while; while the one-man researches are in many cases incomplete, misleading and wasteful. If all those interested would only get together, divide up the work, meet periodically to discuss results and get the reaction not of one, but of several able minds, we might solve many of our problems in one tenth or less of the time now required and at much less expense. This field of insulation is one of the most important of them all, and because of its breadth and depth has been cultivated only in small patches. It is therefore one which promises the most from comprehensive cooperative treatment.

W. J. Foster: After Dr. Whitehead's remarks the thought that came to me, was the difficulty in determining in advance by any laboratory methods what is going to be the effect of usage on different insulations. That is the great difficulty that we are laboring under. We are now getting experience, more and more of it, and we can in the case of any new problem, where we have a difficult machine to construct, decide upon certain experiments; we may go so far as to apply mechanical force to our insulated coils so as to distort them somewhat. It is possible to imitate the distortions that we think the coils will be subjected to in assembling, and then apply our tests again, and find out how much the insulation has been weakened. but after all this is done, we may meet with a surprise in the course of two or three years from some actions going on that we did not anticipate.

Such experiences have emphasized the importance of using insulations that will stand high temperatures. We all see that that is a desideratum—that we must, if possible, have insulations that will stand temperatures that will be met with above what we foresee. At the same time we are faced with the effect of heat expansions, and contractions on the life of insulation. I think without any doubt insulations have proven insufficient due to the time action of heating and cooling.

There are a number of points in these two papers that I would like to discuss, but I will not take up the time to speak of many of them, but along the line of usage there is one point in Mr. Dubsky's paper *viz*: where he refers to the turn insulation. He lays down a rule which perhaps is a good guide and may be a good one to follow, but I should limit its application to certain branches of work. He makes the recommendation that the turn insulation be $1/n$ of the external, that is, in the case of a two-turn coil the insulation should be one-half of the external, in the case of three turns, one third, and in the case of twenty turns, one twentieth. Now, there are other factors than the number of turns, as those of us who are practical designers and builders of machinery know, such as the voltage between turns, which, in the case of a machine of any given capacity, say a 30,000 kilowatt turbo-generator is just as high in the case of a 2300-volt machine as in the case of a 13,000-volt machine, and in that case the rule that Mr. Dubsky suggests would prove inadequate, whereas in the case of a much smaller size machine, say 1000 kilowatts, if we had fifteen or twenty turns of small conductor, and an external insulation very thick, the turn insulation prescribed would probably be altogether too much; it would be too costly, and result in a machine of poorer characteristics in other respects.

Henry W. Fisher: I was very much interested in what Dr. Whitehead had to say because it is difficult for me to conceive how there could be air films of the magnitude described by the authors.

In the determination of costs, we have often measured the thickness of paper as we were applying a definite number of wraps and our data show that it would be almost inconceivable to see how air films having thickness of two or three mils could exist.

In applying high-voltage stress to cables insulated with certain compound, we have found that after the voltage reaches a certain point the resistivity which has previously been gradually decreasing may commence to rise again. This action corresponds to polarization with direct current, but why it should occur with alternating current is difficult to conceive.

It seems rather remarkable that the voltage stress at the point of ionization given by the authors should in no case be very far from the average result of 22 kilovolts per centimeter. Calculations which we made in connection with tests conducted some time ago indicate about the same value as above.

It is very important to use the right kind of compound to saturate paper-insulated cables. If the compound is thin, it may be quite easy to fairly fill the insulation, but if the cable happens to be in a slanting position when in operation, the compound may run out through the voids caused by stretching the lead when the cable was installed, thus leaving vacant spots where ionization may take place with disastrous results.

With fluid compound the draining effect may be such as to cause the disintegration or carbonization of the filler in the middle of the cable, or in other places as is pointed out by Mr. Shanklin in a former paper.

It is important to have cables with low dielectric losses but I would a great deal sooner have the cables filled with a compound of proper physical properties even if by doing so the dielectric loss would be somewhat increased.

R. E. Argersinger: We have been having serious trouble from corona on certain generator coils, resulting in breakdowns and service interruptions. In one case of two generators operating side by side, one was insulated entirely with mica and the other with a composite insulation. Both showed very marked corona under normal operating conditions, the mica, as one might expect, showing more than the other, but, as far as the damage went, the mica-insulated machine suffered very little.

On a number of occasions I have heard the statement made that mica was never attacked by corona, and I was therefore somewhat surprised when one of the speakers this afternoon said he had seen mica insulation badly pitted.

I took one of these mica coils where the slot wrapper had been practically destroyed by the action of the corona, and cut out a section of insulation. From this I sliced off layers from the back side so as to get down as close to the slot wrapper as possible to see how deeply the corona had penetrated. To the best of my ability I could only find that the very outside layer of mica had been affected. That had practically gone with the slot wrapper, but the action apparently had not penetrated into the real body of the mica insulation. The varnished cambric, of course, was severely attacked.

C. Francis Harding: Apropos of the suggestion of Dr. Whitehead and Prof. Adams it may be of interest to learn of some of the information obtained in the midst of a research investigation in which the speaker has been interested. This investigation, which we have been carrying on at Purdue University for about two years, is on the fixation of atmospheric nitrogen by the so-called silent discharge method, a method which was reported by Dr. Hugo Spiel, of Germany, preceding the war. The apparatus used is in principle like Dr. Whitehead's corona voltmeter, with the exception that it was so designed that the volume of air, temperature, pressure and the moisture could be controlled and all variables held constant in each instance. This investigation has led to some interesting results.

The work has not been completed, but there are a few things which may be reported, which may be of assistance to others carrying on research investigations along parallel lines.

It is very evident that in studying the yield of the nitrous-oxide gas, or the nitric acid when this gas has been dissolved in water, that the yield varies greatly with the volume of air supplied, the temperature, the pressure and the moisture. That is, if you hold all the variables constant, except one, and vary that, you get a peculiarly peaked curve not unlike the one which Mr. Osborne showed in his telephone paper; so that one critical velocity of the air, the other variables being held constant; one critical temperature, or very small range of temperature, and a very small range of pressure, determine to a very marked extent the maximum yield of the nitrous-oxide and therefore the nitric acid.

Of course, we have worked with very large volumes, but it is reasonable to suppose that in the small volumes of air included in the insulation similar laws would hold.

In attempting to find suitable material for the conduction of these gases to the absorption chambers, and for the absorption chambers themselves, tests on materials which were subjected to boiling nitric acid before they were used, seemed to indicate that certain materials were not readily attacked, while these same materials in this fixation apparatus were very quickly attacked. That led us to believe that the combination of ozone with the nitrous-oxide was a very much more serious attacking agent than the nitrous-oxide or the nitric acid alone. For instance, in making connections between glass or aluminum tubes, using the different insulating materials such as rubber, varnished cambric, etc., many of these, when subjected to the combination of ozone and nitrous fumes, would disintegrate in a very few minutes time, whereas many of them in boiling nitric acid would not be readily attacked.

We noted also that these critical conditions, combinations of temperature and pressure and volume of air confined in the apparatus, would change very suddenly the nature of the gases coming through the discharge chamber. One instant the brown fumes of the higher oxides of nitrogen, would appear, and the next moment one would get a white fog in their place, with apparently no change in the variables themselves. Just what those white fumes are, we do not know, but they are at least one of the indications of a particular combination of variables in the nitrogen apparatus.

It is hoped that during the next year we may be able to show some very definite results from this series of experiments. They have to do, not with the arc process at all, but with the silent corona discharge between conductors.

Ralph W. Atkinson: I am going to confine my remarks particularly to the paper of Messrs. Shanklin and Matson, and more especially to some differences we have found in our results as compared with theirs. In a great measure, our data are quite parallel to theirs.

In the first place, however, I wish to point to some things which I believe are errors in the paper. In Table IV, in column 4, the value given as 1.624 for several samples should be 1.43, with the corresponding changes in the last column. That actually will emphasize the conclusions that are already drawn. These are cables that were said to show ionization at an unusually low voltage gradient. With the above suggested changes, if the figures I have given are correct, the conclusion will be still more apparent; that is the samples already shown to be bad are even worse than indicated. In Table III near the middle of the fourth column, the value 1.396 should be 1.29, which will change the figures in the last column similarly, and which will bring the data on this cable more in line with those on the others in the same table.

The authors show an ionization curve, stating that it is typical. Reference to the paper published in 1917 by Mr. Shanklin in collaboration with Mr. W. S. Clark shows that the large number of ionization curves there given are strikingly different from this typical one. Part of the difference undoubtedly is due to the fact that fewer points were taken on the old curves and thus sharp turns might be shown as rounded.

However, in some cases, the actual points obtained are inconsistent with a curve of the nature now given as typical. Also though I have tested many specimens showing curves similar to this "typical curve," I have tested a large number of specimens showing curves of quite different characteristics.

Three-phase measurements cannot be made with the same facility or accuracy as can single-phase measurements, but important additional information is given by such tests. On account of the convenience single-phase will continue to constitute an important factor in the tests of three-conductor cable, but conclusions seriously in error may be drawn if the three-phase tests are neglected. For example, we have found that when three-phase voltage is applied to a three-conductor cable, in some instances the beginning of ionization may occur at a materially lower gradient than when single-phase voltage is applied between the three conductors and sheath.

I have had occasion to measure at different parts of the cross-section the voltage gradient in a three-conductor cable. The results are very interesting and instructive, and I hope to extend these tests and to make them available for presentation before the Institute: they have a particular application in the conclusions drawn by the authors from data of single-phase tests. Consider the sample No. 7. A cable of this make-up might normally be used on a commercial circuit of 24 to 26 kv. In the paper given in 1917 by Mr. W. S. Clark and Mr. Shanklin, such a cable was mentioned as having operated successfully at 24 kv. for five years. Now it can be shown that the maximum gradient in such a cable operating on a 24 kv. three-phase circuit, is much in excess of 22 kv. per cm. Therefore, accepting the above and the data supplied by the authors and unless the recently tested cables show ionization at a lower stress than the ones described in the older paper, cables have operated successfully at a gradient much in excess of the ionization point.

The data given by the authors are very interesting and instructive; the theory and conclusions drawn must be given weight and consideration and doubtless afford a partial explanation of the facts presented, though too inadequate fully to serve that purpose. In fact, I consider that the data presented when fully analyzed, as well as tests which I have conducted, show distinctly that the method given for determining the voltage rating of a cable does not give results which entirely accord with operating experience.

Delafield DuBois: What I have to say rather supplements the criticism of Mr. Whitehead. I wish to show why it is that the statement by Messrs. Shanklin and Matson on page 495 "Knowing the effective permittivity of the solid insulation and the voltage at which ionization begins, curves can be calculated showing the stress across gas spaces of an assumed thickness" is only true under very limited conditions.

What might be termed the Ohm's law of the dielectric circuit is that

$$D = k\, g$$

or the flux density is equal to the permittivity times the gradient.

If we have solid insulation intercepted by a layer of air of uniform thickness, of large area as compared with its thickness and running normal to the general direction of the flux, the flux lines, at least near the center of this air space, will be unchanged in direction when passing from the solid insulation and through the air, and the flux density in the air will be the same as in the solid insulation. In this case we may write $k_1\, g_1$ for the air $= k_2\, g_2$ for the solid and as $k_1 = 1 \quad g_1 = k_2\, g_2$ which is the formula upon which both these papers are based. But this is only true under very ideal conditions. It is not true if the film of air is not normal to the general flux direction, as in this case the flux density in the air will be lower than in the solid material. It is not true if the air film is not of uniform thickness. If for instance the surfaces of the solid insulation are rough the flux in crossing the gap will concentrate between opposing projections, giving gradients in the air much greater than indicated by the formula. Finally it is not true if the air spaces are small in area. In this case so much of the flux is deviated around the edges of the air pocket that even in the center the density is reduced. As an extreme case the gradient, instead of being equal to $g_2\, k_2$ would approach g_2, the gradient in the solid dielectric.

In a cable there may be air spaces that comply with the conditions under which calculations as to thickness are possible, but there are surely also many that do not and these would give false results if measured by this method.

L. W. Chubb: I would like to emphasize what Prof. Harding just said with regard to the critical conditions necessary for the formation of corrosive gases. I recall the accidental formation of ozone, in a test, in such volume that it made several men sick. The structure of the corona member, the conditions of the circuit, etc. were noted. Later, we tried to duplicate the conditions but could get only a very small yield of ozone. We do not know just why but under similar conditions of operation and construction these gases will be formed and corrosive action take place in one case, while in other cases of the same kind no trouble will be experienced.

Prof. Harding also expressed the view that it was combination with the ozone that made the gases active. It is my opinion that it is not the presence of the ozone as much as it is the ionized or nascent condition of the gas formed. Platinum, for instance, will not be attacked by hydrochloric acid gas or chlorine. Platinum as corona electrode, however, in these gases will react and form chloride of platinum.

F. W. Peek, Jr.: Fortunately the universal medium in which we are immersed, air, is an insulation and not a conductor. Unfortunately, however, it has not the highest dielectric strength and has low permittivity. Experience has shown that it is almost impossible to rid certain insulations of air films and bubbles in one form or another. On account of its low permittivity, the unit stress, or gradient, on the air films is usually several times what it is on the solid insulation. The strength of air is constant at a given density for large spacings. For small spacings, however, the unit strength increases very rapidly with decreasing spacings. With small spacings the strength may be as great as that for solid insulations. It is still, however, under the disadvantage of taking more than its share of the stress due to its low permittivity.

The breakdown of the air films does not mean that the apparatus will break down immediately; it may take years. it is undesirable because it causes gradual deterioration of the solid insulation. This deterioration may be due to carbonization, mechanical action or chemical action, depending upon the nature of the insulation. Soft rubber, for instance, is almost immediately ruined by the ozone. It appears almost as if it had been cut. Varnished cambric attacked for several years by corona has a honey-combed appearance of mechanical action. The strength of the air films is thus generally the limiting feature. The stresses must be such that the films do not break down.

Mr. Dubsky measured the strength of air films of different thicknesses between solid insulation. These data are of great scientific interest because he has found that the strength of these films surrounded by insulation is the same as it is between metal electrodes. These data are practically applied by estimating the thickness of the largest films in a given type of insulation and so proportioning it that the stress on the air is below the measured breakdown value.

Messrs. Shanklin and Matson have investigated the problem in a different way. They have measured the losses in the insulation. They find that the losses and the alternating-current resistances increase rapidly at a certain point, apparently when the air films begin to break down. These are important data and the papers fit in well together.

I cannot be pessimistic about insulations. High-voltage cables have not been built principally because of lack of demand. There are other limiting features beside voltage. For instance, the leading wattless kv-a. of a high-voltage cable network would be enormous. Transformer bushings operating at several hundred thousand volts are really short high-voltage cables. Transformers have been built up to almost a million volts.

Comfort A. Adams: First, I want to say that I fear Mr. Peek has misinterpreted my statement, which I intended to be something like this—that in the case of high-voltage armature insulation the actual thickness of mica is such that if it were an absolutely solid, homogeneous air-free mass of mica insulation it would stand up indefinitely under voltages several times, those normally impressed.

Now, just a word in regard to the deterioration of insulation under corona attack. It is very difficult to say how much this is due to chemical action, how much to some kind of corona bombardment and how much to the heat resulting from corona. I wish now to express the opinion that the deterioration in mica at least, is due more to corona heat and corona bombardment, than to chemical action, although the latter doubtless plays some part in certain types of insulating materials.

This opinion is based upon both theoretical analysis and experimental evidence.

Speaking first from the theoretical point of view, there are two factors not at first sight obvious which tend to increase the heat produced by internal corona.

a. High frequency of corona discharge. Consider a thin flat air space perpendicular to the electric flux in an otherwise homogeneous wall of insulation. When the voltage gradient in the air space exceeds the critical point, corona is established, and the heat developed is approximately proportional to the current, other things being equal. But if the air space is very thin as compared with the thickness of the insulation wall, the corona current is the displacement current through this wall and is therefor proportional to the voltage gradient in the wall, the dielectric constant of the insulating material and the frequency. However the frequency is not that of the circuit fundamental but that of the discharge itself, which is of the order of hundreds of thousands of cycles per second, as both the inductance and the capacity of the discharge circuit are very low. Thus the corona displacement current and the heat developed is thousands of times greater than if at fundamental frequency.

Also the resulting high-local temperature lowers the initial voltage at which corona begins and causes the corona to last through a larger part of each half cycle. This accounts for the unexpectedly large losses disclosed by the experiments of Shanklin and Matson.

b. Corona Blast. In an air space of any appreciable thickness and reasonable length, the corona current is deflected by the magnetic field at right angles thereto, and is driven longitudinally in the air space against the end, impinging against any exposed edges of the very thin mica films actually fusing them in some cases. These films are peeled up and fused one at a time, gradually increasing the length and thickness of the space and the violence of the action.

Ample evidence backing up these conclusions has been discovered in the insulation of machines which have been in service less than two years.

Chemical action does certainly take place but as this depends upon the presence of air and moisture the access of either to the interior of impregnated masses of insulation must be so slow as to render it practically harmless. Moreover there is as yet no experimental evidence to indicate that this corona-chemical action affects the mica appreciably even after years of service, although it does affect impregnating compounds.

Finally the problem is very complicated and very important from the practical point of view. There is hardly any field of electrical engineering research where the results are so likely to extend the limits or boundaries of practise.

E. E. F. Creighton: Some time ago in making a study of insulation we arrived at the same conclusion as the authors in regard to air and its disagreeable effects. In making the investigation we found it impossible, in ordinary carbonaceous materials, to get rid of the effect of heat, and its deteriorating influences on carbonaceous material, and we were led to make tests on porcelain in which we could eliminate the factor of carbonization. One can get porcelain of all degrees of porosity by different degrees of firing. With porcelain there can be no carbonization to limit and change the effect of the bombardment of the air in the pockets.

Now, by choosing the different kinds of porcelain, the dry-process being one type, with very long spaces, and the wet-process procelain, with longer or shorter air spaces, according to the degree of firing, it is possible to study the air pockets more thoroughly than can be done with carbonaceous insulation.

Unfortunately, the exigencies of war work required us to discontinue our tests, and it has been impossible to put them into shape for publication. We used an Alexanderson alternator with a frequency of about 200,000 cycles. This high frequency of continuous alternating current heats the porcelain rapidly. There was a marked difference in heating that took place due to the difference in degrees of porosity.

J. J. Matson: Relative to Mr. Chubb's statement on the pitting of mica of which Mr. Argersinger also spoke, I will say, that we also in extreme cases found pitting; but as far as our cases go, and we have not had many, we have not found very much pitting. It has been more often, as President Adams said, a corona blast and fusing. We have found the same thing, the creepage of the corona, but we did not touch on that in the paper, due to the fact that it does not affect the beginning and it was the starting of the corona that we were interested in.

Prof. Harding spoke of the color of the material coming from the insulation. We have found different colors in the corona trouble. That is due to the different complex inorganic substances which you have in the coil or in any insulation. The nitric acid will affect the various complex substances in various ways and give different shades or hues, but we have usually found that holds for a given compound in the same spectrum, that is, it will either fade or brown with age.

W. I. Middleton and E. W. Davis: (communicated after adjournment.) Considerable work has been done both in this country and abroad on the effect of potential stress on insulating materials and all experimenters are convinced that it is possible to permanently injure them by overstressing. Just how much stress we can put upon an insulating material without injury is a matter of much discussion. It is possible to stress a material to a point of apparent hurt but if the stress is removed the material will recover its normal condition.

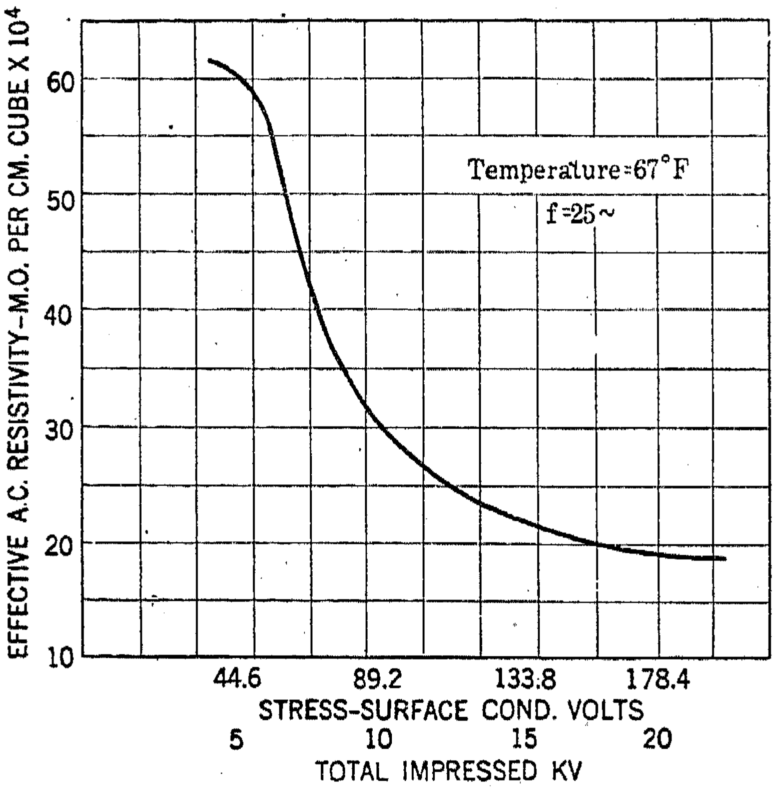


FIG. 1—A-C. RESISTIVITY VS. VOLTS

6 str. B. & S.—7/32 in. wall paper insulated—lead covered.

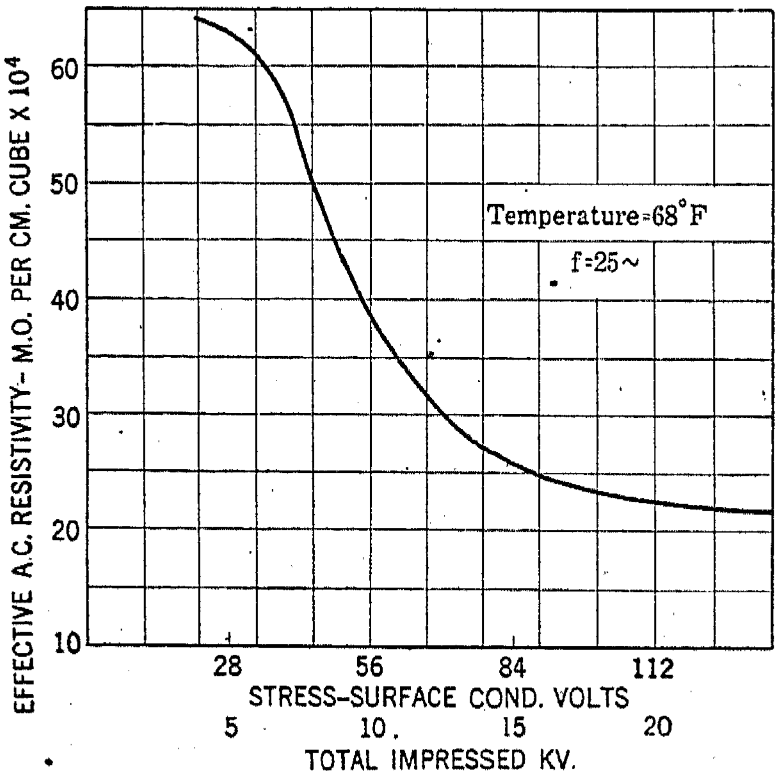


FIG. 2—A-C. RESISTIVITY VS. VOLTS

2/0 str. B. & S.—9/32 in. wall paper insulated—lead covered

In many cases it is difficult to determine what has taken place as the physical properties of the material show no apparent change and it is only by electrical tests that we can detect the effect of stress. In many cases where heat has been generated the effect is plainly visible.

The authors state that ionization takes place at about 57 volts per mil in a well made cable. We are led to believe we should keep the working stress below this value. Good practise tells us this is not necessary as many paper cables are operating successfully at twice this stress. In fact, if we are to advance in the manufacture of high-tension cables, we must go well beyond twice this value.

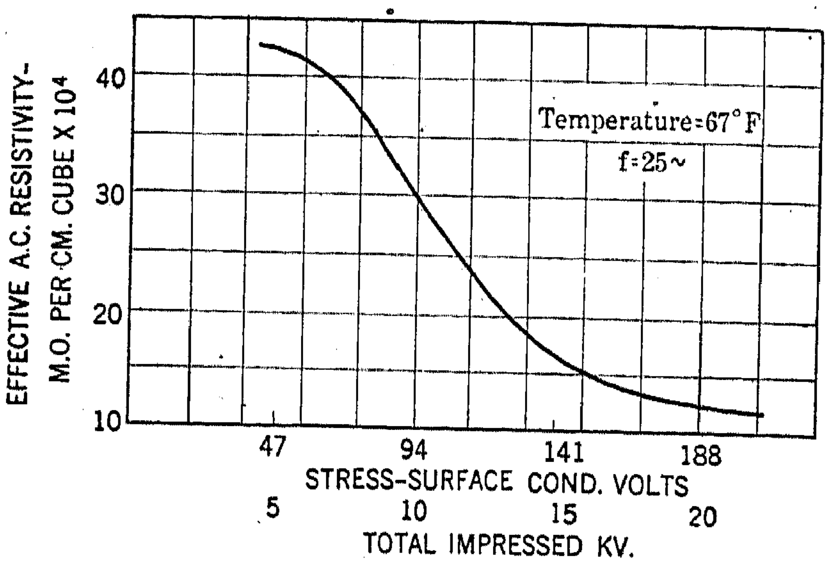


FIG. 3—A-C. RESISTIVITY VS. VOLTS

6 sol. B. & S.—7/32 in. wall rubber insulated—lead covered.

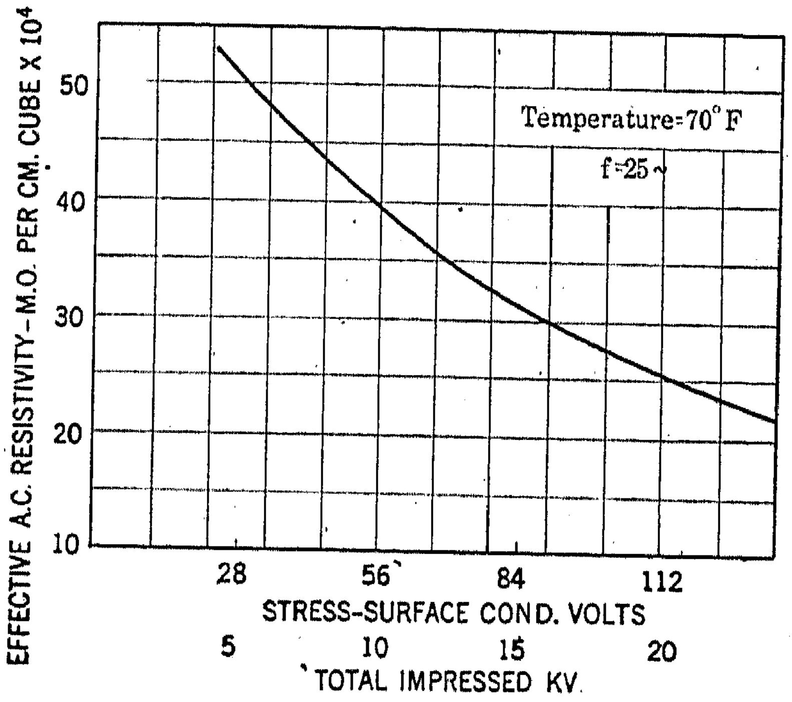


FIG. 4—A-C. RESISTIVITY VS. VOLTS

2/0 str. B. & S.—9/32 in. wall cambric insulated—lead covered.

When we begin to lay down theoretical laws for allowable stresses on cables we obtain results that practise condemns. We must therefore, be careful how far we allow ourselves to be influenced by such laws. For example—there has been in operation for a number of years a 3-conductor No. 2 stranded cable operating at 25,000 volts with a 9/32 in. wall of paper on each conductor and a 4/32 in. jacket. The stress at the surface of the conductor is 108 volts per mil. With these same walls of insulation to keep the stress down to 57 volts per mill, the conductors would have to be 2,000,000 cir. mils, or a 3-conductor No. 2 stranded operating at 25,000 volts would require 46/32 in. or nearly 1 ½ in. of insulating material on each conductor.

We must not let theory carry us beyond the limits of good common sense or away from what we know to be good commercial practise.

The following curves show characteristics inconsistent with the theory of ionization. In Figs. 1 and 2 for paper cables the knee of the curves occurs at the same value of total impressed voltage but widely different values of voltage stress at the surface of the conductor. Fig. 3 for a rubber cable shows a knee at practically the same total impressed voltage as the paper cable. Rubber insulation applied to a solid wire under great mechanical pressure has no occluded air, therefore, this knee could not be due to ionization of occluded gases.

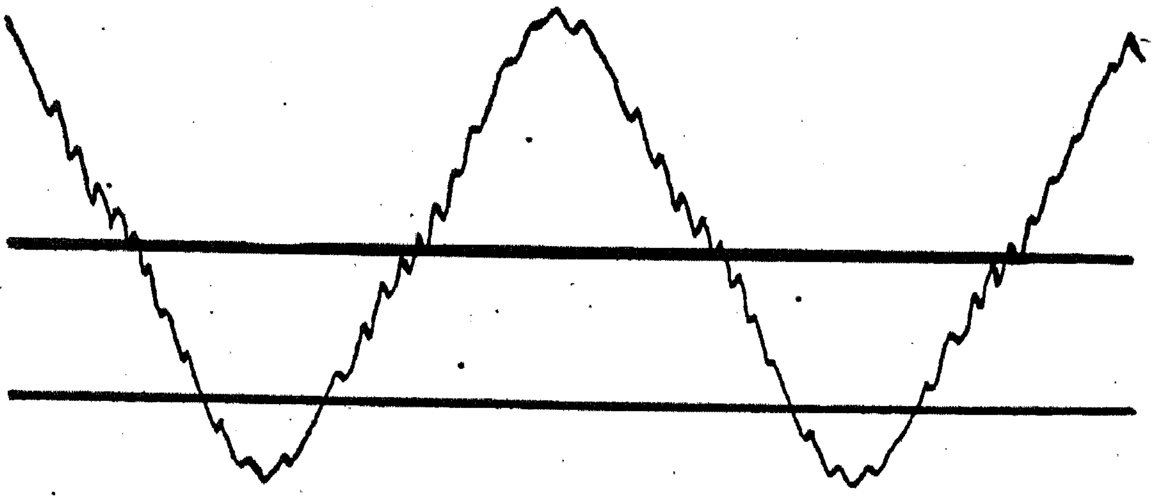


Fig. 5—Voltage Wave—High-Tension Transformer Form Factor = 1.53 $f = 25 \sim$

Fig. 4 shows curve for cambric insulation in which there is no knee. Due to the method of laying up this type of insulation, a cambric cable should have more air entrapped than a well made paper cable.

Fig. 5 shows the voltage wave used for getting the data for Figs. 1 to 4.

F. Dubsky: Regarding Mr. Chubb's comments on the effect of formation of conducting paths caused by chemical action and carbonization due to corona, I wish to point out that when this effect takes place in air bubbles at the flat sides of coils, between the inside wall of the coil insulation and the conductors, it is liable to weaken there the layer and turn insulation.

With regard to Mr. Whitehead's comments I wish to state the following:

The most concerned air bubbles are of comparatively large area, that is of such a shape so as not to distort; practically, the straight line electric field. The thickness of such air bubbles varies, of course, from zero to some maximum thickness and the gradient on air at any point of the air bubble may be determined from the equation

$$g_1 = K \frac{E}{X + X_a K} \qquad \textbf{(12a)}$$

where g_1, K, E and X have the same signification as in equation (**12**) page 515, and X_a is the thickness of the air space at any point of the air bubble.

A lateral dielectric stress can occur only in the narrow air pockets, as in the thread-like spiral air spaces at the supposed butt joint edges of wrapping. In this case the stress lines are partly shunted around through the solid insulation of higher permittivity and thus the dielectric stress in the narrow air space is reduced.

The dielectric strength and the dielectric stress are expressed in this paper in volts per mil rather than in kilovolts per centimeter in order to be more convenient in application for the designing engineer, who is accustomed to express the dielectric strength of insulating materials in volts per mil.

With regard to Mr. Foster's comments on internal coil insulation I would like to emphasize my opinion, that the layer and turn insulation of coils embedded in slots should be determined rather in relation to the insulation to ground than with regard to the normal induced voltage existing between layers and turns. If this is done an internal coil insulation is obtained which will safely meet occasional high-voltage stresses due to surges and lightning.

Presented at the 7th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 21, 1919.

Copyright 1919. By A. I. E. E.

ABNORMAL VOLTAGES WITHIN TRANSFORMERS

BY L. F. BLUME AND A. BOYAJIAN

Abstract of Paper

Mathematical analysis is made of a rectangular wave impinging upon a transformer winding and quantitative values deduced of the resulting internal voltage stresses in terms of transformer constants. It is shown that the conclusions also apply in part to abrupt impulses and approximate idea is given of the reaction of a transformer to high frequencies. The difference between operating transformer with isolated and grounded neutral is shown. Energy losses are not considered in the mathematics although the manner in which the results are affected is pointed out. Finally, theoretical results are compared with impulse and high-frequency tests made in the laboratory.

I.—Introduction

To avoid breakdown of transformers resulting from abnormal voltages, the present practise is to either safeguard the transformer winding by external protective devices which are designed to prevent line disturbances from reaching the transformer, or to insulate the windings to withstand them. A third but less obvious method is so to design the transformer that voltage disturbances entering the winding will not result in excessive voltage concentration.

By showing in what way the transformer constants influence the voltage concentration this paper indicates how a transformer may be so designed as to mitigate the stresses occasioned by a given voltage disturbance.

II.—Conclusions

Theory and experiment both of which are fully described later in the paper confirm the following conclusions.

1. Transformers differ from transmission lines on account of the presence of coil-to-coil capacitance and mutual inductance.

2. Three kinds of disturbances are recognized:

   A—abrupt impulses: B—rectangular waves with long tails: C—high-frequency voltages sustained and damped.

A. ABRUPT IMPULSES

3. The presence of coil capacitance causes the transformer to respond as a capacitance and not as an inductance to abrupt impulses and towards all frequencies* above its lower natural frequencies of oscillations.

4. The voltage distribution within the transformer windings for abrupt impulses and for high frequencies depends upon the factor,

$$\alpha = \sqrt{\frac{C_g}{C_s}}$$

where C_g is the total capacitance of winding to ground and C_s is the total coil capacitance from one end of the coil stack to the other.

5. The greater the value of alpha, the greater the concentration of voltage at the line end and the smaller the voltage in the interior of the winding. The maximum volts per unit length of coil stack at the line end is equal to alpha times the value of the voltage corresponding to uniform distribution. In high-voltage power transformers of disk high-voltage, and concentric-barrel low-voltage windings, the value of alpha varies from 5 to 30.

6. The more abrupt the impressed impulses, or the higher the impressed frequency, the greater is the concentration of voltage at the line ends. The maximum voltage expressed in the preceding paragraph corresponds to a perpendicular wave front or infinite frequency impressed.

7. If the turns are uniformly distributed throughout the windings, alpha is also the factor giving the concentration of the volts per turn at the line end; but on account of extra turn insulation, the usual practise is to considerably reduce the number of turns on the end coils.

As the voltage concentration in the end coils depends upon the value of series and ground capacitances and is independent of the number of turns, it follows that the volts per turn on the end turns is inversely as the number of turns per coil. For this reason it is evident that extra turn insulation will be effective only when it is so designed that the increase in dielectric strength is greater than the increase in voltage concentration.

8. For abrupt impulses and for very high-frequency

---

*See paragraph 15.

oscillations it is immaterial whether the neutral is isolated or grounded.

B. Rectangular Waves with Long Tails

9. If the impulse has a long tail, the phenomena within the transformer will be initially as described in paragraphs 1 to 5 above, followed by a series of damped oscillations, the amplitudes of these oscillations depending not only on the amplitude of the impressed wave but also on the value of alpha, and on whether the neutral is isolated or grounded.

10. In addition to being proportional to the value of the impressed wave the amplitude of oscillation for a given harmonic is smaller the higher the harmonic and the smaller the value of alpha.

11. There is no simple relation between wave length and frequency and on that account waves cannot penetrate into a transformer winding without distortion.

12. As these oscillations occur simultaneously within the transformer windings, the resultant voltage distribution at any instant is obtained by re-combining the harmonics at that instant.

13. For isolated neutral and for alpha equal to 10, a maximum of 2.8 times impressed volts exists from neutral to ground and the volts per turn in the interior of the windings is approximately four times normal.

14. For grounded neutral and for alpha equal to 10, a maximum of 1.5 times impressed volts exists from the windings to ground, and the volts per turn in the interior of the windings are increased to four times normal and at the neutral to 7.5 times normal.

C. High Frequency

15. When the applied high frequency is greater than the natural frequency of the transformer, the reaction of the transformer is similar to its reaction towards abrupt impulses, and paragraphs 3 to 8 apply.

16. Theoretically, by impressing a sustained high-frequency voltage across transformer terminals at a natural oscillating frequency of the windings, the transformer can be made to resonate and indefinite voltages built up limited only by internal losses. However, if the transformer is excited by a transmission line, the voltage built up by resonance is also

limited by the relative values of surge impedance of transformer and line.

17. However in the high frequency tests here reported extreme high voltages were not observed. This is probably due to the following reasons:

(a) The impressed high-frequency was highly damped.
(b) The presence of internal capacitances shunting the windings decreases the rapidity with which the voltage is accumulated.
(c) Internal losses within the transformer windings.

18. The effects of (b) and (c) are greater the higher the frequency of oscillation. For this reason high internal voltages are built up, only at the lower frequencies.

III.—Description of Phenomena

The theory of electrical waves in transmission lines cannot be directly applied to transformers due to the fact that a transformer, unlike a transmission line, possesses capacitance between coils and mutual inductance.

Assuming that the inductance and capacitances are uniformally distributed along the winding as diagrammatically represented in Fig. 1, mathematical analysis can readily be applied, but if the constants are bunched as is the case for transformers in which the high and low voltage windings are interlaced, the problem becomes very complicated.

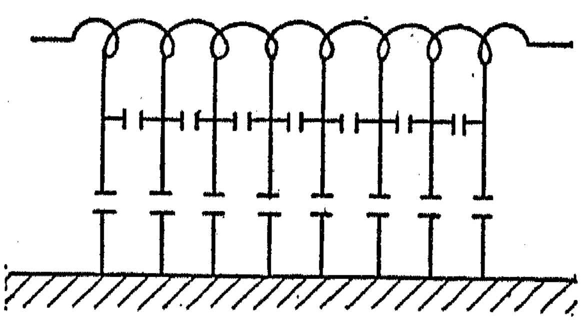


FIG. 1

A good example of a concentric winding transformer to which the conclusions of this report are applicable is given in Fig. 4. This is a three-phase 2500-kv-a. transformer, the 100,000-volt winding of which is Y connected and made up of three stacks of disk coils. It is necessary to assume that the capacitances between turns and layers are very large compared with capacitances between coils and ground to make Fig. 1 applicable.

The purpose of this paper is to show what takes place within the transformer windings under the impact of rectangular waves. Rectangular waves are chosen rather than periodic waves on account of the greater simplicity of the problem, and

---

*See *Abnormal Voltages on Transformers*, by J. M. Weed, A. I. E. E. TRANS. 1915, p. ~~1621~~. 2197 waves of long duration (long compared with the time of a complete oscillation within the transformer) are assumed because the resulting voltages are greater than with the very short waves or impulses.

In general there will be two cases requiring separate treatment depending upon whether the neutral is isolated or grounded.

Although the phenomena with which we are dealing involves in its entirety only a very small fraction of a second, it is nevertheless convenient to divide it into three distinct time periods, each requiring separate treatment. These will be designated; 1, initial period; 2, transient period; 3, final period.*

The initial period is the period from the beginning of the disturbance to the time of maximum applied voltage. The time involved is so small that there is no appreciable growth of magnetic flux and therefore the phenomena may be considered entirely electrostatic without appreciable error. This is followed by a transient period during which there is a continual interchange of energy between the electric and magnetic fields. The energy interchange usually takes the form of highly damped high-frequency voltage and current oscillations. When the oscillations become negligible the final period has been reached in which the winding has again assumed a permanent condition of equilibrium.

TRANSFORMER REACTING AS A CONDENSER

Due to the combination of coil and ground capacitances, the transformer, during the initial period, reacts as a concentrated capacitance. The condensers are rapidly charged and the voltage across them rises to nearly double the voltage of the incoming wave. The time required for charging is so very small that it is safe to assume that twice the voltage of the incoming wave is applied to the terminals of the transformer windings before an appreciable inductive current can be established in the windings. The equivalent or effective capacitance of the transformer, while being charged, is approximately given by equation (**21**),

$$C_{eff} = \sqrt{C_g \times C_s} \qquad \textbf{(21)}$$

where C_g is the total capacitance of the surface of transformer

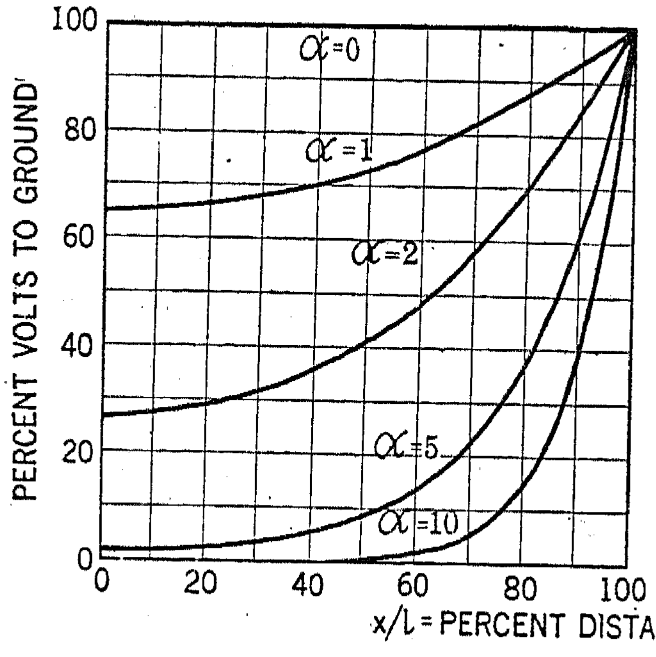
---

*See paper by Karl Willy Wagner entitled "Progress of Waves in Windings." *Elektrotechnik und Maschinenbau.*" Feb. 21, and 28th, 1915.

winding to ground, C_s is the capacitance between the two ends of the winding. The calculation of the latter capacitance is simplified by assuming that the ends of the winding are electrically connected to clamping plates or end rings. C_s is then defined as the capacitance between these plates.

INITIAL VOLTAGE DISTRIBUTION

The voltage distribution throughout the transformer winding when the condensers are charged is given by formula (**11b**) for isolated neutral, and formula (**13b**) for grounded neutral.



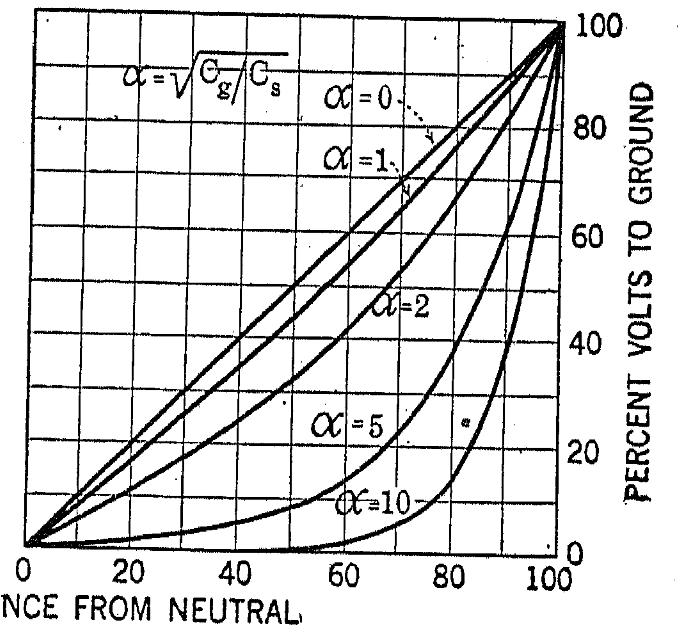


FIG. 2—NEUTRAL ISOLATED

FIG. 3—NEUTRAL GROUNDED

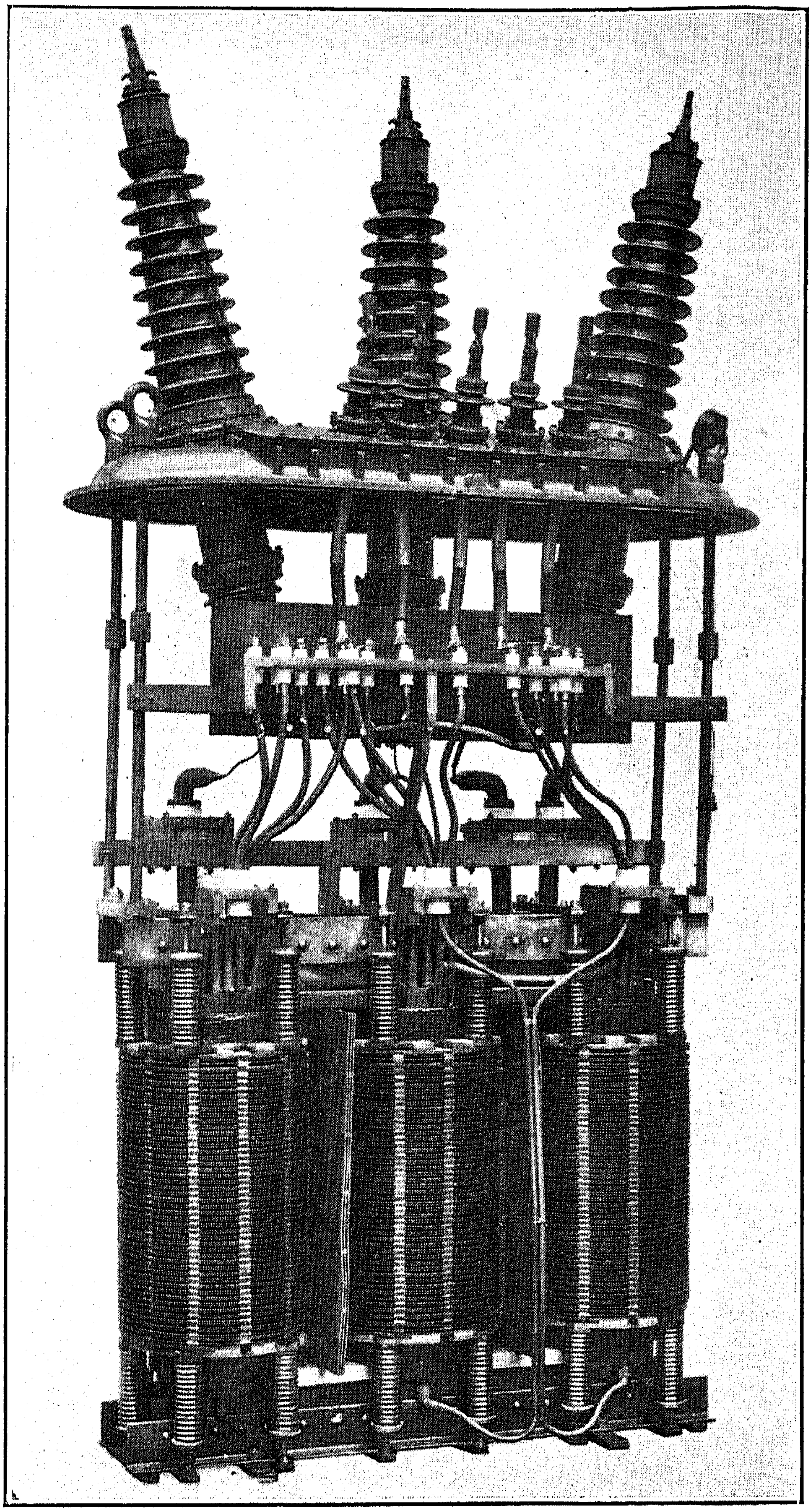
$$e = E_0 \frac{\cosh \frac{\alpha x}{l}}{\cosh \alpha} \qquad \textbf{(11b)}$$

$$e = E_0 \frac{\sinh \frac{\alpha x}{l}}{\sinh \alpha} \qquad \textbf{(13b)}$$

where E_0 is value of voltage to ground on the transformer terminals. This will be either twice the value of the incoming wave, or, if lightning arresters are assumed, the maximum voltage permitted by the protective device.

The value of alpha is the square root of the ratio of the ground capacitance (C_g) to coil capacitance (C_s).

Plotting the ratio e/E_0 as ordinate and the ratio x/l as abscissa gives the system of curves shown in Figs. 2 and 3. For alpha equals zero, that is, ground capacitance zero, the curve is a straight line (uniform voltage distribution). The



[BLUME AND BOYAJIAN]

FIG. 4—THREE-PHASE—60-CYCLE—2500 KV-A.—100,000-VOLT TRANSFORMER

greater the value of alpha the less uniform becomes the voltage distribution and the more is the voltage concentrated at the end of the winding. This is the reason for the modern practise of putting extra insulation on the coils and turns at the line ends.

It should be noted that these curves are based on windings in which the turns are uniformly distributed along the coil stack. On account of the space required by the extra end turn insulation it follows that there are fewer turns per unit length of coil stack at the ends of the windings. The effect of this is to cause a still greater concentration of voltage at the ends.

FINAL VALUE OF VOLTAGE DISTRIBUTION

(a) Eventually, in case of isolated neutral, the voltage of the winding will be brought to a potential of twice the original value of the wave with zero current flowing.

(b) For the case of grounded neutral the final condition will be a straight line with neutral at ground potential and line at twice the voltage of the original wave, with a current in the winding increasing at a constant rate, (assuming no resistance).

TRANSIENT VOLTAGE DISTRIBUTION

The transient by which the voltage distribution changes from the initial to the final value can be analyzed into a complex series of damped oscillations or standing waves at various frequencies, wave lengths and damping factors.

The maximum values of these standing waves are readily obtained by analyzing the initial voltage distribution as given in curves, Fig. 2, into space harmonics with respect to the final voltage distribution as axis of reference. Thus we have for $\alpha = 10$ the following initial values of the standing waves.

NEUTRAL ISOLATED		NEUTRAL GROUNDED.	
Wave length on winding n	Amplitude in per-cent of line to ground voltage E_n/E_0	Wave length on winding. n	Amplitude in per cent of line to ground voltage e/E_0
1/4	125%	1/2	58%
3/4	34%	3/2	23%
5/4	15%	5/2	6%
7/4	7%	7/2	4%
9/4	3%	9/2	3%

These oscillations at various frequencies occur simultaneously within the transformer windings and the complex wave at a given time is obtained by recombining the harmonics at that time (Fig. 5).

The equation of any standing wave in a transformer winding. is

$$e_x = E_n \operatorname{Sin}\left(2\pi n \frac{x}{l}\right)\cos w_n t$$

or (33d)

$$e_x = E_n \operatorname{Cos}\left(2\pi n \frac{x}{l}\right)\cos w_n t$$

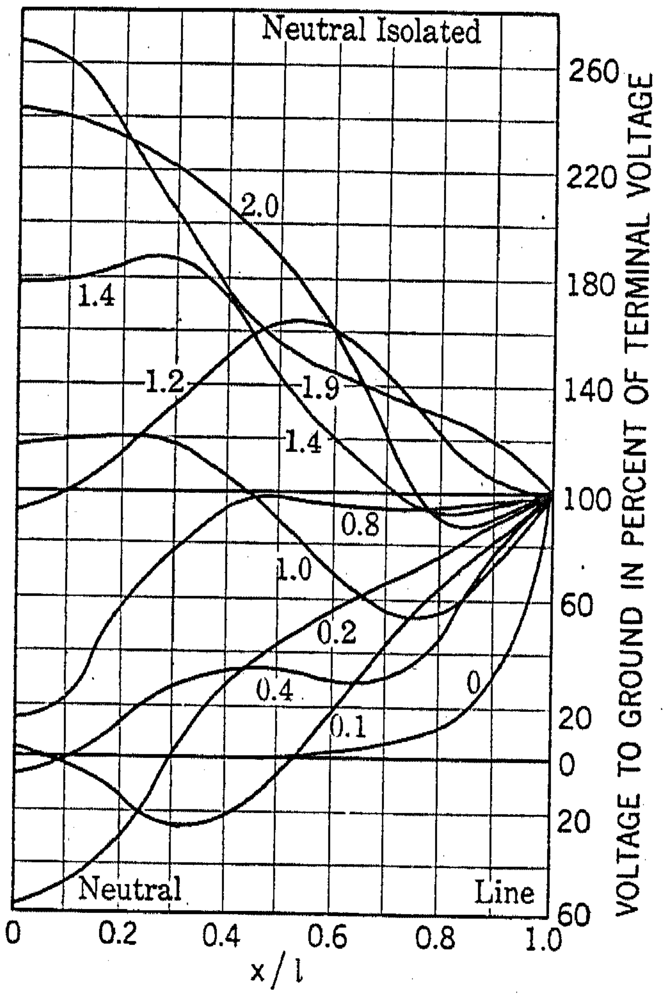


FIG. 5—VOLTAGE DISTRIBUTION WITHIN TRANSFORMER WINDINGS AT VARIOUS TIMES AFTER IMPACT OF RECTANGULAR WAVE

where n is the number of wave lengths per harmonic along the stack. For the fundamental, n is ½ when neutral is grounded, and ¼ when neutral is isolated. The frequency of oscillation is given by the equation:

$$f = \frac{2\pi n^2}{\sqrt{L\left(C_g + 4\pi^2 n^2 C_s\right)}} \tag{35e}$$

These equations are particular solutions of the general differential equations and are derived under the heading of "Mathematical Development."

An inspection of formula (**35e**) shows that the natural frequencies are a function of the wave lengths. For $\alpha = 10$ the variation of wave lengths with frequency is as follows:

NEUTRAL ISOLATED		NEUTRAL GROUNDED	
Wave length	Relative * frequencies	Wave length	Relative * frequencies
n		n	
1/4	1	1/2	4
3/4	8	2/2	14
5/4	20	3/2	28
7/4	33	4/2	42
9/3	47	5/2	56

*These figures are not accurate due to the variation of inductance with wave length as discussed on another page.

This peculiar relation between frequency and wave length is due to the presence of coil capacitance and mutual inductance, and causes a continual change in the form of the complex wave from instant to instant. This is in marked contrast to the phenomena which take place in the transmission line, where a complex wave of any shape will be propagated without distortion, except for the effect of resistance.

Maximum Voltage to Ground and Maximum Volts Per Turn

The considerations of greatest interest in connection with these curves are the maximum voltages to ground and the maximum volts per turn at various points within windings. These are plotted in Figs. 6 and 7. Of course the maximum values of voltages at the various points do not occur at the same instant.

An inspection of these curves shows that, for the case of isolated neutral, the maximum voltage to ground occurs at the neutral and is equal to 2.8 times the voltage from line to ground and for grounded neutral the maximum volts to ground is 1.5 times line to ground voltage. The volts per turn for isolated neutral is 10 times normal at line end, 4 times normal in the major portion of winding, and zero at the neutral. With neutral grounded it is 10 times normal at line end 5 times normal in the major portion of the winding and 7.5 times normal at the neutral.

Surge Impedance

In order to have a convenient means of calculating the maximum value of current associated with each oscillation the ratio of terminal volts to maximum current is given by

$$Z = \frac{E}{I} = \frac{1}{2\pi n}\sqrt{\frac{L}{C_g + 4\pi^2 n^2 C_s}} \qquad (39b)$$

The value of surge impedance calculated for the transformer shown in Fig. 4 is 60,000 for the fundamental, 29,000 for the third harmonic, 17,000 for the fifth harmonic and 12,000 for the seventh harmonic.

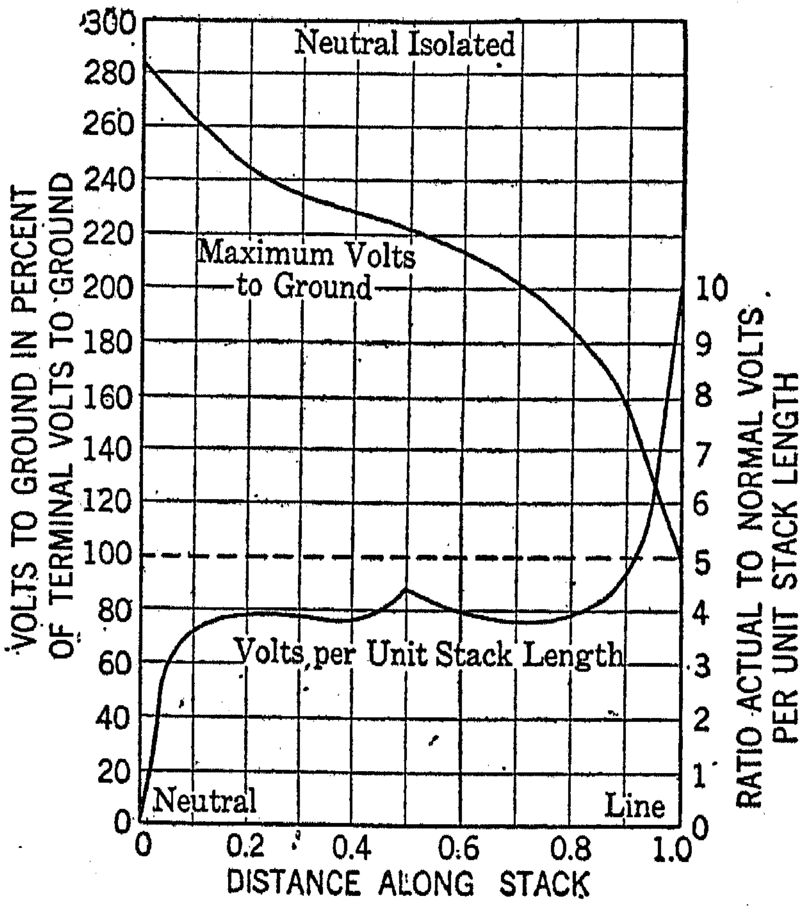


Fig. 6—Voltage Distribution within Transformer Windings Maximum Values after Impact of Rectangular Waves

In the analysis just given it has been tacitly assumed that the voltage E applied across the terminals of the transformer winding is maintained throughout the oscillations. Strictly speaking, this is not true, because during the time when the voltage to ground is increasing in the interior of the winding, the transmission line is delivering current to the transformer windings, and during the time when this voltage to ground is decreasing, the transmission line is receiving current from the transformer windings, or in other words, there is a continual interchange of energy between the transmission line and the winding throughout the oscillation, and as the ratio of voltage to current associated in the rectangular wave in the transmission line is a constant value, this interchange of energy cannot take place without modifying the line voltage. However, calculations of equivalent surge impedance show that in all practical transformers the amount of energy absorbed and delivered by the transformer windings is very small compared with the energy residing in the rectangular wave, and that, therefore, this fluctuation in voltage is negligible in amount.

Again it may at first appear that the axis of oscillation taken in the case of a grounded-neutral transformer, cannot be correct because the final condition for the case of grounded neutral is zero voltage throughout the windings owing to the fact that the windings become a dead short circuit. This is again theoretically true but it has no bearing on the problem because in all practical cases the time taken for the transformer windings to degenerate into a short circuit is so very long compared with the time involved in the oscillation of the fundamental frequency that it can be safely assumed that the transformer winding oscillates through a complete cycle of the fundamental before the voltage applied to the terminals of the transformer windings can be appreciably lowered.

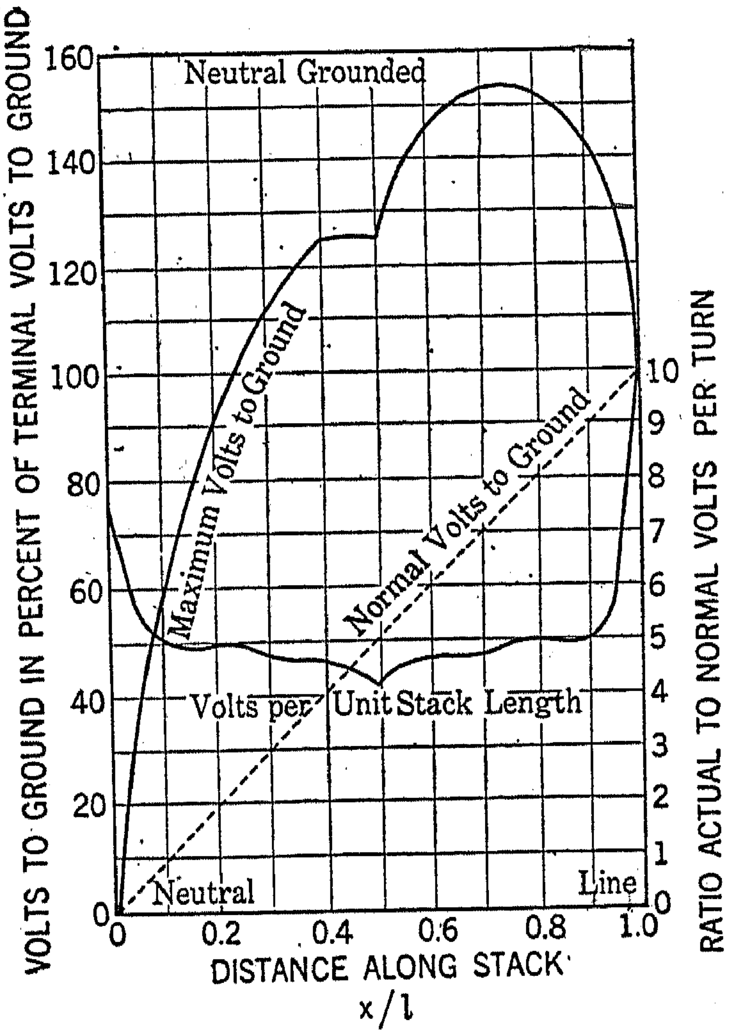


FIG. 7—VOLTAGE DISTRIBUTION WITHIN TRANSFORMER WINDINGS MAXIMUM VALUES AFTER IMPACT OF RECTANGULAR WAVE

NATURAL FREQUENCY OF OSCILLATION OF A WINDING

Formula (**35e**) is useful for the purpose of obtaining the natural frequency of a winding. For a solenoid consisting of a single one-layer coil, the capacitance between turns may be neglected and the formula becomes,

$$f = \frac{2\,\pi\,n^2}{\sqrt{L\,C_g}} \qquad \textbf{(35f)}$$

From this equation we reach the very interesting conclusion that the natural frequency of oscillations are not simple multiples of the fundamentals in the order of 1, 2, 3, 4, etc., but that they are proportional to the square of these values that is 1, 4, 9, 16, etc., (neglecting the variation of inductance with wave length.)

If the capacitance between turns or coils is not negligible the formula reverts to equation (**35e**). The influence of coil capacitance is to make the natural frequencies lower than those given by formula (**35f**). Moreover, the harmonics are not simple multiples of the fundamental in every case.

At very high frequencies the influence of ground capacitance becomes negligible compared with series capacitance and the formula for frequency becomes,

$$f = \frac{n}{\sqrt{L\,C_s}} \qquad \textbf{(35g)}$$

DAMPING FACTORS

Of course the voltage distribution as given in Fig. 5 departs considerably from what occurs in practise on account of dielectric losses, which introduce a damping factor, which is greater the higher the frequency.

It may be, therefore, especially where the order of frequencies progresses as the square of the wave lengths, that before the fundamental has completed one-half of a complete oscillation most of the higher frequencies have become negligible. A knowledge of the dielectric losses would be necessary to determine whether the voltage stresses are appreciably restricted by them.

HIGH-FREQUENCY TRANSIENTS

An idea of the transient phenomena occurring within a transformer winding when sustained high frequency is impressed can best be obtained by considering the applied high frequency to be made up of successive positive and negative rectangular waves, the frequency of which is the same as the fundamental frequency of the transformer winding. During the time of the first half cycle, phenomena are as described above. When the applied wave changes from positive to negative, the voltage to ground within the transformer winding due to the fundamental has reached its maximum value. The effect of this change in applied voltage is to increase the amplitude of the fundamental approximately by twice the amplitude of the applied voltage. In the example given the above fundamental has a value of 1.24 times the terminal applied voltage for the case of isolated neutral. At each cycle this value is increased by twice the value of the terminal voltage, causing the voltage to ground to increase regularly in steps so that the maximum potential to ground at the end of the second half cycle is 4.72, at the end of the third half cycle 7.20 etc.

The above assumes that the internal losses are insufficient to reduce its amplitude and neglects the amplitude of all other harmonics. Of course, if the applied frequency happens to be equal to the frequency of one of the harmonics the building up of voltage by resonance as described above would apply to that harmonic and not to the fundamental.

It should be noted that the amplitude of the oscillations within the transformer winding are not necessarily the amplitude of the impressed voltage, but they depend upon the amplitude of impressed wave voltage and also upon the values given in the table under "Transient Voltage Distribution." These values are dependent upon the value of "α"; the smaller the value of "α" the smaller will be the value of internal oscillations, and moreover, for a given value of "α" the amplitudes of the oscillations are smaller for the shorter wave lengths.

However, when a sustained high frequency, or a damped oscillation, having a frequency equal to a natural frequency of the transformer, is impressed, the resulting stress as previously shown may very greatly exceed those given by the curves, the values being limited by the value of "α", the internal losses, and the damping factor of the impressed wave. For a reliable quantitative analysis an accurate knowledge of all these factors is necessary.

IV.—Impulse Tests on Small High-Voltage Transformer Winding

A 30,000-volt transformer winding made up of a stack of 52 disk coils approximately 15 in. long and 10 in. in diameter was used for these tests. Grounded metal cylinders were placed inside and outside of the stack, to simulate low-voltage winding and tank.

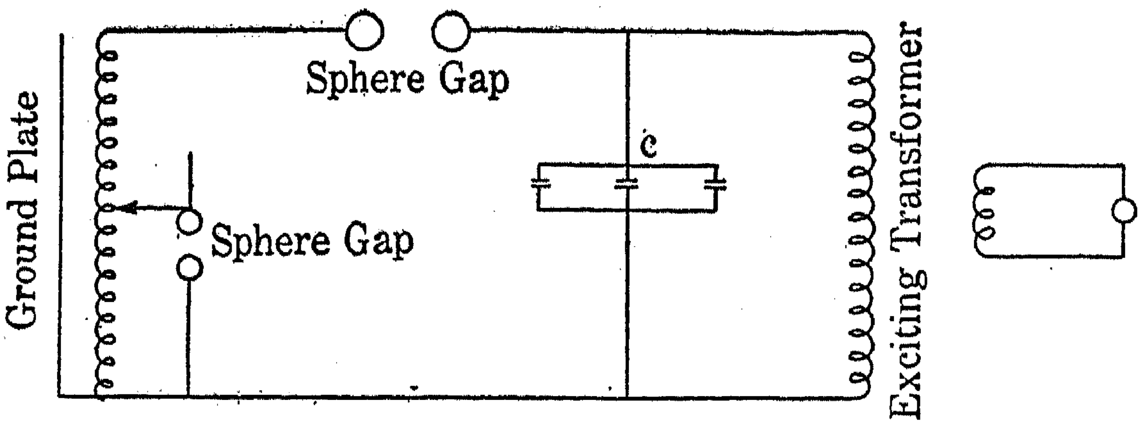


Fig. 8—Impulse Tests on Small High-Voltage Transformer-Connection Diagram

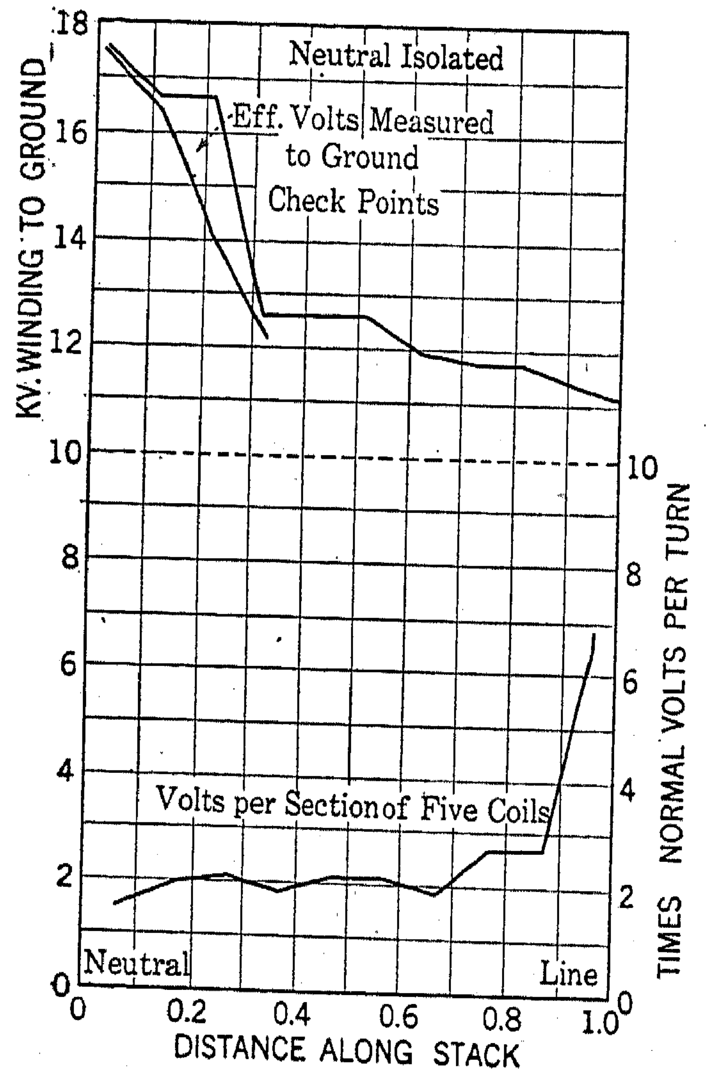


Fig. 9—Impulse Tests on Small High-Voltage Transformer—for Connections See Fig. 8

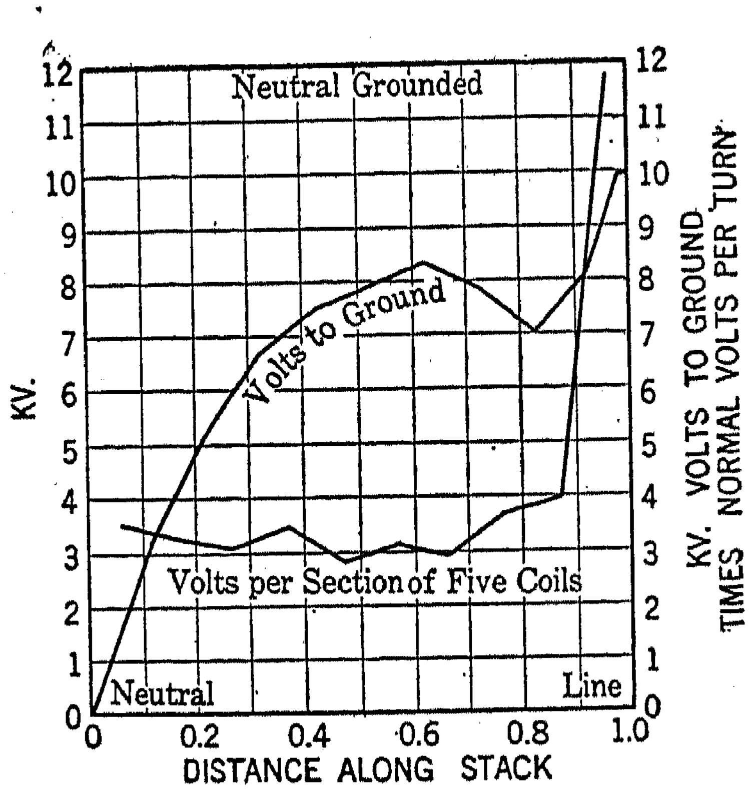


Fig. 10—Impulse Tests on Small High-Voltage Transformer

The connection used in testing is shown in Fig. 8. Voltage was applied from line to ground; in some tests the neutral (one end) of the coil was grounded to the metal cylinders and in the other test isolated. A spark gap A was connected in series with the coil, and a large condenser C in multiple with it. The operation is as follows: As the voltage rises, it charges the condenser C. This voltage also appears across the gap A, a very small part being consumed in the capacitance of the coil. When the gap arcs over, the entire voltage, abruptly applied across the winding, gives rise to a series of oscillations, within the winding. The use of the condenser C (which must be as large as practicable) is to maintain the applied voltage while the capacitance of the coil is being charged and also to allow an easy return path for the high-frequency oscillations set up in the coil. As these oscillations die out before the 60-cycle voltage has appreciably changed, the impressed voltage is in effect a sustained rectangular wave.

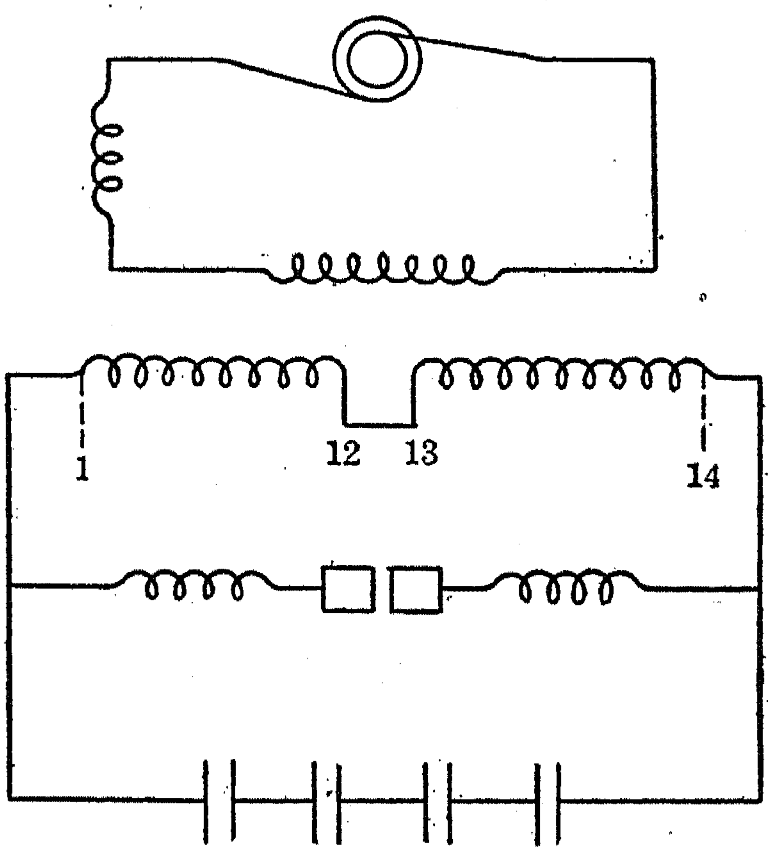


FIG. 11—HIGH-FREQUENCY INVESTIGATION—60 CYCLES—3000 KV-A.—140,000 VOLTS—DIAGRAM OF CONNECTIONS

The voltage to ground and the voltage gradients are plotted in Fig. 9 and Fig. 10.

Constants of the Coil

Ground capacitance $C_g = 12 \times 10^{-10}$ farads.
Series capacitance $C_s = 4.4 \times 10^{-12}$ farads.

$$\alpha = \sqrt{\frac{C_g}{C_s}} = 16.5$$

V.—High-Frequency Tests*

High-frequency tests on the high-voltage windings of 3000-kv-a. 140,000-volt transformer (Fig. 4) compared with theoretically calculated stresses resulting from impact of rectangular waves.

Transformer constants;

C_g = Ground capacitance $= 9 \times 10^{-10}$ farads.
C_s = Coil capacitance $= 9 \times 10^{-12}$ farads.

$$\alpha = \sqrt{\frac{C_g}{C_s}} = 10$$

Connection diagrams for these tests are given in Fig. 11, and high-frequency voltages obtained across taps are plotted in Fig. 12 and Fig. 13. In Fig. 14 is plotted the approximate volts obtained by these tests at any frequency (curve marked by dots). For comparison the volts per turn as theoretically obtained for rectangular waves (See Fig. 6) and corrected for end turn insulation has been plotted on the same sheet (curve marked by crosses.) The theoretical values have been multiplied by two, for the following reason.

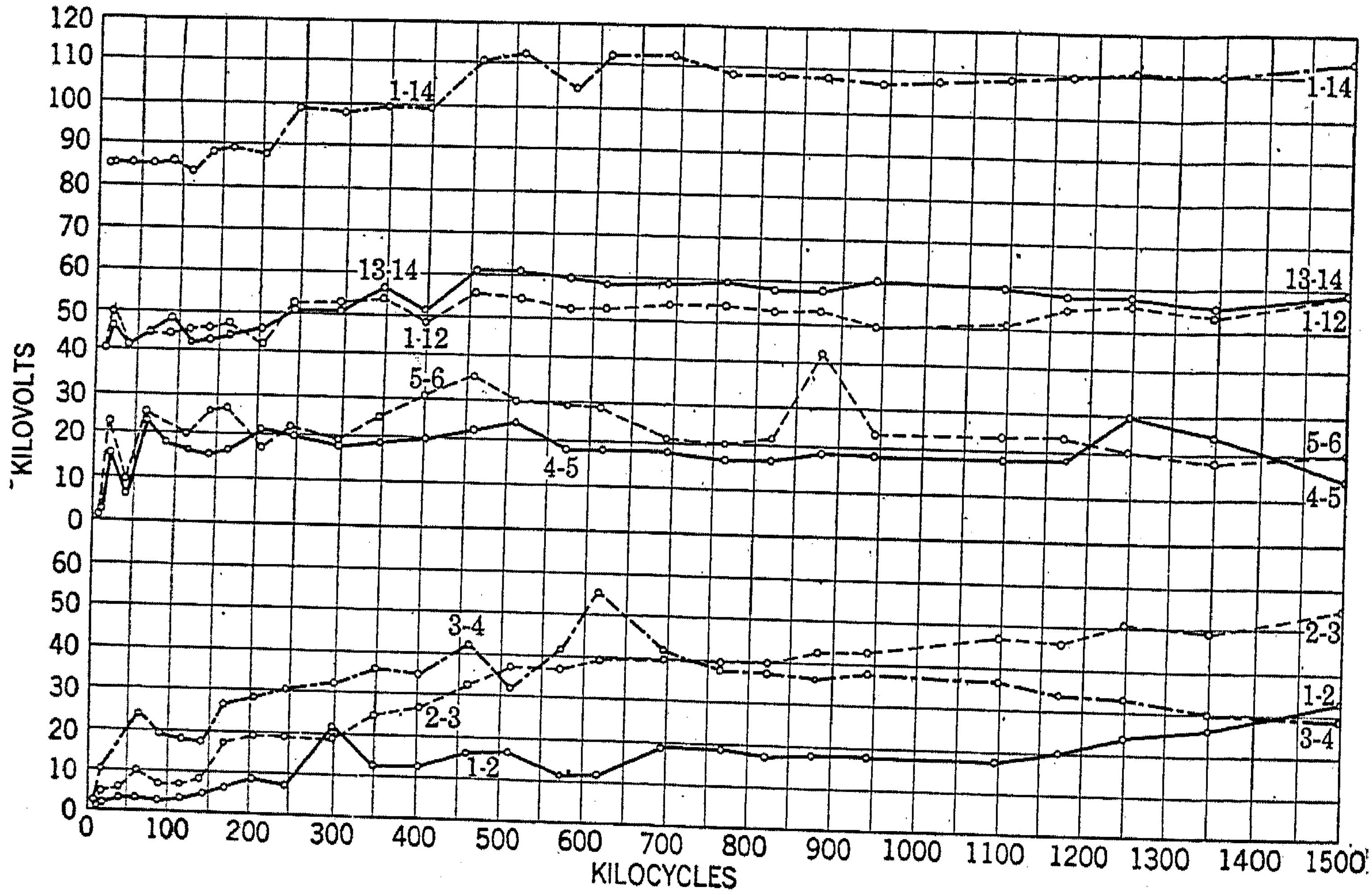


FIG. 12—HIGH-FREQUENCY VOLTAGES MEASURED BETWEEN TAPS

Discharge takes place every one-half wave at the maximum point of the wave. Condensers discharge into the inductive circuit, the maximum value of high-frequency voltage being equal to maximum value of the 60-cycle voltage. One-half wave of the high-frequency discharge is shown in Fig. 15. It is seen that during this half cycle the high-frequency voltage changes from maximum positive to maximum negative. The original 60-cycle voltage distribution, just before the arc-over, is the straight line shown in Fig. 16. Half a high-frequency cycle after the arc-over, the resultant of the 60-cycle and high-frequency voltage distribution is shown by curve "*a*" in Fig. 16, assuming that the frequency is high enough for the wind-

---

*Tests made by J. M. Weed and J. E. Clem.

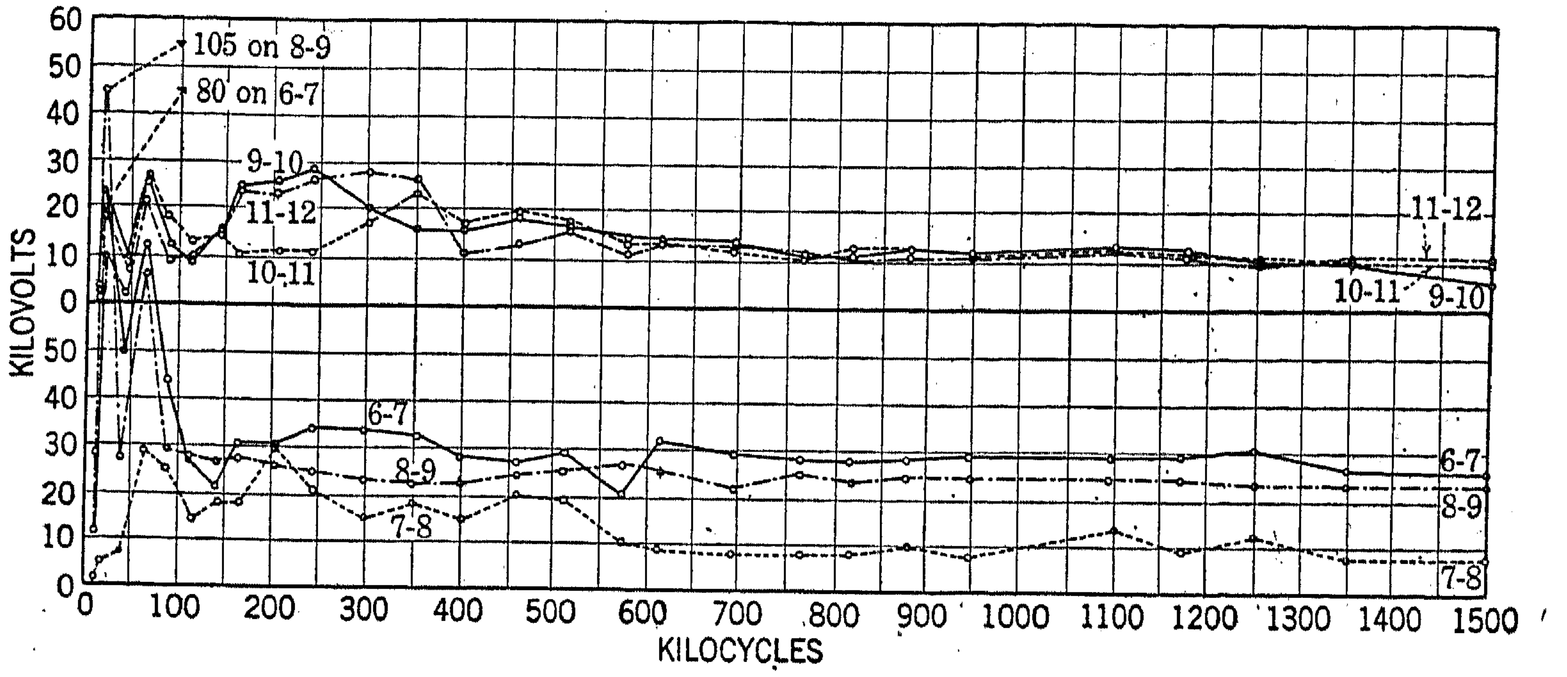


FIG. 13—HIGH-FREQUENCY VOLTAGES MEASURED BETWEEN TAPS

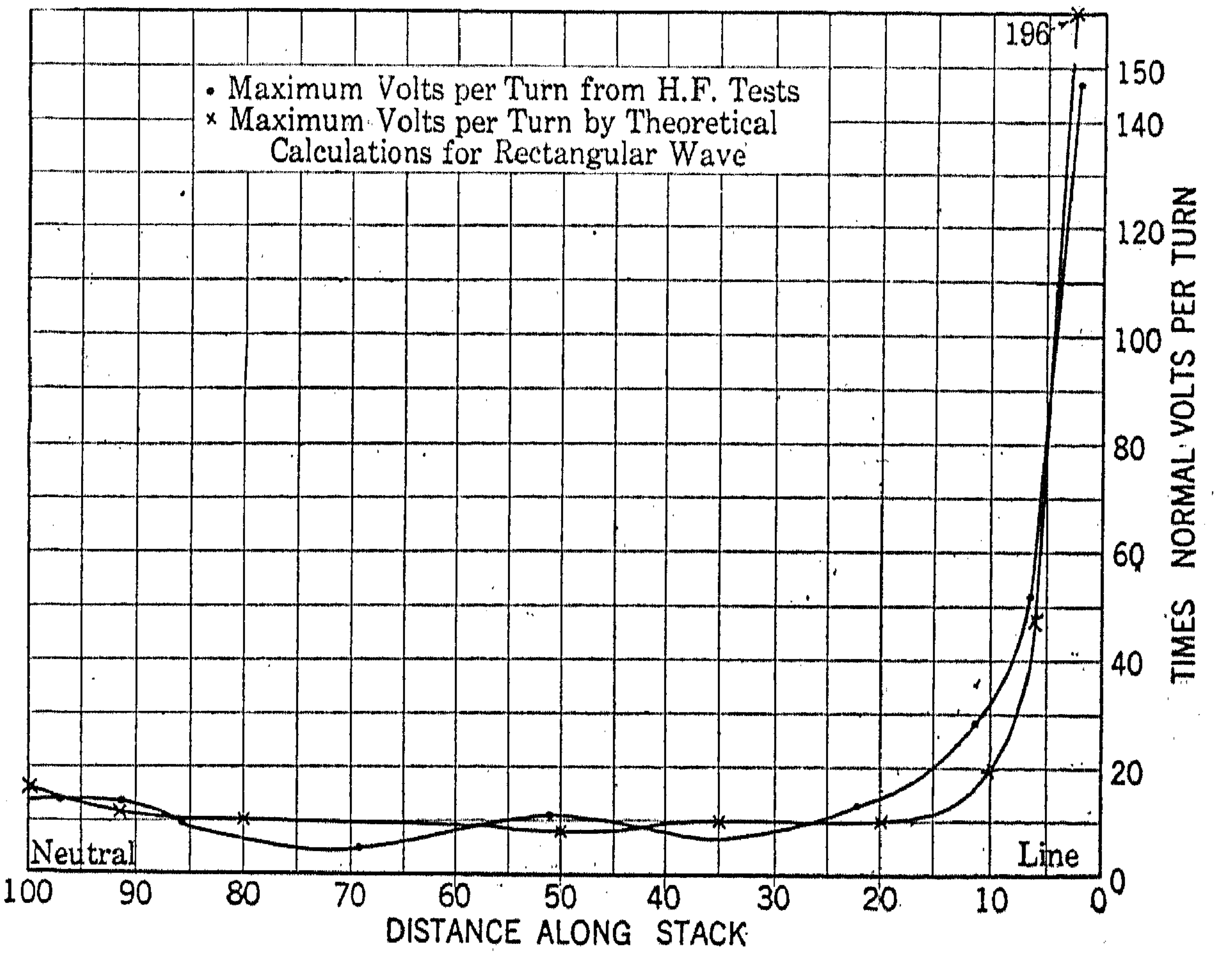


FIG. 14—3000-KV-A.—140,000-VOLT TRANSFORMER

ing to act as condenser for the first half cycle of the high-frequency wave. Thus it is evident that the combination of the high- and low-frequency waves doubles the voltage gradient.

HIGH-FREQUENCY TEST DATA

Taps	Per cent turns	Per cent along stack	Maximum volts per turn times normal
1—2	.19	2.1	147 at 1,500,000 Cycle
2—3	1.2	6.3	52 1,500,000 "
3—4	4.	11.4	28 at 600,000 "
4—5	8.2	16.7	11 at 62,000 "
5—6	13.5	22.5	13.5 at 900,000 "
6—7	30.7	36	5.6 62,000 "
7—8	48	51.5	11.5 62,000 "
8—9	68	69	6 20,000 "
9—10	89	86	9.9 62,000 "
10—11	94	91.5	13 62,000 "
11—12	98	97	14 62,000 "

The volts per turn occurring at ends of windings are the same as if a rectangular wave of voltage 2 E had been applied. For this reason the values for volts per turn as theoretically obtained for rectangular waves have been multiplied by two in order to compare these values with the high-frequency test. The higher the frequency of test, the more nearly should the condenser volts per turn distribution equal twice the voltage distribution caused by a rectangular wave.

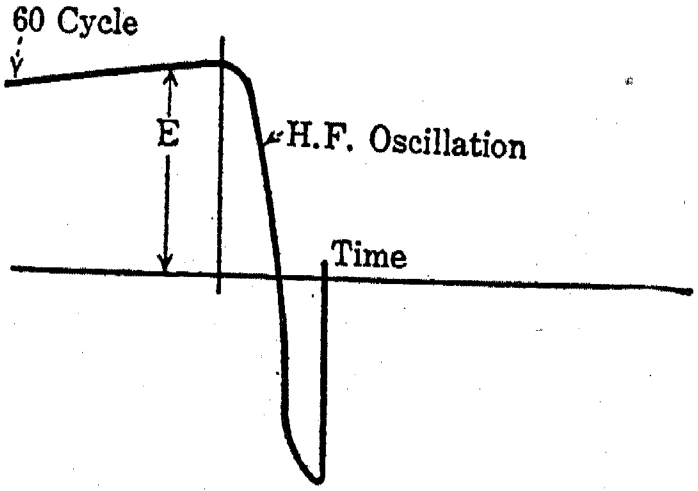


FIG. 15—TIME-VOLTAGE CURVE AT LINE END OF WINDING

V—Mathematical Development

1. Initial voltage distribution in a winding for a steep wave front.

2. Initial effective or equivalent capacitance of a winding for a steep wave front.

3. Time required to charge the initial effective capacitance of a winding.
4. Analysis of transient phenomena following the initial voltage distribution.
5. Surge impedance of an oscillating winding.
6. Example.

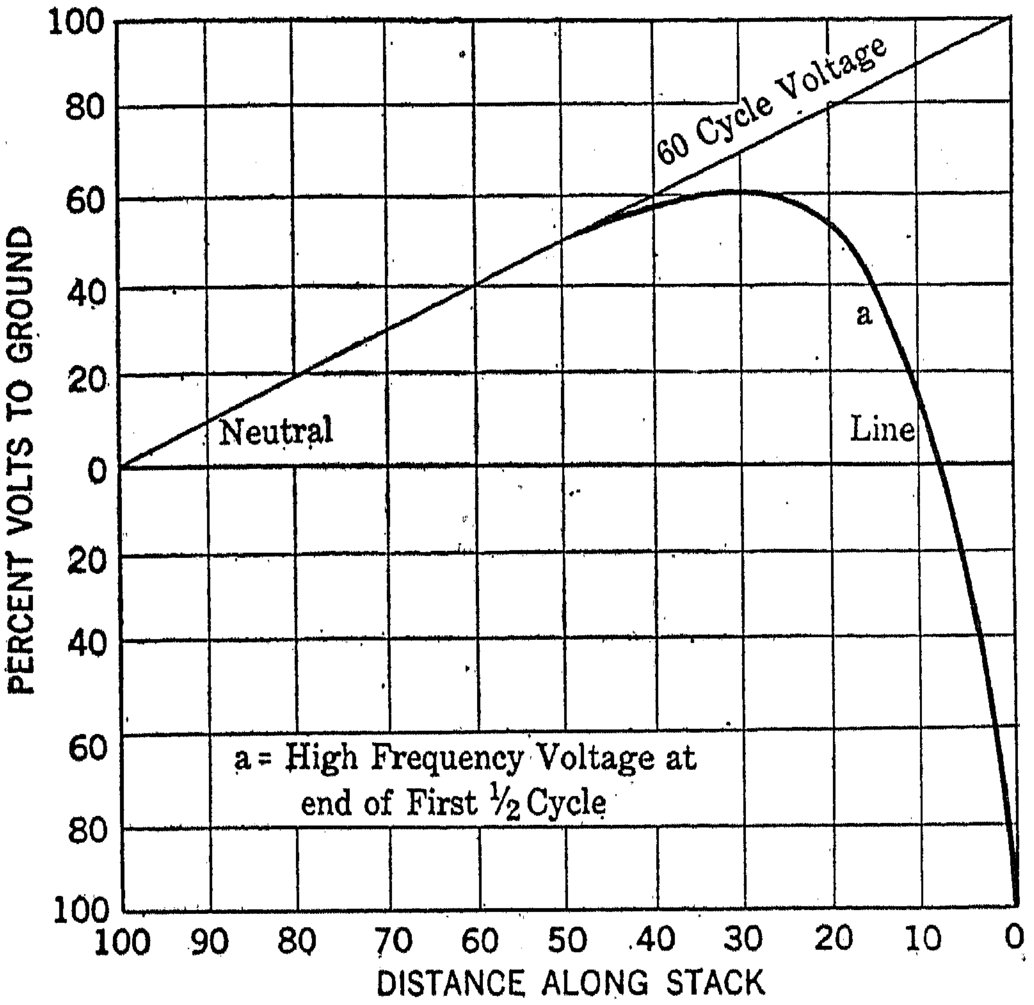


Fig. 16—Space-Voltage Curves for $t = 0$ and $t = \frac{1}{2}$ High-Frequency Cycle

1. Initial Voltage Distribution in a Winding for Steep Wave Fronts

If voltage is suddenly applied to a system of inductances in multiple with a system of capacitances, the initial voltage distribution will be determined by the capacitances. At the instant of impact, the condensers short-circuit the steep wave front. If the wave is maintained, the condensers become charged, and the voltage rises. The time required for this charging is extremely small as given by equation (**22**).

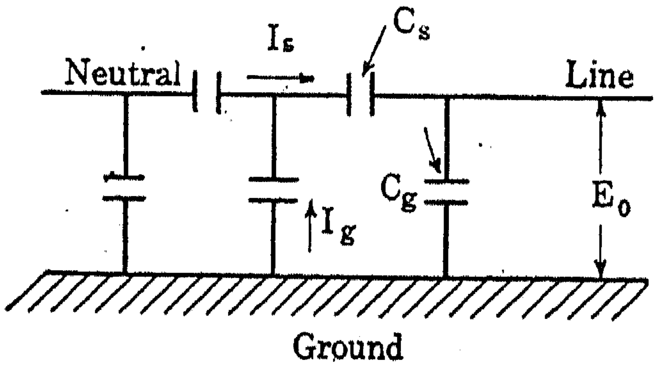


Fig. 17

The voltage distribution, when the condensers are charged, is the same as that when an alternating voltage is applied to the system of condensers with the inductance disconnected. Considering the system of internal and ground capacitances by themselves (Fig. 17), and assuming an alternating voltage of frequency f, $\left(f = \frac{\omega}{2\pi}\right)$, applied, the following relations will be evident.

Let C_s = Capacitance between line end of coil and neutral, shunting the winding.

C_g = Capacitance between winding and ground for a coil length of line to neutral.

l = Coil length line to neutral.

$l\,C_s$ = Capacitance between portions of winding unit distance apart.

$\frac{C_g}{l}$ = Capacitance to ground per unit length of coil

e = Voltage to ground at any point.

$\frac{d\,e}{d\,x}$ = Voltage gradient, volts per unit length of coil.

E_0 = Impressed voltage line to ground.

ω = $2\,\pi\,f$ = angular velocity.

I_s = Current in the coil capacitance C_s.

$$= \omega\, C_s\, l\, \frac{d\,e}{d\,x} \qquad (1)$$

I_g = Current in ground capacitance C_g (total.)

$\frac{d\,I_g}{d\,x}$ = Current in ground capacitance per inch of coil.

$$= \omega \frac{C_g}{l}\, e \qquad (2)$$

The increase per unit distance in the current of the shunt capacitance is the current supplied by the ground capacitance per unit length, that is,

$$\frac{d\,I_s}{d\,x} = \frac{d\,I_g}{d\,x} \qquad (3)$$

Substituting in equation (**3**) the value of $\frac{d\,I_s}{d\,x}$ from (**1**), and

$\frac{d\,I_g}{d\,x}$ from (2), and simplifying,

$$\frac{d^2\,e}{d\,x^2} - \frac{1}{l^2}\,\frac{C_g}{C_s}\,e = 0 \qquad (4)$$

The solution of this equation is of the form

$$e = A\,\epsilon^{px} \qquad (5)$$

Substituting (**5a**) and its derivative in (**4**), and solving for p,

$$p = \pm\frac{1}{l}\sqrt{\frac{C_g}{C_s}} \qquad (6)$$

and

$$e = A\,\epsilon^{px} + B\,\epsilon^{-px} \qquad (7)$$

The constants A and B are determined by the terminal conditions as follows.

Case I.—Neutral Isolated. If the neutral is isolated at

$$x = 0 \quad I_s = 0 \qquad (8)$$

The value of I_s is, from (**1**) and (**7**).

$$I_s = \omega\,l\,C_s\,\frac{d\,e}{d\,x} \qquad (9a)$$

$$= \omega\,l\,C_s\,(A\,\epsilon^{px} - B\epsilon^{-px}) \qquad (9b)$$

Substituting, $x = 0$, and $I_s = 0$

$$0 = \omega\,l\,C_s\,p\,(A - B) \qquad (10a)$$

$$A = B \qquad (10b)$$

Substituting, (**10b**) in (**7**),

$$e = A\,(\epsilon^{px} + \epsilon^{-px}) \qquad (10c)$$

$$= A'\cosh\,(p\,x) \qquad (10d)$$

Now, at $x = l \quad e = E_0$ (**10e**)

$$e = E_0\,\frac{\cosh\,(p\,x)}{\cosh\,(p\,l)} \qquad (11a)$$

$$= E_0\,\frac{\cosh\left(\frac{\alpha\,x}{l}\right)}{\cosh\,(\alpha)} \qquad (11b)$$

where $\alpha = p\,l = \sqrt{\frac{C_g}{C_s}}$ and the neutral is isolated

Case II.—Neutral Grounded. The terminal conditions are

at $x = 0 \qquad e = 0$ (**12a**)

and at $x = l \qquad e = E_0$ (**12b**)

Applying condition (**12a**) to equation (**7**).

$$0 = A + B$$

$$A = -B$$

$$e = A\,(\epsilon^{px} - \epsilon^{-px}). \tag{12c}$$

$$e = A' \sinh p\,x \tag{12d}$$

Applying condition (**12b**) to equation (**12d**)

$$E_0 = A' \sinh p\,l \tag{12e}$$

$$A' = \frac{E_0}{\sinh p\,l}$$

Therefore, $e = E_0 \dfrac{\sinh p\,x}{\sinh p\,l}$ (**13a**)

$$= E_0 \frac{\sinh\left(\dfrac{\alpha\,x}{l}\right)}{\sinh \alpha} \tag{13b}$$

where, $\alpha = p\,l = \sqrt{\dfrac{C_g}{C_s}}$ and the neutral is grounded.

Initial voltage distribution curves for both the isolated neutral and the grounded neutral are given in Figs. 2 and 3.

GENERAL REMARKS ON THE INITIAL VOLTAGE DISTRIBUTION

When the capacitance to ground is negligible compared with the capacitance between coils, that is, $C_g = 0$, the initial voltage distribution is determined by the series capacitances and is a straight line if the capacitance between coils is uniform.

As the capacitance to ground more and more predominates, the curve becomes steeper near the line end, that is, the voltage is concentrated more and more near the line end.

The voltage gradient (volts per unit length) is $e_s = \dfrac{d\,e}{d\,x}$

$$= \frac{\alpha\,E_0 \sinh\left(\dfrac{\alpha\,x}{l}\right)}{l \cosh \alpha} \text{ neutral isolated} \tag{14a}$$

$$= \alpha \frac{E_0}{l} \frac{\cosh\left(\dfrac{\alpha\,x}{l}\right)}{\sinh \alpha} \text{ neutral grounded} \tag{14b}$$

The maximum voltage gradient is at the line end, $x = l$,

$$e_s = \frac{\alpha E_0 \sinh \alpha}{l \cosh \alpha} \text{ neutral isolated} \tag{15a}$$

$$= \alpha \frac{E_0}{l} \tanh \alpha \text{ neutral isolated} \tag{15b}$$

Similarly $e_s = \alpha \dfrac{E_0 \cosh \alpha}{l \sinh \alpha}$ neutral grounded **(16a)**

$$= \alpha \frac{E_0}{l} \coth \alpha \text{ neutral grounded} \tag{16b}$$

For α greater than 3,
Tanh α = coth α = 1

This is true for most transformers, and the voltage gradient at the line end becomes $\left(\frac{\alpha E_0}{l}\right)$ volts per unit length.

If the voltage were uniformally distributed throughout the winding, the gradient would have been $\left(\frac{E_0}{l}\right)$. Therefore, when the total capacitance to ground is very much larger than the coil capacitance between neutral and line, the voltage gradient at the line end is α times the value corresponding to uniform distribution.

It will also be noticed that for values of α greater than 5, the initial voltage distribution for a steep wave front is practically the same for the case of isolated as for grounded neutral.

INITIAL EFFECTIVE CAPACITANCE OF A WINDING FOR A STEEP WAVE FRONT.

In calculating the time of charging the system of coil and ground capacitances by an incoming wave, it becomes necessary to know the effective capacitance of the combination, This is equal to the effective capacitance for alternating voltage.

Representing this effective capacitance by C_{eff}, line current by I, line voltage (a-c.) by E, frequency by "f".

$$I = 2 \pi f C_{eff} E \tag{17a}$$

$$\text{or } C_{eff} = \frac{I}{2 \pi f E} \tag{17b}$$

$$\text{But } I = I_s \text{ at } x = l \tag{17c}$$

$$\text{and } I_s = 2 \pi f C_s l \left(\frac{d e}{d x}\right) \tag{17d}$$

The value of $\left(\frac{d\,e}{d\,x}\right)$ must be taken from equation of initial voltage distribution.

Case I.—Neutral Isolated. By equation (**15b**) at $x = l$

$$\frac{d\,e}{d\,x} = a\,\frac{E}{l}\tanh a \qquad (\mathbf{18a})$$

and $I = 2\,\pi\,f\,C_s \times a\,\frac{E}{l}\tanh a$ (**18b**)

Substituting (**18b**) in (**17b**).

$$C_{eff.} = \sqrt{C_g\,C_s} \times \tanh a \text{, neutral isolated} \qquad (\mathbf{19})$$

Case II.—Neutral Grounded. Solving similarly for the above case, but using equation (**16b**) instead of (**15b**),

$$C_{eff.} = \sqrt{C_g\,C_s}\,\coth a \qquad (\mathbf{20})$$

As stated before, for values of a greater than 3, which is the case with most transformers, tanh a equals one, coth a equals one, and

$$C_{eff.} = \sqrt{C_g\,C_s} \qquad (\mathbf{21})$$

whether the neutral is grounded or isolated.

TIME REQUIRED TO CHARGE THE CONDENSERS TO THE INITIAL VOLTAGE DISTRIBUTION

The time required to charge a condenser by an incoming wave is given by the following formula, modified from one given by G. Faccioli in *G. E. Review*, 1914, page 749.

$$t = 2.3\,C_{eff.}\,Z\,\log_{10}\left(\frac{100}{100 - \%\,E_0}\right) \text{ seconds.} \qquad (\mathbf{22})$$

$C_{eff.}$ = Equivalent massed capacitance of the winding from line to neutral, calculated by formula, (**19**), (**20**), or (**21**).

Z = Surge impedance of the transmission line, and may be taken between 300 and 400.

$\%\,E_0$ = The actual terminal voltage in per cent of the final terminal voltage.

As an example, we will calculate the time necessary to charge the capacitance of the transformer described above to 90 per cent of its final value.

$C_g = 9 \times 10^{-10}$ farads.
$C_s = 9 \times 10^{-12}$ farads
$C_{eff.} = 9 \times 10^{-11}$ by formula (**21**)

Substituting this value of C_{eff} in (**22**) we obtain

$$t = 2.3 \times 9 \times 10^{-11} \times 300 \times \log_{10}\left(\frac{100}{100-90}\right)$$

$= 6.2 \times 10^{-8}$ seconds, which is about one twenty millionth of a second. It is justifiable, then, to assume that the initial voltage distribution by the condensers takes place before the inductance can begin to draw any appreciable current.

ANALYSIS OF TRANSIENT PHENOMENA FOLLOWING THE INITIAL VOLTAGE DISTRIBUTION

Let $I_L =$ Current in the winding.
$I_s =$ Current in the coil capacitance.
$I_g =$ Current in the ground capacitance (total).

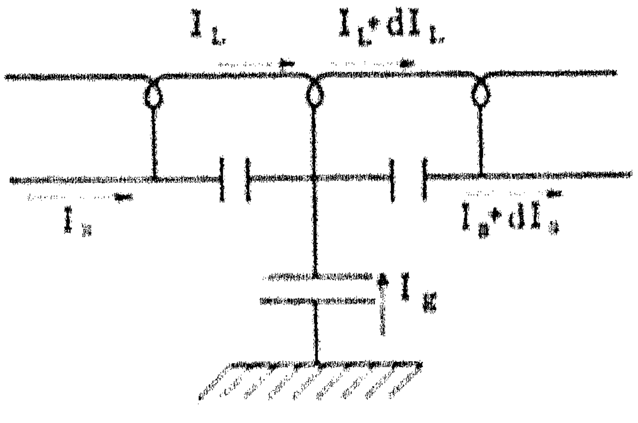


FIG. 18

The following relation will be evident from Fig. 18.

$$\frac{d\,I_L}{d\,x} + \frac{d\,I_s}{d\,x} - \frac{d\,I_g}{d\,x} = 0 \qquad \textbf{(23a)}$$

Equation (**23a**) states that the increase in the currents in the winding and the internal capacitance (as we go towards the line end) is equal to the current contributed by the ground capacitance, which is evident from Fig. 18.

For reasons that will be seen later, we differentiate (**23a**) with respect to t, and obtain

$$\frac{d^2\,I_L}{dxdt} + \frac{d^2\,I_s}{dxdt} - \frac{d^2\,I_g}{dxdt} = 0 \qquad \textbf{(23b)}$$

It will be noticed that equations (**23a**) and (**23b**) are true generally and are independent of what the initial or final voltage distributions may be.

Equation (**23**) involves three variables, I_L, I_g and I_s, and, before it can be solved, it must be expressed in one variable.

Since we are interested mostly in the voltages, we will express the variables in terms of voltages.

Let e = voltage to ground.

e_s = voltage gradient (volts per inch of coil) in the winding.

$= \dfrac{de}{dx}$

N = Total turns (line to neutral)

φ = Total flux linking any turn.

φ_L = Leakage flux linking any turn.

φ_M = Main flux linking the total coil.

B_L = Leakage flux density at any point.

h = Effective length of leakage path.

L = Leakage inductance of coil.

(MLT) = Mean length of turn.

(For a complete list of symbols see the end of this paper.)

To Find the Relation Between I_s and Voltage. The current I_s in the capacitance between coils is equal to the capacitances per inch $l\,C_s$ times the rate of change of voltage per inch $\left(\dfrac{d\,e_s}{dt}\right)$.

$$I_s = l\,C_s \frac{d\,e_s}{dt} = C_s\,l \frac{d^2 e}{dx\,dt} \tag{24}$$

$$\frac{d^2 I_s}{dx\,dt} = C_s\,l \frac{d^4 e}{dx^2\,dt^2} \tag{25}$$

Equation (**25**) can be directly substituted in equation (**23b**).

To Find the Relation Between I_g and Voltage. The current density in the capacitance to ground is equal to capacitance to ground per inch of coil times the rate of change of voltage to ground.

$$\frac{d\,I_g}{d\,x} = \frac{C_g}{l}\,\frac{d\,e}{dt} \tag{26}$$

and

$$\frac{d^2 I_g}{dx\,dt} = \frac{C_g}{l}\,\frac{d^2 e}{dt^2} \tag{27}$$

Equation (**27**) can be directly substituted in (**23b**).

To Find the Relation Between I_L and Voltage. The relation between the magnetizing current (I_L) and the voltage is somewhat more complicated than that of the capacitance currents and voltage, due to the mutual inductance between turns and the presence of leakage and main fluxes. The connecting links between current and voltage to ground at any point x are current at x, ampere turns or magnetomotive force effective in producing leakage flux, flux density at point x, flux producing voltage at point x, volts per turn and volts. See Fig. 19. The mathematical relation between these physical quantities are as follows:

The voltage gradient e_s, *i. e.*, volts per inch, equals number of turns per inch N/l times the rate of change of total flux φ (= main flux φ_m + leakage flux φ_L). Thus,

$$e_s = \frac{N}{l}\,\frac{d\varphi}{dt}\,10^{-8} \tag{28a}$$

$$= \frac{N}{l}\,10^{-8}\,\frac{d}{dt}\left\{\varphi_L + \varphi_m\right\} \tag{28b}$$

The leakage flux (φ_L) linking x is obtained by integrating the leakage flux density (B_L). Thus,

$$e_s = \frac{N}{l}\,10^{-8}\frac{d}{dt}\left\{(MLT)\int_x^l B_L\,dx + \varphi_m\right\} \tag{28c}$$

If we differentiate (**28c**) with respect to x we get rid of the integral sign, and also of the last term because it is independent of x. Thus,

$$\frac{de_s}{dx} = -\frac{N}{l}\,10^{-8}\,(MLT)\,\frac{dB_L}{dt} \tag{29a}$$

The negative sign in (**29a**) is due to x being the negative limit of integration in (**28c**). Of course it could have been assumed of the opposite sign provided other signs were kept consistent with it.

The leakage flux density B_L in (**29a**) is produced by the ampere turns enclosed by the flux.

$$B_L = \frac{N\mu}{lh}\int_0^x I_L\,dx \tag{29b}$$

where $\mu = 4\,\frac{\pi}{10}$ when using dimensions in cm.

$= 3.2$, when using dimensions in inches.

$h =$ effective length of leakage path.

It may be questioned why in (**28c**) we use the limits $\int_x^l$ and in (**29b**) $\int_0^x$. The reason may be stated thus: the flux linking a current is outside of it, and the ampere turns producing the flux are inside of it. Therefore, if the total flux linking a certain point be integrated on the right of that point, the ampere turns producing that flux must be integrated on the left of the point. As a concrete example consider Fig. 19.

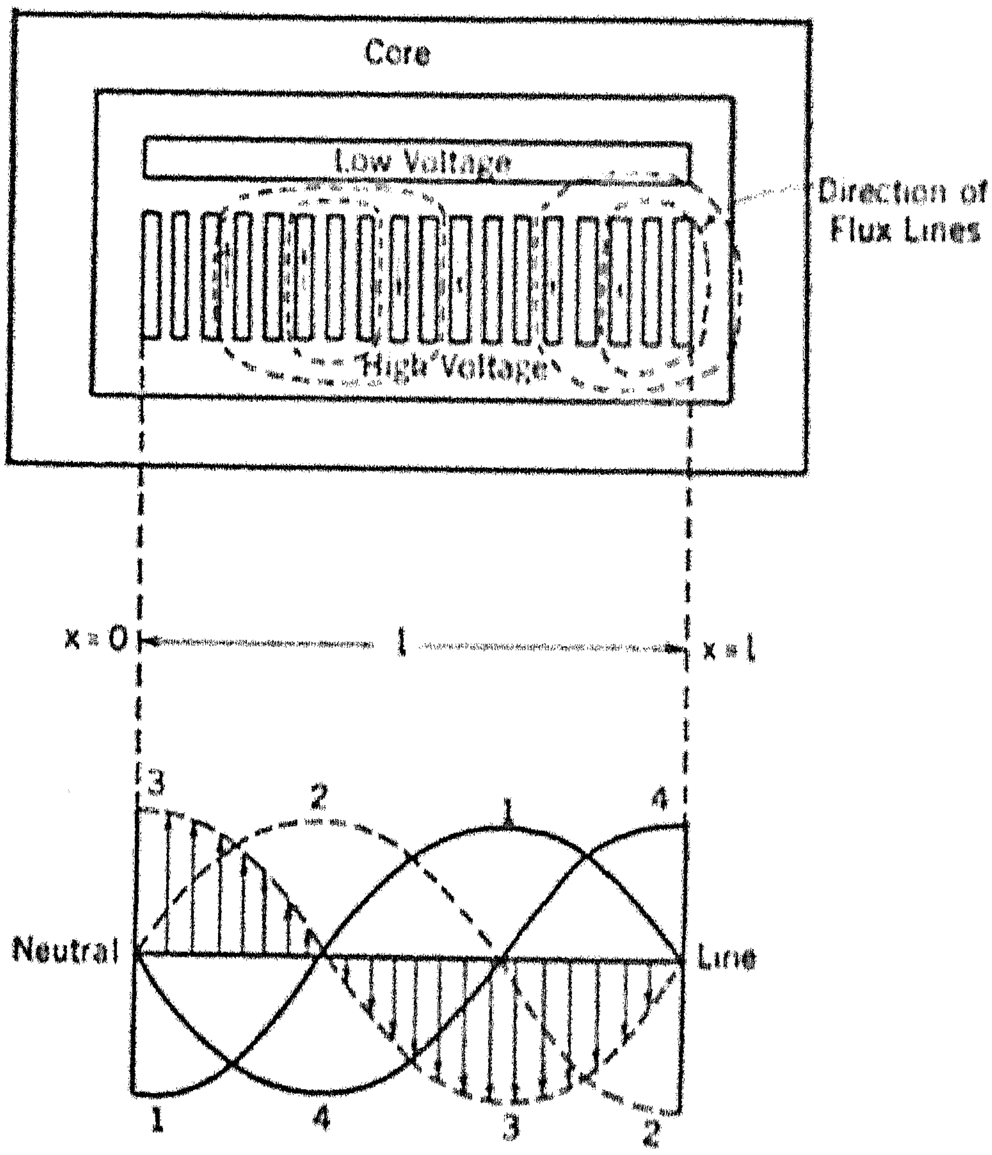


FIG. 19—NATURAL OSCILLATION WITHIN TRANSFORMER WINDING FOR ¾ WAVE LENGTH WITH ISOLATED NEUTRAL AND LINE END CONNECTED TO TRANSMISSION LINE

Curve 1, Volts to ground = E. Curve 2, Volts per turn = DE/DX. Curve 3, Flux density, or Magnetomotive force = D^2E/DX^2. Curve 4, Current = D^3E/DX^3.

The flux producing voltage in a turn at the point x is all the flux from x to l; but the ampere turns producing the flux at x are all the ampere turns from 0 to x.

Returning to the main problem, (**29b**) substituted in (**29a**) gives

$$\frac{d\,e_s}{d\,x} = -\frac{3.2\,N^2\,(MLT)}{l^2\,h\,10^8}\,\frac{d}{d\,t}\int_0^x I_t\,d\,x \qquad \textbf{(29c)}$$

$$= \frac{L}{l^3}\,\frac{d}{d\,t}\int_0^x I_t\,d\,x \qquad \textbf{(29d)}$$

where

$$L = \frac{3.2\,l\,N^2\,(MLT)}{h\,10^8} \text{ (See note opposite)} \qquad \textbf{(29e)}$$

Differentiating (**29d**) with respect to x eliminates the integral sign. Thus,

$$\frac{d^2 e_s}{d x^2} = - \frac{L}{l^3} \frac{d I_L}{dt} \qquad \textbf{(30a)}$$

Remembering that e_s (voltage gradient) is equal to $\frac{d e}{d x}$ (derivative of voltage to ground), (**30a**) becomes

$$\frac{d^3 e}{d x^3} = \frac{L}{l^3} \frac{d I_L}{dt} \qquad \textbf{(30b)}$$

(**30b**) gives the value of $\frac{d I_L}{dt}$, while, equation (**23a**) involves $\frac{d I_L}{d x}$. If we differentiate the former with respect to x and the latter with respect to t, substitution becomes possible. This is the reason why (**23a**) was differentiated with respect to time giving (**23b**).

Differentiating (**30b**) with respect to x and rearranging.

$$\frac{d^2 I_L}{dx dt} = \frac{- l^3}{L} \frac{d^4 e}{d x^4} \qquad \textbf{(31)}$$

(**31**), (**25**) and (**27**) express in terms of voltage the three terms of the fundamental differential equation (**23b**). Making the substitution, (**23b**) becomes

---

NOTE:—In discussing the leakage flux relations, we have assumed the effective length of the path as constant and independent of wave length. This is approximately true for only short waves on disk coils when the radial build of the coils is comparable to the wave length. Otherwise, the formula for L will not be accurate enough for numerical calculations. L, however, can be determined experimentally for various wave lengths as follows. Take one portion of the winding equal to one quarter of the wave length under consideration and short-circuit it. Take an equal portion adjacent to it, excite it as primary, and measure its inductance (= Reactance/377 for 60-cycle test). Multiply this value of inductance by $32\, n^3$ to get the effective value of L for that wave length.

If the winding consists of disk coils, for approximate calculations the effective length of leakage path may arbitrarily be assumed as equal to the radial coil-build plus one-sixth of wave length:

$$h = b + \frac{l}{6\, n} \qquad \textbf{(29f)}$$

$$\frac{d^4 e}{d x^4} - \frac{L C_s d^4 e}{l^2 dx^2 dt^2} + \frac{L C_g}{l^4}\frac{d^2 e}{dt^2} = 0 \qquad (32)$$

(32) is the general differential equation of electrical phenomena occurring in windings, except that it does not take into account resistance and dielectric losses.

A solution of this equation is

$$e = A \cos p\, x \cos \omega\, t. \qquad (33)$$

Substituting in equation (32) gives

$$\omega = \pm \frac{p^2 l^2}{\sqrt{L\,(C_g + l^2 p^2 C_s)}} \qquad (34)$$

These equations show that a transformer is capable of sinusoidal oscillations in space and time at various frequencies and wave lengths.

Determination of the Constants. The voltage distribution may be expressed as the sum of a transient and a permanent component. The permanent component is the final voltage distribution. For a d-c. voltage applied, the final voltage distribution is $N B$ (Fig. 20) for the case of neutral isolated, that is, the whole coil assumes the impressed voltage E_0 against ground. The actual initial voltage being given by the hyperbolic curve $O A B$, the transient component is the difference between $N B$ and $O A B$. Expressing the same relation in another way $O A B$ represents the actual voltage when referred to $O M$ as the axis of reference, but represents the transient component when referred to $N B$ (the line of equilibrium) as the axis of reference. For neutral grounded (Fig. 21) $O B$ represents the permanent or equilibrium line, and $O A B$ (referred to $O B$ as axis) represents the transient component of the voltage distribution.

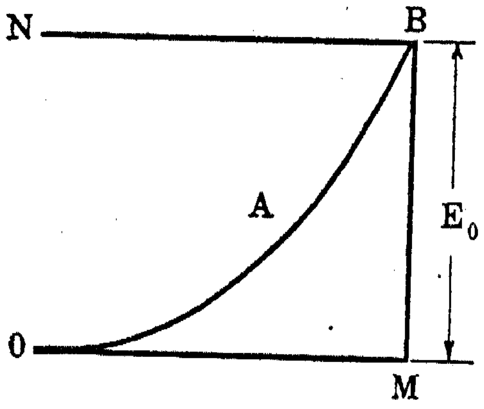


FIG. 20

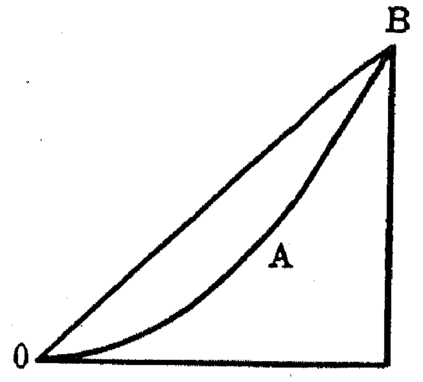


FIG. 21

To try to substitute the hyperbolic formula of initial voltage distribution in equation (33) leads to an indeterminate form, but if analyzed into space harmonics (with respect to the line of final voltage distribution), substitution is readily effected, and the actual voltage to ground is the sum of the harmonics and the voltage of the reference line to ground.

An inspection of Fig. 5 shows that for neutral isolated the initial voltage distribution curve $O\,A\,B$ is a quarter wave length (and its odd harmonics) with respect to the equilibrium line $N\,B$ with maximum voltage at the neutral. An inspection of Fig. 6 shows that for neutral grounded the initial voltage distribution curve $O\,A\,B$ is a half wave length (and its even and odd harmonics). The numerical values of the harmonics are derived by harmonic analysis.

Representing wave length by λ, initially, for neutral isolated,

$$e' = E_\lambda \cos \frac{2\,\pi\,x}{\lambda} \tag{35}$$

and for neutral grounded

$$e' = E_\lambda \sin \frac{2\,\pi\,x}{\lambda} \tag{35a}$$

where $E\lambda$ is the maximum value of the wave.

The value of p in equation (**34b**) is therefore $\frac{2\,\pi\,x}{\lambda}$ and

$$\omega = \pm \frac{4\,\pi^2\,l^2}{\lambda^2 \sqrt{L\left(\dfrac{C_g + 4\,\pi^2\,l^2\,C_s}{\lambda^2}\right)}} \tag{35b}$$

$$= \pm \frac{4\,\pi^2\,(l/\lambda)^2}{\sqrt{L\,\{\,C_g + 4\,\pi^2\,C_s\,(l/\lambda)^2\,\}}} \tag{35c}$$

Now l/λ is the ratio of coil length to wave length and we shall designate it by small n. The longest wave to which the winding will oscillate is a quarter wave length, and $n = 1/4$. Thus n may be looked upon as the order of the space harmonic. If n equals one, the wave length equals the coil length; if n equals two, the wave length is one:half of the coil length, or, there are two complete wave lengths from line to neutral. Substituting n for l/λ,

$$\omega = \frac{4\,\pi^2\,n^2}{\sqrt{L\,(C_g + 4\,\pi^2\,n^2\,C_s)}} \tag{35d}$$

$$f = \frac{2\,\pi\,n^2}{\sqrt{L\,(C_g + 4\,\pi^2\,n^2\,C_s)}} \tag{35e}$$

For long wave lengths (low harmonics, n small) capacitance to ground predominates,

and
$$f = \frac{2 \pi n^2}{\sqrt{L\, C_g}} \tag{35f}$$

For the very short wave lengths (high order of harmonics, n large) the coil capacitance C_s predominates.

$$f = \frac{n}{\sqrt{L C_s}}\,. \tag{35g}$$

Since there are two values of ω, one positive and the other negative, and since cos $(+\omega)$ equals cos $(-\omega,)$ equation (**33**) becomes

$$e' = A \cos (2 \pi n x/l) \cos (\omega_n t) + B \cos (2 \pi n x/l) \cos (- \omega_n t) \tag{33a}$$

$$= A \cos (2 \pi n x/l) \cos \omega_n t + B \cos (2 \pi n x/l) \cos \omega_n t \tag{33b}$$

$$= (A + B) \cos (2 \pi n x/l) \cos \omega_n t \tag{33c}$$

$$= E_n \cos (2 \pi n x/l) \cos \omega_n t \tag{33d}$$

e' is the voltage to the axis of oscillation, and for every harmonic there is a term like (**33d**). The voltage to ground is, for neutral isolated,

$$e = E_0 - E_n \cos (2 \pi n x/l) \cos \omega_n t \tag{33e}$$

and for neutral grounded

$$e = E_0 (x/l) - E_n \sin (2 \pi n x/l) \cos \omega_n t \tag{33f}$$

As the voltage oscillates above and below the line of equilibrium, and as the frequency is different for the different waves, we may assume that at some instant the maxima of all the harmonics at a point will add arithmetically, and that will represent the maximum voltage to ground at that point. That is, maximum voltage to ground equals, for neutral isolated.

$$\max e = E_0 + E_n \cos (2 \pi n x/l) + \ldots \tag{36a}$$

and for neutral grounded

$$\max e = E_0 (x/l) + E_n \sin (2 \pi n x/l) + \ldots \tag{36b}$$

Of equal importance is the voltage gradient (volts per turn). The maximum voltage gradient at a point occurs when maximum voltage gradients of the various harmonics at that point add directly. This gives, for neutral isolated,

$$\max \frac{d e}{d x} = (2 \pi n/l) E_n \sin (2 \pi n x/l) + \ldots \tag{37a}$$

and for neutral grounded

$$\max \frac{d\,e}{d\,x} = E_0/l + (2\,\pi\,n/l)\,E_n \cos\,(2\,\pi\,n\,x/l) + \,.\,. \qquad \textbf{(37b)}$$

It is to be noted that although the higher harmonics may be small in magnitude yet the volts per turn which they produce may not be negligible on account of the volts per turn varying directly with the order of the harmonic. For instance, a tenth harmonic having a magnitude of ten per cent of the fundamental will cause as much strain per turn as the fundamental.

The initial voltage gradient was discussed under equations (**14**,) (**15**) and (**16**).

SURGE IMPEDANCE OF A WINDING

The surge impedance of a transformer winding may be defined as the ratio of voltage impressed to current at line end of winding.

By equation (**23**)

$$\frac{d\,I_L}{d\,x} = \frac{d\,I_g}{d\,x} - \frac{d\,I_s}{d\,x}$$

$$= \frac{C_g\,d\,e}{l\,dt} - l\,C_s\,\frac{d^2\,e}{dxdt}$$

Substituting the general value of e,
that is, $e = E \cos p\,x \cos \omega\,t$

$$\frac{d\,I_L}{d\,x} = -\,\omega\,E\,(C_g/l + l\,C_s\,p^2) \cos p\,x \sin \omega\,t$$

and integrating

$$I_L = \frac{\omega\,E}{p}\,(C_g/l + l\,C_s\,p^2) \sin p\,x \sin \omega\,t \qquad \textbf{(38a)}$$

At the line end, sin $p\,l$ equals one, and the maximum of the current occurs when sin $w\,t$ equals one.

$$\max I_L = \frac{w\,E}{p}\,(C_g/l + l\,p^2\,C_s) \qquad \textbf{(38b)}$$

$$Z = \frac{E}{I_L} = \frac{p\,l}{\omega\,(C_g + l^2\,p^2\,C_s)} \qquad \textbf{(39a)}$$

$$Z = \frac{1}{2\,\pi\,n} \times \sqrt{\frac{L}{C_g + 4\,\pi^2\,n^2\,C_s}} \qquad \textbf{(39b)}$$

It is, therefore, evident that the surge impedance of a winding is not a constant but a function of the wave length, the shorter the wave length the less the surge impedance and hence, for a given voltage the current associated with higher harmonics is greater.

The current surges back and forth into the line, and it is assumed that the surge impedance of the line is low enough not to impede this free oscillation as will be seen in the following example.

EXAMPLE

The constants of the high-voltage winding of the above mentioned transformer are as follows,

$$\begin{aligned} C_g &= 9 \times 10^{-10} \text{ farads} \\ C_s &= 9 \times 10^{-12} \text{ farads} \\ \alpha &= 10 \end{aligned}$$

By formula (**29e**)

$$\begin{aligned} L &= 8 \text{ for } n = 1/4 \\ &= 14 \text{ `` } n = 1/2 \\ &= 20 \text{ `` } n = 3/4 \\ &= 23 \text{ `` } n = 1 \\ &= 27 \text{ `` } n = 5/4 \\ &= 30 \text{ `` } n = 3/2 \\ &= 32 \text{ `` } n = 7/4 \\ &= 34 \text{ `` } n = 4/2 \end{aligned}$$

The initial voltage distribution is, by equations (**11b**) and (**13b**).

$$\% \, e = \frac{e}{E_0} = \frac{\cosh (10 \, x/l)}{\cosh (10)} \quad \text{neutral isolated}$$

$$= \frac{\sinh (10 \, x/l)}{\sinh (10)} \quad \text{neutral grounded}$$

These are plotted in Figs. 2 and 3 together with curves for other values of α.

Analyzing the curve for $\alpha = 10$ into its space harmonics with respect to the corresponding axis of oscillation, we find the following values:

Neutral Isolated.—

Order of harmonic	Number of wave lengths n	Voltage $\frac{E\,n}{E_0}$
Fundamental	1/4	− 125 %
3rd	3/4	+ 34 %
5th	5/4	− 15 %
7th	7/4	+ 7 %
9th	9/4	− 3.5%
11th	11/4	+ 1 %

The frequencies corresponding to these harmonics, calculated by equation (**35e**), are

Order of harmonics	n	λ	ω	f	Ratio
Fundamental	1/4	168 in.	28,000	4,600	1
3rd	3/4	56 "	150,000	24,000	5.2
5th	5/4	34 "	310,000	49,000	10.6
7th	7/4	24 "	480,000	76,000	16.5

Expressing the voltage to ground in per cent of line voltage (line to ground) the complete equation is

$$\% \, e = \frac{e}{E_0} = 1 - 1.25 \cos\left(\frac{\pi}{2} \times \frac{x}{l}\right) \cos \; 28{,}000\, t$$

$$+ 0.34 \cos\left(\frac{3\,\pi}{2} \times \frac{x}{l}\right) \cos 150{,}000\, t$$

$$- 0.15 \cos\left(\frac{5\,\pi}{2} \times \frac{x}{l}\right) \cos 310{,}000\, t$$

$$+ 0.07 \cos\left(\frac{7\,\pi}{2} \times \frac{x}{l}\right) \cos 480{,}000\, t \text{ etc.}$$

The maximum possible voltage to ground at any point at any time is equal to

$$\text{max. } \% \, e = 1 + 1.25 \cos\left(\frac{\pi}{2} \times \frac{x}{l}\right)$$

$$+ 0.34 \cos\left(\frac{3\,\pi}{2} \times \frac{x}{l}\right)$$

$$+ 0.15 \cos\left(\frac{5\pi}{2} \times \frac{x}{l}\right)$$

$$+ 0.07 \cos\left(\frac{7\pi}{2} \times \frac{x}{l}\right) \text{ etc.}$$

This is plotted in Fig. 6.

The maximum voltage gradient is the derivative of the above equation.

$$\text{max. } \% \, e_s = \frac{d\,e}{d\,x} = \frac{\pi}{2\,l} \times 1.25 \sin\left(\frac{\pi\,x}{2\,l}\right)$$

$$+ \frac{3\,\pi}{2\,l} \times 0.34 \sin\left(\frac{3\,\pi\,x}{2\,l}\right)$$

$$+ \frac{5\,\pi}{2\,l} \times 0.15 \sin\left(\frac{5\,\pi\,x}{2\,l}\right)$$

$$+ \frac{7\,\pi}{2\,l} \times 0.07 \sin\left(\frac{7\,\pi\,x}{2\,l}\right) \text{ etc.}$$

This is also plotted in Fig. 6. Calculating the voltage gradient at the line end by this formula we find that it is less than that obtained by equation (**15b**). This means that a sufficient number of harmonics are not included in the above formula. Neglecting the highest harmonics is permissible for the interior of the coil as they may be damped out before the highest possible maximum would be reached, but for the line end it is better to calculate the voltage gradient by the hyperbolic formula. This has been done in the calculation of the above curve.

Neutral Grounded: The initial voltage distribution for this case is practically the same as for isolated neutral (see Fig. 3, $\alpha = 10$). Analyzing this curve into space harmonics with respect to the slant line $\alpha = 0$, we find the following values:

No. of wave lengths on the stack n	$\frac{E_x}{E_0}$	ω	f	Ratio
½ −	.58	84,000	13,000	1
2 × ½ +	.23	230,000	37,000	2.8
3 × ½ −	.13	390,000	62,000	4.8
4 × ½ +	.06	580,000	92,000	7.1

The maximum voltage to ground is

$$\text{max. } \% \, e = \frac{x}{l} + 0.58 \sin \left(\frac{\pi x}{l}\right) + 0.23 \sin \left(\frac{2 \pi x}{l}\right) + 0.13 \sin \left(\frac{3 \pi x}{l}\right) + 0.06 \sin \left(\frac{4 \pi x}{l}\right) \text{ etc.}$$

This is plotted in Fig. 7.

The maximum voltage gradient is the derivative of the above.

$$\text{max. } \% \, e_s = \frac{d\,e}{d\,x} = \frac{1}{l} + \frac{0.58\,\pi}{l} \cos \left(\frac{\pi x}{l}\right) + 0.23 \times \frac{2\,\pi}{l} \cos \left(\frac{2 \pi x}{l}\right) + 0.13 \times \frac{3\,\pi}{l} \cos \left(\frac{3 \pi x}{l}\right) + 0.06 \times \frac{4\,\pi}{l} \cos \left(\frac{4 \pi x}{l}\right) \text{ etc.}$$

This also is plotted in Fig. 7.

Surge Impedance. By equation (**39b**)

$n =$	1/4	1/2	3/4	1	1 1/4	1 1/2	1 3/4	2
Z =	60,000	38,000	29,000	22,000	17,000	14,000	11,500	9,600

Symbols

α = $\sqrt{C_g/C_s}$

A = Constant of integration

B = Flux density, also constant of integration.

C = Capacitance.

C_g = Capacitance to ground (total.)

C_s = Coil capacitance (total), from line end to neutral.

E_0 = Terminal voltage, *i. e.*, line to ground.

E_n = Max. value of the "n" th harmonic to the axis of oscillation.

e = Voltage to ground at any point at any instant.

e_s = Voltage gradient in the winding, volts per unit length, proportional to volts per turn.

f = Frequency cycles per second, $\omega/2\,\pi$.

g = Subscript meaning ground.
I = Current.
l = Length of coil (or coil stack), line to neutral.
N = Number of turns total, line to neutral.
n = Number of wave lengths on the winding for a given harmonic.
ω = Angular velocity, radians per second.
= $2\pi f$
Z = Surge impedance.

DISCUSSION ON "ABNORMAL VOLTAGES WITHIN TRANSFORMERS" (BLUME AND BOYAJIAN), NEW YORK, N. Y., FEBRUARY 21, 1919.

H. R. Summerhayes: I would like to have the author give some additional information on the value of alpha, it goes over one per cent; now, it has occurred to me that in some classes of apparatus, for instance, as induction regulators or in generator coils, that alpha might be less than one, I may be wrong,but it is even conceivable that if the iron is brought close to all of the coils as in a generator or as in a transformer which might have ground shields between the coils, that the value of alpha might be less than one. I would like to know how that would modify the conclusions.

I am interested to notice the difference between grounded neutral systems and those with isolated neutral. Apparently, in most cases the grounded neutral has some advantage in the effect on the apparatus, especially with rectangular waves.

G. Faccioli: I knew by experience and theory that transformer coils—like men—do not always cooperate, so that when an abnormal condition of operation arises some of the coils are left to take the brunt of the disturbance while other coils do not do their share. I also knew, as all the engineers interested in this class of phenomena knew, that the capacity of the transformer winding is the controlling factor of the distribution of voltage when a steep wave front impinges upon the winding but I did not know the definite law governing the relation between the distribution of voltage and the capacity. Now Lord Kelvin said that you never know a phenomenon until you can put it into figures. The authors of this paper have succeeded for the first time in putting this phenomenon into figures and therefore I feel that they have for the first time really explained it, rendering thereby a great service to the art.

C. A. Adams: I feel that we owe a debt of gratitude to those men who are willing to devote their valuable time to the preparation of papers of this kind. Such papers are not popular, and are frequently criticized by many who attend our meetings, but they are of the utmost fundamental and practical importance and constitute the foundation on which the practical designing engineer must build if he is to be sure of his superstructure.

Referring now to this particular subject, we all realize that the mathematical analysis is approximate. It is impossible to state in accurate mathematical terms the complete physical problems, as the conditions are too complex, but when the mathematical analyses have been made as carefully as we know how under the circumstances, and when the conclusions have been checked by careful research work, they become tremendously useful in rationalizing our attack on the practical problem in hand, and in dealing with new problems in the same field.

H. R. Summerhayes: It seems to me that the paper has a bearing possibly upon testing, I think the author mentioned that it might be possible to build transformers in accordance with these principles, which would be strong for the service and cheap, so to speak, but would not necessarily meet the present rules of testing, and I think that is where the paper has important bearing in its possible effect on the rules.

C. A. Adams: You refer to the Standardization Rules?

H. R. Summerhayes: Yes, the rules for insulation testing.

C. A. Adams: That is certainly a very important bearing.

J. F. Peters: (Abstracted by L. W. Chubb.) It has been known for many years that during disturbances high voltages are developed in transformer windings and especially in that part of the winding adjacent to the line. The paper is of considerable value and interest because it affords a method of approximately determining the magnitude and distribution of these abnormal voltages under certain conditions.

The authors have computed and plotted in Figs. 2 and 3 the voltage distribution in a transformer winding for various values of α when a voltage having a sheer front or rectangular wave is suddenly applied to the terminals. By an exactly similar method the voltage distribution in any coil of the stack can be determined. The authors have taken as a unit, one coil length. Now, if we consider one coil and take as a unit, one turn, and consider X along the plane of the coil (at right angles to the length of the stack) then the solution for the coil is the same as for the stack with grounded neutral. C_g becomes the capacitance between the coil under consideration and the next adjacent coil and C_s becomes the capacitance between the inner and outer turns of the coil. The voltage distribution within the coil

$$e' = E_0' \frac{\sinh \frac{a' x'}{l'}}{\sinh a'} \qquad a' = \sqrt{\frac{C_g'}{C_s'}}$$

E_0' = voltage across the coil considered.

e' = voltage distribution in coil.

a' is always greater than zero and in many cases may be greater than a.

Therefore, the maximum voltage per turn may be considerably greater than that given by the slope of the curves in Fig. 2 and 3. In insulating the windings for these abnormal voltages between turns and between coils one has, as in the design of most apparatus, the choice of two evils. The voltage for any coil of the stack is approximately constant irrespective of the number of turns it contains. Therefore, decreasing the number of turns in the line coil in order to get more space for insulation increases the voltage per turn of the coil and one must choose between volts per turn and insulation space between turns.

The information that the paper gives on the effect of grounding the neutral is very interesting. For the example that the authors have computed, the maximum voltage to ground for grounded neutral is 1.5 normal as compared with 2.8 times for isolated neutral. The voltage per turn for the major part of the winding is approximately the same for both. It is four times normal for the isolated connection and five times normal for the grounded connection. The voltage per turn for the line end of the stack is the same for both connections, while for the neutral end, the voltage per turn is greater for the grounded connection, but is less than the volts per turn at the line end. Both ends of the stack are generally insulated the same for reasons of symmetry; therefore there is no additional difficulty encountered in the end turn insulation for the grounded neutral and there is a decided gain in the maximum voltage to ground.

I feel that the conclusions drawn regarding the transient voltage distribution following the initial voltage are not entirely fair. Equations (**33e**) and (**33f**) give these transients only when the damping factors due to resistance and dielectric losses are neglected. When including the damping factors the parts of these equations that are multiplied by E_n are also multiplied by the damping factors. For the high frequencies in these transients the damping is undoubtedly large and by the time all the harmonics have simultaneous maximums the damping factors will have gotten in their good work on decreasing the amplitudes and it is questionable whether the assumptions leading to equations (**36a**) and (**36b**) are justified.

F. Dubsky: The factor alpha for alternator, motor and induction regulator windings, having coils embedded in slots, may be estimated with the following values:

For one single coil $\alpha = 0.7$ to 2 (the larger value for deep and narrow slots).

For one phase of the winding $\alpha =$ (0.7 to 2) times the number of coils connected in series per phase.

Since the factor alpha for one coil is comparatively small it follows that the voltage distribution within a coil can be considered as uniform.

The factor alpha per phase may attain, of course, a considerably large value and produce, therefore, a greater concentration of voltage across the line coil.

Since all coils of the alternator, motor and induction regulator windings are usually wound and insulated alike, their layer and turn insulation should be selected in proper relation to the coil insulation to ground.

F. W. Peek, Jr.: The effects of sustained high-frequency oscillations have been mentioned. Some time ago we made extensive experiments with sustained high-frequency oscillations impressed on transformers. Voltages were measured over a wide range of impressed frequencies by gaps placed across taps brought out from the various coils throughout the winding.

It was found that voltages equivalent to line voltage could be built up across a very small fraction of the winding. The point or points of maximum voltage occurred at different parts of the winding, depending upon the frequency of the applied oscillations. These high internal voltages existed without appreciable increase in voltage across the terminals. A gap arrester is thus not effective for protection against these oscillations. Protection may be obtained, however, by shunting the apparatus by a condenser and resistance in series.

These data are of great importance, in design, to determine the best methods of applying insulation and of building transformers as free from the probability of high internal voltages as possible. Mr. Blume's paper is of importance from both of these standpoints.

Fortunately, there need be but very little fear of conditions producing high internal voltages on a properly insulated, connected, and operated system.

A. E. Kennelly: (communicated after adjournment). The paper is of special interest from the standpoint of artificial electric lines. It brings to our notice a new type of artificial line, and one which deserves consideration from the operating engineer. This new type is an all-capacitance line, with its series elements shunted by inductances. The conditions, however, are such that at the moment of initial disturbance, the inductance shunts are virtually non-existent, by reason of their sluggishness of action; so that the artificial line presented in a high-tension transformer is at first, very nearly a pure all-capacitance line. After the lapse of a certain brief period, the inductance shunts begin to come into play. The artificial line thereafter changes to one of mixed character, with condensers and inductances in series, and condensers in shunt. It is this automatic change in the nature of the artificial line of the transformer with time, that makes the case presented in the paper so interesting.

The theory of all-capacitance artificial lines has been already outlined.* It has been shown that in any pure all-capacitance line, the potential and current distributions are independent of the frequency, and belong to the hyperbolic-sine and hyperbolic-cosine series of numbers, whether the distant end be grounded or freed. From this point of view, the quantity α, defined on the second page of the paper, is the angle subtended by the line, uncorrected for lumpiness, is a real angle expressible in hyperbolic radians, and is the same at all frequencies.

---

* *Artificial Electric Lines,* by A. E. Kennelly, New York 1917, Chapter XVII.

The quantity described in (**21**) as the effective capacitance of the transformer, is the surge admittance of the line, when multipled by $j\omega$, where ω is the impressed angular velocity. Again the quantity defined as $\alpha X/l$, in (**11**) and (**13**), is the position angle of a point on the line. Equations (**13**) and (**14**) merely restate the well known propositions that the potential and current at any point on an artificial line are respectively proportional to the sine and cosine of the position angle at the point. It is to be noted that a value for α such as 10, given in the paper, is a very large real hyperbolic angle. It shows that such transformers may develop large angles, and that alternating-current artificial lines may present real angles, whereas it is very commonly supposed that all such lines develop complex angles varying with the frequency.

No oscillations are possible in a homologous line; of which an all-capacitance line is one type; because the energy is all of one type—in this case electric energy—and the energy must be capable of appearing in more than one type—such as either electric or magnetic energy—in order that oscillations may occur. When the inductance shunts are awakened, and come into action, the oscillations discussed in the paper begin to appear.

It will be of very great interest, and also of great practical importance, to ascertain how long it takes to wake up the inductance shunts, and how gradual the awakening is. It is to be hoped that the authors will attempt to answer these questions experimentally.

L. F. Blume: In reply to Mr. Summerhayes' question regarding the value of alpha (ratio of ground capacitance to series capacitance) this design constant in practise may vary from values approaching zero to very high values. It is possible to have values less than one where the ground capacitance is small compared with series capacitance. However, I doubt if this condition occurs very often in practise. In apparatus other than transformers in the majority of cases, the value of alpha is quite large. In the case of a transmission line the value of alpha is infinity because series capacitance is zero.

Mr. Peters raises the question concerning the initial distribution of potential within a coil. In the type of transformer winding analyzed in our paper, the value of internal capacitance (capacitance from layer to layer) is relatively large, that is, large compared with capacitance from edge of coil to ground, due to the small distance between layers. And since this capacitance is essentially in series with ground capacitance, it can have but little effect on the voltage distribution to ground, and between layers within the coil. However this is not necessarily true for all types of windings and for this reason, it is important that the distribution within the coil should be properly analyzed.

The influence of internal losses on oscillations is undoubtedly of very great practical importance because the insulation and other losses combined, very quickly absorb the energy of oscillation. It should be remembered in this connection that only a small fraction of energy of an impinging wave enters a transformer winding, the majority being returned to the transmission line by reflection. To take care of the energy losses in the theory would have made the mathematics very much more involved. We have only attempted to outline in a rough way the influence of losses in dampening out these oscillations.

Presented at the 348th Meeting of the American Institute of Electrical Engineers, Boston, Mass., March 14, 1919.

Copyright 1919. By A. I. E. E.

SOME POSSIBILITIES OF STEAM RAILROAD ELECTRIFICATION AS AFFECTING FUTURE POLICIES

BY CALVERT TOWNLEY

ABSTRACT OF PAPER

Electricity fills every requirement of railroad service, but as it involves a large investment, electrification has proceeded slowly. Electrification has also been retarded because the problem has been largely considered one of replacing the steam locomotive by the electric locomotive whereas in reality the problem is much broader. It really offers a fundamentally different method of train propulsion because the limitations of the steam locomotive disappear and the strictly limited motive power is replaced by one that is practically unlimited, thereby opening up many possibilities in the methods of railroad operation. While there are a number of different systems of electric traction all of the systems have many features in common and the possibility of unlimited electric power is a characteristic of them all. A brief review is given of electrified sections of railways showing the advantages which have been realized in both the freight and passenger service. Existing electrifications have been operated for a sufficient length of time so that operating statistics are now available, and any proposed undertaking may therefore be predicated on established facts. While electrification will greatly increase track capacity, there is a large railroad mileage which already has more than sufficient capacity, in which case electrification would not be justified. On the other hand, there are so many cases where its advantages are clear and conclusive that when the railroads are able to finance their required electrification it will test the capacity of the electric factories of the country to serve them.

ELECTRICITY now performs every railroad service previously rendered exclusively by steam locomotives and in every case does it better than it was done before. But in order to use electricity a large investment in equipment and installation must be made and electrification has proceeded slowly because railroad executives were not convinced that the advantages to be gained are always worth the cost.

The progress of electrification has also been impeded, first, before the war by the difficulty in financing, due to conditions other than the merits of electrification, and second, since the war began, because every one has been too busy to consider any work that could be deferred and because the government's taking over the railroads has created an unsettled situation not conducive to the investment of new capital for future returns. Now, however, there seems to be ground for hoping that these bars to progress will be removed in the not distant future so that electrification can be again studied on its merits, therefore our consideration of the subject is timely.

In reviewing the past twenty years history of this question, I cannot escape the conclusion that we electrical men, and not our steam road colleagues are responsible for the slow progress made. We have not known enough about either the science or the art of railroading. Our belief in, and our zeal for our own profession has lead us, albeit with entire honesty of purpose, to make more or less extravagant claims as to what we could do and to underestimate the cost of doing it. The inevitable reaction of mind which followed an accurate determination of facts of course disturbed confidence in our judgment. But if at times we have injured the cause of electrification by claiming too much, strange as it may sound, we have injured it a great deal more by not claiming enough. Electrical engineers not having always been railroad men, have been unable to study railroad problems as they should have been studied, that is to say with only real and not with any arbitrary limitations before them. It has been natural for the electrical man to ask the railroad man for a statement of the conditions he was expected to meet. It was equally natural for the railroad man to prescribe the conditions upon which his steam service was predicated. Under these circumstances the problem became largely one of replacing one sort of locomotive with another, and of balancing hoped-for economies in operation and maintenance on the one hand, against fixed charges for the additional investment required, on the other. Right there comes the mistake. A perfectly natural but yet a fundamental mistake, for which no individual or class should be censured but for which the unusual development of the art is responsible. We cannot blame railroad men for not being electrical engineers nor electrical engineers because they are not railroad men but the progress of electrification has had to lag until both should be able to see, each with the eyes of both. It is only by combining the railroad's man knowledge of the fundamental requirements of his service with the electrical man's skill in applying electricity to perform that service that all the possibilities of any specific problem may be developed.

The electrification of a railroad is not simply the substitution of one kind of locomotive for another. It is far more than that. It is the adoption of a fundamentally different method of train propulsion. It is conservative to say that, within the bounds of ordinary practise, electricity can furnish every train with all the pulling power that can be used. The limitations of the steam locomotive in this respect disappear and ruling grades rule no longer. A strictly limited motive power is replaced by one that is practically unlimited.

There are a number of so-called "systems" of electric traction and heavy emphasis has been laid by the advocates of each upon its points of difference from every other. So much has been said about these differences and so little about the points of similarity as to create an entirely misleading impression. It is a fact that there are more kinds and types of steam locomotives in use many times over than there are electric systems. It is a fact that except for the storage battery locomotive, which has but a limited field of application, all electric systems have many more common features than differences. It is a fact that they agree on fundamentals and differ in detail only. Their costs may not be the same, their efficiencies may vary but they all do their work and do it successfully and well. The possibility of unlimited electric power is a characteristic not of any one system but of all. It is due to basic differences between steam and electric equipment. A steam locomotive is a complete independent unit which not only generates but also utilizes its power. The electric locomotive generates no power at all. It is only a translating device receiving energy from an outside and a remote source. The electric power house always having much greater capacity than any one locomotive, can supply ample power for the heaviest train on the steepest grade. The steam locomotive which carries its own power house with it, is limited to the capacity of its one boiler. By the multiple unit principle, as many electric locomotives as may be needed can be coupled together and operated in synchronism by one crew from any cab. Any required tractive effort can thus be exerted without slipping the wheels, without imposing undue strains on the rails or bridges and without increasing the number of engine crews.

The business of a railroad is to transport freight and passengers. I put freight first because on the average it produces 73 per cent of the revenue. Unlimited motive power permits longer trains and higher schedule speeds. On the Elkhorn grade of the Norfolk & Western the schedule speed was doubled. It cuts the operating cost by hauling more cars with the same or a smaller crew. The Norfolk & Western uses two electrics to do the work of three Mallets. These new opportunities at one fell swoop banish many of the railroad's time honored traditions. The traffic possibilities must be studied from a new angle and advantage taken of every facility. It is a new thought to realize that train length is limited not by motive power but by the yard tracks and length of sidings, or that all the trailing tonnage that the draw bars will stand can be hauled. Nor are these new limits fundamental. Sidings can be extended, draw bars can be made stronger, if it pays to do it. In a word electrification opens up tremendous possibilities of increasing the freight capacity of a road and without it being necessary to build additional tracks.

While not as important as freight, passenger traffic likewise comes in for its share in the widened horizon and the vanishing tradition. Unlimited power of course is available but the absence of combustion is another basic advantage. Smoke and cinders disappear. Tunnel operation loses its terrors. Unobscured signals permit normal speeds with undiminished safety. Projects like the Pennsylvania terminal in New York depending entirely on submarine tunnel operation and previously impracticable, become immediately possible. Railroads owning valuable realty in cities can erect buildings thereon, where before smoky locomotives made any structure above the ground level impracticable. The aerial rights are now valuable. Multiple unit operation has in fact made suburban traffic. The rapid acceleration made possible by electric traction has directed attention to the equal value of rapid retardation and has quickened the study of braking accordingly; also of modified coach design to bring about the more efficient loading and discharge of passengers. These combined possibilities secure increased schedule speeds and attract patronage. The people not only get over the line in a shorter time but as a corollary more people get over it in the same time. Again it is seen therefore that in passenger as in freight traffic the ability to do something that could not be done before, rather than to do the same thing at a lower cost is the most valuable attribute of electrification, and again we find a greatly augmented capacity without the need of additional tracks.

It is not my purpose to make an exhaustive comparison of the relative advantages of steam and electric operation. That has been done often and well by others. What I have said about the expanding opportunities for electrified service is by way of illustration to emphasize my plea that the question should always be viewed in its broader aspect and not hampered and restricted within any narrower limitations than properly belong to it.

I am going to assume, then, the broadest possible treatment and to suppose that every electrification project is to have its pros and cons most fully examined. The real and vital question then is, "How far will this lead us?" "To what extent may we expect complete electrification of all our roads?" Parts of a number of them have already been equipped. Many of these are numbered among our prominent roads, successful corporations which have had the advice of the most highly skilled executives and engineers, and which are progressive. The service performed on the electrified sections comprises practically every kind of railroad transportation. The Bluefield division of the Norfolk & Western R. R. in West Virginia is an example of an important coal road operating through the mountains. The Chicago, Milwaukee & St. Paul 440-mile main line, through Idaho and Montana, demonstrates what can be done by a transcontinental carrier on a large scale with through traffic, both freight and passenger. The New York, New Haven & Hartford R. R. 73-mile stretch between New York and New Haven shows how through freight and a heavy passenger traffic can be taken care of on the most congested four track section of an important eastern carrier and what is possible for complicated freight yard operation, while the New York Central and the Pennsylvania out of New York City are splendid examples of our greatest modern passenger terminal electrifications. There are of course many other electrifications, but even if there were not, those named are of a character to command the respect and attention of the railroad world. Now, every one of these projects has been successful. Every one has justified itself. Nearly every one in its present scope represents an extension of the zone initially electrified, the most convincing evidence possible as to what views the operating companies hold regarding these several projects. Railroad officials are generally glad to give others the benefit of their experience so it is reasonably safe to say that operating statistics are available covering long enough periods so that the results to be expected from any proposed undertakings may be predicated on established facts and not upon theories. In the light of present day knowledge, therefore, what answer can we make to the question "Should all railroads be electrified?"

Taken together in 1910 there were in the United States 240,000 miles of railroad main line regardless of the number of tracks. Of this mileage approximately 1250 or one half of one per cent has been electrified or is today in process. The remaining 99½ per cent comprises of course, roads performing every variety of service. They range from the back country branch line built by some over enthusiastic promotor and now perhaps, operated as part of a large system, only because operation cannot be avoided and regularly contributing its annual deficit, up to the most important through arteries of travel upon which the commerce and industry of the nation depend. Every sort of community is served; every kind of railroading has its place in this vast aggregation of effort and the variables in the problem are so numerous and their nature often so profound as to well daunt the courage of one who seeks to formulate them for incorporation in a general statement. Fortunately or unfortunately, depending on the point of view, it has been my lot to have to deal with this electrification problem from both sides. At one period from the standpoint of an intimate affiliation with the development and manufacture of electrical apparatus and at another from that of one charged with official responsibility on the railroad's behalf. I am a thorough believer in the virtues of electrification and an enthusiast about the wonders which it can accomplish but I also have a keen appreciation of the almost infinite variations in the railroad problem and a very wholesome respect for the dollar. I do not believe that all railroads will ever be electrified. I am not sanguine even that all the tracks of any one really big system will be so equipped in our time. It is a question of economics, and the results will not justify the expenditures even when considered with such broad vision as that which guided the Pennsylvania in spending millions to put their passenger terminal in New York City without the prospect of a direct return. Electrification will increase the track capacity. But there are thousands of miles of railroad that have sufficient capacity now, frequently several times over, and where the wildest stretch of imagination fails to picture a future need of this kind. Electrification works wonders in suburban and interurban passenger service. I have ridden for hours across the western prairies without seeing a single town, much less a city where these advantages would count. Electrification effects marked economies in fuel, in maintenance, in labor and otherwise through a long list; but electrification calls for a heavy investment and unless these economies bulk large enough, the interest on such investment will wipe them out and turn the enterprise into a losing venture. I do not believe the cause of electrification is helped by undue optimism on the part of its advocates. Rather should there be an enlightened partisanship, enthusiastic where enthusiasm is justified but tinged with the sober conservatism of the man who has to put his own dollars to work.

There need be no discouragement to the electrical engineer in the views just given, nor to the railroad man who has looked toward the new motive power for salvation. There are so many cases where electricity should be used, where its advantages are clear and conclusive, that once the railroads escape from the financial slough of despond in which they are now wallowing and are again able to get capital for their needs, there will not be enough engineers, there will not be enough electric factories in the country to serve them. Every big system has need of electricity somewhere. For some small roads it may mean the difference between solvency and bankruptcy. I electrified a short derelict line for the New Haven Road between Meriden and Middletown, long before given over into the one-train-a-day-annual-deficit class, and turned it into a good earner.

There can be no rule established. Generalities are sure to be misleading but electrification is now firmly intrenched and successful. It is recognized by railroads generally as an effective agency with great possibilities and one which is particularly valuable for certain specific purposes. Time alone will tell how broad its application is to be but I am confident we can await developments with tranquility assured that the art is in a healthy condition and that progress will be along the right lines.

DISCUSSION ON "SOME POSSIBILITIES OF STEAM RAILROAD ELECTRIFICATION AS AFFECTING FUTURE POLICIES (TOWNLEY). BOSTON, MASS., MARCH 14, 1919.

F. H. Shepard: We all know that electrification is a large but not impossible problem. Every hardshell railroad man can tell you that electric service, wherever it is adopted, always gives that section of the railroad better service.

Now, the transportation problem in the United States is probably the greatest problem we have. The railroads are the greatest employers of labor. The expeditious and regular movement of traffic is a matter which concerns the individual activities of everyone of us. We all know how our industries were halted when our lines of transportation were severely affected, due to the embargoes imposed during the last two years, and especially how serious this was in the case of our war activities.

The development of the United States is dependent entirely upon the freedom of transportation. For the proper development of our industries and for the success of our undertakings it is necessary that the broadest vision be exercised in the expansion of our railroad facilities.

If railroads are to expand in the proper way, it is necessary that the compensation for their product be adequate. Unfortunately we have been carrying on for a dozen years in this country a policy of unwarranted domination and witless regulation which has resulted in the starvation of our railroads.

The major expansion of a railroad is determined by the conditions of the future. No terminal pays for itself the first year, but terminals are necessary to secure proper transportation facilities. Revisions of line call for large expenditures, but they are not always immediately remunerative. If the return to railroad corporations is sufficient to carry them along, and to insure a healthful condition of finance, the railroads are ordinarily keen enough to take account of the situation ahead of them and look into the future. I think this starvation, more than anything else, has been the cause for the halt in railroad electrification.

The question in its entirety is a vast one, and although we will never see all the railroads electrified, none of us can even now say when we will see extensive electrification. However looking at the proposition in general terms, I think it may be said that the cost involved is something of the order of that which would be required to duplicate the present way, or tracks which now exist. That involves an expenditure which, if applied to include all the railroads, would amount to something like $15,000,000,000. This expenditure of course, is unthinkable at the present time, but gives an idea of the size of the problem.

The traffic on the railroads of the country doubles about each twelve years. Electrification can, in any particular situation, double the capacity of a railroad. The cost to move traffic primarily consists of two items: First, the rent for the investment, comprising the returns to the stockholders and bondholders, and second, labor and material which is practically all the product of labor. Now, electricity is a tool, and should be so considered, when analyzing electrification. It is a relatively new tool, unlimited in extent as to the amount of power that can be applied to a train. Mr. Townley shows pictures of locomotives having 4000 continuous h. p., and 7000 h. p. during acceleration. I know of projects which contemplate the use of 8000 h. p. continuously for a single train, also of tests in which the input has reached 8000 h. p. for a single locomotive and 15,000 kw. for a single train. These are relatively large figures.

The use of electricity as power in industry is fundamentally a labor-saving device. The great expansion in England after the Napoleonic wars was due to the use of steam which supplanted labor. Electricity is more flexible and can secure in all industry and in railroad electrification as well, more effective use of labor.

All steam railroad operation has been built up around the steam locomotive. The steam locomotive requires attention at about 100-mile intervals, terminals have been built up, and the application of labor has been determined, by the limitations of steam locomotives, so that the present method of assigning labor has thus grown up and along with it a lot of arbitraries all of which prevent, in the substitution of electricity for steam power, the new tool from showing all the fundamental advantages it should produce.

It is quite conceivable, for instance, that a through train can take its electric locomotive and run 500 miles. The operation of an electric locomotive does not impose the burden upon the operator of that train that is imposed upon him in the case of the steam locomotive. The electric locomotive, properly operated, and adequately designed for the service, will run without frequent attention and at a very low minimum of expense for maintenance and upkeep as well. We all know that a motor in a factory, provided it is oiled and occasionally wiped off, will run almost forever. Your fan motors do the same thing. It is reasonable to picture an electric locomotive performing service in a somewhat comparable way. The electric locomotive does not possess all the variables which influence the proper performance of the steam locomotive—such as the kind of water, the grade of fuel, the condition of the flues, firebox, etc., the skill of the fireman or of the locomotive driver, all of which limit the ability of the steam locomotive to perform successfully, during shorter or longer periods. The steam locomotive is a 100 per cent engine when it comes out of the shop. Its capacity is never as good after it has seen service.

The economies of electrification which will be possible when this new tool is thoroughly developed and understood will be greater than we now imagine. The possibilities of railroad electrification I consider are scarcely scratched.

I have in mind a rather extensive project on which the cost of fuel was actually less than the contemplated cost for electric power, but the other advantages, by a rather more extensive revision of service than is ordinarily contemplated, show an overwhelming advantage for the electric proposition.

I think Mr. Townley has touched a very important point in saying that our vision has been rather too limited as to the possibilities of electric power in electrification, and that great developments will follow a better analysis, based upon the possibilities of revision of railroad service.

W. S. Murray: Epitomizing the paper itself, it seems that its fundamentals are: that we have been prone not to combine the railroad and the electrical man in the past. In the future we must have a composite man, with two eyes, one eye for the railroad conditions and the other eye for the electrical conditions. That is a very important part which has been omitted in the past.

Another point in Mr. Townley's paper is that we must not try to electrify all the railroads in the United States. That is certainly patent, for the various reasons presented by Mr. Townley. He has pointed out that in all forms of electrification, pertaining to terminals, freight, passenger and switching service, in fact in all services the electric locomotive has performed very successfully, and that is significant.

Now, the thing in electrification to which I would like to draw particular attention is this: If we selfishly think of the electrification of one road or two roads, it is very true that we may reap a great many economies from that electrification, but it seems to me that is not the right way to go after this matter. If we are going to make electrification a success in this country we must do district or regional electrification.

In 1914 I had the pleasure of discussing with one of our eminent Institute men a plan of electrification which I think will appeal to you. It comprehended a district of electrification. There are six roads on the west side of the Hudson River, just outside of New York City—the Lehigh Valley, the Central of New Jersey, Delaware, Lackawanna & Western, the Pennsylvania, the West Shore, and the Erie. It seemed to us at that time that there was opportunity offered to work out a Power and Equipment Company with the idea of applying standard equipment to these six railroads, for handling electric passenger, freight and switching service within division zones of the six roads. A power station of 200,000 kw., or whatever capacity might be necessary was contemplated for location central to the roads.

At that time the railroads were much embarrassed, as they are now, in the matter of finanace, but the thought occurred that the railroads could be relieved of the financial burden through the agency of equipment trust bonds to cover the cost of locomotives and multiple unit cars and the contracts for power would be a guarantee for the bonds necessary to build the power station thus making the only cost to the railroads the building of their over head contact systems.

It was contemplated that unit shopping facilities would be arranged to cover the equipments of all the roads and the building of these shops would be merged into the banking syndicate who would underwrite the concern to be known as say, the Power and Equipment Co.

I thought and pondered a great deal over that electrification problem and it rather automatically expanded itself from a district problem into a regional problem. I have given much thought to Secretary Lane's proposed super-power generation, transmission and distribution plan for application in the regional district between Boston and Washington. By such an arrangement is offered the opportunity of propelling all trains in this region by electricity, and at the same time supplying all industrial concerns with light and power. I am most thoroughly in accord with the Secretary's advances in regard to this matter. It contemplates a trunk line transmission system between Boston and Washington, the route of which will lie sufficiently removed from the coast, as a military precaution, and which will include the use of steel transmission towers supporting wires bearing 3-phase current at a potential, say, of 220,000 volts, these lines, probably three or four in number, lying parallel to each other and on private right of way.

The super-power transmission would tie together all of the stations, steam and hydroelectric, which latter might be built on the rivers flowing to the coast between Washington and Boston; and large power houses to be constructed at the mouths of mines. Into this super-transmission trunk line system, would be fed this highly economical power.

You might say, right away—what would be the advantage of such a line, and could we not, on account of the very large stations we have developed, produce power as economically, and why, therefore, go to this investment in the super-power transmission line?

Now, our operating investigations show that with units of 25,000 to 40,000 kw. we can safely depend on being able to produce a kw-hr. for something between 10 lb. and 15 lb. of steam. I venture to say that over that regional district the amount of steam required to produce a kw-hr. as applied to the railroads and to the factories will average 25 lb. to 30 lb. and consequently, there would be offered an enormous saving in the production cost of power. This is not, of course, to preempt the use of any large stations that are already in existence as they, if their efficiencies justified would contribute their quota of power. And even those plants of lesser efficiency than the super power plants to be built would serve two most important functions: *First,* contributing power during the peak hour of service and acting as voltage regulators thus equalizing the potential over the regional zone; and *second,* during the periods of lesser load offering their capacity as synchronous condensers so that the idle current would be absorbed by them, thus permitting the super-power plants to work their great units of forty to sixty thousand kilowatts up to their maximum and economic generating capacity.

A great number of bills, as you all know, have been held up due to the rather disorganized condition in which Congress found itself at the close of its last session, and this bill,together with the others, was sidetracked, but it is hoped that the new Congress will take the matter up just as promptly as possible and appropriate a sufficient amount of funds to make the investigation.

Now, I said a little while ago that I would like to speak of something other than electrification, or at least speak of it in connection with electrification. It is this—and it is the progenitor of electrification—the amount of coal that we are using unnecessarily in this country, and particularly in this so-called North East Atlantic Coast section, is criminal. I was talking with Professor Breckenridge of Yale University, who you know has the chair of Mechanical Engineering there, and he told me that we were mining from 75,000,000 to 100,000,000 tons of anthracite coal per annum, and that in seventy-five years our supply of coal would be gone. He told me, in nearly the same breath, also, that they were already seriously at work trying to develop bituminous coal furnaces for house heating, in order that this situation could be taken care of when the anthracite coal was gone. He followed these remarks by saying—"Murray, you may not know that over a very short period of years we have by leaps and bounds gone up from the mining of 100,000,000 tons of bituminous coal to over 500,000,000 tons."

Now, I ask you to visualize this matter. I know from our Government reports that today 33 per cent of our cargo space is taken up by coal shipments. I venture to say that the percentage in New England is 40 per cent.

Now, what is the reason? A very great deal of it can be put down to improper power application by virtue of burning two pounds of coal where one is only necessary, and probably three pounds. I think I can speak with some authority so far as the railroad end of this matter is concerned, as I have put in ten pretty intensive years in the matter of determining just what the coal consumption was as between steam and electric service, for freight, passenger and switching engines.

Now, what are the figures? With the units in the Cos Cob

Station of the New Haven Road averaging 4000 kw. it was shown that, beyond any peradventure of doubt, the saving in passenger service was 2 to 1, in freight service 2.5 to 1, and in switching service 3 to 1. So, you can see right away, from the railroad side, that the coal saving would be very prodigious, with the super-power transmission, as the energy received into the transmission lines would come, not from units of the size I have mentioned, but from units of ten times that size. In fact, I think I can visualize such a line, and I believe that we all in this room will see this thing some day,—such a super-power line picking out districts which will warrant substations, ranging from 100,000 to 150,000 kw., or maybe 200,000 kw., and distributing to them its highly economic power. I know that is a pretty large order and involves a large sum of money, but in the same breath I want to say we are on the verge of very large things. Do you realize that lack of power is now limiting our industrial expansion on the Atlantic seaboard? And when I say to you that industrialism, which so tremendously depends upon the railroads for shipment of raw commodities, has got much of its cargo space preempted for the carrying of coal, why is it that we cannot with reason view the construction of a common carrier system for power thus lifting the coal off the roads and automatically creating the cargo space for the transportation of raw commodities and finished products.

The world is full of big problems, and I believe there are just as many big men in the world to solve these problems. I want to offer you a slogan, which has come to me, and it is this—We spent billions of dollars for destruction, for preservation; now let us spend billions of dollars for construction, for conservation.

And there comes to me another slogan as I visualize this transmission line which we must back Secretary Lane in building, and that is: The primaries of today will be the secondaries of tomorrow.

We are developing today on the Housatonic River, with special reference to the storage of water, power stations that will contribute over 350,000,000 kw-hr. yearly and this is just a little section, in that tremendous industrial district of the Naugatuck Valley. What does that mean in national conservation? Why, it means an equivalent of 500,000 tons of coal a year. That will be one of Connecticut's contributions to conservation.

During the war I had the privilege and the opportunity of making a report not a great while ago to the Government in the matter of what should be done to take care of the shipping load in the Philadelphia district. The Hog Island situation was such that additional capacity was absolutely necessary in that district at peak load. There is a river up in Pike County, Pennsylvania, called the Wallenpaupack, which we found had a basin which would hold seven billion five hundred million cubic feet of water; the physical conditions offered a draw down of twenty-six feet. The head is 300 feet, and the penstock would have to be about 5500 feet in length. There was opportunity to develop anywhere from a twenty thousand to a one hundred thousand kw. plant. There was 80,000 to 90,000 kw. required at Philadelphia. The rainshed of that river is 240 square miles with something like 31 or 32 inches of rainfall per annum. The cost of that development and the 102-mile transmission line from Philadelphia was $12,000,000., and the cost of a corresponding steam plant of equal capacity at tidewater in Philadelphia, was $13,000,000, and the actual figures which the operating account showed, put to the credit of the hydroelectric station to supply that power in any form wanted, (kw. or kw-hr.,) $1,000,000 per annum. That meant the steam plant had to be amortized at $20,000,000, before it began to compete with the hydroelectric station.

There are quantities of resources in the form of water powers along the coast line between Washington and Boston, and the system proposed could be extended to Portland, Maine, in one direction, and down to Richmond, Virginia, in the other; but just as Mr. Townley has pointed out, the *sine qua non* as to whether or not electrification is applicable—I do not care what form of traffic it may take—whether it is used for transportation or for factories—traffic density is the thing that is going to settle the question. It is by the reliability and economy of the electric drive, over the steam drive, that the dollars are saved, and as has been pointed out by Mr. Shepard, you must consider the fixed charges that the investment must be burdened with, before you can give a concluding answer to the problem.

Now, I want to leave with you this thought. In discussing this matter of the super-power line with a number of engineers, it has been a significant fact that I have not found a single dissenting voice, and everyone has agreed that it is the proper thing to do. Somebody might say—Why, Secretary Lane's plan is all imaginary. Now, that is wrong, and yet I will confess in the same breath that there is imagination in it. I think a man who has not imagination needs a guardian.

No doubt there are in this room many holders of patents. You had to imagine the piece of machinery for which you secured the patent, before you could make it, and so in this case of the super-power line, it had to be imagined—the rest of it is entirely practicable. There is nothing in it that is impracticable. What I have said about the financial return can be proved in advance, before the scheme is put into operation, so, let us make the investigation and find out; do not let us get too far behind England—England is doing just the same thing. I admit England has not anywhere near the area the United Stsates has, but that is not the point. We have got as much area right in this district, this regional district I am discussing with you gentlemen, as England has, and we have the advantage that we can extend that district, and if we have to compete with England, maintain the American standard of wage and living. I am thinking that we must have some super-power scheme of generation, transmission and distribution of power to our factories, by which the necessary economies can be effected and thus put us in position to compete with the world's trade.

I would like to say that this nation owes a job to every returning soldier, be he returning from Europe or from his post in the United States. There are two million sons of America returning from over the seas. Can they not fairly ask—Having you been thinking of us over here while we have been fighting for you over there? Millions of dollars must be spent in factory and in field during this reconstruction period and by these factories, supplied by power from this super-power transmission and distribution line, it will be possible for us to create new national wealth and automatically devise ways and means for the employment of American labor, this will provide us also with a fair answer to the fair question of the returning soldier.

Before I sit down I would like to say just about two words about a paper which I read, written by Dr. Arthur D. Little. The paper is called "Developing the Estate." It is printed in the *Atlantic Monthly* of March. If I could be mean enough to take any more of your time, I would ask that I might read that article to you. What I have said is a patch. What he says covers the field, and I commend that paper to you gentlemen.

Carl Schwartz (read by J. B. Taylor): Ten years ago the impression prevailed quite generally that the highly successful operation of several systems of steam railroad electrification then completed was all that was necessary to open the door for the replacement of steam by electric motive power on a large scale. Few electrical engineers realized, in those days, that the cost of the necessary electrical equipment constituted only a small fraction of the total capital cost of steam railroad electrification. Many other essential items of expense and very large ones indeed, were either overlooked or not fully appreciated; for example, the rebuilding of tracks, the installation of new signal systems, the extensive additions to maintenance facilities, the changes in terminal and switching arrangements, elimination of telephone interferences, etc., etc.

Since that time the question has been in most cases, how can the necessary return be secured for the additional large investment required for electrification? Generally applied, and as matters stand today, electrification would result in an increase in the cost of transportation, because the additional burden of capital expense cannot be compensated for by a corresponding decrease in the cost of operation and maintenance.

It is immaterial to a shipper of freight whether steam or electric motive power is used, nor does a trunk line passenger care what type of locomotive propels his train. Trunk line electrification in general would largely benefit interests other than the railroads, the traveling public or the shipper. Only in the surburban zone of commuters' traffic, by affording increased speed and frequency of operation combined with greater convenience and comfort, does the value of electrification appear to the passenger.

Who, therefore, is to pay the bill?

Purely from a railroad standpoint, electricity has one important advantage over steam motive power, namely, it is a most effective means of increasing the capacity of the transportation machine, and where this applies to the extent of producing additional revenue so as to compensate for the additional investment, electric operation readily justifies itself.

One trouble has been, and it is today, that the balance sheets of the railroads have not shown a favorable relation between revenue and expenditures; otherwise, many roads would have been more favorably disposed towards electrification in spite of the heavy expense involved and notwithstanding animated discussions of the past as to which system was the best to adopt.

Having in mind that, in general, the electrification of steam railroads may be promoted by devising means for reducing capital cost requirements and by simplifying installation as well as operation and maintenance, the writer submits the following points for consideration:

1. The value of a clear line of distinction between:

(a) The generation, distribution and supply of power as a finished product to the transportation machinery.

(b) Railroad electrification proper. This is to cover only means for the operation of the transportation machinery.

Under this classification, item (a), namely, the supply of the necessary power, would be a matter for an industry already well established in the special business of commercial power production with all economic advantages accruing from a pooling of loads of a widely diversified character.

2. The value of standardization of all motive power means along the broadest lines. The writer believes that the time has arrived when this can be done, and that it does not necessarily mean the exclusion of special systems which have proven their merit.

John B. Taylor: Referring to the third paragraph of Mr. Schwartz's discussion, the sentence ending with the words "locomotive propels his train." I can hardly agree with that statement, because I do not think it is correct. I know that many parties, in taking a transcontinental trip, prefer to take the Chicago, Milwaukee & St. Paul, because for a number of miles it is electrified. They have the opportunity to see the mountain scenery free from smoke and nuisances, and that really is a large factor in their determination to take that line.

Mr. Schwartz says at the end of the same paragraph "Only in the surburban zone of travel."

Just why he concedes it in surburban travel and not in transcontinental travel, I do not know.

N. W. Storer: Mr. Townley is right in his statement that the electrical engineers are very largely responsible for the slow progress which has been made in electrification. We are responsible, as he says, in the first place, because we made promises, estimates, which were not fulfilled. It was impossible to fufill them. He is also right in saying that we did not promise enough. I do not believe there is a man in the room here now who can with any degree of accuracy or comprehensiveness say what must ultimately be the result of electrification in this country. I do not think there is anyone with broad enough vision to comprehend all of the advantages and the things which can be developed from a complete electrification of our big systems. These are simply possibilities of years of evolution. They have got to grow, and they will grow just as any other problem of electrification, of power-using apparatus, has grown.

One reason, as I see it, why the electrical engineer has fallen down, so to speak, in his work, is that he has not been close enough to the railway situation—the electrical engineer has seen just one part of the problem, and he has sought to solve that part. Possibly he thought he was solving the whole thing when he undertook to change the motive power. That is the thing that was uppermost in his mind, to get a locomotive or motor equipment which would handle the traffic which the steam locomotive has been doing. Of course, that, we now know, is a very small matter. We must thank our esteemed railway friends for holding back on the job of electrification. We have got to thank them for their conservatism, although it has gone pretty hard with us sometimes, to appreciate its value, but the steam railway man is one who is distinctly from Missouri. He must be shown exactly what he is going to get before he invests any large amount of money in it. There are occasionally men with very broad vision, who will go ahead and undertake things in a pioneer way, but, taken as a whole, the steam railway man must learn through his own experiences.

All who have had any experience in electrifying roads will understand that very thoroughly. We see the mistakes which they are making, we see them attempting to operate electric locomotives on exactly the same basis as steam locomotives have been operated, and we know they are going to have trouble. We tell them so, but it makes absolutely no difference, and they go ahead and operate them, until something breaks down. They have a lot of breakdowns. They have trouble, so much trouble that they begin to appreciate where they were wrong, and they go ahead and take steps to change the situation.

I believe that the whole method of operating the roads, which, as has been pointed out, has grown up around the steam locomotive, will have to be changed. I think that the changes will be so vast that we cannot really understand them at this time, because the power is so different and its application requires entirely different control.

One of the first things that the railway man must understand is the value of proper distribution of his power requirements throughout the day, twenty-four hours a day. With electric power we can on many roads supply power enough at the head of a train or at the head and rear of a train, to handle all of the traffic with one operation per day, perhaps one train would do the whole thing, but everybody knows that that would be a foolish thing to do. It would mean equipping the entire road for that particular maximum peak load, whereas, to get the most economical results, so far as the equipment and the distribution system and the substations, and locomotives, too, are concerned, you would want to distribute them over a very much larger period of time.

When it comes to finding just where the limit is I think we have got to strike a happy medium between the cost of labor, which increases with the number of trains, and the cost of the investment necessary to equip the road and handle the maximum number of trains. It costs money, for instance, a great deal of money, to equip the road to handle the fleets of trains which are sometimes sent out by a western road, maybe loaded with perishable freight, cattle or fruit of some kind. They want to send out a train every ten or fifteen minutes, or every half hour, for six or eight hours, and then the work is done, and they may not have any more freight for another week. The application of electrical operation to such a proposition is clearly out of the question. It would not pay at all to equip the road for such heavy service at such infrequent intervals. The load must be properly distributed. Lines like that, as I take it, are the ones where the conditions are arbitrary rather than real, as Mr. Townley has pointed out.

There is another thing—we are sometimes told that we must be able to empty a yard or clear a track of congestion, and perhaps send a train out every five minutes until the yards and tracks are all cleared. Consider what that means in cost of insurance—that is what it is—simply insurance against delays. Then maybe a delay of possibly four or five or six hours due to a wreck—the traffic piles up—and there are a half-dozen trains on the siding—but why would the avoidance of a delay of a half hour or an hour longer, in getting all of these trains out of the way, be worth double the cost of the substation and overhead equipment supplying that immediate section of line?

These are some of the points that occur to one in considering the electrification of our roads.

There is another point which has been touched upon in the discussion, and that is with reference to a general electrification of all of the roads of the country. I do not think we need to discuss particularly the possibilities of electrifying all of the roads of the country. We have troubles enough trying to electrify one-half of one per cent of the mileage. If we had ten per cent of the roads on our books at the present time, with contracts for electrifying them, we would be overwhelmed with business. It would multiply the necessary numbers of engineers by fifty, or possibly one hundred. All we need to consider at the present time is the most crying necessity in the railroad field—the tunnels, the heavy grades, and the big passenger terminals.

Speaking of some of the advantages in the big terminals, for instance, Mr. Townley mentioned the advantage that could be taken of the aerial rights of the railways, and that has always appealed to me as one of the big things, to enable a railway occupying very valuable ground in a large city to cover their tracks over completely with buildings.

I have felt that it would be quite possible to build immense warehouses over these tracks in the congested parts of the city where the freight could be removed aerially. For instance, is there any reason why freight cars should not be run in on an elevator in one of these buildings right over the track, and shoved up to a higher floor, unloaded where it is not occupying the track at all—run off the track entirely and unloaded—and maybe loaded up with new freight in the same place and quickly sent out?

Our biggest trouble has been in the terminals situation—the terminal congestion around the big seaboards was simply monumental, and it was responsible for the unprecedented delays in the operation of our transportation system and the breakdowns which took place a year ago. Cars were held for weeks, in some cases, where a day should have been sufficient to unload them and send them back over the line to earn more money.

The vision which Mr. Murray has unfolded to our view here today is most inspiring. I wish I were as tall as he is—if I were I feel I might see over some of the intervening space and obstacles in the way and see the day, even clearer than he has pointed out, when this will be an accomplished fact. All of these things which he has pointed out to us are easily within the range of possibility, and what we must look after in the immediate future is the conservation of our resources. We owe it to the future generations to be more economical in the handling of the resources of the country, because in a few years many of these commodities, like coal and iron, will become much more expensive, and they will become so scarce that it will be absolutely imperative to develop other means of supplying the power requirements for the country. One of the greatest things we can do is to exercise all of our ingenuity in saving power. It is said that a public benefactor is one who can make two blades of grass grow where only one blade grew before, and along the same line, if Mr. Murray, or some of the other engineers in that line of work, can make one pound of coal do as much work as two or three pounds can do now, one can see what that means when it is multiplied by the hundreds of thousands of tons of coal which are used every day.

I have been immensely interested in the economies that have been introduced in many of our street railroads, and steam railroads as well, in the saving of power, saving of coal. In the case of the street railroads a saving of ten to twenty per cent in the saving of power consumption has been brought about by different means of operation, introduction of some of these coasting clocks or watt-hour meters or similar devices, something to put a check on the motorman, as by this means they can easily save ten per cent, and possibly in some cases fifteen per cent of the power consumption.

The use of a more efficient form of control, as by controlling the speed, partly by means of field variation, known as field control, will save ten per cent of the power that is ordinarily required to operate street cars and trains on elevated railways, and if it becomes possible, as I believe it will in the near future, to introduce regenerative control in the case of some of these frequent stopping trains, there will be a possibility for another ten, or fifteen, or twenty per cent saving. We have got to be open minded in regard to propositions like that, and must seize every opportunity to bring them about.

The cost of electrification is very large, but it does not look one-hundredth as big to me as it did four years ago. When we consider that Great Britain spent enough per day during the years of the war to have built the Panama Canal in possibly ten days, and our country was spending enough in four or five days to electrify all of the terminals in Chicago, what does the cost of electrification amount to? It is a drop in the bucket, and does not amount to anything.

Our people have become so used to giving, in the last few years especially, that I believe they are going to insist on these improvements, not simply because they are going to save money, but because they are going to require better service, and it is going to be necessary because of the saving in labor. It will increase the productiveness which we must have in order to avail ourselves of the luxuries that are possible in this age.

W. B. Potter (read by S. T. Dodd): Progress in any branch of scientific endeavor would be automatic, if it did not involve some change in well established practise, and usually the physical changes involve an expenditure of money. I believe this is especially true of railroad development. Mr. Townley's reference to these requisites is worthy of careful consideration.

Electrification necessitates no radical change in the handling of transportation but minor changes are usually essential in order to secure the full benefit made possible by the increased power, speed, and continuity of service in the individual unit as compared with steam. The objection is sometimes cited that electrification as compared with steam does not provide equal facilities for handling an emergency congestion of traffic. This is so affected by other conditions as not to be always true, but, granting this truth in some instances, it is an emergency only and should be regarded as such. This condition I believe is more than offset by features of electric railway operation which go far towards removing the possibilities of congestion. Consider one item alone: that the electric locomotive is at its best during coldest weather, under conditions in which the steam locomotive is most limited in its capacity. This feature was especially noticeable during the winter of 1914 and 1915 on the C. M. & St. P. electrified section. During this time there was only partial electrical operation and there were long periods in the Rocky Mountain division when the thermometer stood at 40 degrees below zero. Numerous instances occurred during this cold spell of steam locomotives being frozen up and with their heavy freight trains standing dead on the main line. Under these circumstances the electric locomotives had an opportunity to demonstrate their freedom from temperature limits by repeatedly going out and clearing the line of dead trains and engines.

The investment for equipment is undoubtedly the most influential factor affecting the advancement of electrification. Herein exists the duty and opportunity of the electrical and the mechanical engineer. With the engineer rests the responsibility for devising and utilizing such apparatus and equipment as will insure the best economic return in successful service.

A subject of present consequence and of vital importance in the future is the fact that electrification operated from hydraulic power offers the only known method of conserving our limited supply of coal and oil. Even where hydraulic power is not available a saving of over two-thirds of the fuel, and much of the fuel haulage, can be obtained by the erection of steam power plants suitably located and furnishing electric current from coal burned in modern power houses. An indication of the coal consumption and the traffic lost incident to burning fuel on a small scale in individual units is given by the fact that nearly one-fourth of the coal mined in the United

States is used on the tenders of our steam locomotives, that five per cent of the ton mileage moving over our railroads is occupied with hauling of this coal for railway purposes and that seven per cent of the ton mileage is occupied with hauling this coal again in tenders back of the steam locomotives.

To repeat the statements above in a little more concrete form I may say that the coal and equivalent oil used on steam locomotives in the United States for the year 1914 was equivalent to approximately 140,000,000 tons of coal. The ton miles moved during the same period, excluding the tonnage of locomotive tenders and excluding 75 per cent of the railway coal, amounted to 930,000,000,000 ton miles. If the same tonnage had been moved electrically, basing our figures on a power consumption of 40 watt hours per ton mile, the annual power consumption would have been 37,200,000,000 kw-hr. If this electric energy had been obtained from steam driven power houses with an efficiency of 2.2 pounds of coal per kw-hr., there would have been required 40,000,000 tons of coal instead of the 140,000,000 actually used. This shows a net saving of 100,000,000 tons of coal in one year. This is based on generating the current from steam driven power houses, whereas a still greater saving of coal would have been shown proportionately to the extent we assume hydraulic power to be used.

There are frequently misconceptions as to the amount of power involved in railway electrification. The figures given in the last paragraph form a basis for illustrating this rather emphatically. If the tonnage moved in the United States on steam railroads could have been moved by an expenditure of 37,200,000,000 kw-hr., this would have been equivalent to an average load of 4,250,000 kw. This is not an exorbitant amount as compared with the power stations already installed. The data for the year 1917 indicate that the power station capacity in the United States for central stations, electric railways, and isolated plants, is of the order of 20,000,000 kw. In other words, there is already installed electric power station capacity equivalent to four or five times the power which would be required for operating all the railroads in the United States if the same were electrically operated.

While we agree with Mr. Townley that it is very improbable that all the railroads in the United States will ever be electrified, it is at the same time interesting to note what the power demand of these roads would be and the saving in coal which would result from their electrification.

The further development of central distributing stations with either fuel or water as their source of power, will afford the most direct means of conserving coal. The existence of such large central stations located at strategic points throughout the country, even if primarily installed for railway use, would of itself further develop the use of electricity for various industrial purposes, and by so doing would affect the coal consumption more widely than in the item of railway coal alone.

George F. Sever: I want to bring to you a message from the Pacific Coast where I have been stationed for about seven months. Coal is almost an unknown quantity in that part of the world. One thinks only in terms of oil and water.

There oil is used up and down the Coast for electrical generation and industrial operations, and by the railroads. The railroads use somewhere between 30,000,000 and 50,000,000 barrels of oil per annum, while public utilities in California have used in 1918 about 10,000,000 barrels of oil. Our new commercial shipping fleet is asking for 34,000,000 barrels of oil for the year for carrying our commerce over the seas. Besides these amounts there are about 40,000,000 barrels of oil used for other industries up and down the Pacific Coast.

California itself produces about 110,000,000 barrels of oil per annum, while the whole country, at the present time produces about 350,000,000 barrels. This means in California where much of the oil is used for the operation of the railroads, that hydroelectric development must occur not only for the prospective operation of the railroads over the mountain passes, but also for the operation of the electric public utilities that are using approximately 10,000,000 barrels of oil per annum.

The Oil Administrator for the Pacific Coast, as well as the Federal Oil Administrator, feels that the oil production of the United States will not increase at a high rate. As a matter of fact, they contemplate that it will decrease. The price of oil in 1914 was between 70 and 85 cents a barrel, and at the present time in California, at tidewater, the price is approximately $1.60, and in other parts of the Pacific Coast it ranges from $1.60 to $1.85 per barrel. Every public utility that has available undeveloped hydroelectric power should substitute this for its oil-burning operations, down to a certain limit.

I made careful investigations of hydroelectric possibilities and I advocated developments which, had the war lasted two or three years longer, would have been made to meet the growth due to war needs and those of the different industrial communities. It was clearly shown that over $3,500,000 could readily be saved by the installation of hydroelectric developments in California,

There are three mountain passes of the railroads that can be readily electrified, the Siskiyou in the north, the Sierra Nevada in the east, and the Tehachapi in the south, where both the grades and the loads are heavy.

This country should take every step to conserve fuel oil and the railroads especially should be carefully studied in an endeavor to co-operate in this respect.

Last spring, I made an extensive investigation of the hydroelectric power possibilities in New England in connection with the War Industries Board. At that time there was set forth to the Board the necessity of delivering all the available hydroelectric power in New England into the present transmission trunk lines, increasing and delivering to the same lines from the south and east all the steam-generated kilowatt-hours from the present steam-stations.

The number of tons of coal used by public utilities and manufacturing establishments in New England is now so large, that if all the hydroelectric kilowatt hours available were developed, their substitution for steam generated kilowatt hours would afford slight opportunity for transmitting any hydroelectric kilowatt hours outside of the confines of New England itself.

Charles F. Scott: Mr. Townley has presented a large view of a very large subject. He points out that it is the removal of the limitations of the steam locomotive which is the great thing in railway electrification. This is also the great thing in electrical work of many other kinds; electricity not merely does better what was done before but its great value lies in the new methods possible, due to the removal of prior limitations.

The removal of the limitations in the mechanical transmission of power have made possible the generation of power in new ways, both in great steam power stations and in the large utilization of water power. In its applications to lighting, to power in industries, to commercial heating, it is the new possibility of the effective application by which limitations in other methods are overcome and results of a new kind are secured which bring out the higher values in substituting electricity for other agencies. We have been principally concerned in the past as to how well the electric locomotive could do what the steam locomotive did, usually under some particularly trying situation such as tunnel service, but now Mr. Townley invites attention to the changes in railway operating methods which may bring out the full advantages of electrification.

At the convention at Great Barrington seventeen years ago we were wondering how we could use a voltage above 500 or 600 volts in railway service. Soon the single-phase system appeared, presenting one method, then followed the direct-current and now both are in operation and we can hardly tell which is the direct-current and which is the alternating-current system by the character of the service which is performed.

Another large vision has been presented to us. There has been a change in the attitude of engineers. A number of years ago the American Society of Mechanical Engineers was received by the Mayor of the city where the convention was held and in a fine address he greeted them as a Society of Stationary Engineers, the men who run engines. We have been addressed by a Governor today and what does he say? He says that we are those who minister to the general welfare;

that we have a goal towards which we are working which is the realization of the aspirations of mankind; that we are men engaged in making this world a better place to live in. It is very heartening to be greeted in such terms and it puts upon the engineer a great responsibility.

I wish to propose, Mr. President, the following resolution, and will move its adoption:

RESOLVED, That this meeting heartily endorses the investigation of the power resources and requirements of the North Atlantic coast region which is advocated by Secretary Lane, and that the Board of Directors be requested to consider the passage of suitable resolutions, and the offering of the facilities of the Institute, in support of the project.

George F. Sever: I second the resolution.

(The resolution was put to vote and unanimously carried).

George Gibbs (by letter): I was especially glad to see Mr. Townley bring out a point which needs discussion, that is, the best manner of obtaining coordination between the electrical engineer and the railway operating man, so that a railway electrification may represent all the advantages to be derived from improved motive power and operating methods combined.

In the early days of electric traction (not so long ago, either) railroad men were principally concerned with the question whether electric apparatus was capable of performing heavy railroad service, and electrical engineers were busy weeding out defects and limitations in apparatus. It was during this period that much heated discussion arose among engineers as to the best electric system for general adoption. The settlement of this particular question at the time was, to say the least, premature, as has been demonstrated by the parallel development since of two or more systems having great flexibility. Even now, when it has been conclusively proven that electric traction by more than one system is technically feasible, evolution is still in process, and standardization of system and a discussion of standardization apparatus, except as regards certain general features should, I think, be postponed.

The question has really now changed from one of technical to one of financial feasibility; in other words, railway men wish now to know under what conditions heavy steam railway electrification is a good proposition as regards operating and financial results. There are few railroad companies who can, in these times, face the very heavy expenditures required for electrification except to secure immediate and large operating economies either directly or by an increase in the capacity of the road, or in the stimulation of new business, any one of which factors or all taken together producing added net revenue sufficient to at least pay fixed charges on the new investment. Of course there have been a few cases where absolute necessity determined electrification to make possible an important railway improvement, for instance the underground terminals and tunnel systems in New York City; but aside from such cases, the essential thing is a demonstration of the financial feasibility before general adoption of this modern improvement in operating means can be expected.

As the Institute members doubtless know, the attention of our Government was directed during the war toward electrification, as a possible way of increasing the capacity of the railways for emergency service; also for conservation of our fuel supply. It may be that some important electric projects would in this way have been launched, as war measures, had the war continued, especially as results and not costs were then all-important. But it was quite evident that the Railroad Administration was somewhat at a loss to determine just what it should do in the matter, in view of the rather conflicting views expressed in and out of railway circles by technical and non-technical advisors. This brings me back to the main point brought out by Mr. Townley, as of importance for us to now direct our attention to, viz: we must find some way of attacking the railway electrification problem with a combined technical and transportation knowledge.

I cannot too strongly emphasize the fact that any general electrification of the entire railway system of the country is an economic absurdity by any present electric system. But there are, on the other hand, a number of instances where electric traction is indicated as necessary or advantageous. In some cases the direct savings will be sufficiently attractive to warrant the expenditure, but more often the deciding factor should be the indirect savings produced. The determination of these indirect savings generally involves important alterations in operating methods, facilities, etc., and it is therefore essential that engineers who are called upon to report upon electrification projects should be familiar with transportation methods and possibilities as well as with the electro-technical side of the problem. I regard unfamiliarity with the actual workings of railways as the chief handicap of the purely electrical expert in giving advice to companies who are considering the adoption of this great modern improvement in haulage means.

John Murphy: In Canada, as many of you know, we had a very serious coal shortage extending over two years, and a power shortage in part of Ontario extending over one and one-half years. It was my lot to be fairly close to these two shortage situations, and the question of railway electrification consequently came somewhat forcibly to my attention.

My view of the urgency of railway electrification as a measure of conservation can probably best be summarized by the following statement:

Only three or four per cent of the mechanical energy (represented in the heat value of the coal) is available in the shape of power at the wheels of a steam locomotive. Is it not our duty to try to save a great deal of that 96 or 97 per cent which is now wasted? Railway electrification can save from 50 to 66 per cent of the railway coal when the electrical energy comes from central steam stations. Water power can save it all.

But for the generosity and the wisdom of the U. S. Fuel Administration there would have been much suffering in Canada and many might have died from the cold during the past two or three winters. Our ships, as well as our other transportation facilities could not bring us coal because they were carrying men and materials to Europe to fight the common enemy of humanity. The officials of the U. S. Fuel Administration acting upon the advice of Canada's Fuel Controller, Mr. C. A. Magrath, generously and, as I said, with wisdom, alleviated a great part of the burden that was upon us when they sent us coal. In return, power from our hydroelectric developments came to the U. S.

The railways use such a large percentage of the output of the mines, and they use it so wastefully, that it would seem imperative to compel them to show reasons why their busiest lines should not be electrified at early dates.

H. D. Jackson (by letter): I heartily concur with Mr. Murray in the possibilities of what he terms a super-transmission line between Boston and north of Boston and Washington and south of Washington. The economies to be effected by the installation of such a transmission line will, I believe, very quickly pay for the investment and result in extremely cheap power for railway and other large industrial propositions. This, however, will not solve one of the big problems that Mr. Murray made a point of: that is, the use of coal. One of our largest uses of coal is for heat, although of course its uses for power are very, very great. But in most cases where power is generated coal is used with fairly good economy. I can look out of my window and see in all directions smoking chimneys, and it is these chimneys where the greatest waste occurs. The super-transmission line will in no sense eliminate this waste. In fact, it is my belief that cheap power will result in most cases in an increase in the waste of coal, as heating plants are notoriously wasteful in this way. One of the real items which is before all of us engineers is an improvement in the burning of coal as well as an improvement in the use of power.

C. Townley: Mr. Shepard very happily epitomizes the situation by saying that all steam road operation has been built up around the steam locomotive, and it of course naturally follows—as I have tried to demonstrate—that the removal of many of the limitations of the steam locomotive completely transforms many fundamental questions in railroad operation. As Mr. Shepard has put it "Electricity is a tool" and it becomes at once the duty and privilege of the engineer to make the proper use of it.

Mr. Murray in making some very interesting comments on the possibilities of cooperative action by two or more railroads operating adjacent tracks by reason of which all rails in a given district would be electrified, emphasizes a fact which has now come to be very generally recognized, namely, that it is not economical for a railroad to generate its own electricity. Both the load factor and also the plant or output factor of the load of an electrified steam road are very poor. Therefore not only is the generation of current solely for that load likely to be relatively inefficient but the investment and consequently the fixed charges per unit of output will be extremely high. The cure for this condition is the combination of the railroad load with other loads so that advantage may be taken of the diversity factor thereby made available, an arrangement obviously to be brought about through a power company which supplies energy, not only to the railroad but also to industry.

Mr. Schwartz's communication contains the following statement: "Generally applied—as matters stand to-day—electrification would result in an increase in the cost of transportation." I am reluctant to believe that Mr. Schwartz really means quite that. Not even the most sapient can tell whether the electrification of a given project will result in an increase or in a decrease in the cost of transportation until after the most careful scrutiny of all existing conditions. If, however, Mr. Schwartz has drawn his general conclusions from the results obtained by the New York Central electrification his comment is not surprising. As is well known coincident with electrification the New York Central built their magnificent New York terminal costing many millions of dollars. They undertook extensive enlargement of their terminal trackage including the construction of two train levels and they made other improvements of various kinds, both to their roadway and to their equipment, all designed to take advantage of every opportunity for improvement, but many of the changes in no wise compelled by electrification. No doubt the construction accounts were kept in modern and approved fashion but I do not believe it to be humanly possible to correctly allocate every expenditure and to say that such an item was due entirely to electrification and such another not at all. Mr. Schwartz later says "trunk line electrification in general would largely benefit interests other than the railroads, the travelling public and the shipper". With this statement we can all agree but if as it seems to me probable Mr. Schwartz intended also the inference that trunk line electrification would benefit the traveling public and the shipper and would not benefit the railroad, I feel that the inference should not be passed over. Such a conclusion obviously could not be based on the New York Central operation because that railroad has no trunk line electrification. If it be not based on the New York Central experience, it would be interesting to know on what experience it is based because the two, perhaps most prominent examples of trunk line electrification, namely, that of the Chicago, Milwaukee & St. Paul and that of the Norfolk & Western Railroad, offer a contrary experience. It would be interesting if Mr. Schwartz could give us further information as to the basis of his sweeping generalization.

Mr. Storer's expression of appreciation to the railroads for deferring electrification is pertinent and timely. We have not always felt that way but now we know that too early an attempt to electrify before we really were ready would have brought forth such a multitude of troubles as would have—for a long time—gone far to discourage other roads from contemplated undertakings. In referring to the possibility of handling freight cars by lifting and cross transfer, Mr. Storer has touched upon a question of large possibilities. It is well known that of the time elapsing between the shipment and receipt of freight more is consumed in yards and terminals than in actual transportation. The possibilities of elevation of freight may some day be developed sufficiently to over-shadow in importance the value of electrified transportation itself.

Mr. Potter has given us some impressive figures as to coal consumption. These are always interesting as illustrations.

Major Sever's recital of the tremendous increase in both the use and price of oil once again emphasizes the virtue of economy. It is gratifying to reflect in this connection that the modern power house of today produces a unit of electric energy with about one-half the fuel required some 18 years ago and that the limit of improvement in this direction has not yet been reached nor is it in fact in sight.

Mr. Gibbs succinctly sums up the situation when he says that the question of electrification has now become one of financial rather than of technical feasibility. In other words no one longer doubts that electricity can perform all the transportation duties of steam, the questions for consideration being solely whether a given project will show a satisfactory return on the investment and can the necessary money be obtained. It is extremely gratifying to me to find no dissenting voice and to be able to close with an expression of appreciation for the many distinguished engineers who have been interested enough to discuss my paper.

Presented at the 348th Meeting of the American Institute of Electrical Engineers, Boston, Mass., March 14th, 1919.

Copyright 1919. By A. I. E. E.

ELECTROMAGNETIC THEORY OF THE TELEPHONE RECEIVER

With Special Reference to Motional Impedance

BY A. E. KENNELLY AND H. NUKIYAMA

Abstract of Paper

The theory of the telephone receiver here offered is based upon the motional impedance circle which has been published in various chapters during the last few years. The new theory, which is stated under definite limitations, is a further development; taking into account the m.m.f. produced by the vibration of the diaphragm in the permanent magnetic field. The motional power is thus shown to be derived partly from the testing alternating current and partly from changes in power expended in the magnetic circuit. The motional impedance circle may therefore also be regarded as a power circle, with components along three different axes of reference.

Object of the Paper. It is well known that every telephone receiver possesses a certain difference of vector impedance between the "damped" and "free" conditions, such that, when plotted in polar coordinates, over a suitable range of alternating-current frequency, gives rise to a "motional-impedance locus", which locus is approximately circular, and is known as its "motional-impedance circle".

The motional impedance circle of a telephone receiver affords valuable material for determining the principal mechanical and electrical constants of the instrument.*

In the theory of the motional-impedance circle, as hitherto published, certain phenomena have been left out of consideration for the sake of simplicity; and especially that component of alternating magnetic flux passing through the coils which is due to the displacement of the diaphragm from its zero position when freely vibrating. The result has been that the motional-impedance circle failed to reveal the power expended parasitically (by both hysteresis and eddy currents) in the magnetic circuit. This defect in the theory has already been pointed out by Eccles.†

---

*Bibliography, No. 9.

†Bibliography, No. 24.

It is the object of this paper to supply the above-mentioned omission, and thus to offer a closer approximation between the theory and the observed behavior of the telephone receiver as an electromagnetic motor. It should be pointed out, however, that the theory here presented is still far from being complete, and that much more work, both theoretical and experimental remains to be executed, analysed and reported, before the theory can be regarded as satisfactorily established.

Limitations of the New Theory. The following limitations should be noted in regard to the stage of attainment aimed at in the theory here presented.

1. The alternating current supplied to the receiver is of small amplitude, and of single frequency; so that the magnetic flux passing through the coils and polar surfaces may be regarded as a normal permanent flux, plus a small sinusoidal variation. All the alternating mechanical, magnetic and electric quantities, are therefore assumed to be simple harmonic functions of the time.
2. Throughout the range of frequency considered, the flux in the magnetic circuit of the coils retains the same magnitude and phase with respect to the constant exciting alternating current. This means that the influences of magnetic skin effect and hysteresis in the magnetic circuit remain substantially constant.
3. The proportional effect of a small increment of m.m.f. in the magnetic circuit, due either to a small increase in exciting current, or to a small diminution in air gap at the poles, shall be the same, both in magnitude and in phase.
4. That over the range of frequencies considered, the "force-factor" A, the "equivalent mass" m of the diaphragm, the "mechanical resistance" r of the same, and the "stiffness coefficient" s, remain substantially constant.

The theory is presented with relation to a series of impedance measurements already published* on a particular bipolar Bell receiver "*C*"; so that these measurements offer, in a particular case, a check upon the application of the theory.

Magnetic Circuit and Vibrating System. In Fig. 1, it is assumed that the two poles N and S are permanently magnetized, as though under the action of a constant-current magnetomotive force $\mathfrak{F}_0$ gilberts. Under the influence of the permanent magnetic flux Φ_0 maxwells in the magnetic circuit, the diaphragm becomes flexed from its horizontal plane position and comes to rest at a certain zero position at the same distance from each pole. Since vibrational displacements occur on each side alternately of this reference position of rest, we may reckon a displacement therefrom of x cm. towards the poles as positive, and away from the poles as negative. The total number of turns on both coils together, is taken as N. The active surface of each pole, or of the diaphragm opposite thereto, is taken as S sq. cm., and since the polar gap is small by comparison with the linear dimensions of this surface, we may neglect the magnetic fringe at the polar edges.

---

*Bibliography 17, pp. 422–428 and 449–450.

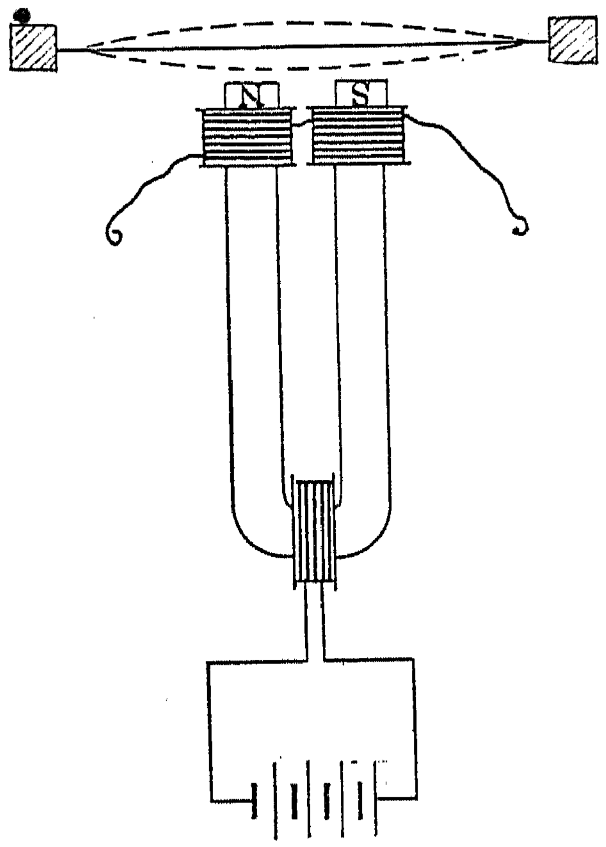


Fig. 1—Simplified Magnetic Circuit of Bipolar Receiver

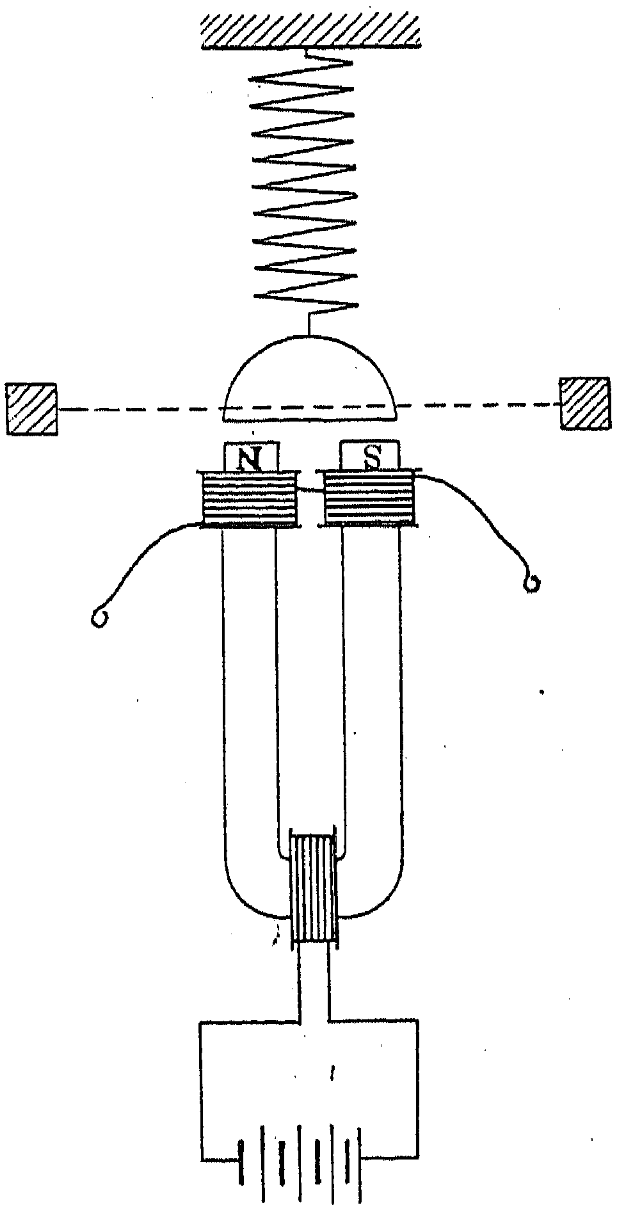


Fig. 2—Simplified Mechanical System of Bipolar Receiver

In Fig. 2, the mechanical vibration system is indicated as simplified into an equivalent mass m grams, under the retractive influence, so far as concerns alternating stresses, of s dynes per cm. of displacement. The frictional resistance force of this system to motion is taken as simply proportional to the velocity of displacement; so that if $\dot{x}$ represents the velocity $\frac{dx}{dt}$ of the mass m, the retarding force is $-r\dot{x}$ dynes, where r is the mechanical resistance of the mass in dynes per unit velocity, or dynes per kine.

The simplification in the vibrational system represented in the transition from Fig. 1 to Fig. 2 seems justified from experimental results.

Magnetomotive Force Damped and Free. When the diaphragm is damped, or prevented from vibrating, a simple alternating current of I r. m. s. absamperes will develop a corresponding simple alternating m.m.f. of $\mathcal{F}_i$ r. m. s. gilberts, in phase with I. This may be represented as in Fig. 3, by the plane vector f_i in the horizontal or reference-phase position. In the receiver analysed, the testing current I was 0.000204 absamperes r. m. s., and taking the number of turns on the two coils together as 1300, the m.m.f. thereby produced would be

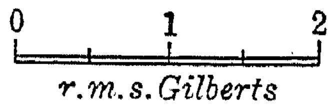


FIG. 3—M. M. F. DIAGRAM AT CONSTANT CURRENT—DIAPHRAGM DAMPED

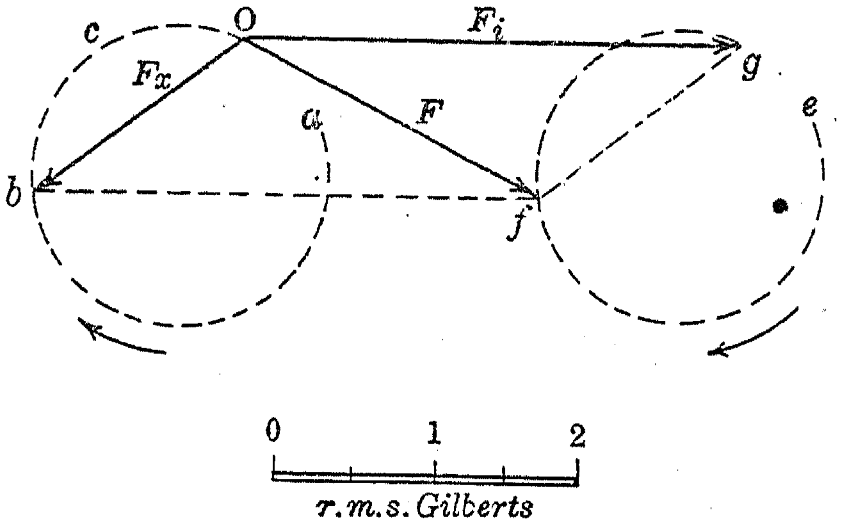


FIG. 4—M. M. F. DIAGRAM AT CONSTANT CURRENT AND 1028 CYCLES PER SEC. DIAPHRAGM FREE

$$4\,\pi \times 1300 \times 0.000204 = 3.334 \text{ gilberts r. m. s.}$$

This is represented, to a scale of 1.0 gilbert per linear cm. in Fig. 1, as a vector 3.33 cm. in length.

When the diaphragm is free, or allowed to vibrate, the same exciting current will produce the same m. m. f. as before; but a new periodic m. m. f. will be developed in the magnetic circuit, owing to the periodic change in diaphragm displacement, and length of airgap; so that the reluctance in the circuit of the permanent magnet will be periodically varied accordingly.

Let $\mathcal{F}_0$ = the m. m. f. of the permanent magnet (gilberts)
$\mathcal{R}_0$ = the reluctance in the permanent magnetic circuit (oersteds)
Φ_0 = the magnetic flux across the airgaps in the circuit (maxwells)

n in the magnetic circuit of the permanent magnet, there be a permanent magnetic flux which, neglecting magnetic age through the surrounding air, will be

$$\Phi_0 = \frac{\mathcal{F}_0}{\mathcal{R}_0} \qquad \text{maxwells} \quad (1)$$

e diaphragm is now caused to vibrate, by reason of independent variations of this magnetic flux due to alternating ent in the coils, the reluctance in each airgap of area S sq. will be varied by $-\dfrac{dx}{S}$ oersteds, if dx is the displacement he diaphragm from its normal position, towards the pole; hat the total change in reluctance will be $-\dfrac{2\,dx}{S}$ oersteds.

ut differentiating (1)

$$\frac{\partial \Phi_0}{\partial \mathcal{R}_0} = -\frac{\mathcal{F}_0}{\mathcal{R}_0^2} = -\frac{\Phi_0}{\mathcal{R}_0} \qquad \frac{\text{maxwells}}{\text{oersteds}} \quad (2)$$

$$\partial \Phi_0 = -\Phi_0 \frac{\partial \mathcal{R}_0}{\mathcal{R}_0} \qquad \text{maxwells} \quad (3)$$

$\partial \mathcal{R} = -2\,\partial x/S$; so that

$$\partial \Phi_0 = \Phi_0 \cdot \frac{2\,\partial x}{S\,\mathcal{R}_0} = \mathcal{B}_0 \cdot \frac{2\,\partial x}{\mathcal{R}_0} \qquad \text{maxwells} \quad (4)$$

s differential change in the flux through the magnetic circuit ch as would correspond to the action of a certain differential ressed m. m. f. $\partial \mathcal{F}_0$, such that

$$\frac{\partial \mathcal{F}_0}{\mathcal{R}_0} = \partial \Phi_0 \qquad \text{maxwells} \quad (5)$$

nsequently, substituting in (4)

$$\partial \mathcal{F}_0 = \mathcal{B}_0 \cdot 2\,\partial x \qquad \text{gilberts} \quad (6)$$

f, then, we know $\mathcal{B}_0$, the normal mean flux density across airgaps, an alternating displacement of ∂x cms. r. m. s. vards the poles at reference phase, will generate an equivalent tor m. m. f. according to (6) in phase with ∂x. This is wn in Fig. 4 by the plane vector $\mathcal{F}_x$. The phase of $\mathcal{F}_x$ with pect to $\mathcal{F}_i$ cannot be assigned until we know the phase of the phragm displacement ∂x, but means for determining this ase difference will be indicated later on. The locus of $\mathcal{F}_x$ is licated in Fig. 4. It is a "displacement-admittance" locus.

At a very low frequency, $\mathfrak{F}_x$ would occupy the plane-vector position $O\,a$. At a very high frequency, it would disappear towards $O\,c$. It would, therefore describe the locus $a\,b\,c$ in the direction of the arrow.

The resultant m. m. f. in the circuit is the plane vector sum of $\mathfrak{F}_i$ and $\mathfrak{F}_x$, and is indicated in Fig. 4 by $\mathfrak{F}$. At low frequencies, the size of $\mathfrak{F}$ exceeds the size of $\mathfrak{F}_i$; but towards higher frequencies, the size of $\mathfrak{F}$ falls below that of $\mathfrak{F}_i$. The effect, therefore, of the vibration of the diaphragm is to change the constant impressed a-c. m. m. f. $\mathfrak{F}_i$ to a correspondingly varied m. m. f. following the vector locus $e\,f\,g$, as the frequency is changed.

In Figs. 3 and 4, the vectors represent, to scale, the approximate values of $\mathfrak{F}_i$, $\mathfrak{F}_x$ and $\mathfrak{F}$, as derived from experimental tests of the selected instrument.

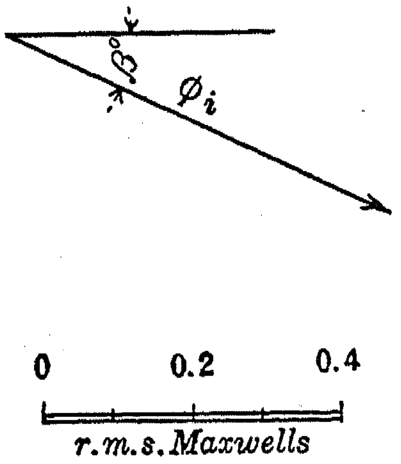


FIG. 5—FLUX DIAGRAM AT 1028 CYCLES PER SEC. — DIAPHRAGM DAMPED

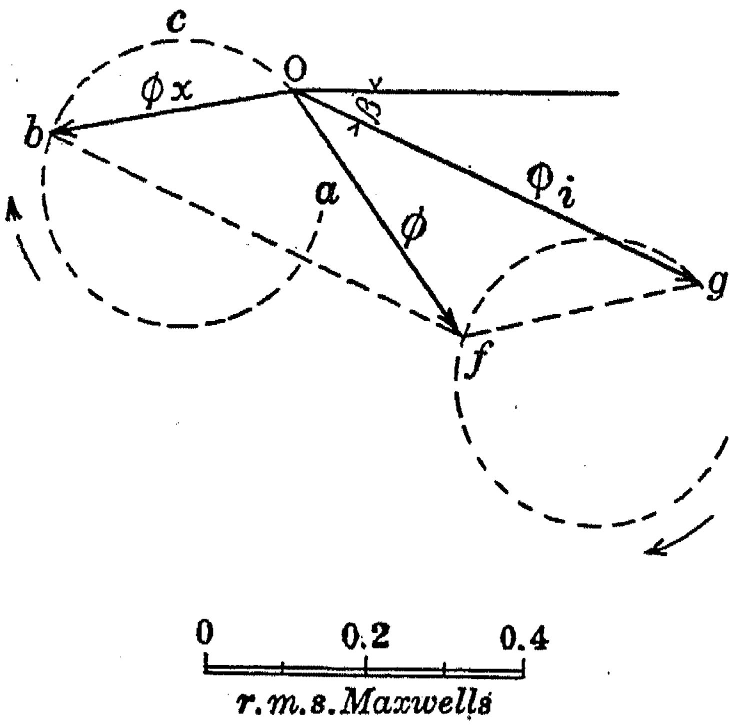


FIG. 6—FLUX DIAGRAM AT 1028 CYCLES PER SEC. — DIAPHRAGM FREE

Magnetic Flux, Damped and Free. In Figs. 5 and 6, the planevector alternating magnetic fluxes are represented with the diaphragm damped and free. When damped, as in Fig. 5, the alternating m. m. f. of $\mathfrak{F}_i$ r. m. s. gilberts at reference phase, produces an alternating magnetic flux of ϕ_i r. m. s. maxwells, lagging in phase $\beta°$ behind the m. m. f. The reason for the lag is partly on account of magnetic hysteresis, and partly on account of magnetic skin effect. This angle $\beta°$ (25.3 deg. in Fig. 5) can be determined from the observed motional-impedance diagram, on the assumption already made that β is half the depression angle of the motional-circle diameter. In certain cases, β is found to exceed 54 deg. although it is, more commonly in the neighborhood of 30 deg.

In Fig. 6, on the assumption already made that the lag β° is the same to both $\mathfrak{F}_i$ and $\mathfrak{F}_x$; *i. e.*, that ϕ_x lags β° behind $\mathfrak{F}_x$, we obtain the fluxes ϕ_i and ϕ_x with their resultant ϕ, as well as the approximate loci over which they travel under change of impressed frequency, β being taken as substantially constant over that range.

Actual observation shows that the angle of lag β_1 between $\mathfrak{F}_i$ and ϕ_i is not, in general, identical with the angle of lag β_2 between $\mathfrak{F}_x$ and ϕ_x, but their sum is 2 β, and the discrepancy is here neglected.*

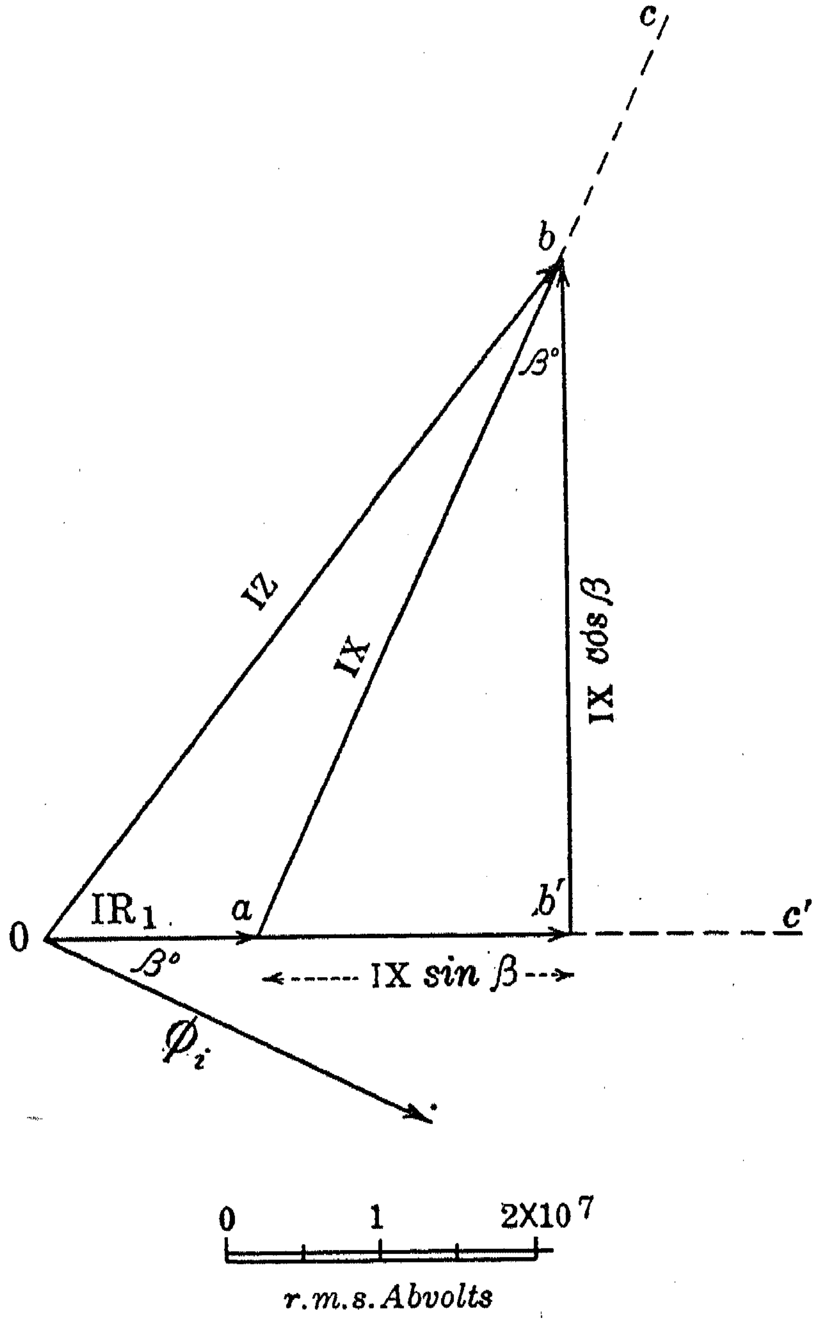


FIG. 7—E. M. F. DIAGRAM AT 1028 CYCLES PER SEC.—DIAPHRAGM DAMPED

Electromotive Forces, Damped and Free. Fig. 7 shows the alternating e. m. f. planevector diagram when the diaphragm is damped. With constant exciting current strength I r. m. s. absamperes, and constant assumed resistance R_1 absohms in the winding, the vector $I\,R_1$ r. m. s. volts, expended in overcoming the winding resistance, will theoretically remain constant at all frequencies. The voltage expended in overcoming the reactance of the winding will be $I\,X$ r. m. s. volts, at a leading slope of $90 - \beta°$; because this voltage must be in leading quadrature to the flux ϕ_i. The resultant voltage $I\,Z$ is that which must be applied to the terminals of the receiver, in order to maintain in its winding the required constant current strength I. The locus of the impressed voltage $I\,Z$ on the preceding assumptions, will therefore be the straight line $a\,b\,c$.

---

*Bibliography 18.

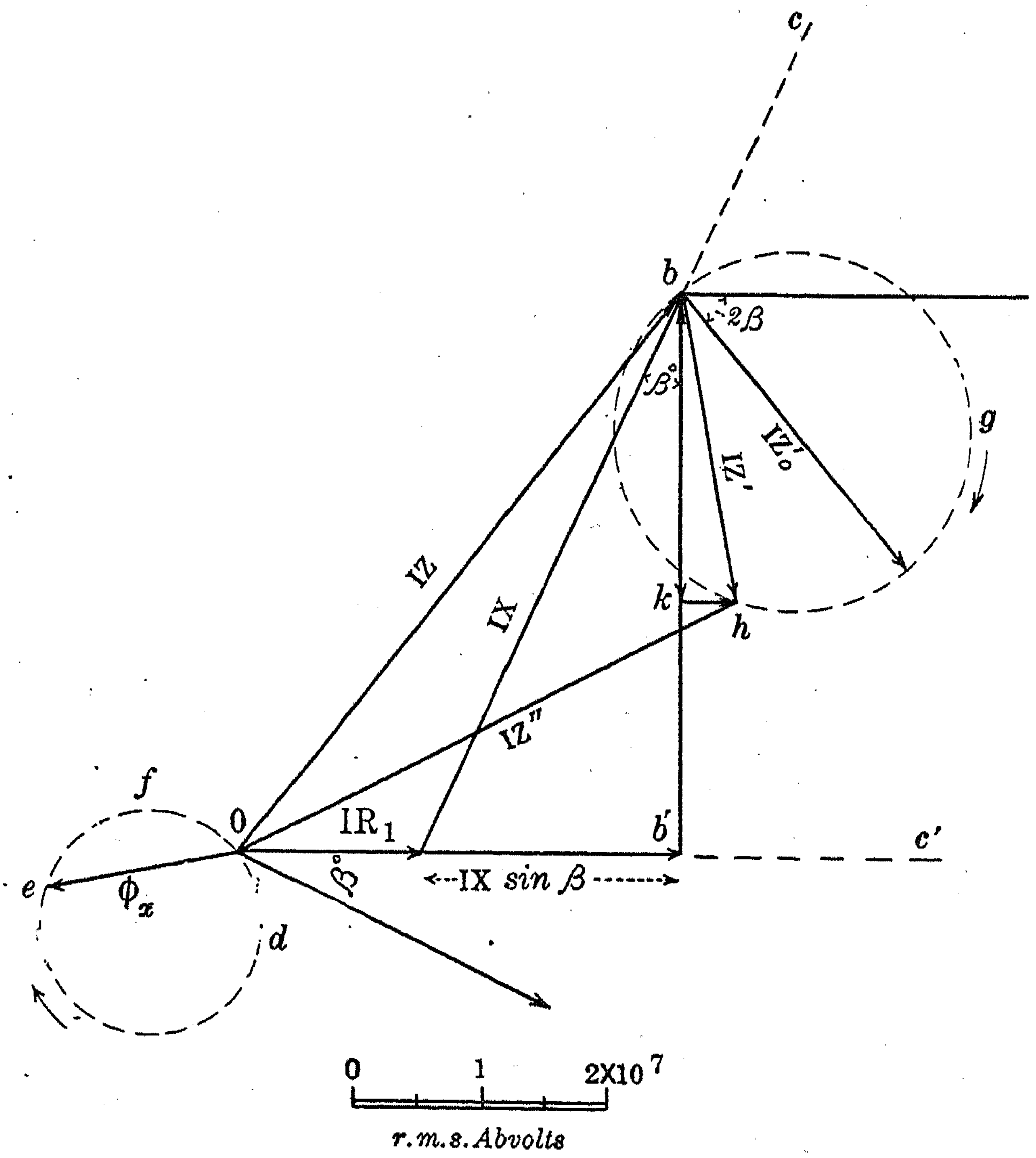


FIG. 8—E. M. F. DIAGRAM AT 1028 CYCLES PER SEC.—DIAPHRAGM FREE

The inductive voltage or voltage expended in overcoming the counter e. m. f. of induction is $a\,b$ on the diagram and is expressed by $j\,I\,\mathcal{L}\,\omega$ abvolts ∠, where $\mathcal{L}$ is a vector inductance of slope $\nwarrow\ \beta°$ as defined by the relation

$$\mathcal{L} = 4\,\pi\,N^2\,\mathcal{P} \qquad \text{abhenries} \angle \quad \textbf{(6a)}$$

where $\mathcal{P}$ is a vector permeance expressed in maxwells per gilbert ∠, or 1/oersteds ∠. The corresponding apparent resistance voltage is $O\,b'$ traversing the locus $a\,b'\,c'$. The apparent reactance voltage is $b'b$. This apparent reactance voltage, as obtained from impedance measurements with damped diaphragm, is

$$I\,|X|\cos\beta = I\,|\mathcal{L}|\,\omega\cos\beta \qquad \text{abvolts} \quad (7)$$

Here the symbol $|\mathcal{L}|$ denotes the size of $\mathcal{L}$ as a scalar.

The apparent inductance $\mathcal{L}$ of the winding should also remain constant. Actually, observations show that the apparent inductance diminishes as the frequency is increased, owing to the influence of changes in magnetic skin effect, which are assumed to be negligible in the stage of telephone theory here aimed at.

Fig. 8 represents the corresponding planevector e. m. f. diagram, with diaphragm free. In this diagram, the additional periodic flux ϕ_x, taken from Fig. 6, is included, and runs over the locus $d\,e\,f$. The vector impressed voltage $I\,Z''$, travels over the straight line locus $a\,b\,c$, as in Fig. 7; but at its end, there is added an e. m. f. locus due to the time variation of ϕ_x, which is a circle $b\,g\,h$. This circle moves parallel to itself, along the lengthening line $a\,b\,c$. The vector $b\,h$ is in leading quadrature to ϕ_x, in the position $O\,e$, and represents the motional impedance voltage $I\,Z'$ at this frequency, or the voltage expended in overcoming the displacement e. m. f. of the diaphragm considered as a motor element. The e. m. f. $I\,Z''$ which must be impressed at receiver terminals, in order to maintain constant current I, is represented by $O\,h$ abvolts. The component $O\,h'$, is the cophase component of e. m. f. and $b'\,k$ the quadrature or reactive component.

Impedances, Damped, Free and Motional. Figure 7, the damped voltage diagram, might also serve as an impedance diagram, if its various vectors are divided by the constant current I, and if, therefore, these vectors are read off to an appropriate scale of absohms. In Fig. 9, the transition is made, however, to a convenient resistance scale. The apparent resistance is $O\,b = R_1 + |X|\sin\beta$ absohms, the apparent reactance $b'\,b = |X|\cos\beta$ absohms, and the impedance with damped diaphragm

$$Z = O\,b = R_1 + |X|\sin\beta + j\,|X|\cos\beta = R_1 + X \quad \text{absohms} \angle \qquad (8)$$

As the impressed frequency increases, the impedance travels over the locus $a\,b\,c$. In Fig. 1 of the paper referred to, (Bibliography 17), the vector damped impedance locus makes an angle of $\beta = 25.^\circ 3$, very nearly, with the reactance axis, between $f = 700 \sim$ and $f = 1500 \sim$.

Fig. 10 shows the corresponding impedance of the instrument with the diaphragm free. The vector $O\,b$ pursues the same path $a\,b\,c$ as in Fig. 9; but at its end, there is added a traveling circular locus $g\,h\,k$. The impedance of the instrument is Z'', and we have the vector relation.

$$Z' = Z'' - Z \qquad \text{absohms} \angle \quad (\mathbf{9})$$

The vector path of Z', plotted separately, is the stationary circle

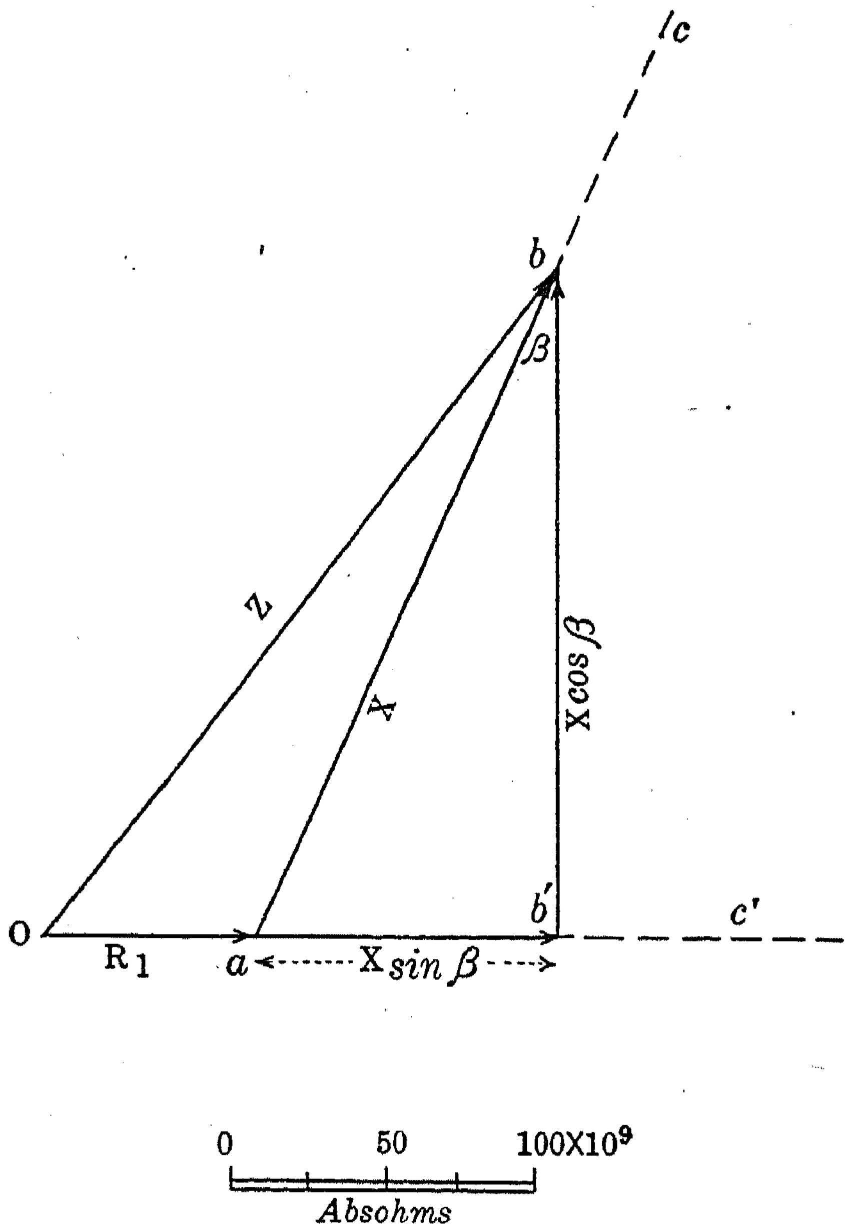


FIG. 9—IMPEDANCE DIAGRAM—DIAPHRAGM DAMPED

$b\,g\,h$, and this circle is obtained from experimental observations at constant current. It passes through the origin of coordinates like the circle of Fig. 27. It has a diametric depression angle, or lag angle, of $2\ \beta$, because the diameter being produced at resonant frequency, the displacement velocity $\dot{x}$ of the diaphragm is then in phase with the flux ϕ_i, which is β° behind the current, and the displacement x is then in lagging quadrature with the velocity or $(90^\circ + \beta)$ behind the current. But ϕ_x is β° behind the displacement x (Fig. 6) and is therefore $90^\circ + \beta^\circ$ behind the velocity $\dot{x}$ or $90^\circ + 2\ \beta^\circ$ behind I. The e. m. f.

$I\,Z'$, Fig. 8, due to the variation of ϕ_x, being in leading quadrature therewith, is then 2 β° behind I, and Z', at resonance, is thus 2 β° behind the reference phase, or resistance axis.

Every telephone receiver has its own particular motional impedance circle, which is very sensitive to changes in temperature or in mechanical adjustment. Motional impedance circles differ:

1. In their size, or diametral length.
2. In their depression angle 2 β.
3. In the distribution of frequencies around the circle.

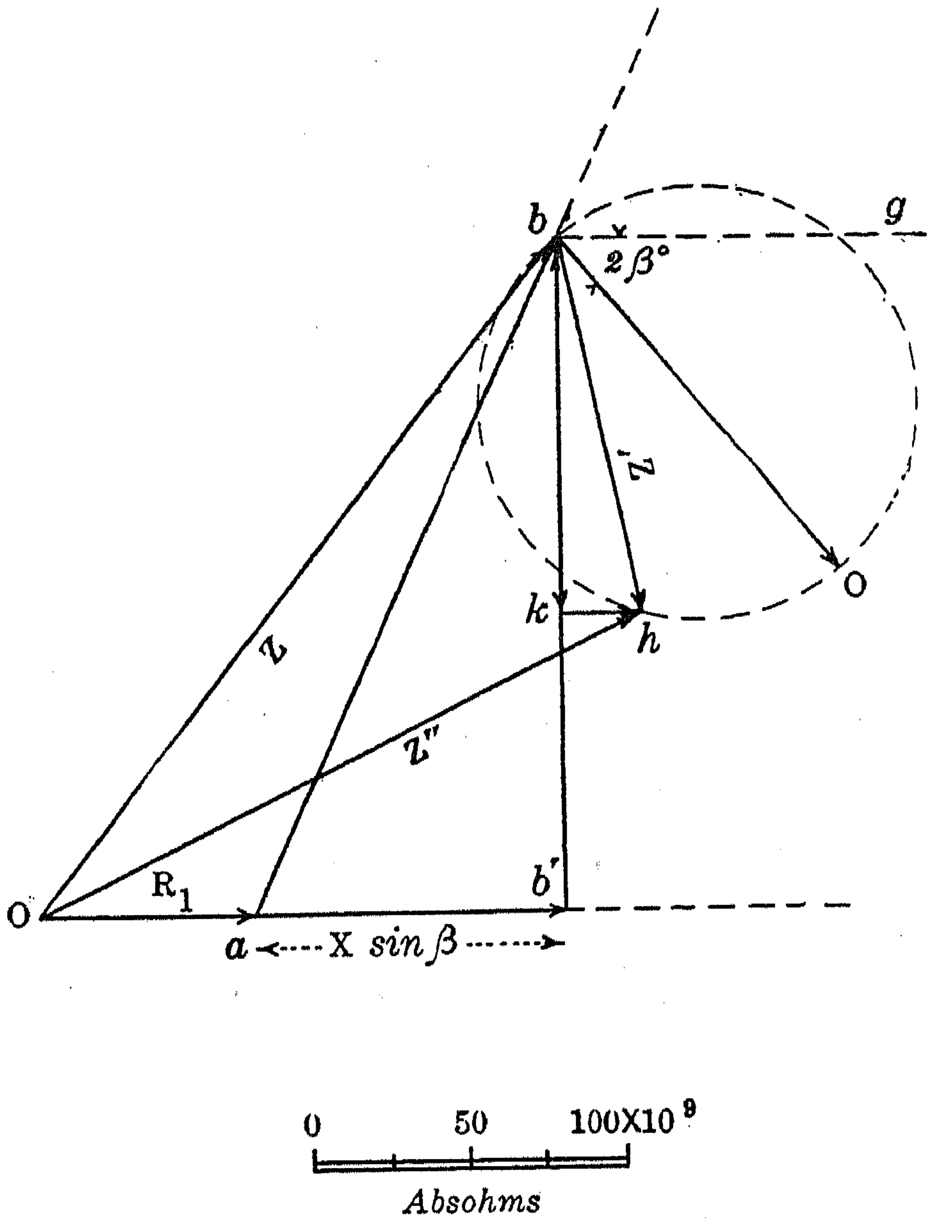


FIG. 10—IMPEDANCE DIAGRAM—DIAPHRAGM FREE

Sharply resonant instruments will pass from the lower quadrantal frequency f_1, to the upper f_2, over half the circle, with a change of only a very few cycles per second. On the other hand, bluntly resonant instruments may require a change in frequency of several hundred cycles per second, to pass over the same range of locus.

Having found by observation the depression angle 2 β of the motional impedance circle, the angle β is known, and the construction of Figs. 9 and 10 may be carried out, with the angle at $b\,b'$ made equal to β. Knowing the apparent resistance $O\,b'$, and the apparent reactance $b'\,b$, at various impressed frequencies, as well as $\beta°$, we can readily compute R_1 and $\mathcal{L}$ from (7) and (8). R_1 should theoretically be substantially constant under the assumption that $\mathcal{L}$ is constant. For the instrument here considered, the following Table gives the values of R_1 as computed from the observed resistances with damped diaphragm, and the angle β assumed constant at resonant value.

Frequency cycles per sec.	Angular Velocity rad. per sec. ω	Measured Resistance absohms at 20 deg. cent. R	Active component of inductive resistance $Rh = \omega \mathcal{L}_0 \sin \beta$	Reactive component of inductive resistance $X = \omega \mathcal{L}_0 \cos \beta$	$R_1 = R - R_h$ absohms
0	0	86.7×10^9	0×10^9	0×10^9	86.7×10^9
429	2694	124 “	41.9 “	89 “	82.1 “
702	4420	145 “	68.8 “	146 “	76.2 “
923.4	5801	162 “	90.5 “	191.5 “	71.5 “
*1015	6378	168.4 “	99.5 “	210.5 “	68.6 “
1306	8202	189. “	128 “	270.7 “	61. “
1507	9460	204 “	147.6 “	314.1 “	56.4 “

*Resonant frequency.

The steady apparent fall in the values of R_1 as the frequency was increased, may be attributed to the influence of changes in magnetic skin effect here neglected.

Electric Power, Damped and Free. With the diaphragm damped, the vector diagram of either Fig. 7, or Fig. 9, might serve as a power diagram, if read off to a corresponding scale of abwatts, or ergs per second. The power diagram is repeated, however, in Fig. 11 to a convenient power scale. $O\,a$ is active power, $I^2 R_1$ abwatts, expended in heating the winding. $I^2 X$ is the vector power $a\,b$, expended in the magnetic circuit. It follows the locus $a\,b\,c$, as the frequency is varied. The active component $a\,b' = I^2 \mid X \mid \sin\,\beta$, is dissipated thermally in the magnetic circuit, in hysteresis and eddy currents. The reactive component $b'\,b = I^2 \mid X \mid \cos\,\beta$ is utilized in alternately storing and releasing energy in the magnetic circuit.

When the diaphragm is freed, the power conditions are indicated vectorially in Fig. 12. $O\,a$, as before, is the thermal power $I^2 R_1$, dissipated in the winding. The power delivered to the magnetic circuit is $a\,h$, which is in leading quadrature to the resultant flux ϕ, see Fig. 6. The active component $a\,h'$ includes the power expended in hysteresis and eddy currents, and also the active mechanical power delivered from the magnetic circuit to the diaphragm. The reactive component $h' h$ includes power utilized in storing and releasing magnetic energy in the magnetic circuit, and also power utilized in storing and releasing reactive mechanical energy in the diaphragm.

Mechanical Velocity, Damped and Free. When the diaphragm is damped, the vibrational velocity of displacement must be zero. When, however, the diaphragm is free, the velocity of displacement $\dot{x}$ will have a definite magnitude and phase. If we assume the vibromotive force $f_i = Og$ r. m. s. dynes, Fig. 13, to remain constant at all frequencies, the locus of the vector velocity $\dot{x}$ will be the circle $O\,a\,b\,c$. This mechanical relation has its complete electric analogy in the simple alternating-current circuit of Fig. 14, where a simple harmonic e. m. f. E, of constant r. m. s. magnitude, and at standard phase, supplies the simple circuit $C\,R\,L$ at successively varied frequencies. All of the resistance of the circuit, assumed constant, is located in the resistor R, all of the inductance in the inductor L, and all the capacitance in the condenser C. At a very low frequency, the impedance of the circuit will

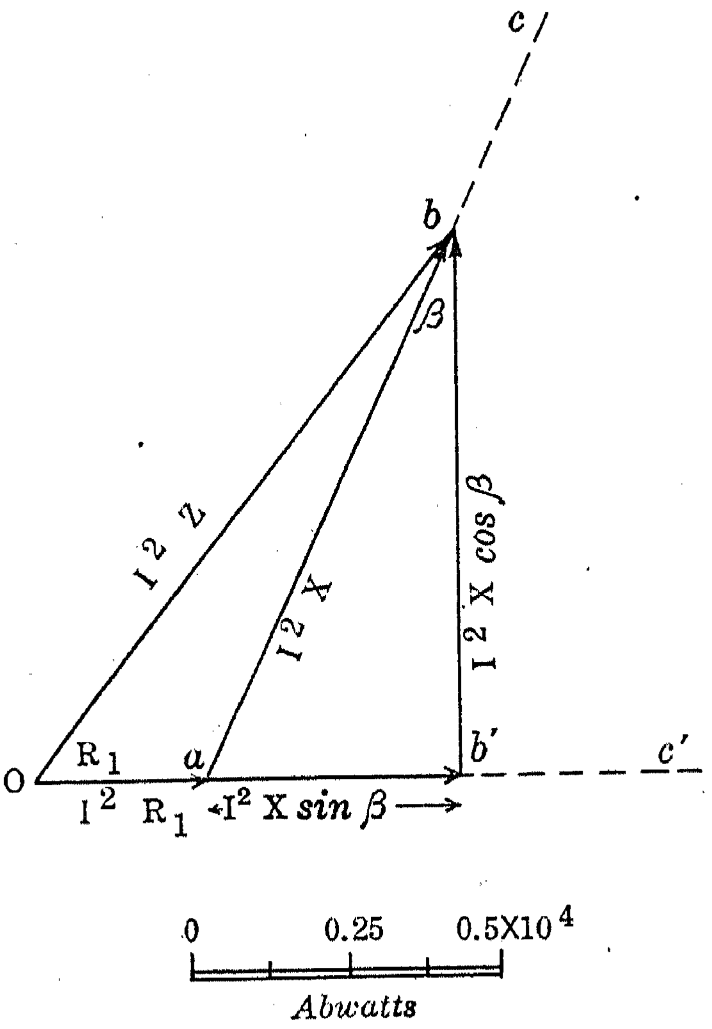


FIG. 11—ELECTRIC POWER DIAGRAM—DIAPHRAGM DAMPED

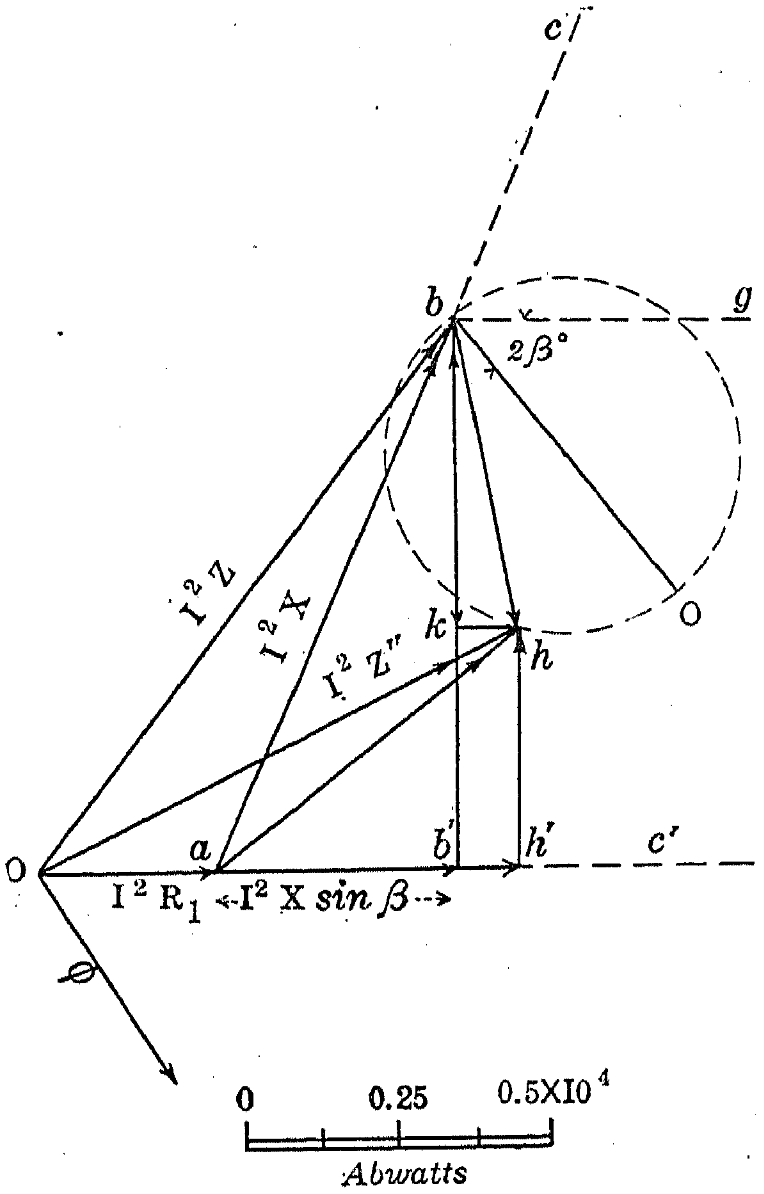


FIG. 12—ELECTRIC POWER DIAGRAM—DIAPHRAGM FREE

be large and entirely due to the condenser C. The alternating current will be very feeble and will lead the impressed e. m. f. by 90°. As the frequency is raised, the impedance diminishes, both in size and in slope. The current strength will therefore increase in magnitude, and come more nearly into phase with E. As is shown in Fig. 15, when the resonant frequency of the circuit is attained, the current will be a maximum at I_0, and will be in phase with E. Finally, as the frequency becomes very high, the current will disappear, and lag 90° behind E. The locus of the vector current will be the circle $O\ a\ b\ c$, Fig.15.

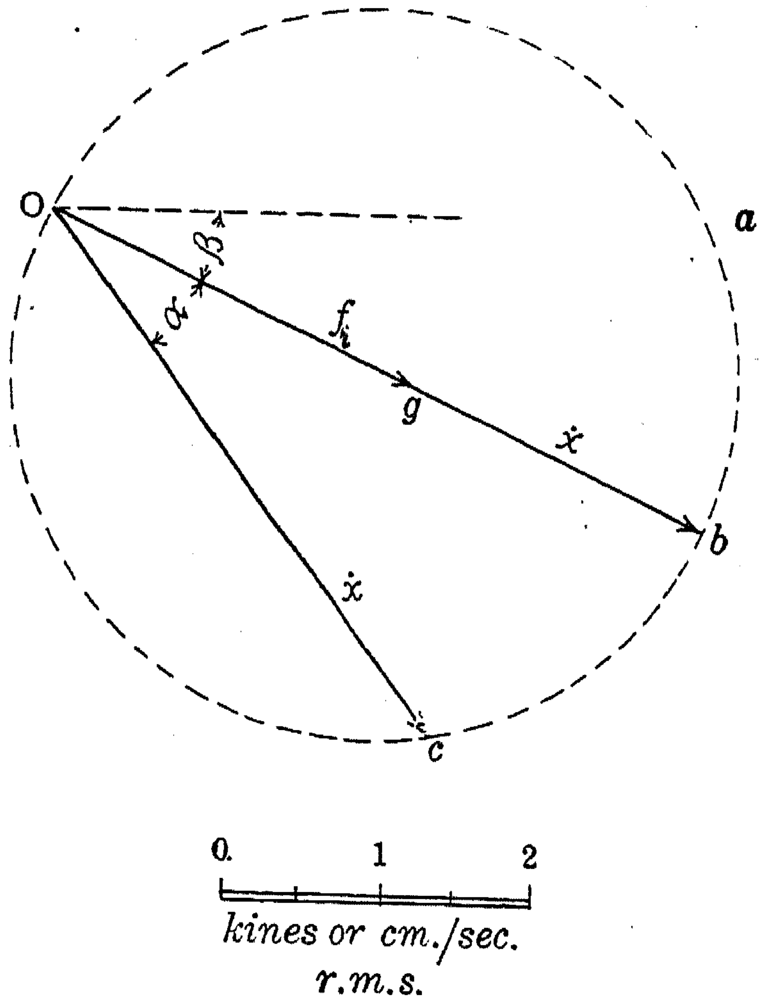


FIG. 13 — VELOCITY DIAGRAM — DIAPHRAGM FREE — CONSTANT V. M. F. AND VARIED FREQUENCY

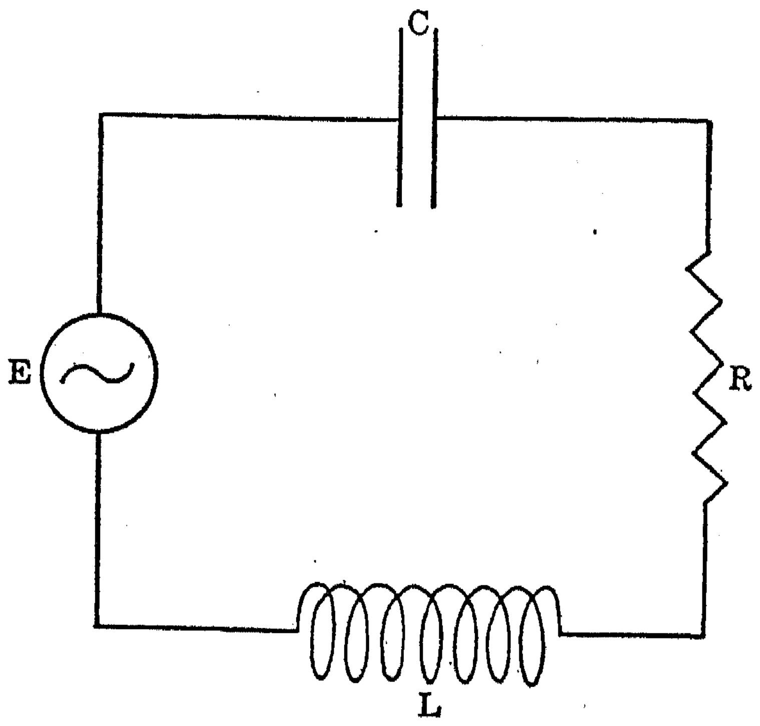


FIG. 14—SIMPLE ALTERNATING-CURRENT CIRCUIT UNDER CONSTANT E. M. F. AND VARIED FREQUENCY

If, in the mechanical system of the diaphragm, the equivalent mass replaces the inductance L, the stiffness s of the diaphragm, replaces $1/C$, and the mechanical resistance to motion r replaces R; also the vibromotive force f_i replaces E; then the vibrational velocity $\dot{x}$ will correspond both in magnitude and phase to the alternating-current strength I. The phase lag α of the velocity is then equal to the slope of the mechanical impedance

$$r + j\,(m\,\omega - s/\omega) \qquad \text{mechanical ohms} \angle$$

Mechanical Displacement. The mechanical displacement of the diaphragm at elongation corresponds, in the electrical system of Fig. 15, to the displacement of electricity at cyclic elongation. It is represented in Fig. 16 by the vector *O c*. This vector is in lagging quadrature to the vector *O c* of Fig. 13, at one and the same impressed frequency. The locus of x is the displacement curve *a b c O*.

Force Diagram Damped and Free. If the diaphragm is damped, any single vibromotive force (v. m. f.) f_i, Fig. 17a, will develop a counter v. m. f. f_c, which will make the resultant force zero; so that no displacement results. If, however, the diaphragm is free, there will be four forces called into action, in similarity to those in the corresponding electrical case of Fig. 15; namely, the impressed single v. m. f. f_i, Fig. 17, in phase with

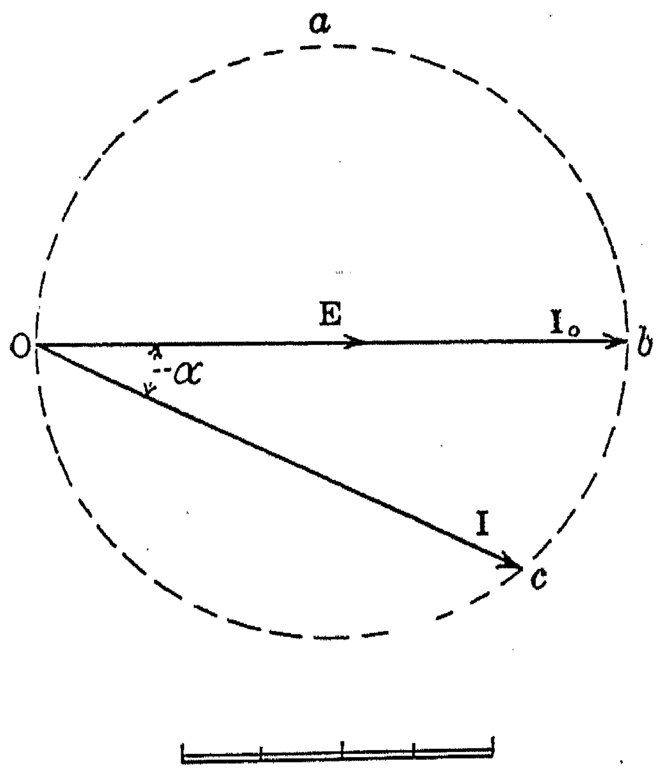


Fig. 15—Current Diagram in Simple A-C. Circuit, Under Constant E. M. F. and Varied Frequency

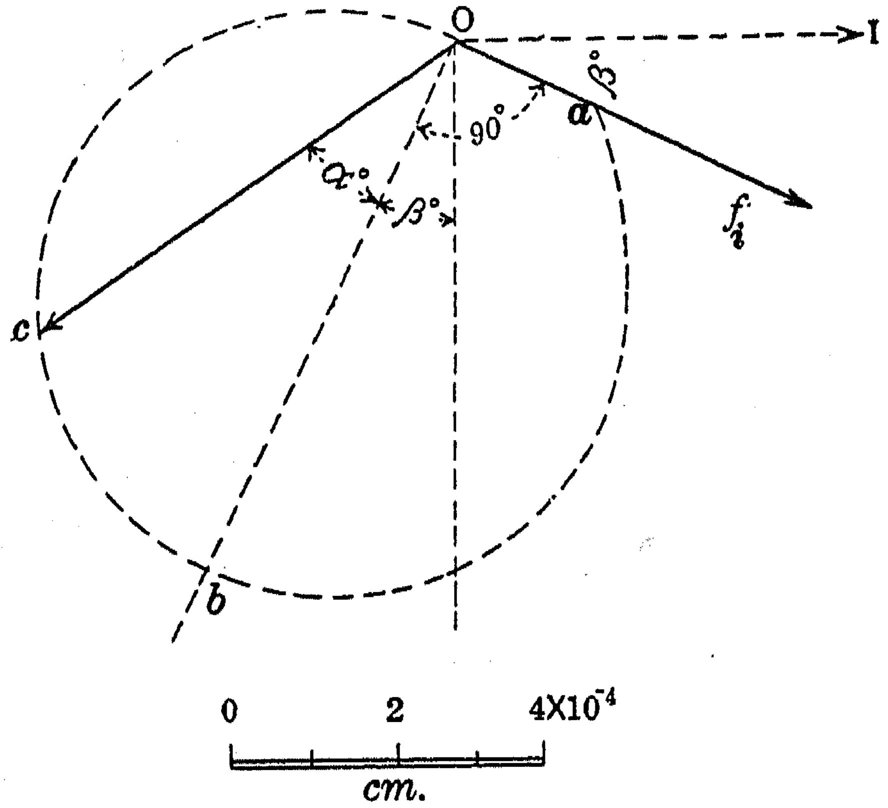


Fig. 16—Displacement Diagram—Diaphragm Free

the flux ϕ_i; the frictional resisting force $-r\dot{x}$ dynes, the inertia force $-j\,\omega\, m\,\dot{x}$ dynes, and the elastic force $j\,\frac{s\,\dot{x}}{\omega}$ dynes. These are represented in Fig. 17 by the vectors *O a, O b, O c* and *O d*. Since *O c* and *O d* must be mutually opposite, their difference *O e*, combined with *O b*, must balance the impressed force *O a*. This relation determines the position and magnitude of the velocity $\dot{x}$. We have already seen that as the frequency is changed from a small to a large value, the locus of $\dot{x}$ is a circle about *O*. At mechanical resonance, $\dot{x}$ will have a maximum size and zero slope, or will coincide with f_i in direction. The displacement x, not indicated in Fig. 17, will always have the direction $o\,c$, 90° lagging behind $\dot{x}$.

The vector force equation is in the rotatable vector diagram of Fig. 17.

$$m\ddot{x} + r\dot{x} + s\,x = f_i \qquad \text{r. m. s. dynes} \angle \quad \textbf{(10)}$$

or since x is a simple harmonic displacement of the type $x_m\,\epsilon^{j\omega t}$,

$$j\,\omega\,m\dot{x} + r\dot{x} + \frac{s\,\dot{x}}{j\omega} - f_i = 0 \qquad \text{r. m. s. dynes} \angle \quad \textbf{(11)}$$

FIG. 17A—FORCE DIAGRAM —DIAPHRAGM DAMPED

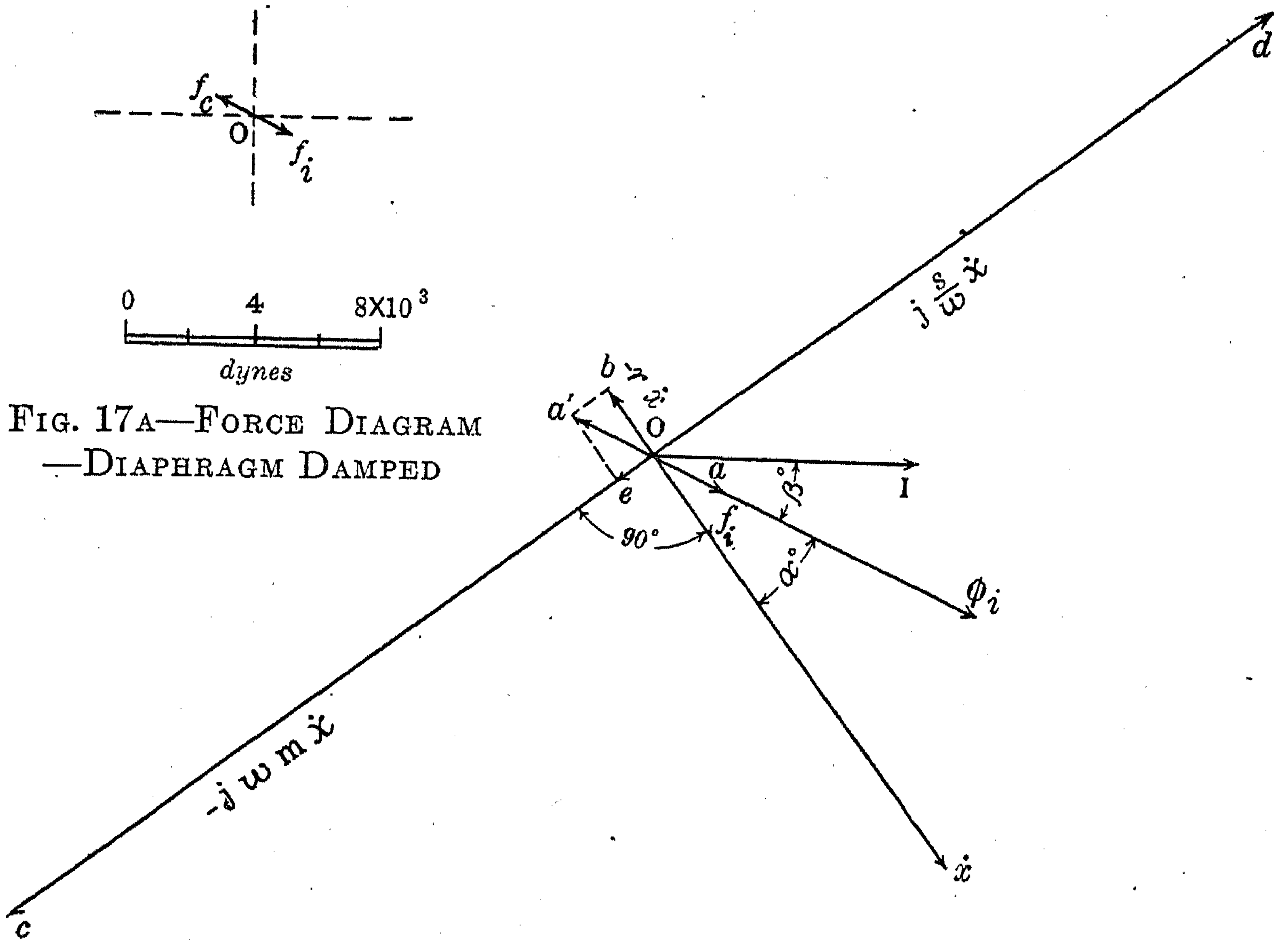


FIG. 17—FORCE DIAGRAM FOR THE IDEAL CASE OF A DIAPHRAGM FREE TO VIBRATE UNDER A SINGLE IMPRESSED V. M. F. f_i R. M. S. DYNES

$$\text{or } -j\,\omega\,m\dot{x} - r\dot{x} + j\dot{x}\,s/\omega + f_i = 0 \qquad \text{r. m. s. dynes} \angle \quad \textbf{(12)}$$

whence

$$\dot{x} = \frac{f_i}{r + j\,(m\,\omega - s/\omega)} \qquad \text{r. m. s. } \frac{\text{cm.}}{\text{sec.}} \angle \quad \textbf{(13)}$$

which corresponds completely to Ohm's law in the vector simple alternating-current case.

The specific magnetic force acting at any time on the diaphragm, towards the magnetic poles, is $\frac{\mathcal{B}^2}{8\,\pi}$ dynes per sq.

cm., where $\mathcal{B}$ is the total magnetic flux density. This flux density comprises the permanent magnetic flux density $\mathcal{B}_0$ gausses, and the alternating flux density $\mathcal{B}$ instantaneous gausses. The total magnetic force is thus

$$\frac{2\,S}{8\,\pi}(\mathcal{B}_0+\mathcal{B})^2=\frac{S}{4\,\pi}(\mathcal{B}_0+\mathcal{B})^2$$

$$=\frac{S}{4\,\pi}(\mathcal{B}_0^2+2\,\mathcal{B}_0\,\mathcal{B}+\mathcal{B}^2)\qquad\text{instantaneous dynes}\quad\textbf{(14)}$$

Since $\mathcal{B}^2$ is very small by comparison with $\mathcal{B}_0^2$, it may be neglected in the summation, and the instantaneous force in the diaphragm is

$$\frac{S}{4\,\pi}(\mathcal{B}_0)^2+\frac{S}{4\,\pi}\cdot 2\,\mathcal{B}_0\,\mathcal{B}\qquad\text{instantaneous dynes}\quad\textbf{(15)}$$

The first term is the permanent force which flexes the diaphragm from its plane position, and which plays no material part in the alternating regime here considered. The second term includes the periodic quantity $\mathcal{B}$, which may be represented vectorially by a vector of size $\mathcal{B}_i$ r. m. s. gausses, lagging $\beta°$ behind the exciting m. m. f. $\mathcal{F}_i$. But $S\,\mathcal{B}_i=\phi_i$ the r. m. s. vector flux in phase with $\mathcal{B}_i$, so that in the damped condition,

$$\phi_i=\mathcal{F}_i\,\mathcal{P}=4\,\pi\,N\,I\,\mathcal{P}\qquad\text{r. m. s. maxwells}\ \angle\quad\textbf{(16)}$$

where I is the r. m. s. exciting current in absamperes, N the total number of turns including both coils, and $\mathcal{P}$ a vector permeance which possesses a lag angle $\beta°$. Consequently from (**15**)

$$f_i=2\,\mathcal{B}_0\,N\,\mathcal{P}\,I=A\,I\qquad\text{r. m. s. dynes}\ \angle\quad\textbf{(17)}$$

or in a bipolar instrument $A=2\,\mathcal{B}_0\,N\,\mathcal{P}$ r. m. s. dynes per absampere ∠ (**18**)

A is a vector force factor with a lag angle of $\beta°$.

We have thus far supposed that only one periodic force f_i acts on the diaphragm. There is, however, a second periodic force introduced by the flux ϕ_x, due to the displacement m. m. f., the magnitude of which is proportional, by (**6**), to the maximum cyclic value of the displacement, and which has the same phase as that displacement.

We have already shown in (**6**) that the displacement m. m. f. is

$$\mathcal{F}_x=2\,\mathcal{B}_0\,x\qquad\text{r. m. s. gilberts}\ \angle\quad\textbf{(19)}$$

This may be considered as similar to the m. m. f. of current

$$\mathfrak{F}_i = 4\,\pi\,N\,I \qquad \text{r. m. s. gilberts } \angle \quad (20)$$

so that a certain number of turns N' multiplied by the r. m. s. vector displacement, would produce the same m. m. f. as the actual number of turns N multiplied by the corresponding r. m. s. vector current. Consequently, in the case of a bipolar receiver

$$N' = \frac{\mathfrak{B}_0}{2\,\pi} \qquad \text{equivalent turns} \times \frac{\text{absampere}}{\text{cm.}} \quad (21)$$

If we consider the ratio

$$\frac{N'}{N} = p \qquad \frac{\text{absampere}}{\text{cm.}} \quad (22)$$

which, in the instrument considered, was 0.168,

$$\mathfrak{F}_x = 4\,\pi\,N'\,x \qquad \text{r. m. s. gilberts } \angle \quad (23)$$

and from (**17**)

$$f_x = p\,A\,x = 2\,p\,\mathfrak{B}_0\,N\,\mathcal{P}\,x \qquad \text{r. m. s. dynes } \angle \quad (24)$$

It may be noted, in passing, that just as in the case of a vector electromagnetic inductance

$$\mathfrak{L} = A/p = 4\,\pi\,N^2\,\mathcal{P} \qquad \text{abhenries } \angle \text{ or ergs per absamp}^2 \angle \quad (25)$$

so

$$p\,A = 4\,\pi\,N'^2\,\mathcal{P} \qquad \frac{\text{r. m. s. dynes}}{\text{cm.}} \angle \quad (26)$$

and is a quantity bearing resemblance to an inductance. From (**25**) and (**26**)

$$A = 4\,\pi\,N\,N'\,\mathcal{P} \qquad \frac{\text{r. m. s. dynes}}{\text{absamp}} \angle \quad (27)$$

so that, in this sense, the force factor A bears resemblance to a mutual inductance.

The additional force f_x, due to ϕ_x, may be introduced into the problem either (**1**) by vector summation with f_i, to form a vector resultant force f, corresponding to ϕ in Fig. 6; or (**2**), by retaining f_i as the sole active force on the diaphragm, and introducing the action of f_x in the form of a mechanical impedance. For practical purposes, the latter method is to be preferred, just as in the simple alternating-current circuit, the effect of a counter e. m. f. of self induction is usually treated as an impedance, instead of being combined vectorially with the impressed e. m. f. in the numerator of Ohm's law.

The force f_x lags behind the displacement x by the angle $\beta°$, according to the relation

$$f_x = p\,A\,x = \frac{p\,A\,\dot{x}}{j\,\omega} = -j\,p\,A\,\frac{\dot{x}}{\omega} \quad \text{r. m. s. dynes} \angle \quad \textbf{(28)}$$

or 90° + $\beta°$ behind the phase of the velocity $\dot{x}$.

In Fig. 18, the force f_x is represented in a particular case with the diaphragm free. Here the m. m. f. $\mathfrak{F}_i$ is taken as of reference phase, with f_i lagging $\beta°$ behind it. This force f_i must now equilibrate four forces; namely, the elastic force $O\,d$, the inertia force $O\,c$, the frictional force $O\,b$, and the new magnetic force $O\,e$, whose components, along the $O\,b$ and $O\,c$ directions, are indicated as $O\,e'$ and $O\,e''$ respectively. With $\mathfrak{F}_i$ at reference phase, the locus of the new force $O\,e$ is a displacement-admittance locus like $a\,b\,c$ in Fig. 4; but considered solely with reference to the velocity $\dot{x}$, $O\,e$ always lags with respect thereto by 90° + $\beta°$. The components $O\,e'$ and $O\,e''$ may be defined as $-\,r'\,\dot{x}$ and $\dfrac{-j\,s'\,\dot{x}}{\omega}$ respectively; where r' is a virtual frictional resistance, and s' a virtual stiffness coefficient.

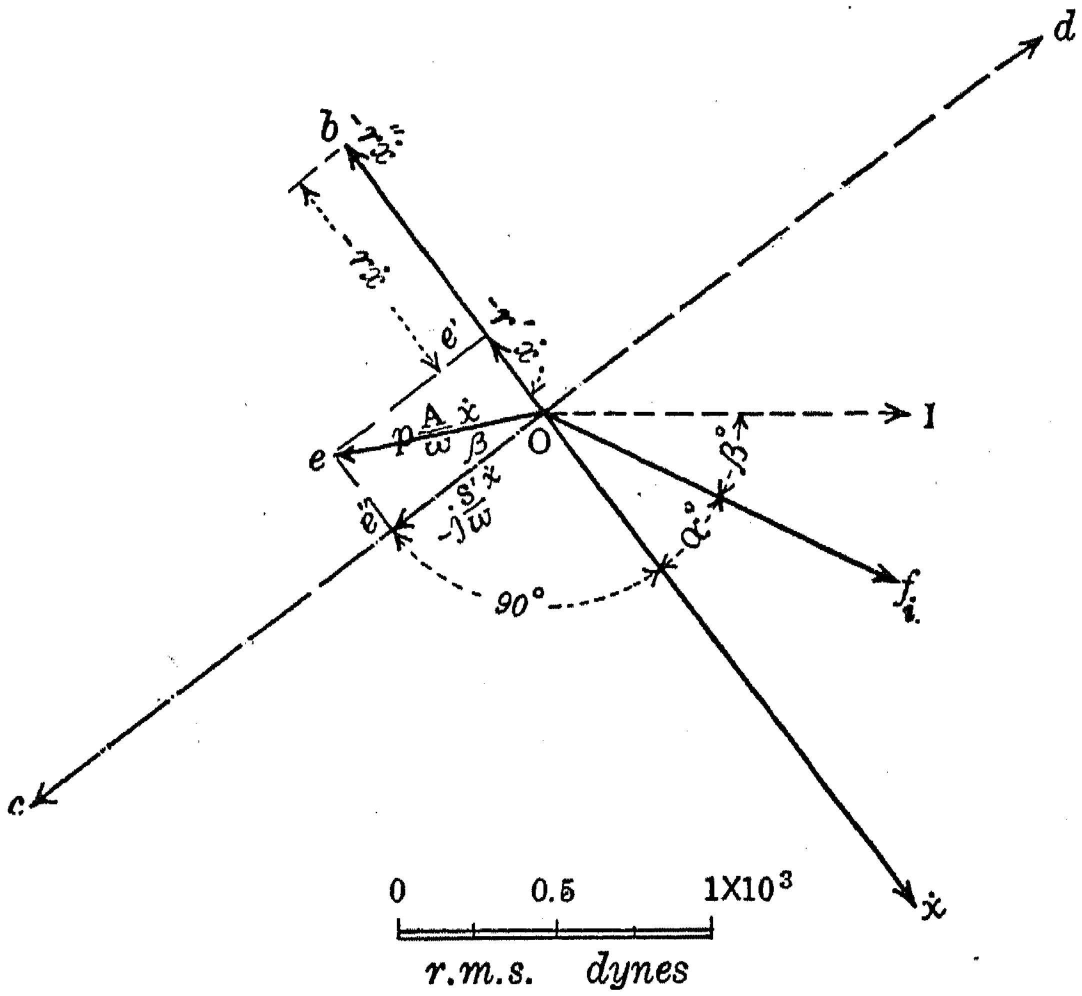


Fig. 18—Force Diagram Including Magnetic Force Due to Displacement—Diaphragm Free Enlarged Scale

The diaphragm therefore behaves as though its frictional resistance were increased by

$$r' = \frac{p\,|A|}{\omega} \sin \beta \qquad \frac{\text{dynes}}{\text{kine}} \qquad (29)$$

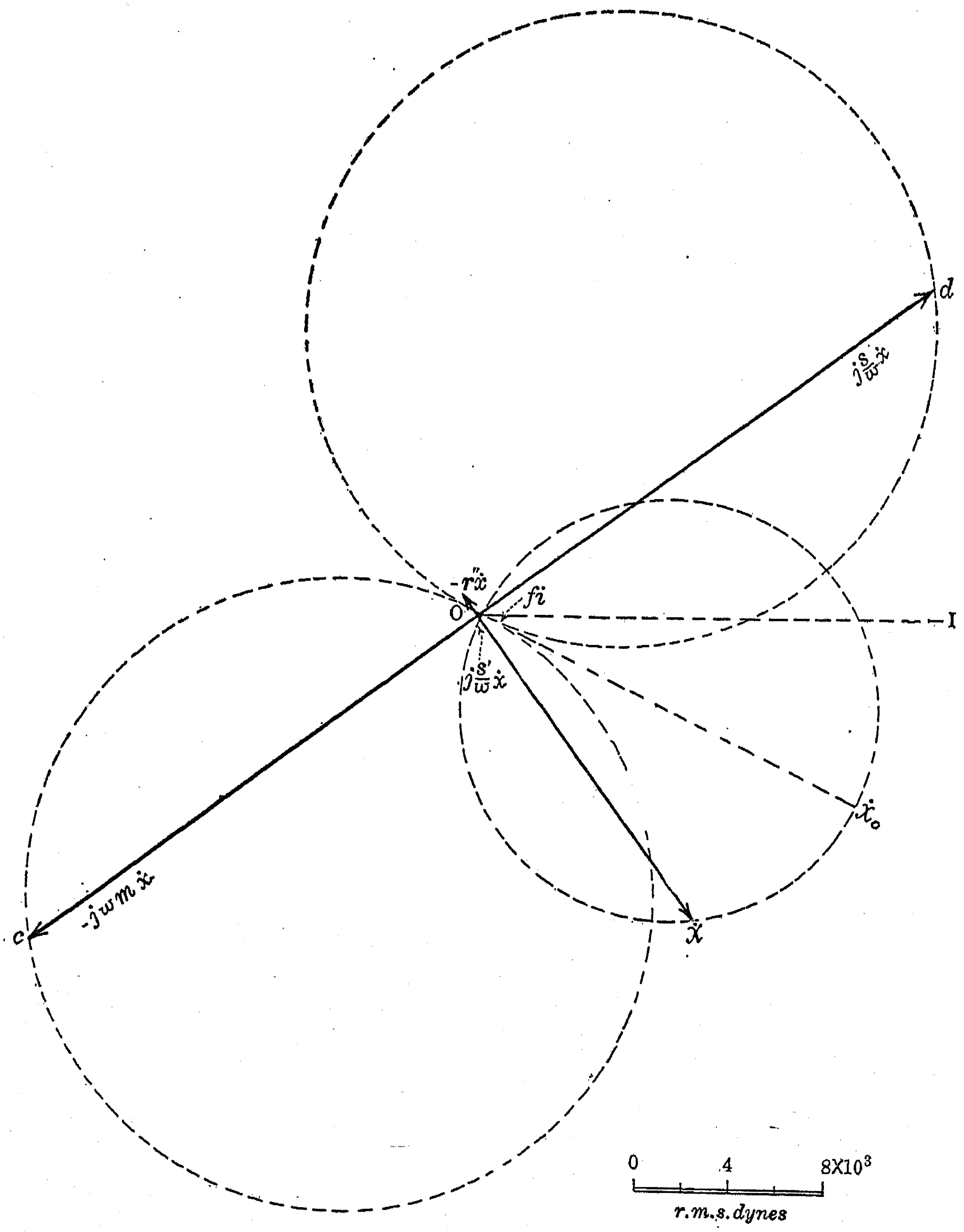


FIG. 19—FORCE DIAGRAM—COMPLETE—INCLUDING MAGNETIC FORCE DUE TO DISPLACEMENT

while the stiffness of the diaphragm is apparently diminished by

$$s' = p\,|A| \cos \beta. \qquad \frac{\text{dynes}}{\text{cm.}} \qquad (30)$$

The values of r' and s' can be assigned, when A, β, and p are known. In the instrument considered, at resonance, $r' = 68.8$ dynes per kine and $r = 199$ dynes per kine, r' diminishing, according to our theory, as ω is increased. Also s' is 9.3×10^5 dynes per cm. as against $s = 376 \times 10^5$. According to our theory, s' is thus only about 3 per cent of s, at all frequencies.

In Fig. 19, the loci of the inertia and stiffness forces, $O\,c$ and $O\,d$, are represented to scale, without including therein the components $O\,e'$ and $O\,e''$, Fig. 18, of the added magnetic force f_x. The directions of motion in the loci, with increase of frequency are all clockwise. The locus of $O\,d$ is a displacement-admittance locus, obtained from the circular locus of $\dot{x}$ by time integration; whereas the locus of $O\,c$ is obtained from that of $\dot{x}$ by time differentiation.

The total mechanical impedance of the diaphragm from (**13**) and (**28**) is

$$z' = r + j\,(m\,\omega - s/\omega) + j\,\frac{p\,A}{\omega} \qquad \frac{\text{dynes}}{\text{kine}} \ \angle \quad (\mathbf{31})$$

The reciprocal of this quantity, or mechanical admittance y' is

$$y' = 1/z'$$

$$= \frac{1}{\sqrt{r''^2 + (m\,\omega - s''/\omega)^2}} \measuredangle \tan^{-1}\left(\frac{m\,\omega - s''/\omega}{r''}\right)$$

$$= \frac{1}{\sqrt{(r + r')^2 + \left(m\,\omega - \frac{s - s'}{\omega}\right)^2}}$$

$$\measuredangle \tan^{-1}\left(\frac{m\,\omega - \frac{s - s'}{\omega}}{r + r'}\right) \qquad \frac{\text{kines}}{\text{dyne}} \ \angle \quad (\mathbf{32})$$

The values of r' and s' are to be taken from (**29**) and (**30**). This vector admittance y', multiplied by the constant impressed mechanical force f_i, due to the constant alternating current, gives the mechanical velocity $\dot{x}$, or

$$\dot{x} = f_i\,y' = A\,y'\,I \qquad \text{r. m. s. } \frac{\text{cm.}}{\text{sec.}} \ \angle \quad (\mathbf{33})$$

and the r. m. s. displacement x by

$$x = -j\,f_i\,y'/\omega = -j\,A\,y'\,I/\omega \quad \text{r. m. s. cyclic cm.} \ \angle \quad (\mathbf{34})$$

The actual maximum cyclic displacement of the diaphragm is then

$$x_m = | x | \sqrt{2} \qquad \text{max. cyclic cm.} \quad (\mathbf{35})$$

Motional Electromotive Force Diagram. Referring to Fig. 16, the vector displacement $O\grave{c}$ is determined by equation (**34**), and the vector velocity Oc in Fig. 13, by equation (**33**), in terms of the motional constants r, r', m, s and s', as well as A. The displacement flux ϕ_x, Fig. 8, is then obtained through (**23**) or

$$\phi_x = \frac{p\,A\,x}{N'} = 4\,\pi\,N'\,x\,\mathcal{P} \qquad \text{maxwells} \angle \quad (\mathbf{36})$$

The e. m. f. produced by ϕ_x in the coils is the motional e. m. f. and is

$$I\,Z' = j\,\omega\,\frac{p\,A\,x}{N'}\,N = j\,\omega\,A\,x$$

$$= A\,\dot{x} = A^2\,y'\,I \qquad \text{abvolts} \angle \quad (\mathbf{37})$$

It may be noted that since $\dot{x}$ is a velocity in kines, the vector dimensions of A are not only expressed by dynes per absampere, as in (**18**) or (**27**), but also by abvolts per kine, as in (**37**).

The vector motional e. m. f. $I\,Z'$, Fig. 8, is in leading quadrature to $O\,e$, the displacement flux. Since in (**37**), this vector is directly proportional to the velocity $\dot{x}$, which has been shown to travel over a circular locus, when, as in (**13**), a single periodic force is impressed upon the diaphragm, the motional e. m. f. $I\,Z'$ must also have a circular locus, as shown in Fig. 8, when the second force f_x is negligible; *i. e.*, when p in (**28**) may be taken as zero. When p is appreciable, the subsidiary constants r' and s' are introduced into the motional equations (**32**) and (**33**). It will be seen that s' merely reduces the stiffness coefficient s of the diaphragm by the same amount at all frequencies; but r' varies with the frequency inversely, as shown by (**29**). The theoretical effect of r' is therefore to distort the locus of the motional e. m. f. $I\,Z'$, from a circle, to an extent depending on the magnitude of $p\,A$, and also upon the range of frequency necessary for the execution of the circle; *i. e.*, upon the bluntness of mechanical resonance. Since in a bipolar instrument

$$p\,A = \frac{\mathcal{B}_0^2\,\mathcal{P}}{\pi} \qquad \frac{\text{dynes}}{\text{cm.}} \angle \quad (\mathbf{38})$$

an instrument which has a relatively powerful permanent magnetic flux density $\mathfrak{B}_0$ in the airgaps, and also large magnetic-circuit permeance, is one in which the force f_x tends to be greatest, by (**28**). This, with a large value of β and a wide range of ω for executing the circle, would give rise to the largest distortion. In practise, the distortion is usually not sufficiently serious to be very noticeable, probably because the range in ω within which half of the circle is executed, is ordinarily only a few hundreds of radians per second. In the case considered, $\omega_2 - \omega_1 = 305$.

It is evident, moreover, from (**37**) that the lagging displacement of vector $I\,Z'$ with respect to I, is that of $A^2\,y'$, and since, at resonance, the slope of y' vanishes (**32**), it will then be the displacement of A^2, or $-\,2\ \beta°$, and $A^2 = Z_0'\,r''$.

At resonance, the size of $I\,Z'$, the motional e. m. f. will, by (**37**) be

$$I\,Z_0' = \frac{A^2\,I}{r''} \qquad \text{abvolts} \angle \quad (\mathbf{39})$$

Motional Impedance. Referring to Fig. 10, the motional impedance executes a circular locus similar to that in Fig. 8, neglecting the disturbing influence of r' at different frequencies. At resonance, its size is

$$Z'_0 = A^2/r'' \qquad \text{absohms} \angle \quad (\mathbf{40})$$

and its slope is that of A^2 or $-\,2\ \beta°$.

From an experimental standpoint, the motional-impedance circle is of primary importance, as, from it, the motional constants may be derived, if the maximum cyclic displacement at resonance

$$x_m = |\,x_0\,|\ \sqrt{2} \qquad \text{cm.} \quad (\mathbf{41})$$

can be measured.

Technique for Deriving the Constants A, m, r'', r', r, s'', s, p and β. (1) The motional-impedance circle is obtained from observations of the apparent resistance and reactance of the instrument, both free and damped, with constant testing current, under adjustably varied frequency. This gives the resonant maximum value Z'_0 and its slope $-\,2\ \beta°$ at ω_0 the angular velocity of apparent resonance; also* the quadrantal values ω_1 and ω_2, or their equivalents. We thus obtain the total damping constant

---

*See technique in Bibliography 17, p. 469.

$$\Delta = \frac{\omega_2 - \omega_1}{2} = \frac{r''}{2\,m} \qquad \text{hyps per sec.} \quad (\mathbf{42})$$

and

$$\omega_0 = \sqrt{s''/m} \qquad \text{rad. per sec.} \quad (\mathbf{43})$$

(2) The amplitude of displacement x_m over the poles at resonance is measured for the measuring current I. Hence

$$A = \frac{I\,Z'_0}{j\,x_0\,\omega_0} = \frac{I_m\,Z'_0}{j\,x_m\,\omega_0} = \frac{I\,Z_0'}{\dot{x}_0} \qquad \text{abvolts per kine} \angle \quad (\mathbf{44})$$

Then from (**40**)

$$r'' = A^2/Z_0' \qquad \text{dynes per kine} \quad (\mathbf{45})$$

Hence by (**42**)

$$m = \frac{r''}{2\,\Delta} \qquad \text{gm.} \quad (\mathbf{46})$$

Also by (**43**)

$$s'' = m\,\omega_0^2 \qquad \text{dynes per cm.} \quad (\mathbf{47})$$

Having ascertained the value of the vector damped inductance $\mathscr{L}$, at resonant impressed frequency, during the measurements of damped impedance, we find by (**25**)

$$p = A/\mathscr{L} \qquad \text{absamp per cm.} \quad (\mathbf{48})$$

We can then compute the value of the resistance component r' of f_x by (**29**) at resonance

$$r' = \frac{p\,|A|}{\omega_0}\cdot\sin\,\beta \qquad \text{dynes per kine} \quad (\mathbf{49})$$

and hence r, neglecting changes in r' with frequency,

$$r = r'' - r' \qquad \text{dynes per kine} \quad (\mathbf{50})$$

Next s' can be computed from (**30**) and s found by the relation

$$s = s'' + s' \qquad \frac{\text{dynes}}{\text{cm.}} \quad (\mathbf{51})$$

The total number of turns N is supposed to be known, from which $\mathscr{B}_0$ is obtainable through the bipolar relation

$$\mathscr{B}_0 = 2\,\pi\,p\,N \qquad \text{gausses} \quad (\mathbf{52})$$

The equivalent number of turns N' for computing the displacement m. m. f. is

$$N' = p\,N \qquad \frac{\text{absamperes}}{\text{cm.}} \tag{53}$$

assuming that N is a pure numeric. The quantities $p\,A$, $\mathfrak{F}_i$, ϕ_i and f_i are then easily found.

Motional Power. If a simple periodic m. m. f. of $\mathfrak{F} = 4\,\pi\,N\,I$ r. m. s. gilberts ∠ at the reference phase of I r. m. s. absamperes, acts upon a leakanceless magnetic circuit of vector permeance $\mathcal{P}$ maxwells per gilbert ∠, the vector magnetic flux thereby produced, neglecting harmonics, which are powerless, is

$$\phi = \mathfrak{F}\,\mathcal{P} \qquad \text{r. m. s. maxwells } \angle \tag{54}$$

The phase of the flux will have the slope $-\beta°$ of $\mathcal{P}$, with respect to the phase of the exciting current and m. m. f. The e. m. f. which will be induced in the coil will be

$$E = -j\,\omega\,N\,\phi = -j\,\omega\,N\,\mathfrak{F}\,\mathcal{P} \qquad \text{r. m. s. abvolts } \angle \tag{55}$$

The power of this e. m. f., expressed electrically, is

$$P = E\,I = -j\,\omega\,N\,\phi\,I \qquad \text{abwatts } \angle \tag{56}$$

the slope of I being taken as zero.* The real component of P will be active power, expended thermally in the magnetic circuit, and the quadrature component will be reactive power expended in cyclically storing and releasing magnetic energy in the circuit. But (**56**) may be written**

$$P = -j\,\frac{\omega\,\mathfrak{F}\,\phi}{4\,\pi} = -\,j\,(f/2)\,\mathfrak{F}\,\phi \qquad \text{abwatts } \angle \tag{57}$$

where $\mathfrak{F}$ is taken as at standard phase, and ϕ has the lagging slope of $\beta°$ with respect thereto. The negative sign attached to the expressions in (**56**) and (**57**) indicates that the power is absorbed in the magnetic circuit.

---

*Bibliography 7.

**The power exerted in a magnetic circuit by a simple alternating m. m. f. $\mathfrak{F}$ r. m. s gilberts, at zero slope, on a simple alternating flux ϕ r. m. s. maxwells ∠, is, in the C. G. S. system,

$$P = \frac{\mathfrak{F}}{4\,\pi}\cdot\frac{d\,\phi}{d\,t} \qquad \text{abwatts } \angle \tag{57a}$$

where the real component is average active power, and the imaginary component max. cyclic reactive power. Here $\frac{d\,\phi}{d\,t}$ corresponds to electric current.

In Fig. 20, the vector m. m. f. $\mathfrak{F}_i$ is drawn to the same reference phase as *I*. The flux ϕ_i, thereby produced, with the diaphragm damped, is indicated as lagging $\beta°$ behind $\mathfrak{F}_i$. The induced e. m. f. *E*, lags $90° + \beta°$ behind $\mathfrak{F}_i$. The vector power *E I*, taken with *I* as of zero slope, is *O a*. Its real component, *O b*, is dissipated. The undissipated or reactive component is *b a*.

In Fig. 21, the r. m. s. m. m. f. and r. m. s. flux are represented without reference to e. m. fs. Their vector product $\mathfrak{F}_i\ \phi_i$, taking $\mathfrak{F}_i$ as of standard phase, would be along the dotted line *O c*. Taking $-f/2$ times this product, as called for in (**57**), we obtain, to a power scale, the vector *O d*, Fig. 22.

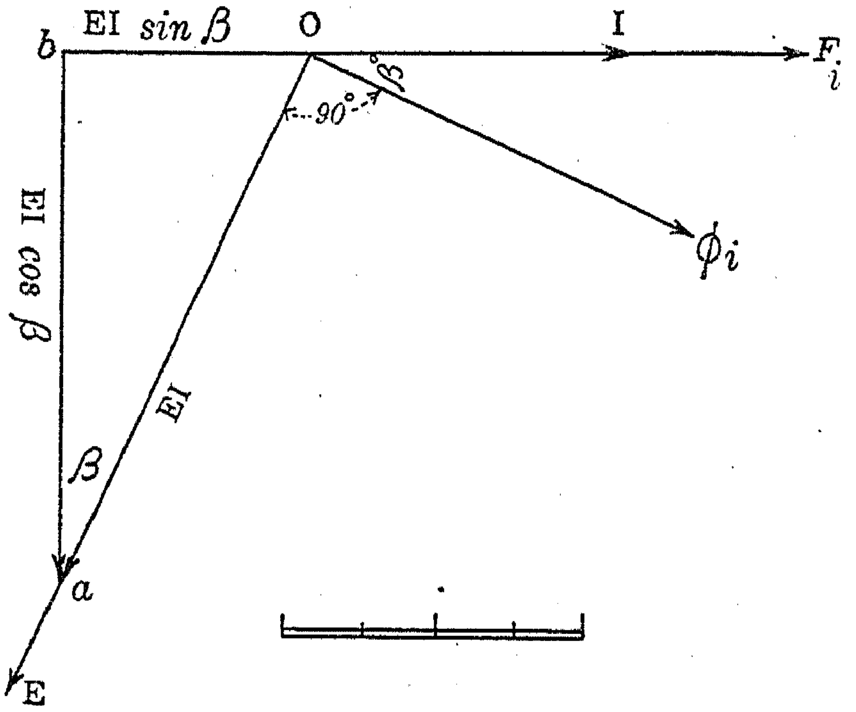


Fig. 20—Diagram of a R.M.S. M.M.F., the R. M. S. Flux Thereby Produced, the Consequent E. M. F. and the Vector Power

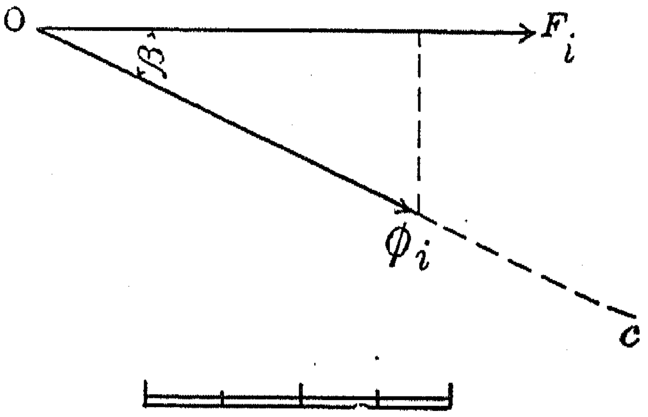


Fig. 21—Fundamental Relations of Power in a Magnetic Circuit Carrying a R. M. S. Flux $\phi\mathfrak{F}_i$, Under an Impressed R. M. S., M. M. F.

The cosine product of $f\ \mathfrak{F}_i/2$ and ϕ_i is reactive magnetic power, and the sine product, or vector product, active power liberated from the magnetic circuit. These conditions are opposite to those in the alternating electric circuit, where the cosine product is active and the sine product reactive electric power. In order to interpret this vector magnetic-circuit power correctly, we may rotate the axis *O X* of reference, through $-90°$, into the position *O A*. The component $Oc = -(f/2)\ \mathfrak{F}_i\ |\phi_i| \sin\beta$ is then the average active power, absorbed by the magnetic circuit, and liberated from it, usually as heat; while the reactive component $O\,b = -(f/2)\ \mathfrak{F}_i\ |\phi_i| \cos\beta$ is power absorbed in storing and releasing magnetic energy.

If, however, we consider the vector power delivered by the electric circuit to the magnetic circuit, it will have the opposite sign to $O\,d$, or will be $O\,d$ reversed, and equal to the vector $O\,d'$, whose components are $O\,c' = +\ (f/2)\ \mathfrak{F}_i\,|\phi_i| \sin\beta$, liberated active power and $O\,b' = +\ (f/2)\ \mathfrak{F}_i\,|\phi_i| \cos\beta$ reactive power engaged in storage.

In Fig. 23, we have the vector magnetic power delivered from the electric circuit with the diaphragm damped. The real component is $O\,a'$ and the reactive component $a'\,a$.

When the diaphragm is free, we have the displacement m. m. f. $\mathfrak{F}_x$ and its flux ϕ_x introduced into the magnetic circuit, as shown in Fig. 24. The resultant of these two m. m. fs. is $\mathfrak{F}$, and that of the two corresponding fluxes is ϕ. This magnetic-circuit vector diagram bears a close resemblance to the electric-circuit vector diagram for the simple case of an alternator driving a synchronous motor through the impedance of a connecting circuit. The m. m. fs. of Fig. 24 then correspond to e. m. fs., flux-frequencies to currents, and their respective permeance vector ratios to admittance ratios. We have

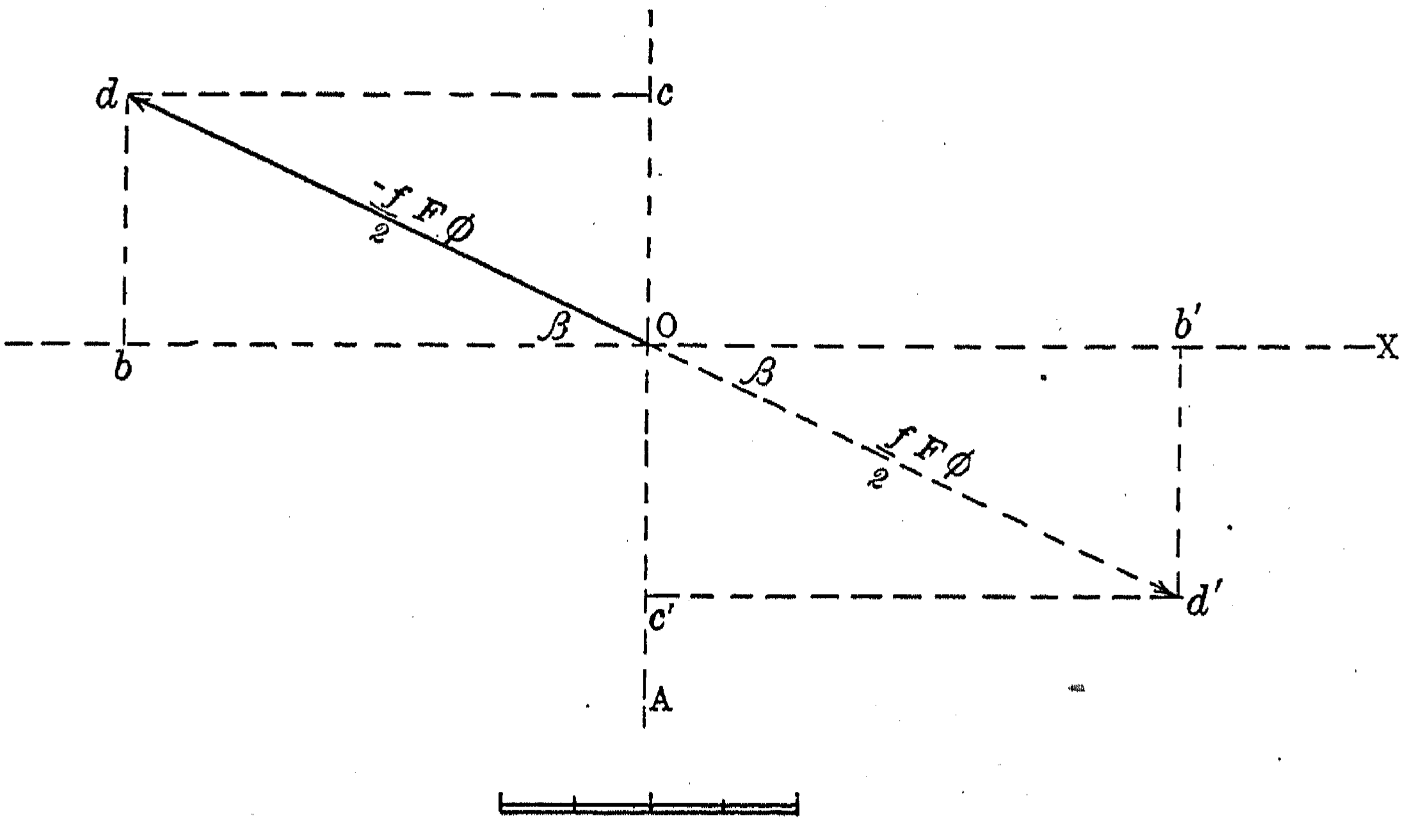


FIG. 22—POWER RECEIVED FROM AND DELIVERED TO A MAGNETIC CIRCUIT CARRYING A R.M.S. FLUX ϕ LAGGING β^0 BEHIND AN IMPRESSED R. M. S. M. M. F. $\mathfrak{F}_i$ AT FREQUENCY f CYCLES PER SEC.

$$\mathfrak{F}_i + \mathfrak{F}_x = \mathfrak{F} \qquad \text{r. m. s. gilberts} \angle \quad \textbf{(58)}$$

If we multiply this equation by the vector resultant flux ϕ, considered for that purpose as of standard phase, we obtain

$$(f/2)\ \mathfrak{F}_i\,\phi + (f/2)\ \mathfrak{F}_x\,\phi = (f/2)\ \mathfrak{F}\,\phi \qquad \text{abwatts} \angle \quad \textbf{(59)}$$

The first term is here the vector power input P_i'' delivered to the magnetic circuit by the exciting current. The second term, reversed in sign, is the vector power P_m liberated from the magnetic circuit into pure mechanical power of the diaphragm, both active and reactive, and corresponding to vector mechanical power given to a synchronous motor in an electric circuit. The term $(f/2)\mathfrak{F}\,\phi$ is the vector power P_r'' expended thermally within the magnetic circuit in overcoming its reluctance. The equation (**59**) may therefore be written

$$P_i'' - P_m = P_r'' \qquad \text{abwatts} \angle \quad (\mathbf{60})$$

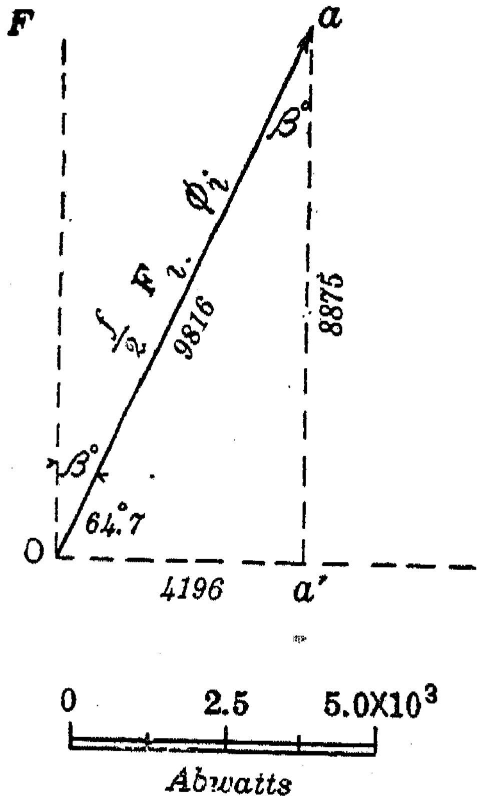


FIG. 23—POWER DELIVERED TO MAGNETIC CIRCUIT AT 1028 CYCLES PER SEC.—DIAPHRAGM DAMPED

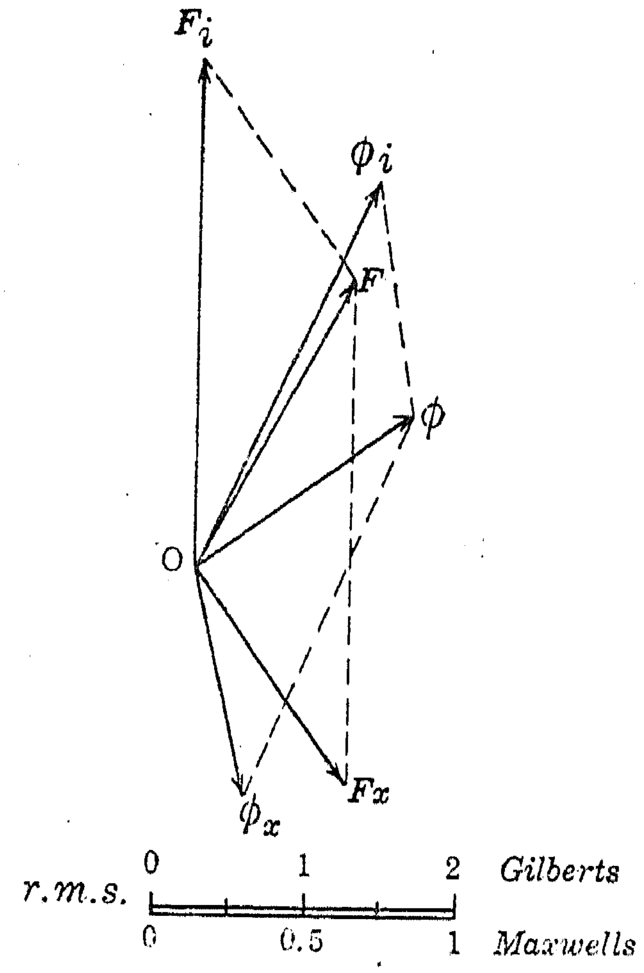


FIG. 24—DIAGRAM OF M. M. F' AND FLUXES IN MAGNETIC CIRCUIT —DIAPHRAGM FREE

In the instrument referred to, these vector values at 1028 ∼ were as shown in Fig. 25, where $O\,b$ is the magnetic input of $P_i'' = 6438\ \underline{/36^\circ.8}$ abwatts, with an active component of 5156 abwatts, liberated both in heat and in mechanical power, and a reactive component of 3856 abwatts, engaged in the reactive storage of magnetic energy. P_m or the vector $O\,a$, is the power expended on the diaphragm mechanically, $3349\ \underline{/0.^\circ 7}$ abwatts, almost entirely in the active form, nearly 3349 abwatts being developed in vibratory motion $\dot{x}^2\,r$, against mechanical resistance r and 40 abwatts in reactive mechanical power; *i. e.*, power consumed in exchanging potential energy cyclically from the elastic to the inertia type. The vector $a\,b$ or P_r'' is the power expended in the magnetic conducting circuit by the total flux ϕ in overcoming the vector reluctance $\mathcal{R}$ of that circuit. At this frequency, it was 4199 $\underline{/64}$°.7 abwatts, with a real component $a\,b'$ of 1795 abwatts, expended thermally in hysteresis and eddy currents. The reactive component $b'\,b$ of 3797 abwatts is expended in maintaining the activity of cyclically storing and releasing magnetic energy of the type $\frac{\mathcal{B}^2}{8\,\pi\,\mu}$ ergs per cu. cm. in the magnetic conducting circuit.

When the diaphragm is damped, the magnetic circuit conditions are shown in Figs. 21 and 23. Here the magnetic input at 1028 ~ is 9816 $\underline{/64}$°.7 abwatts, with no counter m. m. f. of motion. The active component $O\,a'$ is 4196 abwatts, expended entirely in heat leaving the circuit. The reactive component is 8875 abwatts, engaged in maintaining the cyclic storage and release of magnetic energy.

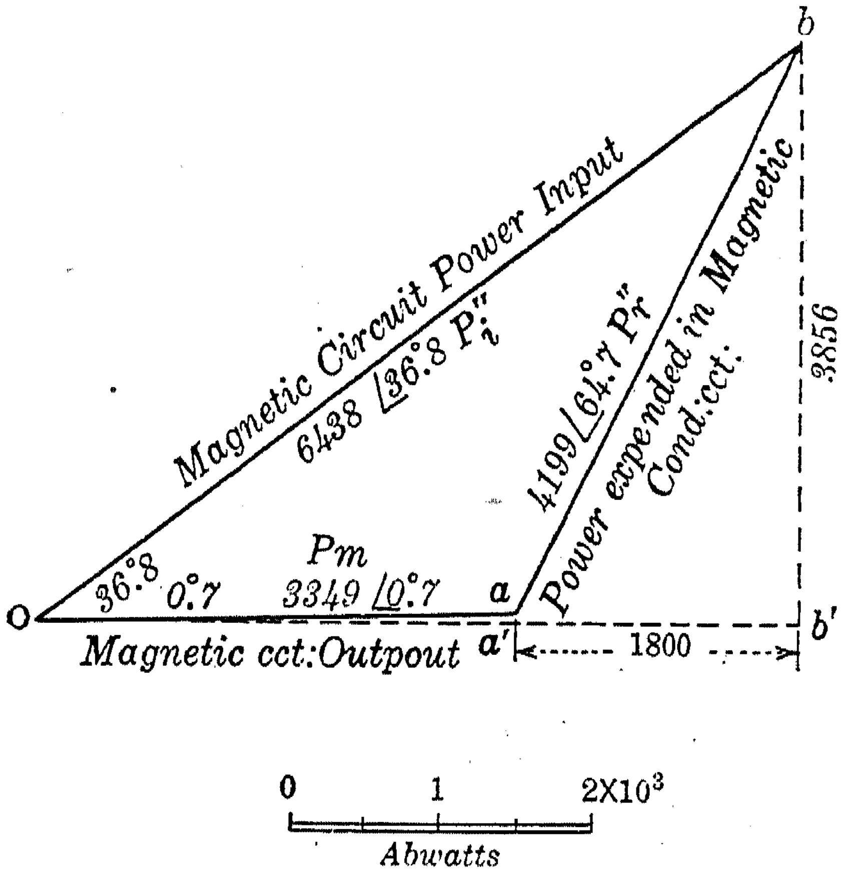


FIG. 25—MAGNETIC CIRCUIT POWER DIAGRAM—DIAPHRAGM FREE

The motional magnetic power diagram is obtained as in Fig. 26 by subtracting the magnetic power input damped P_i, from the magnetic power input free, P_i''. The difference, at the frequency of 1028 ~, is 5108 $\overline{\diagdown}$ 79.°2 abwatts, with an active component of 957 abwatts, representing an increase of the power taken from the electric supply circuit and a negative reactive component of 5018 abwatts, representing a diminution in magnetic power of storage.

It should be observed that the active component $O\,c = 957$

abwatts is much smaller than the power $P_m = 3349$ abwatts, (Fig. 25) put into the diaphragm. The difference of 2392 active abwatts is diverted from the power previously wasted in the magnetic circuit of 4196, Fig. 23, only 1800 abwatts ($a\ b'$ Fig. 25), being wasted thermally when the diaphragm is freed. Consequently, the motional power diagram does not reveal on its real axis all the power delivered to the diaphragm; but only that part which is supplied from the electric circuit. The remainder has to be made up from power saved in hysteretic and eddy loss, due to the reduction in magnetic flux by the motional counter m. m. f., especially towards higher frequencies.

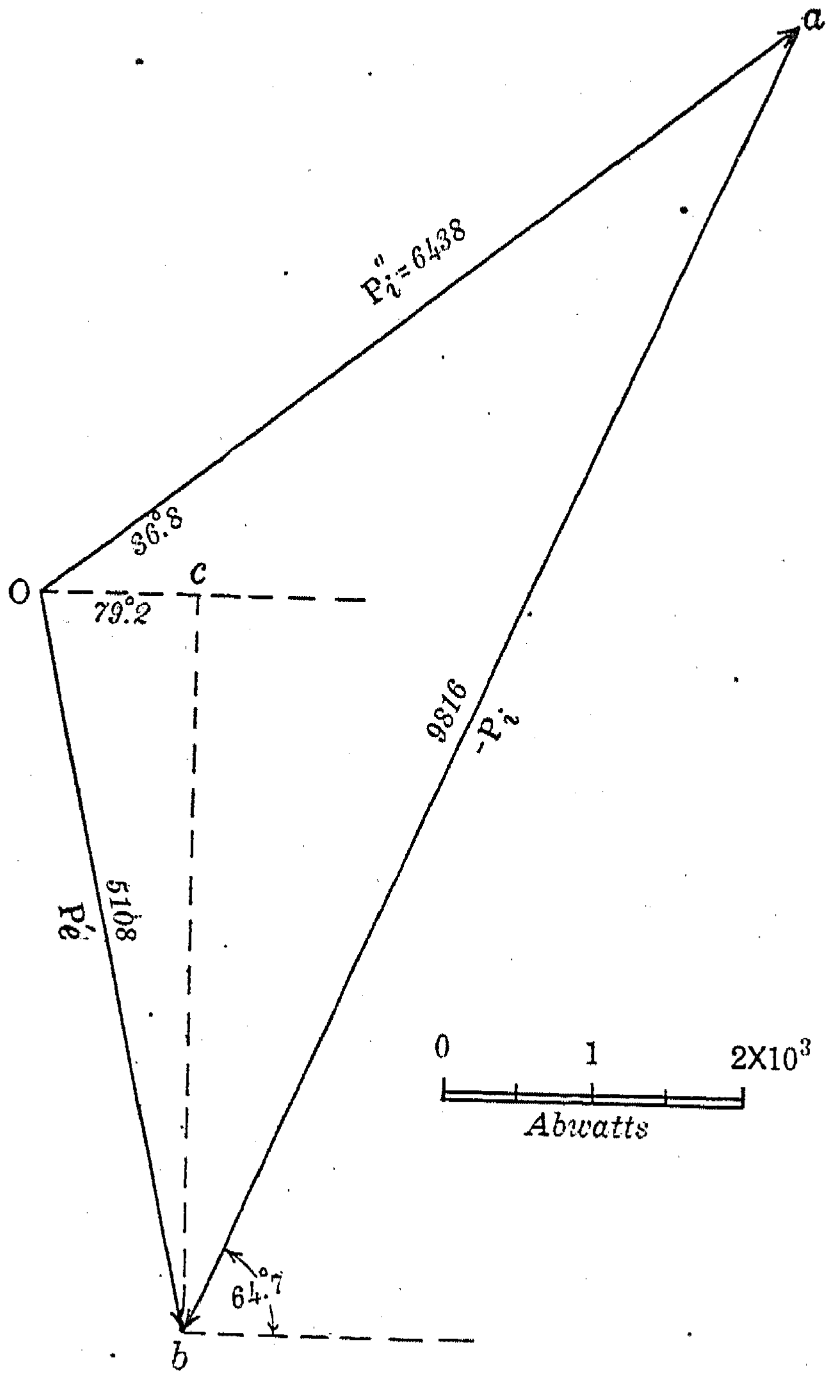


Fig. 26—Diagram of Motional Power in Magnetic Circuit—Diaphragm Free

Turning to Fig. 27, the motional power diagram, the circular locus there represented may be considered as the motional impedance circle of Fig. 10, changed in scale, however, for convenience, in order to permit powers to be read off it. If, however, a fractional scale of power is accepted, the motional impedance circle may be interpreted directly as a motional power diagram. In Fig. 27, *O a* is the vector corresponding in phase to the motional impedance vector at the frequency considered, in this case 1028 ∼. There are three pairs of co-

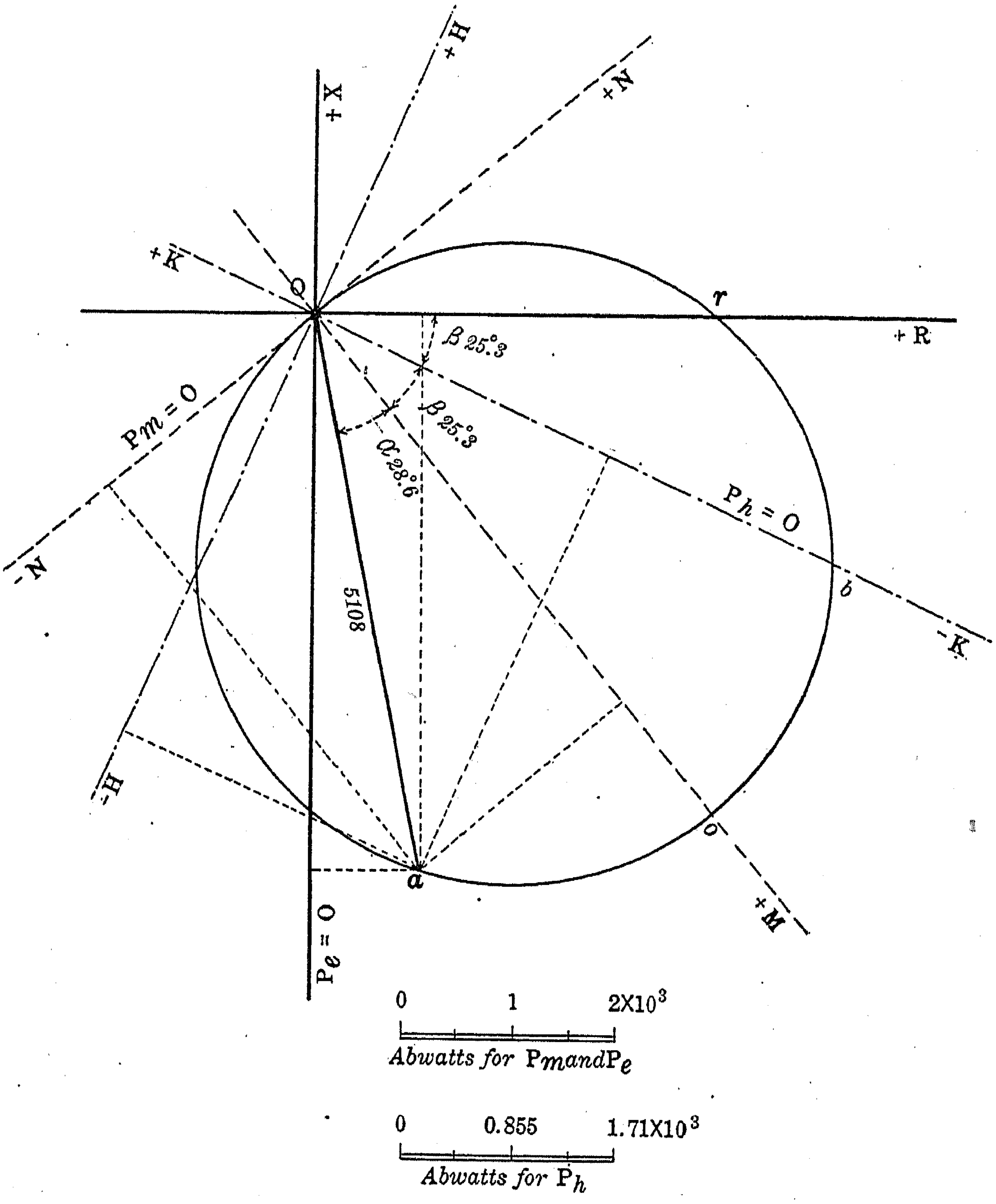


FIG. 27—MOTIONAL POWER DIAGRAM

ordinate axes; namely (**1**) *O R*, at standard phase, for the electric circuit power, and *O X* its perpendicular, (**2**) *O H*, inclined at $-\beta°$ from *O X*, and its perpendicular *O K*, for components of hysteretic loss in the magnetic circuit, and (**3**) *O M*, at a slope of $-2\beta°$ with *O R*, and its perpendicular *O N*, for mechanical power of vibration in the diaphragm.

At any vector motional power $O\,a$, we have the following vector relation

$$P_m'' = P_e' - P_h' \cdot 2 \sin \beta \qquad \text{abwatts} \angle \quad (\mathbf{61})$$

Or, in language, the vector mechanical power P_m'', as read to coordinates $O\,M$, $O\,N$, is equal to the electrically delivered vector power P_e', read to coordinates $O\,R$, $O\,X$, less the hysteretic power P_h', read to coordinates $O\,H$, $O\,K$, multiplied by the coefficient $2 \sin \beta$, which in this case is 0.855: Moreover, this equation is true not only for these quantities as vectors; but also for their respective active components,

$$P_{ma}'' = P_{ea}' - P_{ha}' \cdot 2 \sin \beta \qquad \text{active abwatts} \quad (\mathbf{62})$$

and likewise for the reactive components

$$P_{mr}'' = P_{er}' - P_{hr}' \cdot 2 \sin \beta \qquad \text{reactive abwatts} \quad (\mathbf{63})$$

These propositions, which apply here to the energetics of a telephone receiver, for the particular case of β as the angle separating the systems of coordinates, are true in a general geometric sense for any plane vector, measured to three such sets of coordinates, without restriction of the value of β, and also without restriction as to the locus of the radius vector. This proposition is discussed in the Appendix.

In Fig. 27, we assume a knowledge of Z'_0 absohms ∠, as well as I r. m. s. absamperes the testing current strength, at standard phase. The power expended in this impedance will be $I^2\ Z_0'$ abwatts at the same slope as that of Z_0'. In the case considered, $I = 2.04 \times 10^{-4}$ r. m. s. absamperes, and $Z_0' = 140 \times 10^9 \overline{\diagdown}\, 50°.6$ absohms. The vector power is therefore $(2.04)^2 \times 1.4 \times 10^3 = 5827 \overline{\diagdown}\, 50.°6$ abwatts. This vector $O\,o$ is laid off in Fig. 27, to scale, as the diameter of the motional power circle $O\,r\,o\,a$. At any assigned impressed angular velocity ω, the angle of the corresponding radius vector with respect to the diameter $O\,o$ is

$$\alpha = \tan^{-1}\left(\frac{m\,\omega - s''/\omega}{r''}\right) \qquad \text{degrees lag} \quad (\mathbf{64})$$

In the case considered, with $f = 1028 \sim$, and $\omega = 6460$, $m\,\omega = 5827$, $s''/\omega = 5681$, $r'' = 268$; so that

$$\alpha = \tan^{-1} 0.545 = 28.°6.$$

The vector position $O\,a$, corresponding to $1028 \sim$ can thus be laid down in the circle. Its length is 5108 abwatts. With respect to the *R X* coordinates it is 5108 ⟍ 79.°2. It is 5108 ⟍ 28.°6 to *M N*, and 5108 ⟍ 143.°9 to *H K*. Here the electrical vector is 5108 ⟍ 79.°2 abwatts, the mechanical vector 5108 ⟍ 28.°6, and the hysteretic vector $P_h = P_h' \cdot 2 \sin \beta$

$$= 5108 ⟍ 143.°9 \times 2 \sin 25.°3 = 4369 ⟍ 143.°9.$$

The proposition of (**61**) states the mechanical power of 5108 ⟍ 28.°6, is equal to the electrical power of 5108 ⟍ 79.°2, minus the hysteretic power 4369 ⟍ 143.°9. Moreover, the active mechanical power of vibration, $\dot{x}^2 r''$ is the *O M* component of *O a*, or 4484 abwatts. This is made up of the active electric power component $I^2 R'$, as measured on the *O R* axis, of 957 abwatts, minus the active hysteretic component along the *H* axis, or − (− 3537) = + 3537 abwatts. The mechanical power is therefore mainly supplied by saving in hysteresis loss, and only partly supplied by assistance from the electric circuit. Similarly, the reactive mechanical power $j\,\dot{x}^2\,(m\,\omega - s''/\omega)$ is the *O N* component of *O a*, and measures $-j\,2445$ abwatts. This is equal to the reactive electric component $j\,I^2 X'$ along *O X* or $-j\,5018$ less the hysteretic component measured to its own scale along *O K* = − ($-j\,2591$) = $j\,2591$ abwatts.

Proceeding in the above manner, the power put into mechanical vibration of the diaphragm can be read off the power circle at any assigned frequency, and its component sources, electrical and hysteretic, determined either graphically or by computation. It will be evident that for frequencies below that of the vector *O b*, β° behind *O R*, the hysteretic active power is positive. This means that the hysteretic loss is increased at these frequencies, so that the electric circuit has to supply not only all the mechanical vibration power, but also the extra hysteresis and eddy loss due to increased magnetic flux. At the frequency of *O b*; there is no additional hysteretic loss, because the flux in the magnetic circuit has the same size, with the diaphragm either damped or free. At this particular frequency, all the active mechanical power is supplied directly from the electric circuit. At frequencies above that of *O b*, the active mechanical power is partly supplied by hysteresis saving. At the vector point *c*, where the circle cuts the reactance axis, there will be no extra power supplied by the circuit to maintain the mechanical output of the diaphragm; but all of this power will be paid for by saving in hysteresis loss, when the diaphragm is released.

The total active mechanical power developed in the diaphragm will be

$$P_m'' = \dot{x}^2 (r + r') = \dot{x}^2 r'' \qquad \text{abwatts} \quad (65)$$

and the active power put into vibration

$$P_m = \dot{x}^2 r \qquad \text{abwatts} \quad (66)$$

Similarly, the active power put into motional hysteresis loss,

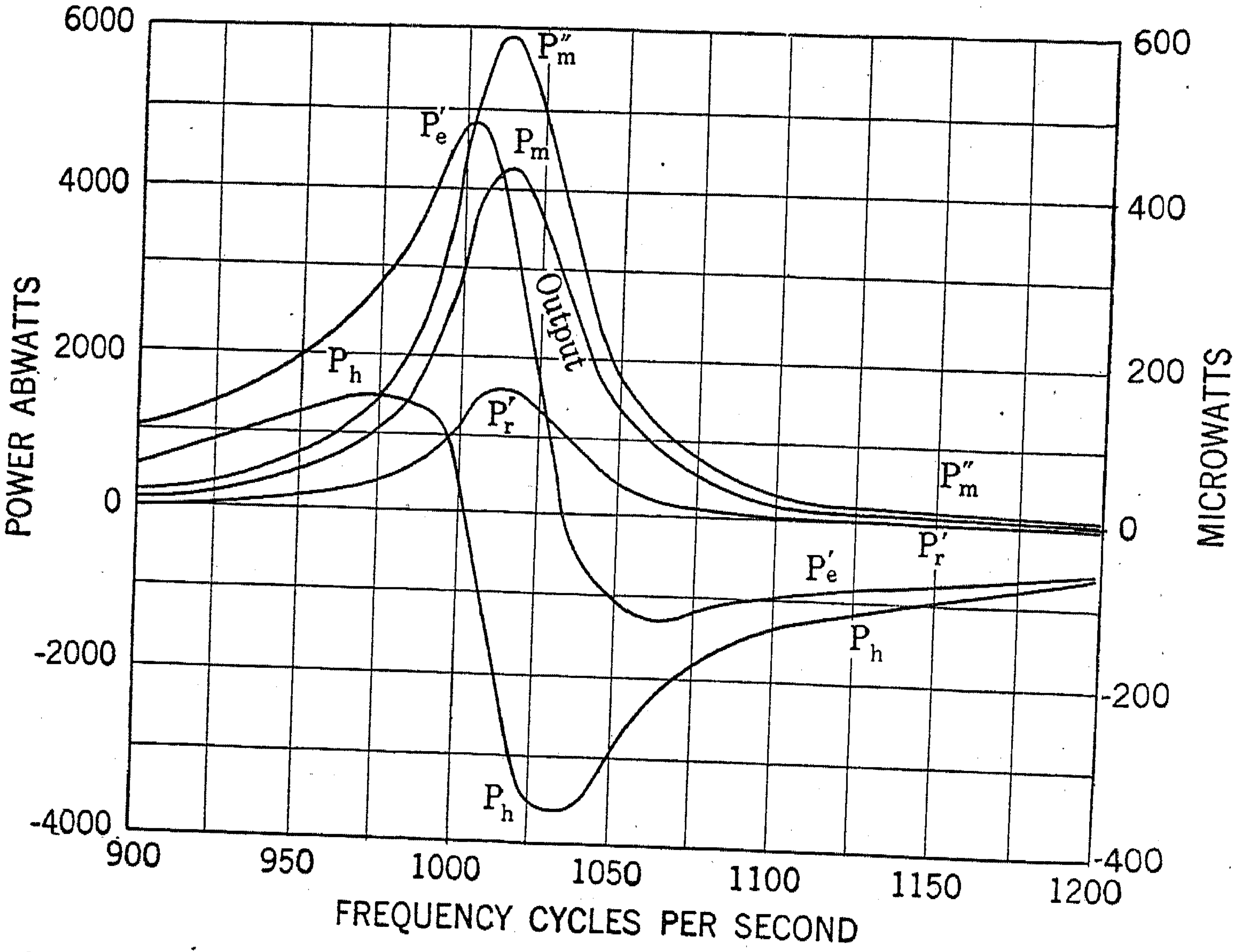


Fig. 28—Magnetic Circuit Motional Power Disbtribution at Different Frequencies. P_m'' Gross Mechanical Active Output P' Active Electric Power Increases P_h. Change of Hysteretic Power. P_r' Active Mechanical Power Expended Against r' in Displacement. P_m Net Active Mechanical Output Free. $P_m'' = P_e' - P_h = P_r' + P_m$.

by reason of the vibration of the diaphragm in the magnetic field will be

$$P_r' = \dot{x}^2 r' \qquad \text{abwatts} \quad (67)$$

The mechanical efficiency of the diaphragm, as a motor, at resonance, is therefore

$$\eta_0 = \frac{r}{r''} = \frac{r}{r + r'} \qquad \text{numeric} \quad (68)$$

In the case considered, $\eta_0 = 0.74$.

Strictly speaking, according to our theory, r' is a function of ω, see (**29**), but unless the resonance is very blunt, the changes in r' and in η, with change in frequency, are not likely to be important.

Fig. 28 shows the distribution of the various active powers, in the instrument considered, over a range of frequency from $900 \sim$ to $1200 \sim$. Here $P_m = \dot{x}^2 r$ is the active mechanical power output in sound and in overcoming internal mechanical bending friction of the diaphragm. $P_r' = \dot{x}^2 r'$ is active mechanical power expended in the magnetic circuit in hysteresis and in eddies, by reason of the vibration of the diaphragm in the permanent magnetic field. $P_m'' = \dot{x}^2 r'' = \dot{x}^2 (r + r') = P_m + P_r'$ is the total active mechanical output of the diaphragm. P_e' is the extra active electric power taken from the electric circuit and put into mechanical vibration when the diaphragm is freed. The saving of active damped power put into the magnetic circuit when the diaphragm is freed is $-P_h$. Consequently

$$P_m'' = P_e' - P_h.$$

The following tabular statement gives the computed analysis of the constants of the particular instrument considered, partly taken from the Kennelly-Affel paper of 1915, already referred to, and partly as developed in this analysis:

TABLE OF NUMERICAL DATA CONCERNING ANALYZED RECEIVER

MECHANICAL	
Area of each pole, cm × cm.	1.14×0.199
Distance separating poles, cm.	0.686
External diam. of diaphragm, cm.	5.52
Diameter of clamping circle, cm.	5.0
Thickness of diaphragm over japan, cm.	0.031
Weight of diaphragm, gm.	4.397
Number of turns including both spools	1300
ELECTRICAL	
D-c. res. at 20 deg. cent. absohms R_1	86.7×10^9
Inductance at resonant frequency $\mathfrak{L}_0$ abhenries.	3.65×10^7
Current through receiver I r. m. s. absamp.	2.04×10^{-4}
Resonant frequency apparent, f_0 cycles per sec.	1015
Resonant angular vel: ω_0 radians per sec.	6378
Motional impedance circle diameter Z_0' absohm.	140×10^9
Slope of permeance at resonant frequency β. degrees	$-25^\circ.3$

Max. amplitude of vibration at resonance x_m cm	10.35×10^{-4}
Effective cyclic amplitude vibration at resonance x_0 cm.	7.32×10^{-4}
Max. cyclic velocity at resonance $\dot{x}_0$ r. m. s. cm. per sec or kines	4.666
Quadrantal angular velocities ω_1, ω_2 radians/sec.	$\begin{cases} 6227 \\ 6532 \end{cases}$
Total decrement Δ hyps. per sec.	149
Force factor A dynes /absampere or abvolts/kine.	$6.12 \times 10^6 \overline{\diagdown}\, 25.^\circ 3$
Total mechanical resistance r'' dynes/kine	268
Equivalent mass in grams	0.902
Total stiffness coefficient s'' dynes/cm.	36.7×10^6

RECOMPUTED ELECTROMAGNETIC DATA

Ratio of numbers of turns p absampere/cm.	0.168
Mechanical resistance of magnetic reaction r' dynes/kine	68.8
Pure mechanical resistance r dynes/kine	199
Stiffness coefficient of magnetic reaction s' dynes/cm.	0.928×10^6
Pure mechanical stiffness coefficient s dynes/cm.	37.63×10^6
Equivalent number of turns per cm. displacement N' gilberts/cm.	218
Normal mean polar flux density $\mathcal{B}_0$ gausses	1369
Normal total polar flux Φ_0 maxwells/pole	311.0
Equivalent permanent magnetic reluctance $\mathcal{R}_0$ oersteds	$0.582 \angle 25.^\circ 3$
Equivalent permanent magnetic permeance $\mathcal{P}_0$ (oersteds)$^{-1}$	$1.718 \overline{\diagdown}\, 25.^\circ 3$
Equivalent permanent m. m. f. gilberts	181
M. M. F. of testing current (2.04×10^{-4}) r. m. s. gilberts $\mathfrak{F}_i$.	$3.334 \angle 0^\circ$
Flux in magnetic circuit due to current ϕ_i r. m. s. maxwells	$5.73 \angle 25.^\circ 3$
Mechanical force due to current I f_i, r. m. s. dynes, $v\, m\, f$	$1249 \overline{\diagdown}\, 25.^\circ 3$
Magnetic power input at 1028 ~ with current I, P_i abwatts,	$9816 \angle 64.^\circ 7$ $= 4196 + j\, 8875$
Total mechanical impedance at $f = 1028$, z' dynes/kine	$305.2 \angle 28.^\circ 6$
Mechanical velocity at $f = 1028$. $\dot{x}$. r.m.s. cm/sec.	$4.091 \angle -53.^\circ 9$
Cyclic mechanical displacement at $f = 1028$. x r. m. s. cm.	$6.332 \times 10^{-4} \overline{\diagdown}\, 143.^\circ 9$
M. M. F. due to displacement, at $f = 1028$ and for current I, $\mathfrak{F}_x$ r. m. s. gilberts	$1.735 \overline{\diagdown}\, 143.^\circ 9$
Resultant m. m. f., at $f = 1028$. $\mathfrak{F}$ r. m. s. gilberts	$2.174 \angle -28^\circ$
Flux due to displacement, at $f = 1028$. ϕ_x r.m.s. maxwells	$2.980 \overline{\diagdown}\, 169.^\circ 2$

Resultant flux at $f = 1028$. ϕ r. m. s. maxwells. 3.757 ⟍ 53.°2
Magnetic input diaphragm free, at $f = 1028$.
P_i'' abwatts ....6438 ∠ 36.°8
=5156 + j 3856
Free magnetic power, or total power absorbed in
magnetic circuit, at $f = 1028$ ~, abwatts ∠ P_r'' =4199 ∠ 64.°7
= 1795 + j 3797
Mechanical output power of diaphragm, P_m at
1028 ~ abwatts ∠...................... = 3349 ∠ 0.°7
= 3349 + j 40
Total mechanical power of diaphragm at 1028 ~
P_m'' abwatts ∠ = 5108 ⟍ 28.°6 = 4484 − j 2445
Motional electric power of diaphragm at 1028 ~
P_e' abwatts ∠ = 5108 ⟍ 79.°2 = 957 − j 5018
Hysteretic power at 1028 ~ P_h abwatts ∠
= 5108 × 0.855 = 4369 ⟍ 143.°9 = − 3537 − j 2591
Mechanical efficiency of diaphragm at apparent
resonance, η_0 numeric = 0.74
Net efficiency of receiver, freed, and at apparent
resonance numeric, = 0.4
Maximum motional impedance per unit of continuous-current resistance Z_0'/R_1 numeric ∠ = 2.04 ⟍ 50.°6
Force factor per root of continuous-current res.

$$A/\sqrt{R} \quad \frac{\text{dynes} \sqrt{\text{absohms}}}{\text{abvolts}} \angle = 23.36 \diagdown 25.°3$$

Effect of a Changing the Number of Turns in Receiver of given Winding Space. If the windings of a given receiver are allowed to occupy a definite constant volume, and also if the ratio of bare to covered wire diameter is kept constant over the range of wire diameters selected for the winding, so that the same volume and weight of copper will exist no matter what the size of wire; then it is evident that if we reduce the wire diameter one-half, the number of turns N will be quadrupled and the d-c. resistance R_1, at constant temperature, increased sixteen fold. The effect of this will be to increase N, A, $\mathcal{F}_i$, and ϕ_i, 4 times, leaving $\mathcal{F}_0$, $\mathcal{B}_0$, $\mathcal{P}$, and β, unchanged. The quantities $\mathcal{L}$, Z, Z_0', P_e, and P_{m0}, will be all increased 16 times, (neglecting variations in r') or in direct proportion to the resistance. Consequently the ratio of active mechanical power output $\dot{x}^2 r/I^2 R_1$, where R_1 is the d-c. resistance of the instrument, will remain substantially unchanged for all values of R_1 within the range for which the copper in the winding space remains constant. On this theory, changing the winding of a receiver, other things remaining the same, increases the mechanical output at resonance in proportion to the resistance, for a fixed exciting current strength, but for a given amount of $I^2 R_1$ power loss in the winding, this mechanical power output remains substantially constant.

Since Z_0' the maximum or diametral value of the vector motional impedance increases, according to this theory, directly with the d-c. resistance of the winding, different receivers should have their diametral impedances Z_0' reduced to a common basis of comparison by taking the ratio Z_0'/R_1. The sensitiveness of a receiver is proportionate to this ratio. The ratio is independent of the strength I of the testing current. An instrument of large diametral impedance per unit of d-c. resistance is therefore an instrument of inherently large electromagnetic sensitivity, no matter what the resistance of the winding may be. This ratio calls for no other measurements than that of the motional impedance circle or the essential sector thereof, and the d-c. resistance at normal temperature.

Similarly, the force factor A should be divided by $\sqrt{R_1}$, in order to bring it to a common basis of comparison among various receivers.

It should be observed that the diametral or maximum size of the motional impedance Z_0' is not produced at the frequency of strict mechanical resonance of the diaphragm. True mechanical resonance is produced when the angular velocity is such that the mass reactance $m\,\omega$ is numerically equal to the elastic reactance s/ω. In the motional-impedance circle, however, the apparent resonance and diametral impedance are produced when $m\,\omega = \dfrac{s''}{\omega} = \dfrac{s - s'}{\omega}$. The fictitious component s', due to the displacement flux, thus slightly lowers the apparent resonance, or causes the maximum velocity, displacement and impedance to occur at a frequency a little lower than that of true mechanical resonance. In the case considered, mechanical resonance would be produced at $\omega_0 = \sqrt{s/m}$ $= \sqrt{37.63 \times 10^6/0.902} = 6459$ radians per sec.; corresponding to 1028∼; whereas the apparent resonance would be produced at $\omega''_0 = \sqrt{s''/m} = \sqrt{36.7 \times 10^6/0.902} = 6379$, corresponding to 1015∼. This displacement of resonance must occur in the motional impedance circle, whatever method of including the effects of the displacement flux φ_x is adopted. Apparent resonance occurs at the frequency of mechanical resonance as modified by magnetic reaction, which acts in such a manner as to reduce the stiffness coefficient of the diaphragm. The angle of displacement $h\,b\,o$ in the motional impedance circle, Fig. 10, between the vector of true mechanical resonance and apparent resonance, is equal to the angle α between ϕ and ϕ_i at the angular velocity ω_0' of true mechanical resonance; because at ω_0' the velocity $\dot{x}$ is in phase with the resultant flux ϕ.

A bibliography of the subject, without pretensions as to completeness, is appended, in so far as relates to the motional-impedance aspect of telephone-receiver theory.

It may be noted that the maximum mechanical output $\dot{x}_0^2\,r''$ at apparent resonance is entirely active power, and is numerically equal to $I^2\,|Z_0'|$ abwatts or the square of the testing current multiplied by the size of the diametral impedance. This constitutes a simple method of determining the maximum mechanical output of the diaphragm, including frictions and hysteretic loss due to vibration. The gross efficiency of the receiver at apparent resonance is therefore $\dfrac{I^2\,|Z_0'|}{I^2\,R_0''} = \dfrac{|Z_0'|}{R_0''}$.

In this case, this efficiency reaches $\dfrac{140}{257.3} = 0.544$. This must be multiplied by the $\eta_0 = 0.74$ to give the net efficiency of the receiver at apparent resonance of 0.40. This is the ratio of active mechanical output of the diaphragm, including internal frictions and acoustic delivery, to the active electric power consumed at terminals, with the diaphragm free. With the receiver pressed against the ear, the condition is intermediate between damped and free.

SUMMARY OF RESULTS

1. The theory presented in the earlier papers mentioned in the bibliography were lacking in not taking into account the m. m. f. and flux, of diaphragm vibratory displacement in the magnetic field.

2. The present theory purports to include the above mentioned elements, under the form of an apparent increment r' in mechanical resistance, and an apparent diminution s' in the stiffness coefficient, the latter being independent of the frequency, but the former varying inversely as the frequency.

3. The effect of r' is to introduce a slight distortion of the motional impedance and power diagrams from the strictly circular locus, assuming the force-factor A and other constants

to remain substantially the same at all frequencies; but the distortion is not serious from a practical standpoint.

4. The m. m. f. of vibratory displacement may be expressed in terms of a coefficient N', a fictitious number of turns. The ratio p of N'/N is a constant of the instrument.

5. The depression of the motional-impedance circle diameter is $2\ \beta°$ where $\beta°$ is the lag angle of the damped magnetic flux behind the exciting current.

6. The power analysis depends directly upon the m. m. f. and flux diagram, without any necessary reference to the e. m. fs. produced. The power in the magnetic circuit is then found to be similar to the power in a simple alternating-current circuit, containing an alternator, a synchronous motor, and a simple connecting impedance.

7. The observed motional impedance circle may be converted into a corresponding motional power circle, by a suitable alteration in scale of linear dimensions. By introducing three sets of coordinate axes, mutually displaced by $\beta°$, the power output P''_m, can be read off in terms of the electric power input P'_e, and the hysteretic power input P_h. The same relation holds for active and for reactive components.

8. At the slope $-\ \beta°$ in the power circle, the mechanical power is all electric. At the slope $-\ 90°$, it is all hysteretic.

9. The active power mechanically expended in the magnetic circuit by the diaphragm as a motor, against hysteresis and eddy currents $\dot{x}^2\, r'$, can be separated from that expended against friction and air resistance $\dot{x}^2\, r$. The mechanical efficiency is $\dfrac{r}{r + r'}$, which varies somewhat with the frequency. It tends to increase slightly with frequency, according to this theory.

10. The behavior of a particular instrument analysed some years ago in a preceding paper is investigated in detail, at a particular frequency, in accordance with the theory.

11. The gross efficiency of a receiver at apparent resonance is $\dfrac{|Z'_0|}{R''_0}$ or the ratio of the motional impedance diameter to the apparent resistance of the instrument at this frequency with the diaphragm free.

Appendix

Proposition that the Plane Vector

$$|\rho_m|\ \overline{\backslash\alpha} = |\rho_e|\ \overline{\backslash 2\ \beta + \alpha} \qquad - 2 \sin\ \beta\ |\rho_m|\ \overline{\backslash 90^\circ + \alpha^\circ + \beta}$$

In Fig. 29, *O A* is a planevector, or geometrical complex quantity of any convenient size ρ, taken at standard phase, or zero slope. It may be denoted by ρ_A · *O B* is a similar plane vector of the same size, distinguished by the symbol ρ_B. Its slope is $-\ 2\ \beta^\circ$. *O c* is a vector denoted by ρ_c of size ρ and drawn perpendicular to *O d*, the bisector of the angle *A O B*.

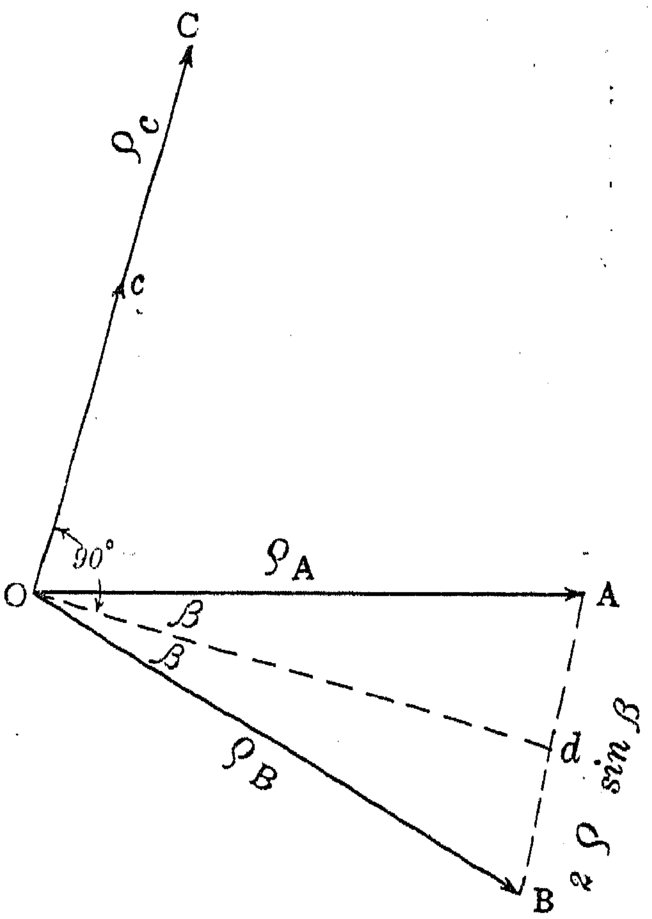


FIG. 29

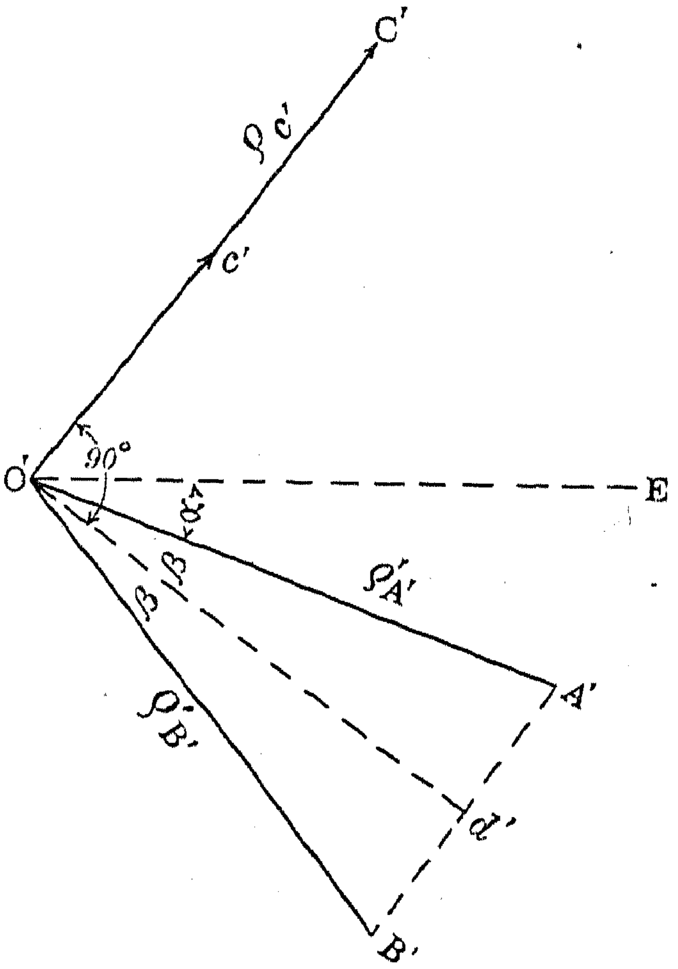


FIG. 30

Then, because *B A* expressed vectorially is $|\rho|\, 2 \sin\ \beta\ \underline{/90^\circ - \ \beta}$ and is equal to the vector *O c*, we have the vector relation

$$\rho_B = \ \rho_A - 2 \sin\ \beta \cdot \rho_c \qquad \text{length units} \angle \quad (\mathbf{69})$$

If now, as in Fig. 30, we rotate the whole vector system in the plane of reference through the angle $-\ \alpha^\circ$ with respect to *O E*, a fixed line of reference, the effect will be the same as though the vector system remained fixed in the plane; but the line of reference, *O E* was rotated through an angle of $+\ \alpha$. This rotation of a reference line cannot, however, affect the vector relation expressed in (**69**) when those vectors are taken to the new reference line.

But the three vectors of (**69**) may be regarded as one and the same vector ρ, Fig. 31, expressed each with respect to three initial lines $O''m$, $O''e$, and $O''h$, and then denoted by ρ_m, ρ_e and ρ_h respectively. Thus in Fig. 32,

$$|\rho_m| \backslash\overline{\alpha} = |\rho_e| \backslash\overline{2\beta + \alpha} - |\rho_h| \, 2 \sin\beta \backslash\overline{90^\circ + \alpha + \beta} \quad \text{length units } \angle \quad (\mathbf{70})$$

Since this relation holds for the vectors themselves, it must also hold for their real components, and for their imaginary components respectively.

The power circle of Fig. 27, corresponds to Fig. 31; while

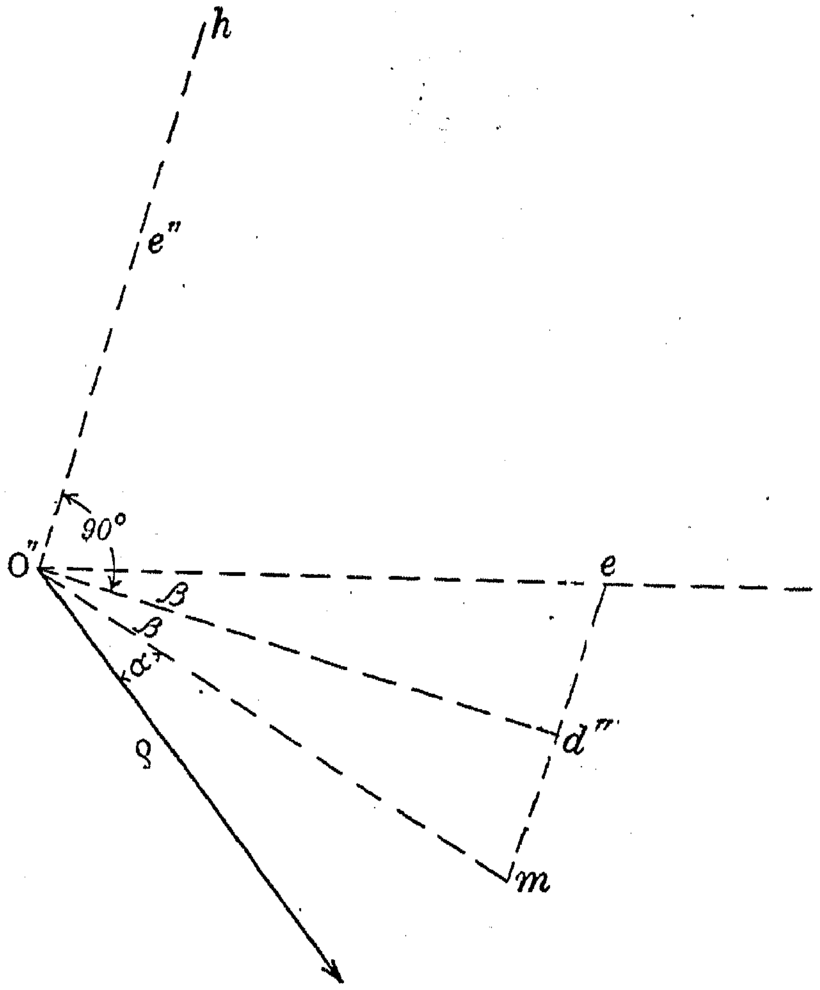


FIG. 31

the interpretation of (**62**) and (**63**) are referable to the construction of Fig. 32.

Referring to Fig. 24, in the analogy of a simple a-c. circuit in which a generator of $\mathfrak{F}_i$ r. m. s. abvolts terminal e. m. f. supplies an external circuit of fixed admittance $\mathcal{P}$ mhos ∠, with a current of ϕ_i r. m. s. absamperes. To the condition of diaphragm damped, corresponds the analogous condition of zero c. e. m. f. in the synchronous motor. To the condition of diaphragm free, corresponds the analogous condition of e. m. f. $\mathfrak{F}_x$ r. m. s. abvolts in the synchronous motor armature, which would produce the current of ϕ_x r. m. s. absamperes if operating independently in the circuit, at assumed constant frequency.

The vector change in electric power P'_e given by the alternator to the circuit when the diaphragm is suddenly freed, will be the power exerted by the e. m. f. $\mathfrak{F}_i$ on the current ϕ_x. The vector mechanical power P_m'' given to the motor by the current ϕ_i will be the power of ϕ_i on $\mathfrak{F}_x$.

Converting these electric powers into their analogous magnetic powers, and remembering that in the magnetic circuit a vector a-c. m. m. f. $|\mathfrak{F}| \angle \gamma°$ r. m. s. gilberts, producing a vector alternating flux $|\phi| \angle 0°$ r. m. s. maxwells, develops a vector power in the magnetic circuit of $|(f/2)\ \mathfrak{F}\ \phi| \angle\ 90° - \gamma°$ abwatts, of which the real part is average active power, and the imaginary part is maximum cyclic reactive power, we have

$$P'_e = |(f/2)\ \mathfrak{F}_i\ \phi_x| \angle\ 90° - (90° + \alpha + 2\ \beta)$$

$$= |(f/2)\ \mathfrak{F}_i\ \phi_x|\ \overline{\diagdown}\alpha + 2\ \beta = |(f/2)\ \mathfrak{F}_x\ \phi_i|\ \overline{\diagdown}\ \alpha + 2\ \beta \quad \text{abwatts} \angle \quad (\mathbf{71})$$

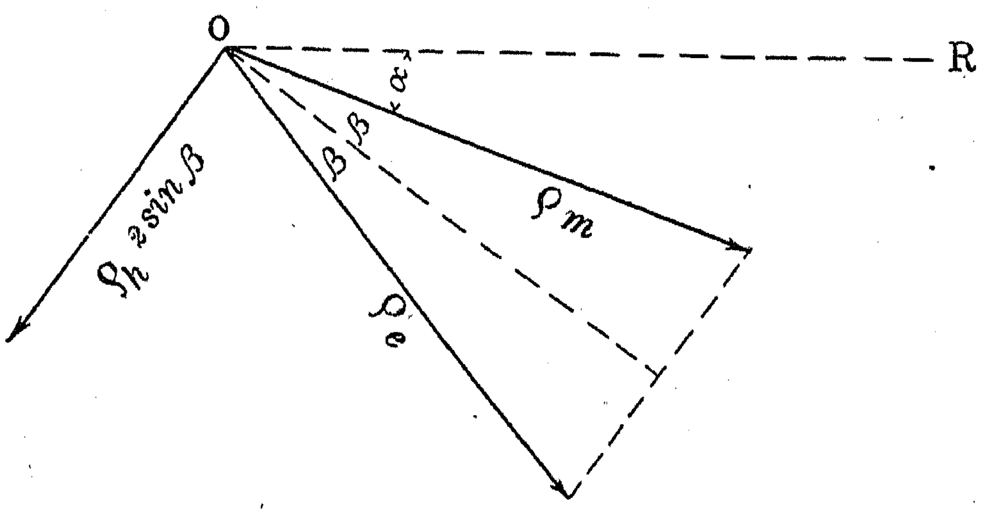


Fig. 32

where $90° + \alpha$ is the angle between ϕ_i and $\mathfrak{F}_x$. This vector power has a circular locus, because $|(f/2)\ \mathfrak{F}_x|\ \overline{\diagdown}\,\alpha$ has a circular locus.

Also

$$P_m'' = |(f/2)\mathfrak{F}_x\phi_i| \angle 90° - (90° + \alpha) = |(f/2)\mathfrak{F}_x\phi_i|\ \overline{\diagdown}\,\alpha \quad \text{abwatts} \angle \quad (\mathbf{72})$$

This vector mechanical power must also have a circular locus under varying frequency.

Consequently,

$$P_e' - P_m'' = |(f/2)\ \mathfrak{F}_x\ \phi_i|\ \overline{\diagdown}\,\alpha + 2\ \beta - |(f/2)\ \mathfrak{F}_2\ \phi_i|\ \overline{\diagdown}\,\alpha \quad \text{abwatts} \angle \quad (\mathbf{73})$$

and this by (**70**) is

$$P_h = |(f/2)\ \mathfrak{F}_x\ \phi_i| \cdot 2 \sin\ \beta\ \overline{\diagdown}\,90° + \alpha + \beta \quad \text{abwatts} \angle \quad (\mathbf{74})$$

a vector hysteretic power having also a circular locus.

List of Symbols Employed

For simplicity of numerical treatment and universality of application, the units employed are those of the C. G. S. magnetic system.

A	force factor (dynes per absampere ∠* or abvolts per kine ∠)
α	angle of mechanical effective impedance; *i. e.*, mechanical impedance including magnetic displacement reaction (degrees)
$\mathcal{B}_0$	magnetic flux density in the airgaps due to permanent magnet (gausses)
$\mathcal{B}_i$	r. m. s. magnetic flux density due to current (r. m. s. gausses ∠)
$\mathcal{B}$	instantaneous alternating flux (gausses)
β	mean angle of lag between magnetic flux and m. m. f. (degrees)
C	capacitance of a condenser (farads or abfarads)
$\Delta = \frac{r''}{2m}$	total apparent damping constant of a diaphragm (hyp. radians per sec.)
E	r. m. s. e. m. f. in a simple alternating-current circuit (volts or abvolts ∠)
$E = I Z'$	motional e.-m. f. (r. m. s. abvolts ∠)
$\eta = r/r''$	mechanical efficiency of diaphragm, corresponding to armature efficiency in a synchronous motor (numeric)
η_0	mechanical efficiency of diaphragm, or friction and acoustic mechanical resistance to total mechanical resistance, at apparent resonance (numeric)
$\mathfrak{F}_i$	m. m. f. due to constant exciting alternating current (r. m. s. gilberts ∠)
$\mathfrak{F}_x$	m. m. f. due to vibratory displacement of diaphragm in permanent magnetic field (r. m. s. gilberts ∠)
$\mathfrak{F}$	resultant vector m. m. f. in magnetic circuit (r. m. s. gilberts ∠)
$\mathfrak{F}_0$	m. m. f. due to permanent magnet (gilberts)
f	impressed frequency (cycles per sec.)

---

*The sign ∠ after a unit signifies that the quantity is a plane-vector or complex quantity as distinguished from a scalar or simple numerical quantity.

f_0,	impressed frequencies of apparent resonance (cycles per second)
f_c	reactive mechanical force of clamping (dynes $\angle$)
f_i	vibromotive force on diaphragm due to alternating current I alone (r. m. s. dynes $\angle$)
f_x	vibromotive force due to vibratory diaphragm displacement in permanent magnetic field (r. m. s. dynes $\angle$)
I	strength of constant small alternating testing current (r. m. s. absamperes)
$j = \sqrt{-1}$	
L	Self inductance of an electric circuit (abhenries or henries)
$\mathcal{L}$	Self inductance of a receiver winding with diaphragm damped (abhenries $\angle$)
m	Equivalent mass of a diaphragm (grams)
N	Number of turns in the winding of a bipolar receiver, including both coils (numeric)
N'	Equivalent number of turns for expressing a displacement m. m. f. in cm-turns (absamperes per cm. or gilberts per cm.).
P_e, P_e', P_e''	Electric power supplied to receiver with diaphragm damped, vector change in supplied electric power due to motion with diaphragm free, and total supplied electric power with diaphragm free. (abwatts or ergs per sec. $\angle$)
P_h	Increase in magnetic circuit power loss due to motion of diaphragm (abwatts $\angle$)
P_h'	Apparent increase in magnetic circuit power loss due to motion of diaphragm, as measured in the motional power circle diagram along the hysteretic axis before applying change in power scale abwatts $\angle$
P_i, P_i''	Damped magnetic power input (abwatts $\angle$), and input power from electric to magnetic circuit with diaphragm free (abwatts $\angle$)
P_m, P_m''	Mechanical output power of diaphragm against pure mechanical res. r (abwatts $\angle$) and total mechanical output power of diaphragm against total resistance of $r'' = r + r'$ (abwatts $\angle$)
P_{ma}, P_{ea}, P_{ha}	Active or dissipatory components of mechanical, electric, and hysteretic power (abwatts)

P_{mr}, P_{er}, P_{hr}	Reactive or non-dissipatory components of mechanical, electric and hysteretic power (j abwatts)
$P_r' = \dot{x}^2 (r' + j s'/\omega)$	Motional power or power expended against reactive force of diaphragm displacement (abwatts ∠)
P_r''	Free magnetic power, or vector power of resultant m. m. f. $\mathcal{F}$ on resultant magnetic flux ϕ, or total absorbed in magnetic circuit (abwatts ∠)
$p = N'/N$	Ratio of equivalent displacement turns to exciting turns (absamperes per cm.)
$\mathcal{P}$	Permeance of magnetic circuit (oersteds^{-1} ∠ or maxwells per gilbert ∠)
ϕ_i	Magnetic flux due to constant exciting current (maxwells ∠)
ϕ_x	Magnetic flux due to displacement x in magnetic field (maxwells ∠)
$\phi = \phi_i + \phi_x$	Resultant magnetic flux (maxwells ∠)
R_1	Continuous-current resistance of receiver winding (absohms)
R	Apparent resistance of receiver to alternating current with diaphragm damped (absohms)
R_h	Increment of apparent resistance due to hysteresis and eddy currents, with diaphragm damped (absohms)
R'	Resistance component of motional impedance Z' (absohms)
R''	Total apparent resistance of receiver with diaphragm free (absohms)
R''_0	Total apparent resistance of receiver at apparent resonance (absohms)
r	Mechanical resistance of diaphragm including internal frictions and acoustic or air resistance (dynes per kine)
r'	Hysteretic resistance of magnetic circuit to motion of diaphragm, or apparent increase in friction of diaphragm due to displacement flux losses in magnetic circuit. (dynes per kine.)
$r'' = r + r'$	Total mechanical resistance to motion of diaphragm (dynes per kine)

R_0	Reluctance of magnetic circuit to permanent flux (oersteds)
S	Surface area of one pole neglecting magnetic fringe. (sq. cm.)
s	Effective stiffness coefficient of diaphragm to vibrational displacement (dynes per cm.)
s'	Apparent diminution in stiffness coefficient due to displacement in magnetic field. (dynes per cm.)
$s'' = s - s'$	Total effective stiffness of diaphragm including magnetic displacement (dynes per cm.)
t	Time elapsed from a certain epoch (seconds)
X	Reactance of a receiver winding with damped diaphragm (absohms $\angle$)
X'	Reactive component of motional impedance Z' (j absohms)
X''	Reactive component of receiver impedance Z'' with diaphragm free (j absohms)
x	Displacement of diaphragm towards poles from normal flexed position (r. m. s. cm. $\angle$)
x_0	Displacement of diaphragm at apparent resonance (r. m. s. cm. $\angle$)
$x_m = \|x_0\| \sqrt{2}$	Maximum cyclic displacement of diaphragm at apparent resonance (max. cm.)
$\dot{x}$	Velocity of diaphragm r. m. s. kines $\angle$ or r. m. s. cm. per sec. $\angle$
$\dot{x}_0$	Velocity of diaphragm at apparent resonance (r. m. s. kines $\angle$)
$\ddot{x}$	Acceleration of diaphragm (r. m. s. kines per sec. $\angle$)
$y = 1/z$	Mechanical admittance of a diaphragm when the magnetic reactive force due to displacement is neglected (kines per dyne $\angle$)
y'	Apparent mechanical admittance of diaphragm, including magnetic reactive force due to displacement (kines per dyne $\angle$)
Z	Impedance of a receiver winding with diaphragm damped (absohms $\angle$)
$Z' = Z'' - Z$	Motional impedance of winding (absohms $\angle$)
Z''	Impedance of winding with diaphragm free (absohms $\angle$)

Z_0'	Motional impedance at apparent resonance, and at diametral or maximum size. (absohms $\angle$)
z	Mechanical impedance of a diaphragm with reactive magnetic force neglected (dynes per kine $\angle$)
z'	Effective mechanical impedance of diaphragm including reactive magnetic force (dynes per kine $\angle$)
$\omega = 2\pi f$	Impressed angular velocity (radians per sec.)
ω_0'	Impressed angular velocity at true mechanical resonance (radians-sec.)
ω_0	Impressed angular velocity at apparent mechanical resonance (radians per sec.)
ω_1, ω_2	Lesser and greater quadrantal angular velocities (radians per sec.)
r. m. s.	Contraction for root-mean-square.
$\|Z\|$	Size or mere numerical value of a vector quantity Z.

Bibliography

(1) Lord Rayleigh, "Theory of Sound", 1894, Vol. 1, pp. 366, 473.

(2) Wien, *Annalen, d. Phys.*, Vol. 4, 1901, p. 450.

(3) Henri Abraham, "Rendement Acoustique du Téléphone", *Comptes Rendus, Acad. d. Sc.*, 144, 1907.

(4) Henri Poincaré, "Étude du Récepteur Téléphonique", l'Éclairage Électrique, Vol. 50, Feb. 16, Feb. 23, Mar. 9, Mar. 16, 1907., pp. 221–372.

(5) A. E. Kennelly and Walter L. Upson, "The Humming Telephone". *Proc. Am. Phil. Soc.*, Vol. 47, pp. 329–365, July, 1908 .

(6) John B. Taylor, "Telegraph and Telephone Systems as Affected by Alternating-Current Lines", TRANS. A. I. E. E., Oct., 1909, Vol. 28, Part II, p. 1169–1215.

(7) A. E. Kennelly, "Vector Power in Alternating-Current Circuits", TRANS. A. I. E. E., Vol. 29, pp. 1233–1267, June, 1910.

(8) G. W. Pickard, "Telephone Receiver Efficiency". *Western Electrician*, May 6, 1911, Vol. 56, p. 899.

(9) A. E. Kennelly and G. W. Pierce, "The Impedance of Telephone Receivers as affected by the Motion of their Diaphragms", *Proc. Am. Ac. Arts and Sc.*, Vol. 48, Sept., 1912, pp. 113–151; also *The Electrical World*, N. Y., Sept. 14, 1912; also *British Assoc. Adv. Sc.* Report Dundee Meeting, 1912.

(10) Chas. F. Meyer and J. B. Whitehead, "The Vibrations of Telephone Diaphragms", TRANS. A. I. E. E., Vol. 31, part II, 1912, pp. 1397–1418.

(11) Geo. D. Shepardson, "Equivalent Frequency of Telephone Currents", A thesis in the library of Harvard University, Cambridge, 1912.

(12) H. A. Affel and O. C. Hall, "Telephone Receiver Characteristics", Thesis, *El. Engg. Dept., Mass. Inst. Tech.*, June, 1914.

(13) Augustin Guyau, "Le Téléphone Instrument de Mesure", Paris, *Gauthier-Villars*, 1914.

(14) R. L. Jones, "Simple Vibratory Systems and their Impedance Analysis", Unpublished Memorandum, Engg. Dept., Western El. Co., 1914.

(15) L. Bouthillon and L. Drouet, "Étude Expérimentale du Récepteur, Téléphonique," *Comptes Rendus*, Vol. 158, June 2, 1914, p. 1568, also *La Révue Électrique*, Oct. 16, 1914, p. 294-295.

(15a) A. Guyau "The Telephone Receiver" *Journ de Physique*, June, 1914, Vol. 4, pp. 480–492.

(16) A. E. Kennelly and H. O. Taylor, "Explorations over Vibrating Surfaces of Telephonic Diaphragms Under Simple Impressed Tones", *Proc. Am. Phil. Soc.*, April, 1915, pp. 95–136, *Bulletin No. 7, Research Div. El. Engg. Dept., Mass. Inst. Tech.*

(17) A. E. Kennelly and H. A. Affel, "The Mechanics of Telephone-Receiver Diaphragms, as Derived from their Motional Impedance Circles", *Proc. Am. Ac. Arts & Sc.*, Nov., 1915, Vol. 51, pp. 422–482, *Bulletin No. 8, Research Div., El. Engg. Dept., M. I. T.*

(17a) B. S. Cohen and J. G. Hill "Long-Distance and Cable Telephony", *Inst. P. O. Elec. Engrs.* 1916.

(17b) "Notes on the Electromagnetic Theory of the Telephone" by R. L. Wegel, an unpublished memorandum of the Western Electric Co.

(18) A. E. Kennelly and H. O. Taylor, "Some Properties of Vibrating Telephone Diaphragms", *Proc. Am. Phil. Soc.*, April, 1916, Vol. 55, pp. 415–460, *Bulletin No. 11, Research Div., El. Engg. Dept., M. I. T.*

(18a) M. Latour "Theory of Telephone Receiver" *La Lum. Elec.* Vol. 33, June 17th and July 22, 1916.

(19) Geo. D. Shepardson, "Telephone Apparatus", Appleton & Co., 1917.

(19a) B. S. Cohen "Telephonometry" *The Electrician*, Vol. 77, May 26th and June 2nd, 1916, pp., 244–245 and 277–279.

(20) H. O. Taylor, "Motional Impedance Circle of the Telephone Receiver", *Proc. Radio Club of America*, June, 1917.

(21) Henri Tournayre, "Théorie du Téléphone" *L'Industrie Électrique*, July 25, Aug. 10, 1917.

(22) A. E. Kennelly, "Some Experiments on the Effects of Changes in Diaphragm Thickness on the Characteristics of a Telephone Receiver", *Journal of the Franklin Institute*, November, 1917, Vol. 184, pp. 723–726.

(23) John Mills, "Radio Communication", McGraw-Hill Book Co., 1917, pp. 26-35.

(24) W. H. Eccles, "Wireless Telegraphy and Telephony", D. Van Nostrand Co., 1918, pp. 347–350.

(25) Marius Latour, Note sur les Pertes dans les Tôles de Fer aux fréquences élevées. *Révue Gén. de l'Él.* Vol. 3, April 13, 1918, pp. 539–543.

Presented at the 348th Meeting of the American Institute of Electrical Engineers, Boston, Mass., March 14, 1919.

Copyright 1919. By A. I. E. E.

THE ABSOLUTE MEASUREMENT OF THE INTENSITY OF SOUND

BY ARTHUR GORDON WEBSTER

Abstract of Paper

This paper includes a description of a series of acoustical researches extending over a period of twenty-eight years. The properties of vibrating bodies and the subject of elastic hysteresis are discussed. Two fundamentally important instruments for the absolute measurement of sound have been developed and the theory given. The first is the standard of sound, called the phone, which is capable of reproducing at any time a sound of the simplest character and which permits the output of sound to be measured in watts of energy. The second is an instrument called the phonometer for measuring a sound in absolute measure. This instrument is now practically as sensitive as the human ear. Two essential features are the small damping of the vibrating system which results in extreme sensitiveness, but at the same time in great selectiveness, and the capacity for being tuned. Each of these instruments is fitted with a variable volume resonator and tuning over a range of about two octaves is accomplished by varying the volume of the resonator and by changing the tension of the wires to which the piston is attached. The determination of the space distribution of sound and of the effect of disturbing bodies, and the measurement of the reflecting coefficient of surfaces have been accomplished. The phonotrope is a third instrument designed and used to find the direction of a source of sound, for example a fog signal.

THE profession of acoustical engineering is a new one that has arisen during the last few years. The late Professor Wallace C. Sabine, of Harvard University, attracted the attention of the public to the very great importance of physical measurement and thoroughly scientific design in the construction of auditoriums which until his time had remained an entire mystery to architects.

The writer has been interested for many years in the problem of the measurement of sound in absolute units, involving the design of an instrument which is capable of determining, at any point in space, the pressure in the air wave as a function of the time. In order to carry out this object three things are necessary.

First, the construction of a standard of sound which will enable a given sound of the simplest possible character, namely, that in which the pressure varies as a simple harmonic function of the time, to be reproduced at any time, and which permits an output of sound to be measured in watts of energy. This problem has been solved by a number of persons including Professor Ernst Mach and Dr. A. Zernov of Petrograd.

Second, the invention of an instrument capable of measuring a sound of the simple harmonic character described, also in absolute measure; that is to say, in such a way that the amplitude of pressure variation may be measured in absolute measure, as dynes per square centimeter, or in millionths of an atmosphere. This problem has also been successfully treated by a number of investigators, among whom may be mentioned Professor Max Wien, Lord Rayleigh and Lebedeff.

Third, a step in the process remains which is perhaps equally important as the first and second. Given the invention of the proper source of sound, which I have called a phone, because it is a sound and nothing else, and a proper measuring instrument, which I call a phonometer, there still remains the question of the propagation and the distribution of sound in space between the phone and the phonometer. Any measurements made in an enclosed space will be influenced by the reflection from the walls, and even if we had a room of perfectly simple geometrical form and were able to make the instruments of emission and reception work automatically without the disturbing presence of an observer, it would still be impossible to specify the reflecting power of the walls without a very great amount of experimentation and complicated theory. Nevertheless, this is exactly what Sabine did. He did it, however, by employing as a receiving instrument the human ear. Those who have made experiments upon the sensitiveness of the human ear for a standard sound will immediately doubt the possibility of making precise measurements by the same ear at different times, and particularly of comparing the measurements made by one ear with those made by another. Nevertheless, Sabine attained wonderful success and was able to impart his method to pupils and colleagues who carried on his methods successfully.

To proceed with the question of disturbing objects, one should take his phone and his phonometer to an infinite distance from all objects, which is manifestly impossible. The plan which I followed was to attempt to get rid of all objects except an infinite space covered with a surface the reflecting coefficient of which could be measured. I found the proper weather and the proper space on the golf links of a country club where conditions appeared to be favorable. Several days were spent in determining the reflection coefficient. This was done about ten years ago, whereas with the instrument available to-day it could be done in a very few minutes.

The history of my investigations is briefly as follows: In 1890 I proposed to use a diaphragm made of paper or some similar substance which should be placed, shielded on one side, at the spot where the sound was to be measured. Upon the center of the diaphragm was cemented a small plane mirror. In close juxtaposition and parallel with this was the plane side of a plano-convex lens which viewed in the light from a sodium flame was to give Newton's rings, or as we now call them, interference fringes. Of course, when the sound falls upon the diaphragm, the fringes vibrate rapidly and disappear from sight. I then determined to use a stroboscopic arrangement which would permit the viewing of the fringes as they slowly shifted their position. All sorts of difficulties occurred. The light from the sodium flame was not strong enough to permit the fringes to be seen through the stroboscopic device. It was not at that time easy or perhaps possible to make an electric motor that should drive a stroboscopic disk at a constant speed. One of the first difficulties was to devise a method of controlling a synchronous motor. This was accomplished by means of interrupting the current by a tuning fork.

Later I made the acquaintance of Professor A. A. Michelson and of his remarkable optical interferometer, and I immediately saw that one of the difficulties which I had met, namely, the difficulty of adjustment of the lens so that it should not strike the vibrating plate, could be overcome at once, as the two mirrors in the Michelson interferometer could be any distance apart. Also the trouble due to the faintness of the light disappeared, for in Michelson's instrument white light could be used, and it was possible to use gas, incandescent or arc light with good effect.

In 1896 I put the subject in the hands of a student who produced an instrument which was successful. The examination of the substance suitable for a membrane or a diaphragm consumed much time, and all organic substances such as parchment, paper or anything of that sort had to be discarded because it was impossible to keep the tension constant under the changes of moisture in the air. Metal was found to be quite out of the question on account of the difficulties due to thermal expansion. We next decided upon thin glass diaphragms which we afterward found were used by phonograph companies. A slight change of temperature, however, such as was made by breathing upon the diaphragm, would sometimes produce a change of pitch of several tones, so I next decided to clamp the glass diaphragm upon a ring sawed out of plate glass and cement it on with silicate of soda. I finally found that it was possible to get mica so flat that it was superior to the best possible glass, and this gave me the best results up to that time.

Let me now state briefly the qualities which the diaphragm must have in order to be successful. It is well known to physicists that a flat diaphragm can vibrate in an infinite number of modes which the physicist calls normal vibrations. In the simplest mode the diaphragm vibrates so that every point moves in the same direction. The natural period of this vibration is the lowest that the diaphragm can have. Next is a mode in which the central portions of the membrane move in one direction while all those outside of a certain circle, which we call a nodal line, move in the opposite direction. Other vibrations have radial, as well as circular, nodal lines on the opposite sides of which the diaphragm is moving in opposite directions. When a mass is screwed to any point of the diaphragm its motion is entirely changed. And any apparatus that may be attached to the diaphragm and required to do work, such as the carbon button of the telephone, or anything that interferes with the free response of the diaphragm to the variations of the air pressure will alter the action in such a way as to make it extremely difficult of interpretation. My idea, then, was to make a diaphragm which did not do any work at all, but merely carried the mirror which moved practically by itself.

Now, in spite of the great variety of forms and vibrations, it can easily be shown that under proper circumstances the diaphragm will move essentially as a whole, and may, therefore, be compared to a single body which has three characteristic constants: First, its mass; second, its stiffness, defined as the force required to produce a unit deflection of its central point; and third, and most difficult to control, the coefficient of damping, which is defined as the force of resistance that will be exerted from any cause when the velocity of the body or the central point of the diaphragm is unity. This damping must be due to a large number of causes. In the first place, the resistance of the air will cause damping. In the second place the energy of the sound waves that are radiated from the vibrating diaphragm will cause damping. In the third place and by far the most important, the bending of any body is resisted by elastic forces and we therefore have the phenomenon of elastic fatigue. In order to settle the question "what is the best material from which to make tuning forks", which has been variously answered as tool steel, bronze, bell metal, quartz, etc., I perceived that it was necessary to have an exact method of experimentation.

In striking a steel bar supported by a string at two points it will be observed that the high overtones emitted by the bar rapidly die away, and it occurred to me that if the bar were thrown into one normal vibration there would be a rate of damping for every normal vibration, and that by stating the rate of natural damping the elastic hysteresis could be studied. This was demonstrated experimentally by my assistant Mr. James L. Porter with very successful results as far as the experiment went. The mathematical theory has been lacking up to the present time. Two theories are possible, first the theory of solid viscosity, and second the theory of heredity, or elastic hysteresis. The attempts to compute the results of our experimental measurements are now approaching completion, and we soon shall be able to give a theory of elastic hysteresis. My conclusion is that fused quartz is the best substance from which to make tuning forks.

This digression from the investigation is made in order to show the very great difficulties that this subject presented.

The illustrations (shown in the lecture, but [not] reproduced) show the apparatus as it was built several years ago. It was mounted upon a heavy stand made of bronze, covered in at the back by a heavy bronze cover, through which protruded through air-tight fittings the shafts turning the screws of the interferometer adjustments, three in number. Upon the front of the instrument was attached a resonator properly tuned and at the side was a small incandescent lamp with a straight filament arranged horizontally, an image of which was projected by a lens upon the first miror of the interferometer. A telescope was focused upon this image, giving an image in the reticule of the hor straight filament. This was crossed by the vertical interf fringes. The objective of the telescope was carried b prong of a tuning fork which oscillated vertically anc combined with the horizontal oscillation of the fringes c the vibration of the diaphragm, resulted in a figure in the of an ellipse. On tuning the two sounds nearly togethe ellipse could be caused to go through its various phases as s as desired. At the moment when the ellipse degenerated ir inclined straight line the reticule, which was tilted by a server, was brought into line with the interference banc the tangent of the angle was read off on a straight scale. pressure in the sound wave was then proportional to number read off on the tangent scale. The apparatus used in this form for a good many years, a precursor of it l shown at the Congress of Physics at Paris in 1900.

Professor D. C. Miller's phonodeik, which is a very be ful instrument, was designed for quite other purposes t the phonometer of the speaker. It was intended to ph graph sound curves of any sort and by means of it Profe Miller has obtained most beautiful reproductions of spe and sounds of musical instruments. Unfortunately, instrument does not adapt itself in the least to abso measurements. One objection to Professor Miller's instrum is that it employs a spring made of rubber material. This spr has to be calibrated whenever measurements are taken, anc is well known that the properties of soft rubber are far fr constant.

The speaker next undertook to devise a better mode of re ing the amplitude of vibration of his diaphragm. In the fi place, the mirror which was rotated by the diaphragm in P fessor Miller's apparatus, instead of being carried on an axis jewels was placed on a thin steel torsion strip which could made at far less expense and could be rapidly adjusted. T straight filament lamp was now viewed through a telescope in which the mirror focused the filament in the reticule of the ey piece. A magnification of about 1200 to 1500 is used. Tl instrument was also used photographically just as the old i strument had been used to photograph the motion of the inte ference fringes. Photographing moving interference fringe however, was attended by many experimental difficulties.

A new instrument in the form described was made in 1913, fo

Professor Louis Vessot King,* of McGill University, who had a commission from the Canadian Government to make experiments upon fog signals and had secured permission to investigate the great siren at Father Point on the St. Lawrence. Professor King spent the whole summer at Father Point in making the first survey of an acoustical field ever made. The Canadian Government had furnished him a large steamer with which he went out every day, blew the "criard" as the natives call it, the very perfect siren which can be heard thirty miles away in good weather, and explored the gradient of sound under all meterological conditions, studying the wind, the temperature of the air and of the water, etc. This spurred the speaker on to undertaking the same thing in this country. It was impossible to get from the Lighthouse Board the loan of a steamer, but permission was given to go on any lighthouse steamer going along the coast of Maine. The first step was to tune the phonometer for the different pitches, and at that time the only method of tuning the diaphragm was to load upon it small pieces of wax. This, although it worked perfectly, seemed very clumsy, and not an engineering method. It was therefore decided to redesign the whole instrument so that it could be tuned continuously. It occurred to me that this could be accomplished by furnishing potential energy to the diaphragm by means of a string parallel to it, and bearing against it by means of a bridge, and tunable like a violin string. The very first attempt showed this to be entirely successful. Then the idea immediately presented itself that if a string could be tuned continuously why use the energy of bending at all and why not get entirely rid of all difficulties of elastic fatigue. Since that time the diaphragm has been entirely discarded and replaced by a stiff disk supported upon three strings in tension. The disk is made of aluminum or mica and is carried by a little spider made of aluminum containing three clamps to hold the wire. The wire is made of steel and is under tension about steel pegs, two of which are turned roughly by means of a screw driver, the other by means of a lever actuated by a micrometer screw. We have now a very perfect instrument at least ten times as good as the one Professor King had.

A few figures will show the advantages in the reduction of the damping. In a system of one degree of freedom the amplitude of the vibration for the most perfect tuning is inversely proportional to the quotient of the damping coefficient k divided by the mass m. When we began with the good glass diaphragm we had a value of k something like 150. In 1913 the mica diaphragm lent to King had reduced this to 30. With the phonometer as built now we have reduced this to 1.34. In all these cases the mass was about the same so that so far as the sensitiveness goes, the instrument is more than one hundred times as good as it was ten years ago. The instrument now competes in sensitiveness with the human ear and may be tuned to pitches varying by perhaps two octaves. If I strike a tuning fork and allow its amplitude to die away, a person looking into the eye-piece of the instrument cannot say with certainty whether he can see the sound or hear it the longest. Unfortunately, however, this very great sensitiveness obtained by a small damping is attended with a very great disadvantage. The instrument is extremely selective and if the tuning varies by a very small amount the amplitude falls off very greatly.

---

*King, Phil. Trans. Ser. A. Vol. 210, pp. 211-293-1919.

Within the last three months I have devised a plan of doing away with this, and making an artifcial ear. For, as you know, the human ear has a wonderful sensitiveness, not for all sounds, but to sounds of a frequency of perhaps 30 per second up to perhaps 30,000 per second, attaining a maximum sensitiveness for frequencies of from 1000 to 1500. We know how this affects the telephone and how the damping of the telephone and microphone disks enables the response to be carried over a large range, so that they answer very well for the frequencies involved in speech. Nevertheless, the sounds of *s*, *f*, *t*, and certain others, on account of the very high harmonics involved, are most difficult to transmit telephonically.

Having described the construction of the diaphragm, the new piston with wires, and the method of reading the vibration by means of the inclined mirror which is also reduced to absolute measure by means of the interferometer temporarily attached, or more simply by displacing the diaphragm by a micrometer screw, I come to the theory of the instrument. The resonator into which the air enters, and the hole which is now constituted by an annular opening around the disk, furnishes through the movement of the air in and out an additional degree of freedom. We are then in the possession of a system with two degrees of freedom, statically coupled by means of the increased pressure in the resonator when the piston is forced in. The theory of such a system under the action of the periodic force is well known and has been treated by Professor Max Wien in great detail. If the frequency in 2 π seconds be n and if the mass be m, I shall call the stiffness minus the product of the mass and square of the frequency the uncompensated stiffness or mistuning. If this were equal to zero for a system of one degree of freedom we should have resonance. In the case of two degrees of freedom I plot the mistunings horizontally and vertically, and the amplitude of response as a third co-ordinate. I thus get a surface that may be described as two mountain peaks with a pass between them, the summit of the pass being when both mistunings are zero; but this is by no means the highest point of the region. To attain either summit we must mistune both systems by a certain amount which is porportional to their two dampings. The addition of the resonator to this instrument may produce an increase of sensitiveness of fifty times. You now see, I hope, how we have obtained an engineering instrument, everything in which is measurable in absolute measure. It is obvious that the piston may be made the diaphragm of the telephone, that the instrument may be used by the psychologist, the engineer, the physiologist, for instance in a stethoscope, and in many other applications.

I turn now to the source of the sound, the phone. With the advent of the tuneable diaphragm came the new phone. It is very light. The amplitude of the diaphragm is measured by a microscope. I use a hot wire vacuum tube as a source of alternating current, tuneable at will, and tune the phone to it. We thus have in small compass a very perfect set of acoustical instruments.

The third portion of my investigations involved a determination of the coefficient of reflection of the ground. In order to accomplish this the phone is set at a convenient height and the phonometer at a convenient distance. The latter is then moved back and forth at the same height, when it is immediately found that interference between the phone and its image in the ground sets in, producing a variation of the intensity of the sound. Different curves have been plotted for different coefficients of reflection. When the reflection is zero or the ground is acoustically perfectly black we have a rectangular hyperbola. The existence of the minimum is obvious to the most unskilled observer. We found the coefficient of reflection of grass, or gravel surface, to be about 95 per cent. I may say that the whole measurement of the instruments of the two ends and the transmission between checks up with an accuracy of probably better than two per cent. With this apparatus all sorts of experiments have been performed. By attaching to the phonometer a long glass tube or antenna it has been made possible to explore all sorts of places, such as the field within a horn or a tube lined with absorbent substance. The theory has been always completely verified. In order to examine the transmission of sound a piece of substance is clamped between two heavy cast iron rings cemented against a hole in a brick wall which will exclude all sound except that that comes through the fabric. The transmission through doors, windows, walls, and telephone booths may be carried out very quickly, and the coefficients of absorption and reflection determined.

Finally, there has resulted an instrument for determining the direction of a fog signal blowing in the fog which I call a phonotrope; since the heliotrope turns toward the sun, the radiant of heat, so the phonotrope turns toward the radiant of sound. This instrument consists of two equal horns bringing the sound to the opposite side of the same disk. It is arranged to be rapidly tuned to the whistle, and when the whistle blows the band of light spreads out; the whole instrument is then revolved until the band reduces its width to zero when the whistle is directly ahead. This instrument was taken to Pensacola to see whether it would determine the direction of an aeroplane in the night. It was found to be as sensitive as the ear, but owing to Doppler's principle, the continual coming and going of the aeroplane changed the pitch so as to put it out of tune. A new modification that I have devised will obviate this I hope.

I have now given you briefly and without any mathematics an account of the principles which I think must always be involved in any measurements of sound. I have always been very anxious to join forces with Professor Miller and to calibrate his instruments so as to render his wonderful results of serious interest to the physicist. I am also glad to co-operate with all my colleagues whether engineers, physicists, physiologists or physicians. It will be a great pleasure to me to know that this apparatus may be of use in solving any of the multitude of questions that confront us.

The plates annexed clearly show the three instruments and several damping curves.

The following list of papers *read* is annexed. Only those marked with an asterisk have been printed, except in abstract.

PAPERS ON SOUND

1. B. A. A. S. Toronto, Aug. 1897. "A New Instrument for Measuring the Intensity of Sound".

2. A. A. A. S. Boston, Aug. 1898. "A New Instrument for the Measurement of the Intensity of Sound".

3. A. A. A. S. Washington, Jan. 1903. "A Portable Apparatus for the Measurement of Sound".

4. American Phys. Soc., New York, April 1902. "An Apparatus for the Quantitive Study of Sound."

5. Am. Phys. Soc., New York, April 1902. "Vibrations of Rotating Wires, Spindles and Shafting."

6. Nat. Acad. Sciences, Boston, Nov. 1906. "Acoustic Measurements".

7. Nat. Acad. Sci., New York, Nov. 1907. "Rayleigh's Disk as an Absolute Measure of Sound".

8. Am. Acad. Arts & Sci., Boston, May 1908. "Absolute Measurements of Sound".

9. Am. Phys. Soc., New York, Oct. 1908. "Distribution of Sound from the Megaphone".

10. National Acad. Sci., Washington, April 1910. "On the Distribution of Sound from the Megaphone or Speaking Trumpet".

11. Nat. Acad. Sci., Washington, April 1910. "A New Method for the Study of Elastic Hysteresis."

12. B. A. A. S., Sheffield, Aug. 1910. "A Complete Apparatus for the Measurement of Sound".

13. Am. Phys. Soc., Washington, Dec. 1911. "Elastic Hysteresis in Metal Bars".

14. Am. Phys. Soc., Washington, Dec. 1911. "Another Instrument for Photographing Sound".

*15. Am. Acad. Arts & Sciences, Dec. 1910. "The Wave Potential of a Circular Line of Sources".

16. Am. Math. Soc., Poughkeepsie, Sept. 1911. "The Wave Potential of a Circular Line of Sources".

17. Am. Phys. Soc., Cleveland, Jan. 1913. "Some Points Concerning Absolute Measurements in Sound".

18. Am. Phys. Soc., New Haven, March 1913. "Forced Vibration of a Circular Plate".

19. Nat. Acad. Sci., Baltimore, Nov. 1913. "A New Portable Phonometer".

20. Nat. Acad. Aci., Baltimore, Nov. 1913. "The Transmission of Sound Through Porous Materials".

21. Am. Phil. Soc., Philadelphia, April 1913. "Some Observations of the Transmission of Sound Through Walls."

22. Am. Physical Soc., Washington, April 1914. "A New Phonometer".

23. Am. Phys. Soc., Philadelphia, Dec. 1914. "A New Standard Phone and Phonometer for any Pitch".

24. Nat. Acad. Sci., New York, Nov. 1915. "Experiments and Theory of Conical Horns".

25. Nat. Acad. Sci., New York, Nov. 1915. "Instruments for the Measurement of Sound".

26. Nat. Acad. Sci., New York, Nov. 1915. "An Instrument for Finding the Direction of a Fog-Signal".

*27. Am. Phys. Soc., Columbus, Dec. 1915. "Mechanical and Acoustical Impedance, and the Theory of the Phonograph".

28. Am. Phys. Soc., Columbus, 1915. "Impedance of Conical Horns".

29. Am. Phys. Soc., Columbus, Dec. 1915. "The Phonotrope, a new Instrument for Finding the Direction of an Acoustical Ray".

*30. Am. Math. Soc., Cambridge, Sept. 1916. "On Acoustical Impedance, and a Theory of Horns".

31. Am. Acad. of Arts and Sci., Boston, Nov. 1916. "Practical Tests of a New Phonotrope".

32. Nat. Acad. Sci., Baltimore, Nov. 1918. "An Automatically Played Brass Instrument".

33. Am. Phys. Soc., Baltimore, Dec. 1918. "A Mechanically Blown Wind Instrument".

DISCUSSION ON "ELECTROMAGNETIC THEORY OF THE TELEPHONE RECEIVER" (KENNELLY AND NUKIYAMA) AND "ON THE ABSOLUTE MEASUREMENT OF THE INTENSITY OF SOUND" (WEBSTER). BOSTON, MASS., MARCH 14, 1919.

John B. Taylor: Certainly the matter of tone that Prof. Webster spoke of, the acoustical analysis of musical instruments, is intensely interesting. We have been trying to discover why a bassoon, which is a very complicated instrument, is absolutely unwritten up in a technical way. It is very gratifying to hear Prof. Webster say that he can measure these sounds and noises in quantitative terms.

If Prof. Webster had been at the last meeting of the Institute in New York City, when there was a discussion on telephone interference, and this new interference standard, after all, is a matter of noise, going into the Institute Standardization Rules, he would have been able to contribute something instructive. A question was asked at the meeting about the quantitative measuring of noise in the telephone field, and I do not recall that the answer to that question was very illuminating.

T. E. Shea: According to Dr. Kennelly's theory, I believe, the action of a telephone receiver may be likened somewhat to that of a single-phase transformer, except that the secondary circuit generates oscillating mechanical power instead of oscillating electrical power. The authors of the paper have, in effect, combined the equivalent vector diagrams of the mechanical circuits with the vector diagrams of the electrical circuit; and the resulting diagrams they have obtained are similar to those ordinarily used in explaining the theory of the transformer.

But the wide range of frequencies considered has introduced several new elements, regarding which the authors have made assumptions. One of these assumptions is that the current passing through the receiver is constant. Another is that the angle of lag of the flux behind the impressed magnetomotive force, called by Dr. Kennelly the hysteretic angle β, is also constant.

I would like to ask Dr. Kennelly if, when the current through the receiver varies over a wide range, as it does in practise, the hysteretic angle β would not also vary greatly. I realize that Dr. Kennelly has not attempted to push his theory beyond the limits of constant current, but I think the impression has been given that the angle β is independent of the other variables of telephone receiver theory.

B. A. Behrend: I wish to call attention to the fact that the term "motional impedance" was first suggested by our honorary member, Mr. Oliver Heaviside, to whom we also owe the terms "inductance", "reactance", and "impedance".

R. L. Jones: Professor Webster has described and shown three exquisitely designed acoustical instruments which should be very serviceable in certain types of sound researches. Both the phone and the phonometer are characterized by very sharp tuning, however, and although tunable, an appreciable period of manipulation is required to adjust precisely to a new pitch. In much work, as for example in many studies of speech and music, we are interested in a whole range of frequencies or a flow of rapidly changing pitch. In such cases we need a sensitive phonometer of uniform frequency characteristics. Such apparatus is provided by a properly designed and calibrated combination of electrostatic telephone transmitter and vacuum tube amplifier, as has been described in recent volumes of the *Physical Review.*

The paper by Professor Kennelly and Mr. Nukiyama is primarily a pictorial treatment of the electric and magnetic relations of the telephone system by means of vector diagrams. I believe it will broaden the consideration if we discuss the matter algebraically for the case of a relatively simple receiver.

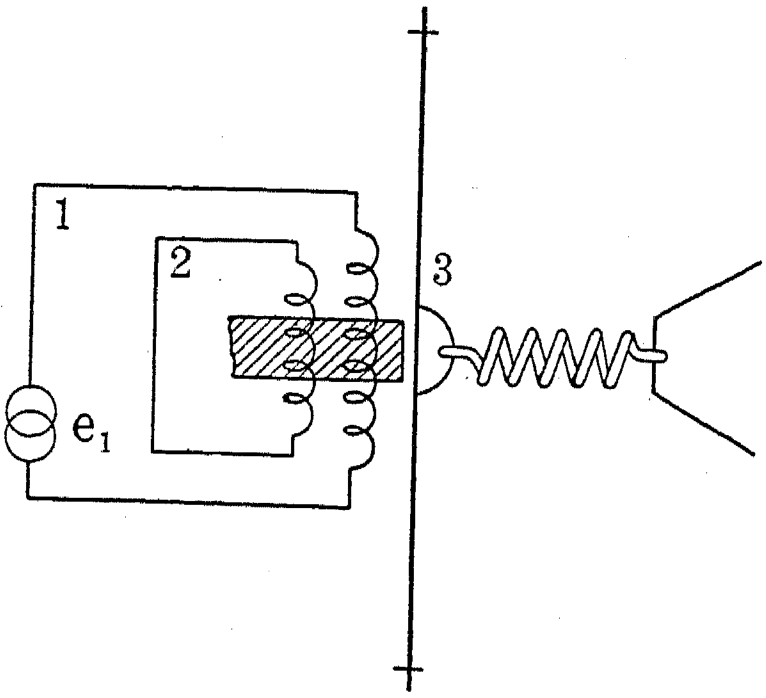


Fig. 1—Simple Receiver with Eddy Currents in the Core

Let us assume that straight line laws hold and in general make the other usual simplifying assumptions just as the authors do. In addition assume that for the frequencies under consideration hysteresis is negligible compared to eddy current losses and that the eddy current losses in the diaphragm are negligible compared to those in the cores. From experience these assumptions appear reasonable as a first approximation.

Fig. 1 then shows the condition schematically where 1 is the driving circuit, 2 the schematic equivalent of eddy current circuits in the cores, and 3 the mechanical or diaphragm circuit. The following nomenclature will be used:

a = inductance or mass
b = resistance, electrical or frictional
c = stiffness
Z = impedance, electrical or mechanical
$\mathring{x}$ = current or velocity

Subscripts refer to the networks of the coupled system, 1 the driving circuit, 2 the eddy current circuit, and 3 the diaphragm or mechanical circuit.

Then from the figure we may write down the well known force equations. There is an electromotive force, e_1, let us say, working in the driving circuit, none in the eddy current circuit, and no external forces working on the diaphragm.

$$\begin{aligned} e_1 &= Z_{11}\overset{\circ}{x}_1 + Z_{12}\overset{\circ}{x}_2 + Z_{13}\overset{\circ}{x}_3 \\ 0 &= Z_{21}\overset{\circ}{x}_1 + Z_{22}\overset{\circ}{x}_2 + Z_{23}\overset{\circ}{x}_3 \\ 0 &= Z_{31}\overset{\circ}{x}_1 + Z_{32}\overset{\circ}{x}_2 + Z_{33}\overset{\circ}{x}_3. \end{aligned} \tag{1}$$

The relations of the mutual impedances appearing in these equations are as follows:

Z_{12} and Z_{21} are the mutual impedances between the driving and eddy circuits

Z_{13} and Z_{31} are the force and reaction factors between the the driving circuit and the diaphragm

Z_{23} and Z_{32} are the force and reaction factors between the eddy circuit and the diaphragm

$$\begin{aligned} Z_{12} &= Z_{21} \\ Z_{13} &= -Z_{31} \\ Z_{23} &= -Z_{32}. \end{aligned} \tag{2}$$

The mutual impedances of a transformer are of the same sign since flux due to current in either winding is opposed by that due to induced current in the other winding. In considering the mutual relations between the receiver diaphragm and either the driving or eddy current circuits, however, it is seen that whereas flux due to diaphragm motion induces currents which set up an opposing flux, flux due to current in the driving winding produces a motion of the diaphragm which tends to increase the flux still further. Hence in the receiver the force and reaction factors are of opposite sign.

By means of (**1**) and (**2**)

$$Z_1 = \frac{e_1}{\overset{\circ}{x}_1} = Z_{11} - \frac{2\,Z_{12}\,Z_{13}\,Z_{23} + Z_{12}^2\,Z_{33} - Z_{13}^2\,Z_{22}}{Z_{22}\,Z_{33} + Z_{23}^2} \tag{3}$$

If the diaphragm is damped so that $\overset{\circ}{x}_3 = 0$, then from (**1**) and (**2**)

$$Z_{1d} = Z_{11} - \frac{Z_{12}^2}{Z_{22}}. \tag{4}$$

By subtraction the motional impedance of the receiver,

$$Z_{1m} = \frac{\left(Z_{13} - Z_{23}\dfrac{Z_{12}}{Z_{22}}\right)^2}{Z_{33} + Z_{23}\dfrac{Z_{23}}{Z_{22}}}. \tag{5}$$

If there were no eddy currents this would reduce to

$$Z_{1m}' = \frac{Z_{13}^2}{Z_{33}} \quad . \quad . \qquad (\mathbf{6})$$

the form familiar to us as the square of the force factor divided by the mechanical impedance of the diaphragm. Comparing (**5**) we see that the force factor with eddy currents has the value

$$B_{13} = Z_{13} - Z_{23} \frac{Z_{12}}{Z_{22}} = b_{13} - b_{23} \frac{j\, \omega\, a_{12}}{b_2 + j\, \omega\, a_2} \qquad (\mathbf{7})$$

$$= \left(b_{13} - b_{23} \cdot \frac{\omega^2\, a_{12}\, a_2}{b_2^2 + \omega^2\, a_2^2} \right) + j\, b_{23} \cdot \frac{\omega\, a_{12}\, b_2}{b_2^2 + \omega^2\, a_2^2} \qquad (\mathbf{8})$$

The effect of eddy currents is evidently to decrease the force factor with increasing frequency, and to cause a lag.

Substituting B_{13} in (**5**),

$$Z_{1m} = \frac{B_{13}^2}{Z_{33} + Z_{23} \dfrac{Z_{23}}{Z_{22}}} \quad . \qquad (\mathbf{9})$$

Substituting primary constants for Z's it may be seen that Z_{1m}

$$= \frac{B_{13}^2}{\left[b_3 + \dfrac{b_2\, b_{23}^2}{b_2^2 + \omega^2\, a_2^2} \right] + j \left[\omega \left(a_3 - \dfrac{a_2\, b_{23}^2}{b_2^2 + \omega^2\, a_2^2} \right) - \dfrac{c_3}{\omega} \right]} \qquad (\mathbf{10})$$

Centering attention on the denominator it may be seen that the eddy current may be considered as increasing the mechanical resistance by an amount equal to

$$b_2 \cdot \frac{b_{23}^2}{|\, Z_2^2 \,|} \quad .$$

Similarly it may be considered as decreasing the effective mass by an amount

$$a_2 \cdot \frac{b_{23}^2}{|\, Z_2^2 \,|} \; .$$

or as increasing the elasticity by a corresponding factor. Physically this second factor merely changes the apparent mechanical reactance.

Now let us turn to a consideration of efficiency relations in the simple receiver with eddy currents in the core. Writing down the power output of the diaphragm divided by the power input to the driving winding we have

$$\mu_{13} = \frac{|\overset{\circ}{x}_3{}^2| \,.\, b_3}{|\overset{\circ}{x}_1{}^2| \,.\, B_1} \qquad \textbf{(11)}$$

where B_1 is the apparent or free resistance of the driving circuit.

Solving for $\dfrac{\overset{\circ}{x}_3}{\overset{\circ}{x}_1}$ from (**1**) and substituting we have

$$\mu_{13} = \frac{\left|\left(Z_{13} - Z_{23}\dfrac{Z_{12}}{Z_{22}}\right)^2\right| \,.\, b_3}{\left|\left(Z_{33} + Z_{23}\dfrac{Z_{23}}{Z_{22}}\right)^2\right| \,.\, B_1} \qquad \textbf{(12)}$$

By further substitution from (**5**) and (**7**)

$$\mu_{13} = \frac{|Z_{1m}{}^2|}{\dfrac{|B_{13}{}^2|}{b_3} \,.\, B_1} \qquad \textbf{(13)}$$

This expression is the true overall efficiency of the receiver at any frequency in terms of the motional impedance, the free resistance, the force factor and the mechanical resistance. If we multiply this last expression by the ratio of the apparent mechanical resistance (**10**) to its true value, we get

$$\mu_{13}' = \frac{|Z_{1m}{}^2|}{\dfrac{|B_{13}{}^2|}{b_3 + b_2\dfrac{b_{23}{}^2}{|Z_2{}^2|}} \,.\, B_1} = \frac{|Z_{1m}{}^2|}{|Z_{1m/0}| \,.\, B_1} \qquad \textbf{(14)}$$

Since the first factor of the denominator, the ratio of the square of the force factor to the apparent mechanical resistance, is the motional impedance at resonance, at resonance this reduces to

$$\mu_{13/0}' = \frac{|Z_{1m/0}|}{B_1} \qquad \textbf{(15)}$$

This is what Dr. Kennelly calls the "gross efficiency at apparent resonance." It is very readily determined from the motional impedance diagram, and is susceptible of fairly precise determination. On the other hand, it applies simply to a single frequency, and is a value that, for the comparison of different receivers, is generally applicable only in case the receivers are quite similar in their natural frequencies and ranges of response. In the case of telephones for wireless telegraphy, one is interested in response at substantially a single frequency and for such instruments this apparent efficiency taken in conjunction with the natural frequency gives a very good measure of the relative performance of different types. It may be of interest to note that for nearly a year past the U. S. Signal Corps specification for wireless telephones has had a requirement stated in terms of these quantities. In the telephone problem, however, we are interested in the whole range of speech frequencies running from 200 or 300 cycles up to 3000 or above. In receivers for reproducing speech, therefore, whether transmitted by wire or by radio, we cannot depend upon the relative measures given by expression (**15**), but must apply (**13**) to the whole range of frequencies, or get some other equivalent measure.

To retransform μ_{13}' to μ_{13}, we have to multiply by the ratio of the true mechanical resistance to the apparent value. The latter appears in the denominator of (**10**). This ratio,

$$\eta = \frac{b_3}{b_3 + b_2 \dfrac{b_{23}^2}{|Z_2^2|}} \qquad (\mathbf{16})$$

as pointed out by Dr. Kennelly, measures the vibratory efficiency of the diaphragm, showing the fractional part of the total vibratory power that is of a mechanical nature.

These results are a small part of the consideration given to this subject in the Western Electric Research Laboratories at New York. It may be of some interest to cite one or two experimental results for the same case as that assumed.

An experimental receiver of the ordinary type was constructed in which the eddy current effects were reduced to a minimum. The cores were soft iron in a form which is the equivalent of very fine laminations and for the diaphragm silicon steel was chosen on account of its high specific resistance. The case was made of Portland cement, this being chosen because we wished to avoid eddy currents in the case and to use a material with about the same coefficient of expansion as iron so as to keep the airgap constant with temperature.

In addition to the driving winding a 100-turn coil was put on close to the pole tip which could be short-circuited at will to represent an eddy current circuit. Measurements on this circuit enabled us to predict its effect on the impedance of the driving circuit. In accordance with the foregoing formulas the predictions could be checked by measurement. To cite simply one piece of data, with the 100-turn coil open, 2 β was about 4 deg., with it closed, 25 deg. The corresponding computed value was 23.5 deg. A second model had the artificial eddy circuit wound as a twin winding, in parallel with the driving circuit. With this arrangement for eddy currents the lag angle 2 β went from a very small angle to 130 deg., and the motional impedance was diminished from 31 ohms to 4 ohms.

A. G. Webster: This is what I call practical engineering. I should like to point out, as a matter of history, that the first problem of this sort solved was solved by James Clark Maxwell in his famous paper in 1865. Of course there are here three lines of equations instead of two. You know how Maxwell understood the impedance, as we call it now, because of the invention of the word of Oliver Heaviside—that the presence of the secondary increases the resistance of the primary and decreases the inductance. We all know that now, but we did not know it then. Of course, then came Lord Rayleigh, who described the reactions of any mechanical system.

Harold S. Osborne: Engineers who are dealing with power transmission rather than with telephone work may feel more at home with the motional impedance diagram of the telephone receiver if they can consider that it is not an entire stranger but the first cousin of an old friend. I refer to the circle diagram of the induction motor in which, as in the telephone receiver, the secondary moves. In the circle diagram of the induction motor, as indicated in Fig. 2, the secondary current corresponding to a given load is represented by a chord of the circle *O I*. The secondary current is a measure of the reaction of the moving secondary on the magnetic flux of the primary winding. Similarly, in the motional impedance diagram of the telephone receiver, the reaction of the moving diaphragm on the fixed winding is represented by a chord of the circle *O V*, (Fig. 3).

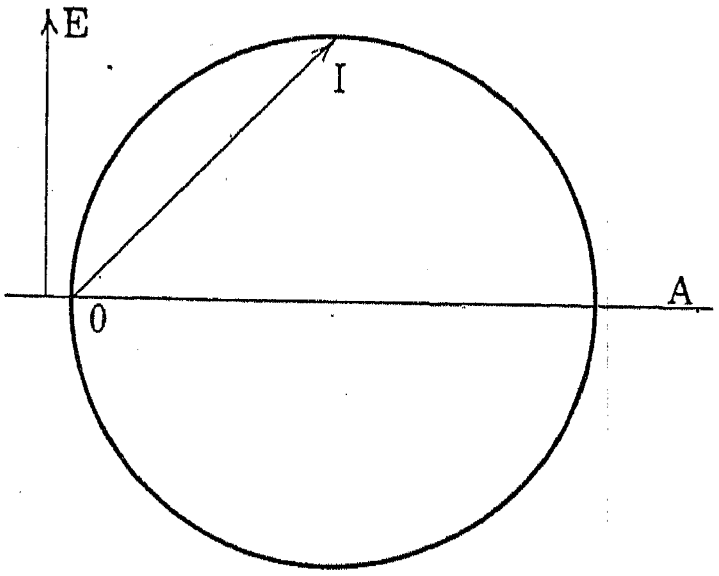


Fig. 2.

There is a further close relationship between these two diagrams. The variable element seems to be different in the two cases because in the case of the induction motor we are considering variations of load, and in the case of the telephone receiver variations of frequency. However, in the induction motor variations in load mean variations in frequency of the secondary current, and one starts with zero frequency at point *O*, and goes half way around the circle to infinite frequency at point *A*. The other half circle corresponds to negative frequencies. In the case of the telephone receiver the law of variation in frequency in passing around the circle is, of course, different. In this case, starting with zero frequency at point *O*, one passes around the circle and at infinite frequency completes the circuit, arriving again at point *O*.

The difference in the use of the circle diagram for the induction motor and for the telephone receiver illustrates one difference between the problems of the power engineer and of the telephone engineer to which I should like to call attention. In the case of a power system, one is interested in studying the system under conditions of constant impressed e. m. f. and varying loads of a given frequency. In the case of the telephone system the load is, in general, constant but the frequency is varying.

The importance of a very wide range of frequencies in telephone work is, I think, an old story even to engineers who are not engaged in telephone work. Over thirty years ago Oliver Heaviside pointed out the effect of variation in efficiency of the telephone receiver with frequency, and the effect of that on the telephone quality. In those days people were still marveling over the fact that this wonderful little instrument could reproduce speech, and that they were astonished at what seemed to them, perfection of articulation, and by the fact that one could even recognize the voice of different persons. However, Mr. Heaviside stated that to the critical observer the telephone quality was very far from perfect. As he expressed it, "the disk does not do it, for want of deadbeatness." The fact that telephony is practicable, he claimed, is not so much a triumph of the electrical art, as it is a triumph of the human brain, which has been trained for years, and, in fact, for centuries, in interpreting the very much distorted sounds that come to it.

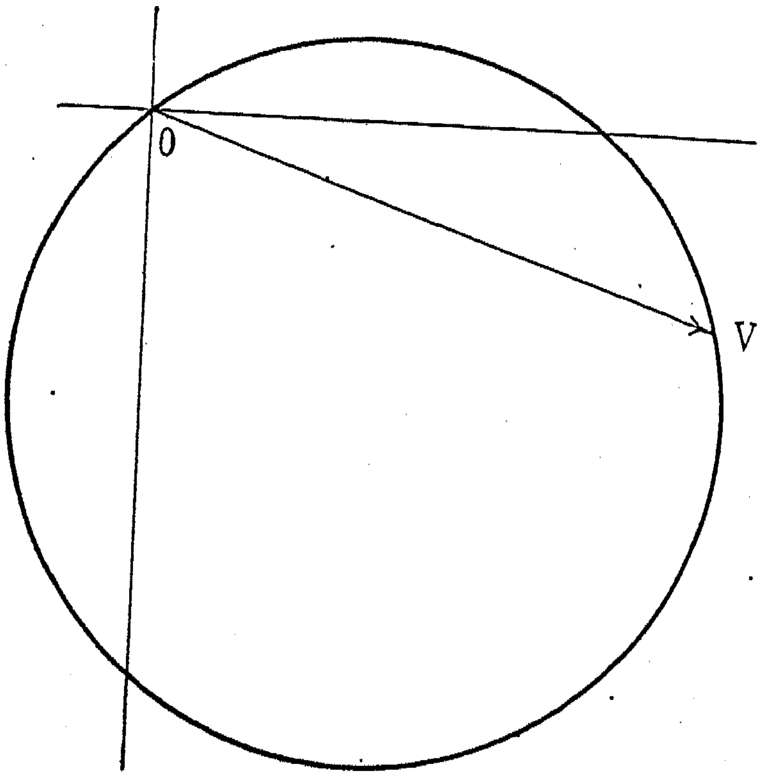


Fig. 3.

I do not believe that there is a general appreciation of the very large amount of distortion of speech which takes place in an ordinary room. Prof. Webster has told us that the reflection from the earth is 95 per cent. From a wooden floor or walls of a building it must be very great. The reflected waves cause tremendously large variations in the magnitude of the different components of the speech sounds, and large differences in different parts of the room. In addition, there is the effect of room noises. Beyond that, there are personal factors which perhaps cannot be called merely physical matters, such as slovenliness of speech on the part of the talker, and defective and wandering attention on the part of the listener.

Now, although large advances have been made since Heaviside's day, the telephone receiver still is very far from a perfect reproducer of the energy supplied to it. However, when one considers the distortions to which ordinary speech is subject, and the fact that our ears through training are capable of understanding this distorted speech very well, it is not so surprising that we should be able to detect the distorted speech of the telephone circuit. It is probable that the difference in quality of conversation between listening in a room like this, and listening an infinite distance from everything, as Prof. Webster puts it, is much greater than the difference between listening in the room and listening over a telephone circuit.

The instruments and methods which Prof. Webster has shown to us are certainly of the very greatest interest to any one who has to deal with acoustic problems. However, electrical engineers, from the prejudice of their training, very much prefer to read their results on electric instruments when it is possible, rather than to observe them through a microscope as mechanical displacements.

Mr. Jones has spoken of the fact that, in many cases, we are very much interested in measuring sound with an instrument which is not highly selective, but which, on the contrary, responds so as to give as nearly as possible a faithful reproduction of a complicated wave shape, that is, an equal response to the different harmonic components of the wave. A good solution of this problem has been discussed in two papers which were printed in the *Physical Review* in July, 1917, which I would like to call to your attention. These are the "Thermophone as a Precision Source of Sound," by H. D. Arnold and I. B. Crandall, and "A Condenser Transmitter as a Uniformly Sensitive Instrument for the Absolute Measurement of Sound Intensity," by E. C. Wente. By means of the instruments described in these papers it is possible to set up a system for transforming acoustic into electric waves and vice versa with an efficiency of conversion which can be made sensibly uniform throughout the range which is important in telephonic work. Of course, these instruments give very small responses, but in these days of vacuum tube amplifiers that is not a difficulty.

Mr. Taylor has spoken of the possibility of using Prof. Webster's instruments to study the characteristics of interfering currents in telephone circuits. However, for such complicated wave shapes as those or for the study of the characteristics of telephone speech, instruments giving a uniform response, such as are described in the papers just mentioned are very much more valuable than instruments which are sharply tuned to particular frequencies.

A. G. Webster: I was very much interested in these papers when I heard them read. I went to New York with some qualms as to whether I had been anticipated. When I saw the amplifying tube I said, "That let's me out." Who knows whether the amplification of that tube is the same for all these frequencies. I said to Major Whitehead last year in his laboratory—"I have seen this wonderful description of the amplifier. Does it do it?" I will not tell you what he said, but those of you who have heard of experiments of that sort, will, I am sure, say that the electrical engineer is mistaken. We know that this is an exact thing, no hysteresis or eddy current. This is a perfection of simplicity, and therefore I recommend it. As to its being selective, it is selective. You can tune it to any one of these sounds and make a photograph and analyze it.

I believe I can give more satisfactory answers to all of these telephone engineer's queries than can be got by the instruments he gets up himself. They are handy, no doubt, and all that. I remember Lord Kelvin seeing one of my instruments several years ago, and he said "It was important that sound could be measured by electrical reading apparatus." I do not do it that way.

A. Press (communicated after adjournment): "May I draw the attention of the authors to a work of mine which antedates that of M. Marius Latour on "Hysteresis and Eddy Current Losses in Iron Plate." This paper appeared in *The Electrician* of London for July 23, 1915. It not only gave a solution of the plate problem but it also indicated a general method for solving problems in which what amounts to a difference of phase between H and B occurs. Since then a simpler method has been developed for dealing with all the harmonics set up by iron in any electrical circuit, the manuscript of which is now in press. Distortions and resonances due to hysteretic harmonic components can be very effectively handled by the methods indicated.

A. E. Kennelly: In reference to the set of equations which Dr. Jones has so clearly explained, it is interesting to observe how his algebraical and our graphical presentations of the behavior of the telephone receiver converge towards similar conclusions.

In response to Mr. Shea's question, the angle β is the angle between the vector damped impedance locus and the reactance axis of coordinates. Strictly speaking, this angle β is a function of the impressed frequency, and increases with increasing frequency. Nevertheless, it usually varies but little within the range of frequency included between the quadrantal points of the motional impedance circle. Such variations as occur outside of those limits have relatively small disturbing influence.

In Fig. 1 of the Kennelly-Affel paper* (Bibliography 17) appears the locus of the damped impedance of the particular telephone receiver which is analysed in this paper presented today. In its Fig. 4 is the motional impedance circle. The depression angle 2 β of the circle's diameter (see also Figs. 10 and 27 of todays paper) is approximately 50°.6. In Fig. 1, the angle β which the locus *ABD*, between *B* and *D*, makes with the reactance axis is very nearly 25°.3. The fact that this angle β as measured on the diagram of the original paper is very nearly half the angle of diametric depression of the motional impedance circle was only discovered after the paper here presented was written, and serves as a check upon the provisional theory. A complete theory, however, would call for a recognition in the change of β with impressed frequency.

We are all indebted to Dr. Webster for his exhibition here of this ingenious acoustic apparatus. The fact that he has been able to reduce the damping coefficient to so low a value as has been mentioned is a very valuable achievement in itself.

Presented at the 349th Meeting of the American Institute of Electrical Engineers, in Joint Session with the Illuminating Engineering Society, New York, April 11, 1919.

Copyright 1919. By A. I. E. E.

PRESENT STATUS OF INDUSTRIAL LIGHTING CODES

BY G. H. STICKNEY

Abstract of Paper

In order to protect workers from accident and eye-strain, industrial lighting codes have been adopted in four states and in Federal establishments. Similar action is under consideration in several other states and there is prospect of extension throughout the country.

Investigation and experience indicate the need of government regulation of factory lighting. When adopted by industrial commissions under authority granted by Legislatures, the codes become in effect state law. Variations in the codes as adopted are less than might appear, some features being experimental. The existing codes correspond in essentials to the Illuminating Engineering Society code, on which they are based.

Mandatory regulations are necessarily limited in function to the assuring of safety. Higher standards are essential for efficient production. Popular education in which electrical and illuminating engineers can cooperate, is an important feature of future development.

Introduction

THE industrial lighting codes are expressions of the "Safety First" movement in terms of legal regulations directed to the lighting of factories. The prime function of these codes is the safeguarding of life, limb and vision of industrial workers. No argument should be necessary to enlist the support of all good citizens to such a humanitarian effort. In addition, the codes seem likely to teach practise which will enhance the earning power, not only of the workers themselves but also of the industrial plants. This incidental result seems likely to be of great economic importance in the coming period of international competition. Sufficient progress has been made in the application of these codes to indicate that such regulations will ultimately be in force throughout the country. Through their relation to lighting practise, these codes seem destined to have a broad influence on electrical engineering, and it is the purpose of this paper to inform electrical engineers regarding the scope and tendencies of these codes, as interpreted from a close association with the work and with those responsible for it.

Industrial lighting codes, based on the Illuminating Engineering Society's code, are now in force in the states of Pennsylvania, New Jersey, New York and Wisconsin. In several other states similar codes have been drawn up and are under consideration for adoption. In others, bills looking to the adoption of the code have been introduced before the legislatures. As a war measure, the Advisory Commission of the Council of National Defense, through its Committee on Labor, appointed a sub-committee known as the Divisional Lighting Committee, to encourage the movement. The sub-committee, working through state representatives, has approached the authorities in each of the states to encourage the adoption of such codes. As the result of this activity, progress has been made toward the introduction of an industrial lighting code or a safety code, by the Departments of Labor, Industrial Accident Commissions, or other agencies charged with the enactment and enforcement or regulations in regard to factory lighting.

Through cooperation of the Bureau of Standards and the Federal Inspectors of Safety, a lighting safety code was adopted during the war, in all Federal establishments, including arsenals, munition plants and navy yards.

NECESSITY

The necessity for such regulation is well explained in the introduction to the new edition of the Wisconsin code, which reads as follows:

> Insufficient and improperly applied illumination is a prolific cause of industrial accidents. In the past few years numerous investigators, studying the cause of accidents, have found that the accident rate in plants with poor lighting is higher than similar plants which are well illuminated. Factories which have installed approved lighting have experienced reductions in their accidents which are very gratifying.
>
> Of even greater importance, poor lighting impairs vision. Because diminution of eyesight from this cause is gradual, it may take the individual years to become aware of it. This makes it all the more important to guard against the insidious effects of dim illumination, of glaring light sources shining in the eyes, of flickering light, of sharp shadows, of glare reflected from polished parts of the work. To conserve the eyesight of the working class is a distinct economic gain to the state, but regardless of that, humanitarian considerations demand it.
>
> Finally, inadequate illumination decreases the production of the industries of the state, and to that extent, the wealth of its people. Factory

managers, who have installed improved illumination, are unanimous in the conviction that better lighting increases production and decreases spoilage.

Illuminating engineers have recognized for a long time that inadequate lighting was responsible, in a greater or less degree, for industrial accidents, and that good lighting was a potent means of accident prevention. Extensive data, accumulated by John Calder[1], R. E. Simpson[2], and others provide convincing proof of this condition.

The loss of visual power, through improper lighting, cannot be so readily recorded, and hence is not so directly substantiated by actual data. It is, however, recognized as a serious menace by oculists[3] and others. Because it incapacitates the skilled worker just when he should be most valuable, and further, because the danger is not readily recognized by those immediately responsible for industrial lighting, the necessity for governmental supervision is even greater for this purpose than for the prevention of mechanical accidents.

In spite of the recent general improvement in industrial lighting practise, the increasing need of ample assurance in this matter is evident from the present tendencies of manufacturing, among which may be mentioned the following: the grouping of larger number of employees; the extension of all night processes; increasing use of dangerous machinery; intricacy of processes and specialization of workers requiring sustained visual application.

HISTORICAL

Practically all the codes in force or under preparation are based on the "Code of Lighting Factories, Mills and other Work Places" of the Illuminating Engineering Society[4], and conform closely to it in all essentials.

This code was prepared by the cooperative action of the Committee on Lighting Legislation, under the chairmanship of Mr. L. B. Marks, and the former Committee on Factory Lighting, under the chairmanship of Professor C. E. Clewell.

The Illuminating Engineering Society was called upon in 1913 to assist in formulating the lighting section of the labor law of New York State. It was found impracticable then to incorporate definite lighting specifications in the law, and so provision was made in the law for the later adoption of such regulations, by the Labor Commission. In preparation for the Commission's requirement, work of drafting a code was

---

1. For references, see Bibliography.

undertaken. The problem proved a very difficult one and many modifications were found necessary, even after its first publication. Various societies, many engineers and state commissioners pointed out desirable changes, which were incorporated from time to time. Even at the present time, although the code is a very valuable working instrument, it is considered, by those who have had most to do with it, to be under development and liable to slight changes and amplifications, with the development of the art and with increased experience in application of the various provisions.

In 1916 at a meeting in Philadelphia[5], Dr. J. P. Jackson and Colonel L. T. Bryant, industrial commissioners of Pennsylvania and New Jersey, respectively, became interested, and after introducing some practical improvements, their respective commissions took action which resulted in the adoption of the code in the summer of 1916. The New York Commission adopted the code with some modifications and put it in force July 1st, 1918, some of the features being tentative for a year.

The Wisconsin Commission began issuing lighting orders in 1913, but these orders did not contain any specification as to intensity of illumination on the work. After thorough investigation the orders were superseded by the present code, which was issued in May 1918.

The author has just received word, as this paper goes to press, that the legislature of the State of Oregon has passed a bill establishing an industrial lighting code.

SCOPE

In the states which have adopted codes, the action has been by industrial or labor commissions, under the authority granted them by legislature, to promulgate rules in the interest of safety of industrial workers. The codes are, therefore, backed by the state police power.

Since the function of the Commissions is limited to insuring safety, the codes require only such lighting as is necessary for that end. Under these circumstances the codes cannot demand the higher standards of illumination desirable for economical production[6]. That the intensities specified are minimum limits, consistent with safety rather than good practise, is recognized, and in connection with all the codes, higher values representing more desirable practise are recommended.

It has seemed to be the experience of labor commissions that at least 90 per cent of the manufacturers cordially desire to comply with their regulations. The principal difficulty so far encountered in applying the codes, has been to make clear to the layman just what is required and how it can be secured. It has, therefore, been found expedient to publish educational sections supplementary to the codes proper. Such sections explain the principles involved and make suggestions for securing and maintaining good installations. The recent revision of the Wisconsin code treats rather comprehensively of the selection and location of lighting equipment, illustrated by charts.

By joint action of the Pennsylvania and New Jersey Commissions, a course of lectures[7] on the interpretation of the codes was given to their inspectors at the University of Pennsylvania in the spring of 1918. A similar lecture[8] was given to factory inspectors of the Board of Labor & Industries of Massachusetts in October 1918. Doubtless this will be repeated and the method applied elsewhere. The New Jersey Commission has held meetings of contractors and manufacturers, to assist in the educational work.

In all the codes, the employment of expert engineering or architectural advice is recommended.

SPECIFICATIONS

The principal lighting specifications common to all states which have adopted the code, are intensity, glare limits, and distribution. It is also usual to require provision of emergency lighting, watchman's circuits and certain switching provisions. In some cases these features are omitted from the codes, being provided for in the law or elsewhere.

A comparison of the codes adopted or under preparation shows a remarkable uniformity in the specification of these factors. There is some variation, especially as regards experimental features, but this is of a nature likely to help rather than retard the development of codes toward excellence and uniformity.

In drawing the specifications, it has been constantly recognized that they must be kept simple and practicable so that conformance could be determined without recourse to lighting experts or bulky and expensive instruments. On the other hand, it is desirable to make the specification as definite as possible, minimizing the part left to individual judgment and thus avoiding controversy and prejudice.

DAYLIGHT

While it has been considered desirable to require that new buildings and extensions be so constructed as to provide adequate daylight, it has been found very difficult, if not impracticable, to lay down a definite rule, and in most of the codes it has been assumed that the purpose was accomplished if suitable artificial light were required where and when daylight is unavailable.

Daylight is subject to a wide variation throughout the hours of the day, seasons of the year, with changes in weather and for different directions of window exposure, etc. In any building the natural lighting may be affected by surrounding construction on property not under the control of the owner. A workroom having ample daylight when erected may be darkened by later construction. In a dense manufacturing center, *e. g.*, downtown Manhattan, it would be impracticable to realize so good a condition, especially on lower floors, as might be reasonably required of a suburban factory. Some reasonable provision can undoubtedly be reached, which will at least require the minimization of glare and provide for wide distribution of light, by the use of suitable awnings, shades, refractive windows, etc.

The Wisconsin Commission has adopted the following rule in this connection:

> Order 2111. Natural Light. Windows, skylights, saw-tooth or other roof lighting construction of buildings shall be arranged with reasonably uniform bays and the glass area so apportioned that at the darkest part of any working space, when normal exterior daylight conditions obtain, (sky brightness of 1.5 candle-power per sq. inch) there will be available a minimum intensity equal to twice that of Order 2112 for artificial light.
>
> Awnings, window shades, diffusive or refractive window glass shall be used for the purpose of improving daylight conditions or for the avoidance of eye strain wherever the location of the work is such that the worker must face large window areas, through which excessively bright light may at times enter the building.
>
> Note. The intensity requirements for adequate daylighting are much higher than those for adequate night lighting, because in general, under daylight conditions, the light coming to the eye from all the surroundings in the field of vision is much brighter than at night, and hence a correspondingly more intense light must fall on the object viewed.

It is the almost universal experience of illuminating engineers and others, that it is possible to operate satisfactorily with a lower intensity of artificial than natural illumination.

The reason for this has not been satisfactorily explained; but with daylight the relatively high illumination of walls and other surrounding objects, compared to the illumination on the work, is undoubtedly a contributory cause.

In the earliest draft of the Illuminating Engineering Society's code, a ratio of 3 to 1 between natural and artificial lighting requirements was specified. Later this was changed to 2 to 1, and still later, in the absence of definite data in confirmation of the necessity of higher daylight intensities, artificial lighting was required when daylight intensities fell below the values specified for artificial lighting. The Wisconsin code and one or more of those under preparation, have adhered to the ratio of 2 to 1, which figure has also been used in the Illuminating Engineering Society's School Lighting Code. Further experience is apparently necessary before any final relation can be determined.

It is probable that in different conditions of daylight and of artificial light, the relative value varies. It is not likely, at least in the near future, that actual foot candle measurements will become the criterion for turning on the lighting, so that an accurate determination of this ratio does not seem to be of immediate importance.

The Illuminating Engineering Society, Pennsylvania, New Jersey and New York codes are essentially artificial lighting codes and do not attempt to specify daylighting. On the other hand, serious conditions of glare are not infrequent in the natural lighting of workrooms, and the extension of the code to mitigate such conditions seems to have considerable merit.

From a hygienic standpoint, sunlight has desirable features distinct from its relation to vision, so that it is desirable to have daylight available wherever possible.

There are some processes which inherently require the exclusion of daylight. Moreover, in the large cities workrooms, such as engine rooms, pipe shops, etc. have been established in basements, apparently without serious harm to the workers. On a larger scale, workrooms have been established above ground, which require artificial lighting throughout the day.

Unquestionably, all visual requirements can be provided with proper artificial lighting. It is, therefore, doubtful if a mandatory requirement of daylight for all places is justifiable. One of the codes under preparation, but not yet adopted, meets these conditions by simply specifying the lighting features necessary without distinction as to whether the light is natural or artificial.

INTENSITY SPECIFICATION

In all the codes which have come to the author's attention, intensity is specified in foot-candles. While at first some difficulty was anticipated in explaining these values and providing for their measurement, the development of an inexpensive and simple "foot-candle meter" has facilitated the application of these specifications.

For yards, passageways, aisles and stairways, the intensity on a horizontal plane, at or near the floor level, is assumed for measurement.

For various operations, the intensity is required at the work and would naturally be measured in the plane of any surface requiring vision of the character indicated. In all cases the limits are given as minima, not averages.

It has been generally recognized in selecting the intensity levels that there are no definite critical points, where a variation of a small percentage, one way or another, would cross a definite line between safety and danger. In determining the various intensity levels, lighting experts have drawn on their experience, supplemented by special investigation. It inspires confidence to note that the American values correspond rather closely to those determined independently in England after a very exhaustive investigation under governmental auspices[9].

The intensity specifications naturally fall into two groups: those required over general spaces and those required for work operations. The former are primarily for the prevention of accident, while the latter have the added element of conservation of vision, especially in the higher intensity classes.

A lower standard is allowed out-of-doors than indoors, on account of the difference in the usual character of surroundings and the nature of travel. The large out-of-door areas must, for economic reasons, have a low intensity requirement, and it would not be reasonable to demand much more than ordinary street lighting or moonlight intensities. The value of 0.02 foot-candle, therefore, appears in all the codes. For interior lighting a minimum of 0.25 foot-candle has been required for all traversed spaces.

New York and Wisconsin have included a classification to cover elevators, washrooms, dressing rooms, etc. at 0.5 foot-candle. This has not appeared in the earlier codes, but seems to be a useful addition.

The lowest intensity for manufacturing under the Illuminating Engineering Society, Pennsylvania, New Jersey and Wisconsin codes is 1.25 foot-candles. New York has adopted 1 foot-candle for the corresponding class, *i. e.*, rough manufacturing. While this value is, in the opinion of some engineers, lower than desirable, there is doubt if there is evidence to warrant the standardization of the fractional value.

The New York Committee has also adopted a new level of 0.5 foot-candle for the handling of materials and other very rough operations, and this step is being included in some of the codes in preparation in other states.

The higher steps are the same in all codes being located at 2.3 and 5 foot-candles, according to the fineness of vision required by the operations.

While it is true that there are operations requiring more than 5 foot-candles, they are exceptional and usually performed by highly skilled specialists. As such individuals are likely to be able to protect themselves, it has not seemed necessary for governmental authorities to take action.

Exceptions arising from the nature of a few processes are necessary to the illumination intensity requirements, as they appear in most of the codes. The omission has been due to the belief on the part of certain commissions that the exceptions were so infrequent and obvious that they can be taken care of readily and thus avoid misunderstanding on the part of others not entitled to exception.

One of the codes under preparation includes a class of zero intensity, while another lists the exceptions in a note following the intensity rule. This reads:

> "Note: Some exceptions to the Intensity Rule:
>
> a. There are some operations that are performed in comparative darkness, as for example, photographic processes in the dark room.
>
> b. There are some operations that are best observed by their own light as in the parts of the process of working glass.
>
> c. Some operations are best observed by the "silhouette" method of lighting in which the work is seen against a lighted background in a comparatively dark room, as in some processes of working with dark threads and lamp filaments.In all such cases in which work is of necessity carried on in comparative darkness, special precautions should be taken to properly safeguard the workmen.

A limiting feature of the intensity specification is the rather indefinite descriptive classification of the operations assigned to the various levels. The need of more definite descriptions is well known to all who have undertaken the work of drawing up specifications. It is sometimes important that an inspector or a manufacturer be able to tell without question within what class any particular operation falls. This, however, is not so serious as a superficial consideration might indicate, being somewhat ameliorated by the fact that manufacturing economy should dictate a much higher intensity than required by the codes.

Many efforts have been made to render the specifications more definite, but the problem is much more difficult than appears at first. There are many degrees of fineness in the same type of process as carried on in different shops or even in the same shop. Lathe work in machine shops, for example, varies in fineness all the way from very rough work down to watchmakers' fineness. An article is made by one manufacturer, say to 0.01 inch of accuracy, while another making nominally the same article works to 0.001 of an inch or finer.

Several proposals have been made, among which may be mentioned:

a. To make the intensity table simply an index of intensity levels, each indicated by an arbitrary designation (a, b, c, etc.) without any description of the process covered. Supplement this with a complete list of all manufacturing processes, using the symbol to designate the particular class to which each process is assigned.

b. To supplement the present classification with a detailed list of standardized processes.

The latter scheme (b) seems to have some advantages provided it is not extended too fast so as to lead to confusion through repeated changes. The New York Commission has adopted tentatively for a year, a list of this sort, but has found it necessary so far to use in many cases the qualifying terms "rough work", "medium work" "fine work", which make the specification in such cases but little more definite than the general classification.

The first plan (a) has some ardent advocates, but has been objected to on the ground that such a list implies an accuracy of description that does not really exist. Another objection is that the space occupied by such a list seemed likely to overemphasize the intensity specification as compared with other code specifications. It seems quite probable that this method will be tried out in one state, in which case its relative merit will be better determined by experience.

In general, it seems desirable for the ultimate good of all that most states adhere to the broad descriptive specification, while a few experiment with the extensions and determine their practicability. They will then be more readily able to adopt any method which shall have been found especially meritorious.

GLARE LIMIT SPECIFICATION

The establishment of proper limits for glare is probably more important than even the matter of intensity. While the provision of sufficient light is fundamental, it is also better understood and has a more obvious relation to production. It seems to be generally agreed that more employes suffer from glare than from insufficient light. Certain lighting conditions are readily recognized as glaring, while others are equally recognized as free from objectionable glare. But when it comes to drawing a definite limit between danger and safety, our present inability to measure or accurately define objectionable glare makes it necessary to utilize a qualitative rather than a quantitative specification.

The I. E. S. Pennsylvania and New Jersey codes simply require that lamps be "suitably shaded to minimize glare," with an explanatory note that glare from lamps or unduly bright surfaces produces eyestrain and increases accident hazard. This is supplemented by the requirement (under "Distribution of Light on Work") that "sharp contrasts of intensity" on the work be avoided.

The New York code requires that "exposed bare lamps, located less than 20 ft. above the floor, shall be provided with shades, reflectors, diffusing glassware or other accessories, to eliminate or minimize glare." Sharp contrasts of intensity on the work must also be avoided.

The Wisconsin Code goes a little farther and says,

> Lamps suspended at elevations above eye level less than one-quarter their distance from any position at which work is performed, must be shaded in such a manner that the intensity of the brightest square inch of visible light source shall not exceed seventy-five candle power.
>
> Exception: Lamps suspended at greater elevations than twenty feet above the floor, are not subject to this requirement.
>
> Note: (a) Glare from lamps or unduly bright surfaces produces eyestrain and increases the accident hazard.
>
> The brightness limit specified in this order is an absolute maximum. Very much lower brightness limits are necessary in many interiors illuminated by overhead lamps, if the illumination is to be satisfactory.

In some cases the maximum brightness should not exceed that of the sky (1.5 to 3.0 candlepower per square inch).

Note: (b) Where the principal work is done on polished surfaces, such as polished metal, celluloid, varnished wood, etc., it is desirable (but not mandatory at present) to limit the brightness of the lamps in all downward directions to the amount specified in this order.

For local lighting the Wisconsin code establishes the limit of 3 candle power per square inch, whenever visible from any working position. This lower limit is selected "because the eyes are more sensitive to strong light received from below."

There is a difference of opinion among illuminating engineers as to the proper values of candle power per square inch to be applied to this specification, while some think a better principle of specification can be determined. It has been suggested that glare be specified in terms of contrast of brightness, perhaps expressed in ratios. Some experimental work has been done along this line and at least two experimenters have made up experimental instruments for measuring ratios of brightness. It is recognized that there is a considerable range in the permissible contrast for different processes for different directions, especially of elevation, and for different intensities.[10]

It therefore seems likely that sometime in the future the codes will have some sort of glare classification corresponding to the intensity classification. But advance in such a direction cannot be made until further investigation and development has provided a reliable basis.

Considerable space has been devoted to the engineering limitations of the present glare specifications, with the view of interesting experimenters and engineers in the solution of the problem. On the other hand, the present specifications must not be regarded in any way as a failure. They are practical, working specifications which will do much to improve lighting conditions. They provide the means of eliminating practically all conditions which constitute a serious menace, and while the lack of a defined limit may in some cases be taken advantage of by either the manufacturers or the inspectors, such instances are likely to be quite exceptional. In reality the present specification is more definite than many existing legal requirements in force regarding matters other than engineering.

DISTRIBUTION OF LIGHT

The New Jersey code reads: "Lamps shall be installed in regard to height, spacing, reflectors or other accessories, so as to secure a good distribution of light on the work, avoiding objectionable shadows and sharp contrasts of intensity."

The distribution of light is partly taken care of in the intensity and glare limit specifications. That is, the requirement that intensity shall not fall below a certain minimum, coupled with economic considerations, tends to avoid a wide variation in intensity, while the latter also requires the elimination of dense shadows on the work. As worded in most codes these rules do not call for the elimination of bright striations of light, or, in brightly lighted rooms, of shadows which may interfere with vision but still have an intensity above the minimum limit.

The distribution of light rule covers these omissions, by calling attention to the relation of lamp equipment and location to light distribution. The lighting effects referred to, if objectionable, are so obvious that they are readily recognized when attention is called to them. Therefore, the rule seems likely to accomplish its purpose without much difficulty.

In the Wisconsin code a further requirement is made that the local lighting shall be supplemented by a moderate intensity of overhead lighting, except when reflection from light colored surfaces produces a general illumination. This renders more definite a feature than is implied by the general requirement of the rule.

Emergency Lighting

Panics, which have occurred in crowded rooms when the lighting has failed at the time of a fire, explosion or other emergency, have emphasized the necessity of avoiding any probability of darkness at such a time. There is, however, a wide variation in the actual needs of different workrooms. For example, where a large number of women workers occupy an upper floor surrounded by dangerous conditions, inflammable materials, etc. continuity of lighting is exceedingly important. But where a small number of employes are scattered over a large ground floor area, with convenient exits and without dangerous conditions, safety can be assured without an elaborate provision for continuity of lighting. Moreover, such provision as would be necessary in the first instance, might be prohibitively expensive for the second.

It has, therefore, seemed necessary to make only a general provision in the code, with the expectation that rulings would be made to cover specific cases, or that the detailed requirements would be supplied later by those having suitable experience. It is more a problem of safety engineering than of illuminating engineering.

The general requirement of the rule, common to most of the codes is: "Emergency lamps shall be provided in all work space aisles, stairways, passageways and exits. Such lamps shall be so arranged as to insure their reliable operation when through accident or other cause the regular lighting is extinguished." In some states the rule has been expanded to include instructions for insuring reliable operation—separate mains or sources of energy being required.

For example, the New Jersey code requires—

> Emergency lighting systems, including all supply and branch lines, shall be entirely independent of the regular lighting system and shall be lighted concurrently with the regular lighting system and remain lighted throughout the period of the day during which artificial light is required or used.
>
> Emergency lighting systems shall be supplied from a source independent of the regular lighting system wherever possible. This source of supply and controlling equipment shall be such as to insure the reliable operation of the emergency lighting system when through accident or other cause the regular lighting system is extinguished. Where a separate source of supply cannot be obtained for the emergency lighting the feed for emergency lights must be taken from a point on the street side of all service equipment. Where source of supply for the regular lighting system is an isolated plant within the premises an auxiliary service of sufficient capacity to supply all emergency lights must be installed from some outside source, or suitable storage battery; or separate generating unit may be considered the equivalent of such service.

It is apparent that the question of insuring the source of supply is one which is open to considerable discussion, involving, as it does, the reliability of the generating station and various classes of supply lines. Not only do the characteristics of service received from electric central stations and isolated plants have to be considered, but in some cases gas service and other illuminants.

It is evident that conditions of reliability vary considerably in different instances and that leeway must be allowed according to the merits of the particular cases. Where central station service is employed, it has usually been assumed that independent wiring to the street main provides a sufficient safeguard.

There is a tendency in some quarters to require the emergency lighting to be in operation whenever the regular lighting is in use. This comes from the apprehension of deterioration of unused equipment as well as the anticipation of failure to put the emergency lighting into operation instantaneously. In many cases simultaneous operation may involve an excessive expense. The provision of an automatic system, with reasonably frequent inspection and tests, should provide ample protection.

Where the system is not electric the automatic control may present more of a problem.

Most of the codes do not specify what intensity of lighting shall be provided. In New York State the emergency lighting is provided for in the law, while the code specifies one-quarter foot-candle as the intensity to be provided. This value, which might be implied from the intensity table, is being considered for inclusion in other state codes. It seems to be the reasonable value to require for exits, stairs, hallways and passageways, and also for the main aisles of large workrooms. It might, however, involve an unnecessarily high investment in lines and equipment, to provide such an intensity in all work space aisles.

SWITCHING AND CONTROLLING APPARATUS

This provides for the location of controlling switches so that at least pilot or night lights may be turned on at the main point of entrance.

Some commissions have considered it important that watchmen and others should be able to go about safely without lanterns, while others believe that the carrying of a lantern is sufficient. The former certainly has advantages, but may in some instances be unnecessarily expensive.

It would appear that the rule in this form is satisfactory on the assumption that an industrial commission would make exceptions where unnecessary hardship is shown.

ENFORCEMENT

Attention has been called to the readiness and willingness of the industries to comply with commission regulations, if they are understood.

Other limitations of the code have already been discussed, but none of these has presented more of an obstacle than the fact that many of those by whom the regulations must be applied are not versed in the principles of light and illumination, and have but little idea of the qualities and quantities necessary for the definition of lighting conditions.

Every effort has been made to simplify the rules and make them understandable to others than engineers. Some of the commissions have included simple definitions of terms used. Some have attached explanatory treatises and articles on methods of designing lighting installations. But there is still a need for more popular education. The possibilities of rendering the codes more definite and accurate in the future depend in a considerable degree on such education.

Undoubtedly the commissions can take care of their inspectors but the small manufacturers, foremen and others responsible for the construction and operation of industrial lighting installations need assistance.

The codes and their appendixes will undoubtedly provide effective mediums, but the help of those professionally related to the lighting practise is needed. It is earnestly hoped that the individual members of the American Institute of Electrical Engineers and the Illuminating Engineering Society, will inform themselves regarding the principles of the code specification and not only offer constructive criticism but also help in the educational effort of applying the regulation.

Conclusion

The author is inclined to look with apprehension upon laws or regulations emanating from the professions whose business they affect most directly. There has sometimes seemed to be a tendency in such legislation to favor the profession. Such tendencies are presumably more due to the professional viewpoint or prejudice, than to any intent to be unfair.

Having been connected from the first with the committees responsible for the illuminating engineering features, it is only fair to say that the danger of prejudice has been anticipated and carefully guarded against.

Especial care was exercised in the make-up of the committees and the solicitation of criticism, not only to represent the viewpoint of every phase of illuminating and other engineering, but also the light user and light purchaser. The engineer, the scientist and the practical constructor and operator were all included.

On the other hand, it is even more dangerous to legislate regarding such a technical subject without the guidance of the profession. In lighting we have numerous examples of such legislation as, for example, in the headlight laws for railways and automobiles. Many of these laws are not effective or reasonably enforcible, their meaning is not clear, while they not only impose unnecessary hardships, but have in some cases prescribed a dangerous rather than a safe condition.

The industrial lighting codes, besides avoiding both of these dangers, have been so formulated as to encourage uniform action throughout the various states of the Union. The continuation of the code development along the present lines is therefore of importance not only to those directly interested in the electrical and illuminating phases, but to the country as a whole.

In discussing the codes an attempt has been made to emphasize the principles involved rather than the details. Changes in details may be expected, but any considerable change in principles seems unlikely. No one is more conscious of the limitations of the code in its present form than the illuminating engineers who have been active in its development, but all who have had anything to do with the work regard it as a valuable working instrument. The author feels safe in saying that they consider it highly important to any state that it be adopted and actively applied as soon as possible. The greatest possibility for future development is through the experience and popular education following its enforcement.

The author has tried to avoid the expression of personal opinion as far as practicable. He has sometimes found it desirable to express his understanding of the views held by committees and their members, but this has been informal and unauthorized and should be so understood.

He herewith acknowledges with thanks the helpful assistance in the way of comment and criticism of Mr. L. B. Marks and Professor C. E. Clewell, to whom perhaps more than to any other individuals we are indebted for the initiation of these constructive regulations.

The author also acknowledges the assistance of Mr. J. A. Hoeveler and others.

BIBLIOGRAPHY

1. "The Mechanical Engineer and the Prevention of Accidents," by John Calder, *Journal* American Society of Mechanical Engineers, February, 1911,

2. "Illumination as a Safety Factor in Industrial Plants" by R. E. Simpson, *Transactions* of Illuminating Engineering Society, Vol. 9, No. 5, page 459. Also *Electrical World*, April 18, 1914, page 858.

"Illumination and One Year's Accidents" by R. E. Simpson, *Transactions* of Illuminating Engineering Society, December 30, 1915, page 868.

"Relation between Lighting Curtailment and Accidents" by R. E. Simpson, *Transactions* of Illuminating Engineering Society, Vol. 13, No. 8, page 429. Also *Electrical World*, November 16, 1918, page 935; *Central Station*, November, 1918, page 156, National Electric Light Association *Bulletin*, October, 1918, page 536.

"Relation of Industrial Lighting to Safety" by C. E. Clewell, *Electrical World*, June 16, 1917, page 1153.

3. "Illumination and Eyestrain" by E. M. Alger, *Transactions* of Illuminating Engineering Society, March 1913, page 130.

"Conservation of Vision" by E. M. Alger, *Illuminating Engineer*, February 1912, page 618.

"Hygiene of the Eye" Wm. Campbell Posey, A. B., M. D. (published by J. B. Lippincott Co., Philadelphia and London).

4. "Code of Lighting Factories, Mills and Other Work Places, *Transactions* Illuminating Engineering Society Vol. 13, No. 4, page 256.

5. "Lighting Legislation" L. B. Marks, *Transactions* Illuminating Engineering Society, February 1916, page 40.

6. "Productive Lighting Intensities" W. A. Durgin, *Transactions* Illuminating Engineering Society, Vol. 13, No. 8, page 417. Also *Electrical Review*, November 2, 1918, page 686.

7. "Factory Inspectors Study Industrial Lighting" by C. E. Clewell, *Electrical World*, July 6, 1918, page 11.

8. "Instructional Session for Special Guidance of State Factory Inspectors, held in Boston, October 31st, 1918" *Electrical World*, November 9, 1918, page 904.

9. First Report of the Departmental Committee on Lighting in Factories and Workshops, Home Office, British Empire, London, England, published by Eyre and Spottiswoode, Ltd., East Harding Street, E. C., London, England.

10. "Effects of Brightness and Contrast on Vision" by P. G. Nutting, *Transactions* Illuminating Engineering Society, Vol. 11, No. 9, page 939. *Electrical World*, February 17, 1917, page 326.

DISCUSSION ON "PRESENT STATUS OF INDUSTRIAL LIGHTING CODES" (STICKNEY), NEW YORK, N. Y., APRIL 11, 1919.

C. A. Adams: While the subject of "Safety Codes" is perhaps not as broadly interesting to the rank and file of our membership as some suppose, it is one of those things which are being forced upon our attention more and more as time goes on and shows the necessity for taking into account in a serious way the comforts and safety of the user of the things which we produce, or at least have some part in producing and installing.

The Safety Code movement, while in some branches it has been in existence quite a number of years—twenty years or thereabouts—is really only in its infancy. At first we did things as a matter of course because we were interested in the job, and finally we have been forced to take into account the interests of the users, but in so doing we meet with certain other serious difficulties, difficulties in establishing the codes and difficulties in enforcing them, I think it is here that not only electrical engineers and illuminating engineers, but engineers in general, may take considerable pride unto themselves in that they secured the provision that men trained, in some degree, in engineering, must be employed in these matters.

The great difficulty in the establishment of codes is to not only get them right, but to get the agreement and consent of all those that are involved in them, so that in a matter of this sort it is not only things which have to be considered, but it is the public and users, it is labor unions and labor, broadly speaking. So that I think that those who have to do with this work, those who have given so much time and thought to it, may feel to a considerable degree comforted, if they do not get much money out of it, in that they are doing a real and vital public service.

I am sure that we all appreciate the amount of disinterested and very serious effort that has been put on a paper of this sort, and back of that the work which has been done on the codes themselves. There are one or two things that Mr. Stickney has said that drew my attention. For instance, in the early part of his paper he said the manufacturers or employers were glad to adopt these codes, because as a rule they meant economies. It would seem natural that a manufacturer would be glad to adopt such codes in the interests of the workers, even though they did not mean economies. I recall a serious blow which I received one day in discussing certain safety devices with a large manufacturer when he objected to the introduction of safety devices on a very great number of machines he had, explaining that it was cheaper to allow the accidents to occur and pay the damages. Fortunately, that is not the attitude of many of our manufacturers at the present time, but there are still a few who take that point of view.

C. E. Clewell: The importance of legislation along the lines on which Mr. Stickney has so ably presented the subject, seems to me to be evidenced by the interest which has been taken in the several states where these codes have recently been enacted into law. When the subject is viewed from the purely legislative side it would seem that the responsibility for drafting these regulations in such a way that they will be understood both by the manufacturers and the factory inspectors, who are intimately associated with the enforcement of the codes, must rest almost entirely with the state departments concerned.

Mr. Stickney has pointed out the very interesting fact that the various labor departments where these codes have recently been put into effect have depended very largely upon the Illuminating Engineering Society for guidance in drafting these regulations in such a way that they will be suitable to the understanding of those concerned.

I have met and talked intimately with quite a number of factory inspectors, both in Pennsylvania and New Jersey, and have received the general impression that these codes, as obvious as they may appear at first glance, contain matters which, to some extent, are new and the situations created are different from the kinds of things which these inspectors have been accustomed to handling in the past, with other regulations governing the factory equipment. On this account it seems to me that just as soon as there is placed in the hands of an inspector a set of regulations which he is called upon to enforce, and at the same time in the enforcement of which he must depend upon his own judgment, that the thing tends to fail of its purpose in so far as the inspector is not posted on the fundamental principles which are back of these regulations.

The demand which has been placed on the Committee on Lighting Legislation of the Illuminating Engineering Society by these various safety departments, in their requests for something definite, has resulted in a method of procedure which, as far as I can see, is the only method which could have been followed by these committees in the development of the codes as Mr. Stickney has outlined.

Three things stand out in looking over Mr. Stickney's paper: First, that the enforcement of these codes and their ready acceptance on the part of the different labor departments and manufacturers, calls for a certain amount of education; second, that the inspectors who are really the go-betweens, as far as the labor department and the manufacturer is concerned, should have in their hands suitable devices by which they can check up their own judgment when they come to these specifications as to quantity of light and glare which Mr. Stickney has emphasized, and third, that it is almost essential, at least from the standpoint of those who are most intimately concerned with these regulations, to reduce as far as possible the element of judgment on the part of the inspectors, and one of the suggestions which has been made has been the compilation of more elaborate lists of various kinds of work and of the intensities corresponding to the same, so that an inspector may know readily how to place in its proper group any given class of work with which he may be confronted in his rounds of inspection.

The first of these points, the matter of the presentation of the principles contained in the codes, has been handled by a supplementary statement in addition to the regulations proper, which contains much information on lighting and the fundamental principles which govern these regulations, and another item, and what seems to me to be a vital point, is the matter of gathering these inspectors together—because the inspectors as they run are not a fixed group, due to changes which are going on all the time— and a recurrence of these gatherings could be planned, at which the inspectors would receive instruction along the lines of these fundamental principles at first hand. The physical measurement has been cared for, to some extent, by the foot-candle meter. As to the matter of covering all the requirements that come up, we must remember that with long lists of working operations and accompanying tables of intensities, the attention of the inspectors and the manufacturers is likely often to be drawn primarily to the matter of quantity, when there is quite a good deal of feeling on the part of those observing the conditions in the industries, that the quantity of light is not necessarily the most important item, and that the elimination of glare is by and of itself often a more important thing to attack than the matter of quantity.

Another item which I notice Mr. Stickney has dwelt upon, and which is somewhat different from these other items involved in the application of the code is the movement on the part of the New Jersey authorities toward the establishment of a Museum of Safety, so that the manufacturers throughout the state can see assembled in one structure, typical types and examples of lighting which will give them some idea of the kind of things they should watch for in systems of this kind.

The Lighting and Illumination Committee of the Institute, under whose auspices this meeting has been held, is sensible of the opportunity which such a general session has offered for discussion of this topic, and I think we all realize, after hearing Mr. Stickney's paper, that he has not only emphasized the scope and necessity of these codes, but has brought out in an interesting way the relations which they bear both to the manufacturers and to the engineering profession generally.

E. B. Rosa: I wish to emphasize particularly the educational value of this class of work and of the codes after they are issued. It seems as though the educational value would be much greater than the legal value—the benefit to those who are glad to avail themselves of these codes is greater, probably, than the benefits to those who have recourse to that law. I do not mean by that, that they should not be adopted by legal authority, but I mean to emphasize the very great educational value, first to those who take part in the preparation of the code—it stimulates the study of and education in the subject, and it is of educative value to those who participate in the preparation of the codes; in the second place it is of very great value to engineers to know about these things; and in the third place it is of great educational value to employers, managers and superintendents, those who are responsible for the lighting and the care of manufacturing plants and other places where lights are needed; and finally, of educational value to the employes, to those who are working in these factories.

While we speak of the compilation as a code, and think of the legal force of it, and the necessity of getting it into form suitable for adoption by legal authority, I think, after all, the educational value of the code is properly more important than anything else.

It is well not to put such codes into force too soon. We believe, in the Bureau of Standards, that these codes—lighting codes or whatever they are—shall be put out and given a trial, and those who are concerned with them given considerable opportunity to become familiar with them, so that the various features of their application may be developed before it is attempted to enforce them strictly. In other words, leave the matter of enforcement to those who are responsible for their enforcement—until they have full opportunity to become familiar with the codes and adapt themselves to the conditions.

C. B. Auel: The lighting codes, as is stated in the paper, are an expression of the "Safety First" movement in terms of legal regulations, and with their object everyone without exception will be in full accord. In the main they are excellent; but, regardless of their excellence they, unfortunately, represent individual states only and we shall accordingly probably have inflicted on us eventually as many different lighting codes as there are states in the Union, thus swelling unnecessarily the ever increasing burden of manufacturers and others, especially those doing an inter-state business, in trying to keep abreast of them. Another thing that should be borne in mind is that safety codes can be turned out, comparatively speaking, over night and are also capable of amendment in the same manner, and as they always involve expense while sometimes missing the object sought, they should be most carefully considered from every viewpoint, before being promulgated. Still another feature that should be mentioned is that all codes are practically a series of instructions to the employers and largely ignore the fact that there are frequently equally important instructions which should be issued to the employes, if the codes are to be successful. Mr. Stickney remarks that the experience of State commissions shows at least 90 per cent of manufacturers cordially desire to comply with their regulations. This is, of course, as it should be, but what manufacturers want to see is about 90 per cent of such commissions agree among themselves and if this can be accomplished, more nearly 100 per cent of manufacturers instead of only 90 per cent will agree with them.

Surely the lumens are no different out in Wisconsin from what they are in New York, or even in Massachusetts, nor should the requirements for good lighting be any different, and it ought accordingly to be quite possible to get the American Engineering Societies' Standards Committee, in conjunction with the Bureau of Standards at Washington, to formulate a single "League of Nations" code for general use throughout the country, and this without any state losing its sovereign rights.

It is certainly pitiful to see a great big nation like this which prides itself on its breadth of view, its vision, and on its being in the front rank of every forward movement, wasting such vast amounts, both in time and money, in having its work done piecemeal, the same problems solved, the same duties duplicated in various parts of its domain, as is evidenced in so many ways, for example, by our State laws including the safety codes. Through the efforts of the American Society of Mechanical Engineers, we have secured a Boiler Code which bids fair to become universal, so why can we not corral these lighting codes, which are commencing to be turned loose and turn them into one code for the community at large?

Referring to the Pennsylvania code, as published, and with which the speaker is best acquainted, the preamble starts off with paragraph entitled "Petitions" showing how to proceed in case one desired to appeal from any of its provisions. The one appeal the speaker would make, as already pointed out, and in so doing he believes he pleads for many others, is "O Lord, deliver us from all State codes." He speaks more feelingly in this case because immediately under the paragraph on "Petitions," is one entitled "Penalties," with fines and imprisonment staring you in the face. Having been duly warned as to the consequences impending for violation of the code or failure to carry it out, there follows a paragraph entitled "Application," which, for literary fog, can hardly be surpassed. It reads, "All Safety Standards of the Industrial Board in force at the adoption of this Standard, or adopted after this Standard, applicable in any way to the Lighting Industry (What is the Lighting Industry by the way?) or the operation of the Lighting Industry, or to the apparatus used in or around (how large a radius is included in 'around,') or in connection with the Lighting Industry, shall be considered as applying to the

Lighting Industry, and to such operation and app
It is quite clear from this (and it is the only clear thin
it), and taken in connection with the penalties, whe
of those having anything to do with industrial ligh
Pennsylvania, are going to land.

The code itself is short as far as its technical side
cerned and the appendix contains some very good m

It is apparent from the wording that effort has been
allow as much latitude as possible in the matter of int
elimination of glare, and spacing of lamps. As poin
by Mr. Stickney, this is due to the fact that suitable
have not been determined which will accomplish the
result and without working unnecessary hardships o
chiefly concerned.

Under the paragraph on "Intensities Required" is
table of the minimum as well as the desirable intensi
quired for different situations and classes of work
Stickney states that "every effort has been made to s
the rules and make them understandable to other tha
neers;" but, it has been the speaker's observation that
inating engineers have apparently been recommending
for factory lighting altogether too low, based largely no
on a lack of knowledge of actual working condition
example, it takes as good light to read a micrometer, a
divided rule, or a blue print, in the shop, as it does to
slide rule or a tracing in the office. Therefore, even
work be classed as rough, but involves the doing o
things as have been enumerated, then the lighting
be arranged on a higher base than the work itself would s
require. Again, the loss of light from dirty and old
and reflectors is glossed over too briefly, and the novice
has to learn by more or less costly experience that in o
maintain the proper degree of illumination he must a
wide margin over and above the values set down in table

The paragraph dealing with "Shading of Lamps" gi
idea of what may be considered as "Glare" or the va
a "minimum of glare". It is a matter of common kno
that a light source of high intrinsic brilliancy within the
of vision not only prevents comfortable and accurate
but may cause permanent injury to the eye if allowed t
tinue any length of time. The eye, due to the posit
the sun at its maximum intensity, has become accusto
bright light from overhead, yet the retinal fatigue res
from snow underfoot on a bright winter day is suf
evidence of its weakness in this respect. When lam
mounted sufficiently high, the shading of the lights
important except to distribute it properly, but whe
mounting height is lowered this feature is one of very
importance, and the paragraph in question should acco
be given more consideration than some of the other se
which have possibly been over-emphasized.

Still, again, the paragraph entitled "Distribution of Light on Work" reads "Lamps shall be so installed as regards to height, spacing, reflectors or other accessories, as to secure a good distribution of light on the work, avoiding objectionable shadows and sharp contrasts of intensity." This taken in conjunction with the intensities given in the tables, would seem to compel the use of overhead lighting exclusively and thus to eliminate localized lighting, whereas, much shop work is of such nature that anything short of individual lamps will prove quite unsatisfactory.

If the paragraph relating to emergency lighting, not only in the Pennsylvania but in the New Jersey code, really means what it says, namely, that every establishment where workers are employed, regardless of what it is, where or how built, or for what purpose it is used, must be equipped with two entirely independent systems of lighting, then we may as well expect to have incorporated in due course in this, that, or the other code, a provision that all machine tools shall have an extra base, that all tables and seats shall have extra sets of legs, that all buildings shall have an extra foundation, except of course in Wisconsin, where no foundation at all will be permitted, since the lighting code in that remarkable state distinctly says every room must have daylight.

Again, a literal interpretation of the Pennsylvania Code would imply that an emergency system should provide an intensity equal to that of the regular system with an equal number of lamps used. The other extreme would be to furnish an emergency lamp merely as a marker in each working space, aisle, or exit, according to the custom in theatres and public buildings—to serve as a guide rather than to provide illumination. While the latter would prove somewhat inadequate, a duplicate system would be quite unnecessary. It would seem that a rather low intensity such as that specified for yards and roadways, should satisfy every requirement for emergency lighting.

The code further requires that emergency lights shall operate concurrently with the regular system, meaning presumably that the emergency lights must be automatically turned on whenever the regular system is in general use. This would be unreasonably burdensome in a great many cases where plants are already wired and equipped for adequate ordinary lighting, without provision for concurrent and independent emergency lighting, and would involve the use of considerable power unnecessarily. Such plants would be compelled not only to rewire their buildings with separate emergency circuits, but to increase their lighting load and equipment beyond those limits which have already been found to give satisfactory illumination. In some classes of service where emergency lights are really necessary, it has been customary to control such circuits by relays actuated by the regular circuit;

the relay operating and causing the emergency circuit to be lighted when, for any reason, the operation of the regular circuit is interrupted. This feature avoids the need of using power simultaneously and unnecessarily in the emergency circuit. The reliability of the emergency circuit may be insured by periodical tests just as fire alarm boxes and equipment are inspected and tested.

Such a radical requirement as "Emergency Lighting" should, it seems to the speaker, certainly be made advisory rather than mandatory, at least until the various states shall have agreed upon a plan which has proven itself by actual experience to be feasible.

M. G. Lloyd: The author of the paper made a reference to the work of the Federal Safety Engineers, and perhaps a word in explanation of that would be of interest. The country had not been long in the war before the Government officials recognized the necessity of conserving human resources. In the summer of 1917, through the initiative of the U. S. Employes' Compensation Commission, there was started a survey of the conditions with respect to safety in the Government war plants, that is to say, the arsenals and the navy yards. As a result of the conditions disclosed in that survey it was decided to appoint safety engineers at each one of these plants, and that was done. The safety engineers, soon after their appointment, arranged a series of meetings at which they got together and drafted some rules to be enforced in their respective plants, with the idea of preventing accidents. Before they got very far along in this work they called upon the Bureau of Standards to assist them in drawing up these rules and codifying them, and in that way the Bureau took a hand in the work and in conjunction with them drafted a series of Federal safety standards which were utilized at these plants.

Had the war continued longer than it did, it is probable that an attempt would have been made to have these safety standards applied to a larger field, that is to say, in munitions plants operated by private companies. The obvious method of securing such enforcement in plants not directly controlled by the Government would have been through the incorporation in Government contracts of proper clauses requiring conformity with the safety rules. With the war ending as it did, the matter, of course, was not pressed further.

The requirements for lighting which were incorporated in these safety standards were included in a set of Standards for Building Construction. These Federal Safety Standards have never been printed as yet. They were simply gotten out in mimeograph form for distribution among the Federal Safety Engineers for their own use and for the use of contractors supplying equipment for the use of the Government, who had to conform with certain of the requirements of these rules. In these Federal Lighting Standards the code of the Illuminating Engineering Society was closely followed, but there were one or two items that were somewhat different. The author has referred to the introduction of the requirement for a half-foot-candle in the codes of New York and Wisconsin for certain places, such as toilets and wash rooms, and that provision was put in these Federal standards over a year ago, and that intensity was also applied to elevators and elevator landings, an item which has not been included in the Wisconsin code.

There was also a requirement covering daylight illumination as well as the artificial lighting, and there is a paragraph I want to call attention to, on account of Mr. Auel's remarks with regard to the effect of dirt, which rapidly decreases the illumination intensities, if it is not taken care of. That clause I might read, as follows: "Artificial lighting equipment and windows shall be periodically cleaned and kept in order so that the illumination intensities do not fall below the minimum value specified. Artificial lighting equipment shall be so installed as to be readily accessible for cleaning." It is recognized that that is a very important item in obtaining suitable values of illumination.

After these Federal Standards were completed we found that there was considerable demand from the outside for copies of them, showing that there are many persons who want some guide to follow for safe practises in private plants. As has already been mentioned this evening, there are many employers who have to be forced to provide suitable conditions, but there are many others who are always seeking information as to how they can better the conditions in their plants.

As a result of this demand it was concluded at the Bureau of Standards that it would be desirable for us to proceed to elaborate these Federal Standards so as to make them more complete and a more suitable series of codes for general use. We have started that work, which has been referred to by our presiding officer, and have four national industrial safety codes now under way.

In considering the requirements in the different industries from the safety standpoint, it may be desirable to know what suitable illumination intensities would apply in the different industries, and I am consequently very much interested in the attempts that have been made in New York State and in Ohio to determine the suitable values in separate individual industries. I hope to see considerably more work done in that direction.

Another point where lighting standards touch the work of the Bureau of Standards is in the National Electrical Safety Code, which has been worked upon at the Bureau for a number of years, and of which a tentative edition for trial application has been in print for over two years. We are now engaged in revising that code and attempting to make it more complete and improving the provisions wherever possible. The present edition of the National Electrical Safety Code has some rather general requirements for illumination. For instance, with reference to generating stations and substations, there is a requirement for an emergency source of illumination as it is one of the places where it is considered necessary that complete darkness should not result from any disturbance with the usual lighting system. An exception is made to that requirement in the case of automatic substations, which are, of course, in condition to operate without an attendant.

There are certain limitations to illumination in storage-battery rooms. For instance, there is a requirement that sockets in that location shall be of the keyless variety, with the idea of preventing any spark which might ignite hydrogen gas and cause an explosion, since such gas is liable to be present in a room of that description. The requirements, however, for intensities of illumination are not very definite; they are covered by such terms as "good" or "sufficient" or "adequate." In the next edition of the Code we hope to get information which will guide those responsible for providing suitable illumination in that class of rooms. We called upon the Illuminating Engineering Society for help in this connection, and I want to express our appreciation of the co-operation which we have received. A committee of the Society carried out an investigation and has made a report giving some values of illumination which they consider suitable requirements to insure safety in such situations. They have recommended, as usual, a minimum requirement and given also a value representing what is considered good modern practise.

ILLUMINATION VALUES FOR POWER PLANTS.
Approved by the Council of the Illuminating Engineering Society, November 4, 1918.

	Minimum Foot-candles	Modern practise Foot-candles
Switchboard instruments, gauges, switches, etc.	1	2 to 4
Switchboards with no exposed live parts	½	1 to 2
Storage battery room	½	1 to 2
Generating room, boiler room, pump room (at machinery or exposed live parts)	1	2 to 4
Stairways and passageways (measurements made at floor level)	½	1 to 2
Stairways and passageways (measurements made at floor level) where there is moving machinery, exposed live parts, hot pipes, etc.	1	2 to 4

The illumination values to be those for the working surfaces. (Measurements to be made on the vertical, horizontal or intermediate plane, as the case may be.)

Any traversed space to be illuminated to not less than one-quarter of a foot-candle minimum (measured at floor level).

I might call attention to the fact also, in view of Mr. Auel's comment, that in the Electrical Safety Code there is provided a set of operating rules which the employee is required to follow in his work in order to do his share in avoiding possible accidents.

Before closing, I want to mention the Oregon law, to which the author referred, and a copy of which I have here in hand. This law, as he mentioned, does not include any definite quantitative values for illumination, but confers upon the Commissioner of Labor the duty of specifying them. It makes general requirements that places shall be adequately and properly lighted; it covers requirements to minimize glare; it provides for suitable distribution of the light, and has a requirement for emergency lamps. There is also the usual requirement for the control of the lighting system. There is a rather unusual provision, however, in empowering the Commissioner of Labor to specify the minimum intensity. This section of the law states that the Commissioner is authorized to establish minimum values. It then goes on to say that he shall be guided in so doing by the recommendations of the Illuminating Engineering Society, but before the specified values can become effective, he is required on his own initiative to appoint a commission of three persons, one to represent manufacturing interests, one to represent electrical workers, and one to be an electrical engineer. This Commission is to hold hearings and it is expressly stated that this Commission shall have power, independently of the Commissioner of Labor, to establish, rearrange or readjust the schedule of limiting values and rules as above set forth, that is, as designated by the Commissioner himself; in other words, there is a court of review, we might say, provided to approve or amend the schedule provided by the Commissioner of Labor, supposedly with the recommendation of the Illuminating Engineering Society.

There is one other point about the Oregon law that seems to me rather important. Most of the State commissions which have acted in regard to the lighting code have general powers for making requirements along the line of safety for factory workers. In the case of Oregon, however, this is a particular separate law, conferring this power on the Commissioner of Labor, to make these requirements. It is of interest to observe that in this law there is no reference to safety, and that apparently the Commissioner is given powers to determine minimum values upon any basis that he may consider fit, not necessarily upon the safety basis. That would seem to open the way for him to make requirements which would be higher than would be ordinarily deemed sufficient merely to provide safety.

Ward Harrison: I do not believe any of us fail to sympathize with Mr. Auel's view that he would like to see a National

Code covering the whole subject. At the same time, I think he will acknowledge that the situation as far as the Lighting Code is concerned, is far better than in the case of the tax laws, or almost any other kind of State legislation with which he may be familiar. Then too, as Mr. Stickney has said there would be an emphatic dissent at the present time from any intimation that we knew all about lighting, and just how universal and permanent legislation on the subject should be framed. There are a great many things that we do not know. The various states in the supplements to their respective codes—most of the differences in the codes are in the supplemental matter—can try out different plans and thus find which one works most equitably and effectively. We shall make more progress in this way than if all were exactly uniform from the beginning.

When the subject of lighting codes was first proposed in some states the attitude of a number of large employers was that of opposition. I know of one case where they said, "We have competent engineers to take care of our lighting; let the other fellow do likewise. This Code will be nothing but one more law added on to all the rest of the industrial regulations. Why should we further it?" At this point one of his associates from another plant replied, "We must not forget that the interests of the large and small manufacturer, so far as labor is concerned, are identical. It is possible that your plant is so well lighted throughout that the eye-sight of your workers will never be impaired and that no accidents will happen attributable to this cause. Yet, you must not forget that the entire community is your labor supply. If the small plants are poorly lighted and if as a result of that there are maimed workers or workers with impaired vision, these men, when in the course of time they go to work in the larger plant, will certainly reduce your average efficiency. We are in favor of having all plants in this state properly lighted."

On the general question of lighting intensities it might be said that, up to this time we have been for the most part in the same condition in which people found themselves a century or more ago in regard to heating. Then they were dependent upon open fires, and temperature-measuring instruments were a rarity. If you stood close to that fire it almost burned you up and yet other portions of the room were often below 40 deg. fahr. Very nearly the same effect is had in lighting by the use of bare drop lamps with no general illumination.

At the time thermometers came into general use people not only began to appreciate what the temperature variations were, but to demand temperatures all over the house which were comfortable; the foot-candle meter and similar measuring instruments should likewise lead to the same conception of lighting intensities. Unfortunately, we have no satisfactory instrument for measuring glare, as yet; in fact, as the author has stated, we do not know just what we want to measure. However, decided progress is being made along these lines and in two or three years we may well expect some form of glare meter which will measure glare almost as readily as the footcandle meter measures illumination. When this is accomplished we shall be more nearly ready for a national code.

John Vogt: In New York State we have a code which we think is about as good as any, and this evening we have not received as many knocks as some of the other states. I believe if these other states had conferred upon the different Departments of Labor, law or rule making power there would be more uniform codes promulgated.

Back in 1897 the labor law of New York State contained provisions for proper lighting of factories and the proper lighting of aisles, stairways and stair halls. In 1903 provisions were added to provide for adequate light in basements and mercantile establishments where women and children were employed. In 1912 and 1913 forty-six sections were added to the labor law as a result of an investigation by the State Factory Commission and gave authority to the Industrial Board of the Labor Department to make suitable regulations as to standards for temperature, ventilation, lighting, etc. About two years ago the Industrial Committee on Lighting was formed and we got up this code to regulate the lighting of factories. Many sections of the labor law relate to the proper sufficient and adequate lighting of stairs, stair halls, stairways, workrooms, storerooms, elevator entrances, elevator cars, washrooms, toilet rooms, etc. These words "sufficient, proper and adequate" meant nothing to the ordinary inspector who is called upon to enforce the provisions of the law. In those intensities which we have worked out we thought it best to follow the Illuminating Engineering Society's Code, and in addition thereto to make a large number of photometric tests ourselves. We have a corps of engineers in the department which is capable of doing this kind of work, and more than ten thousand lighting tests were made for the Code Committee. Since then, there have been more than five thousand tests made, so that these tests play a very important part on the amount of light which is specified in our Code.

One thing that interested me in this paper was relative to the education of inspectors and the suggestion that the inspectors might not be able to carry out the work. Our inspectors are regularly drilled, regularly given lectures. Every two weeks inspectors of our department are called together, and have been for the last three months in the Metropolitan Hall, and given lectures in connection with the American Museum of Safety in matters of this kind. We have particularly given four or five lectures to the factory inspectors from the practical standpoint of our law, not alone on illumination, but sanitation and safety, the removal of dust, fumes and gas. We realize that the Code as it stands to-day is embryonic. It is partly mandatory, but not mandatory insofar as other hearings to be held in the various states, relative to the amounts of light to be provided at processes of manufacture are concerned.

To give you some idea of what has been done in the past year as to orders issued and compliances secured in the regular work of the Department, I have gone over the statistics and find that during the month of July, 1918, when the Code went into effect, relative to that part which is mandatory, that is, the intensities required in washrooms, toilet rooms, aisles, passageways, storerooms, etc., leaving out the part which will be mandatory after July 1, 1919, after the hearings are held at which manufacturers are invited and others express their opinion at these hearings, which are to be held in the cities of Buffalo, Rochester, Utica, New York and some other points, as to whether we are right or not, or whether the Code is proper, the inspectors issued during July 105 orders relating to lights and secured three compliances; in August of 1918, 135 orders issued and 67 compliances secured; September, 174 orders issued and 278 compliances secured; October, 64 orders issued and 88 compliances secured; November, 546 orders issued and 156 compliances secured; December, 446 orders issued and 205 compliances secured; January, 1919, 608 orders issued and 517 compliances secured; February, 765 orders issued and 50 compliances secured, a total of 3,290 light orders issued and 1,569 compliances secured, and only 92 waivers made. These statistics include the work only up to March 1st. Our statistics are not ready for the later months. These orders were issued out of a total of 56,236 inspections of the regular type and 4,791 special inspections. This gives you some idea as to the character of work and the quantity of work that the Department of Labor, now the Industrial Commission, has done.

Total orders issued up to March 1st, 3,290 and compliances secured, 1569, that is, about twice the number of orders issued as compliances secured, giving about 1500 or 1600 compliances still to be secured. I have not had time to segregate these orders to determine whether they were for glare or for improper distribution, etc. From what I can remember, most of them were for the elimination of glare or minimizing of glare.

Many other orders have since been secured, no doubt, but I have only given statistics up to the 1st of March. These orders go out, and the parties upon whom they are served are given a definite time in which to comply with the law. Manufacturers may ask for an extension of time. In that case the order still is pending. In the case of all orders that are pending, compliance will be insisted upon, and if the ordinary method does not secure it the Court will be resorted to. We have a capable legal bureau and these things are threshed out in Court if necessary.

W. P. Hurley: It seems to me that if the codes in themselves are in advance of the popular education, it is the work of the promotion of these codes that needs to be taken in hand, in order to make them of the greatest efficiency. We may get ourselves into the position of the states of Wisconsin and Massachusetts, with their insurance laws, where they passed useful and effective laws, providing for state insurance, but they fell flat because the popular education was not sufficiently advanced on the subject for the people to make use of them.

One of the most interesting points, that has not been touched on here to-night, is how to bring these codes before the public, and it seems to me we must first appeal to the self-interest of the user, that is, the manufacturer, and next the self-interest of those who can with profit to themselves bring the codes to the attention of those manufacturers, and keep up this education, otherwise the burden must be on the Society as a scientific body, and on the Labor Bureaus of the different states. Even if the different states handle it, it means the popular education of the users, and this work will have to be undertaken by them. It would mean that the inspectors, even, must be educated to tell when the law is violated, and they would have to organize to the extent of telling the men how to design installations, which is usually considered beyond their province.

The present time is particularly ripe for the education of all the interests concerned in the lighting industry—by that I mean the manufacturers of the incandescent lamps and the fixtures and the wiring contractors, as well as the plant departments themselves of these manufacturers. It is particularly suitable at this time because we have the gas filled lamp practically in its infancy, as far as application is concerned. The educational work should be with the contractors and dealers. This would be found to react at once in favor of the Code, because it would give them a legal standing and something to work by that could be published and made use of.

W. T. Blackwell: This subject has been considered from practically every phase but there is one point which I would like to bring up and that is the instrumentality through which we are to enforce the code. We have at the present time codes in four or five states, therefore, we must take into consideration how the lighting measures in the codes are to be enforced. The Department of Labor inspectors, upon whom it is encumbent to enforce the requirements of the codes, are as a rule appointed through Civil Service examinations. We as a Society have fathered these codes—therefore, we should take the initiative in taking measures toward the end that the inspectors may receive proper information which will enable them to do their work properly. Last year a series of lectures were arranged at the University of Pennsylvania, under Professor Clewell, for the training of the inspectors of the States of Pennsylvania and New Jersey. Owing to the short period covered by these lectures only a meagre idea of the requirements of good lighting could be touched upon.

Many of the codes have a monetary penalty for non-compliance with requirements and in consequence an inspector who is not thoroughly familiar with lighting is therefore not in a position to determine whether the prevailing conditions in a plant come within the minimum requirements of the code and therefore hesitates about placing a violation on the installation. It seems to me that having evolved the code we should bend our efforts toward the education of the inspectors as has been suggested by the previous speakers. This perhaps can best be done by placing in the hands of the inspectors some form of a manual which would be comparable to the National Electrical Code or the Fire Prevention Manual adopted by the National Board of Fire Underwriters, and also the National Fire Prevention Bureau.

The foot-candle meter provides a ready means of checking up the intensities of illumination, provided the inspections are made at night, but if the inspector is to use this instrument for measuring daylight intensities it will then be necessary to have the instrument equipped with a daylight screen. This, of course, does not offer an insurmountable obstacle but it occurs to me that this instrument offers the best solution to settling differences of opinion which may arise between the inspector and the superintendent of a plant. It would therefore seem desirable that the use of footcandle meters properly fitted to the requirements of the situation should be encouraged in those states where codes are in force.

In order to thoroughly understand the situation and the conditions under which such inspectors work, we must place ourselves in their position or otherwise I fear that the lighting legislation now incorporated in the Labor Codes of several states will become inoperative through lack of enforcement. I, therefore, hope that the Society will take some measures toward formulating an inspection manual for the use of the inspectors in those states having codes.

E. G. Perrott: A point in connection with the Code which I think should be amplified, is the question whether the minimum of illumination should be stated in the Code, or whether it should be left to the discretion of the Labor Department, or the Commissioner who enforces the code.

I wish to mention something which might be considered as paralleling in its operation the lighting codes, that is, the building codes of the various cities. We have in Philadelphia a Building Code under which we are suffering, due to the fact it was adopted in 1854, while it was suitable for that period, from the standpoint of the knowledge which existed at that time, whereas the knowledge of building construction has advanced to such an extent that it no longer applies. The law is a State law and cannot be changed by local authority, cannot be changed by anybody that has anything to do with building construction, and as it requires State action, it is a very difficult matter to get the State Legislature to repeal old acts and reframe them to suit new conditions.

It appears to me that in the state of the art as it is to-day we are in a much better position if we omit from the Code itself the actual illumination we desire for the various parts of the building and leave that to the commissions or to the labor departments to establish as has been done, I believe, in the state of Oregon.

T. F. Foltz (communicated after adjournment): Although there has been a lighting code in force in the State of Pennsylvania for nearly three years, it is only during the past year that definite arrangements have been made by the Department of Labor and Industry to apply its requirements in more than an advisory way. This was due largely to war conditions which affected to a large extent the industries in this State and made difficult the detailing and training of inspectors for this work.

With the revision of the code about a year ago and the course of excellent lectures given to the inspectors last year at the University of Pennsylvania through the courtesy of Professor Clewell, the real work of applying the provisions of the code commenced. These lectures were supplemented by talks at inspectors' meetings of the different districts into which the state force of factory inspectors is divided. It soon became apparent, however, that it was more practicable to detail for this work, men who by training and aptitude, could be drilled into the work of judging and advising on good lighting arrangements for industrial plants. These men are being given special instructions on the subject of factory lighting and on the use of the recently developed compact and convenient foot-candle meter with which each man is equipped. Although the training of these men is not yet completed, results would indicate that we are on the right track.

Thus far it has seemed inadvisable to use the police powers to secure compliance with the provisions of the lighting code; the method used, therefore, is one rather of education and co-operation. It has been found that the average employer, especially the one who does not have a business big enough to have experts on his staff, knows little or nothing about good and bad lighting; although experience shows that he is, on the whole, anxious to correct conditions when he is shown how to do so. He is conversant with requirements for machinery guarding, dust and fume removal, etc., and these requirements can be laid down in concrete form as to dimensions, materials, etc. To him, a requirement for a 5-foot guard is clear, but he has yet to learn the meaning of an illumination intensity of 3 foot-candles. As a means of educating the employer, there was issued with the last edition of our lighting code, an appendix containing much of the information in the Illuminating Engineering Society "Code of Lighting Factories, Mills and Other Work Places."

Some employers can be won over best by the money argument. When they are shown the ridiculously low cost of a good lighting system, as compared with the saving they can make in better and greater output, less spoilage, etc., they regard it as a shrewd business proposition to comply. The means of gaining compliance must be determined after a careful analysis of local conditions as the inspector will find them; and our experience would indicate that compliance with our requirements secured through co-operation and education produces more lasting results than that obtained by the exercising of mandatory powers.

A strict interpretation of the legislation which gives this department its authority would seem to limit its requirements to those just sufficient to prevent accidents to the workers. Illumination adequate for this purpose, obviously, is not enough for satisfactory and efficient work. However, those powers might be interpreted to include the authority to call for conditions that would reduce eye strain, industrial fatigue, etc. Such a broad interpretation, then, would permit requirements for lighting which would be sufficient, not only to prevent accidents, but also to result in truly adequate illumination for all purposes.

Mr. Stickney's paper is a most comprehensive analysis of this subject. He not only shows that considerable progress has been made on the subject of lighting legislation, but he also frankly admits that certain phases are still susceptible to much improvement. For instance, the subject of glare, an element most difficult of treatment in a code, is still in need of further study. To our mind the treatment of glare is as important, if not more so, than that of illumination intensities. It is futile to provide a sufficient quantity of illumination unless the lights are so disposed and equipped as to minimize glare. The requirement that lamps under a height of 20 feet be specially equipped, does not begin to cover the case as there is nothing said about intrinsic brilliancy, size of unit, contrast of brightness, or whether or not the light source is within the field of vision. Limiting the candlepower per square inch of light source does not cover the case as a certain light source in a comparatively dark field may be objectionable, whereas, the same light source in a light field may not be annoying. Thus far, the California requirement takes into account the greatest number of factors affecting glare, particularly that referring to the light source being within field of vision. To cover the case adequately, a rule or formula would have to take into account the intrinsic brilliancy of the light source, contrast of brightness, distance and angle at which the light source is viewed, and the nature of the work done. It is obvious that the complexity of such a rule or formula would militate against its adoption. It is indeed unfortunate that some mechanical means to evaluate glare has not yet been developed to the extent that has the meter for measuring foot-candle intensities.

No attempt is made in our code to give detailed requirements as to the extent and means of securing emergency lighting. Local condition will have to determine whether the emergency lighting will be from an independent lighting plant, separate transformers, separate fuses, from a separate power circuit, or from gas or even oil lamps. The manufacturing processes and the nature and occupancy of the building are also determining factors. The inspector is compelled to exercise fine judgment after making a careful survey of local conditions.

Much credit is due the Illuminating Engineering Society and its individual members for the rapid progress recently made on the subject of lighting legislation, and it is noted with considerable satisfaction that they intend to make further progress along this line. The Pennsylvania Department of Labor and Industry will welcome developments that will increase the practicability of its lighting code.

James R. Cravath (communicated after adjournment): Like other legislation, industrial code lighting legislation is largely a problem of education. First, the experts formulating such codes must thoroughly educate themselves on the conditions to be met. Next, they must pass this information along in usable form to the non-technical legislator and user. Before such codes become thoroughly effective, those responsible for the enforcement of the law must be educated as well as those upon whom the law must be enforced. Such education means that the factory inspectors and superintendents, operating under the law, must understand something of the illumination unit with which they are dealing. Fortunately simplification of instruments for measuring illumination has recently placed this matter on a much more satisfactory basis than heretofore, so that education in what a foot-candle of illumination really means can be carried on a sufficiently large scale to accomplish the desired results within a reasonable time.

Specifications with regard to glare, as the author indicates are rather difficult to formulate at this time, when one considers the prime reason for such legislation is safety. As far as. safety goes, lighting conditions are tolerable which would by no means come up to reasonable hygienic requirements. For example, a glaring light source may be far enough above the line of vision not to interfere with clearness of vision and so it would be exempt under any "safety first" regulations. However, it might be a very bad source of eye fatigue or strain. As this would pertain to the health of the workers, it would also be a justifiable subject of legislation even though not necessary for safety from accidents. The Wisconsin code, which as the author says, is the only one giving definite glare limit specifications, is nevertheless very lenient. Seventy-five candlepower per square inch is over 3600 millilamberts or considerably brighter than an acetylene flame. The difficulty of formulating glare regulations is of course great. Our knowledge of what glare limitations are desirable is probably greater than our knowledge of how to lay down such limitation in a code which must take in such an infinite variety of conditions, as an industrial lighting code.

G. H. Stickney: The extensive discussion which this paper has evoked is a gratifying indication of the wide interest in the subject. The general unanimity of opinion expressed seems to indicate a remarkable agreement among engineers with the line of procedure. There is considerable opportunity for debate as to how far certain specifications should go. I am convinced that more experience in applying the codes is the only reliable basis for determining these questions very much further.

We can all agree with President Adams that manufacturers should welcome safety provisions for humanitarian reasons, independent of ordinary factors of economy. While the Commissions have reported a broad attitude on the part of manufacturers, it is undoubtedly true that the enthusiastic acceptance of the codes by manufacturers will be encouraged by the realization that the codes make for economy rather than expense. Thus, the educational work in which Prof. Clewell and Dr. Rosa have expressed so much interest, will be even more important than might otherwise appear.

The whole trend of the discussion is to emphasize the need of wider popular education in regard to the factors of illumination. It would seem as if there were a need of going even further than anything which has been projected. We may well urge that the values and characteristics of light be taken up more thoroughly in the teaching of physics, not only in colleges, but also in high schools, and that the units of light might even be treated in connection with the teaching of measurements in arithmetic. We can hardly expect that the relations of light quantities will be as broadly studied as the common measures of length, volume, weight, etc., but they might very reasonably be included in the training of all technical students.

I note Mr. Auel is opposed to States' rights. This principle certainly has made the code problem a much more difficult one in this country than it was in England, where there was but one government to deal with. The difficulty, however, as regards lighting codes is a small matter compared with the corresponding confusion in other classes of state law, as has been pointed out by Mr. Harrison.

The question of Federal *versus* State enactment is certainly too big a problem for the Committee on Lighting Legislation to undertake to secure modification. The committee has wisely accepted the existing methods of law making and has done the best it could under these conditions. That there should be a general uniformity of action in all states is of course important, and in this connection it is very fortunate that the Committee has been able to establish a leadership which keeps it in touch with the activities in the different states and enables it to exert a strong standardizing influence.

As pointed out in the paper, the variations in the code so far adopted are not fundamental and are largely confined to undetermined factors, where the experimentation, if properly followed up, should lead to progress followed by uniformity of enactment. All the commissions encountered have seemed to be composed of remarkably sincere, able men, seeking the best interests of the people.

It is true that in one or two instances there seemed to be a tendency on the part of state committees to introduce variations as evidences of individual accomplishment. It does not seem likely, however, that such variations will seriously compromise the ultimate status of the codes. In some ways the variation may be of benefit if the leadership can be maintained. At any rate it cannot be entirely avoided.

No report of hardship has come to me in regard to the enforcement of the Pennsylvania Code and I believe that a fair and common sense interpretation of the rules is likely to be made by the Commission.

In regard to the question of cleaning lighting equipment, it is my recollection that a foot note appeared in at least one of the early drafts of the Illuminating Engineering Society's code, pointing out that the values of required intensity were minima; that lighting equipment should be cleaned and that in the design of lighting installations, at least 25 per cent allowance should be made for depreciation. I believe this note was transferred to the Advisory Section at the suggestion of a State Commissioner. At that time there was a tendency for explanatory notes to accumulate in the code proper and it seemed advisable to eliminate them as far as possible, confining the rules to the conditions to be maintained and treating the methods of securing them in the Advisory Section. The low intensity values have been explained in the paper. These values have been criticised by some as being too high; by others as being too low. They seem to be about right as safety provisions, in view of the present state of the art.

The practicability of the intensity specification was greatly enhanced by the foot-candle meter, which has been mentioned by several in the discussion. The instrument devised by

Dr. Sharp provides a simple inexpensive means by which reasonably accurate measurements can be made by relatively inexperienced observers. Such observers are liable to make greater errors in reading the more complex, though more accurate higher priced instruments.

The present foot-candle meter is being distributed at cost by lamp manufacturers in the belief that a better understanding of intensity will be favorable to the development of the art. It is understood that other illuminometers are under development by instrument makers which, in addition to the present type, should result in a wide distribution of suitable instruments.

Referring to Mr. Perrott's comment regarding the flexibility of intensity values, it should be noted that in every instance so far these values have been adopted by the Commissions and not by act of Legislature. The Commissions have the authority to make changes from time to time. Oregon is the only state in which the Legislature has written any of the definite rules.

I think we are all agreed that the codes should be so authorized that the Commissions can make changes in the rules as necessitated by the advance in the art and without recourse to legislation.

The question raised by Mr. Cowles and others, in regard to the emergency circuits, is an important one. If the specification as written in some of the codes is too literally interpreted, there seems to be considerable possibility of inflicting unnecessary and burdensome expense, especially upon small industries. It is, of course, impracticable to write a specification for each individual set of conditions. Commissioners with whom I have discussed the question seem to think it preferable to specify for the dangerous conditions and allow exceptions where the duplication of service is shown to be unnecessary. Personally I think it would be desirable to recognize such exception in the rule and make provision so that those entitled to exceptions will understand that they can secure them without difficulty. Consideration should be given in each case to the necessity of the emergency lighting, the liability of failure of lighting service, its extent, its relation to other emergencies and the practicable provisions which can be made to avoid undue hazard. It seems reasonable that wherever emergency lighting is necessary it should be separated from the power circuits in such a way that an overload on the power circuit cannot cause entire failure of the lighting.

The need of more definite glare specifications has been emphasized by Mr. Foltz and others. The Committee has been keenly aware of this need. It has been suggested that rules be made for each type of lighting equipment. On account of the variety of reflecting equipment used with the incandescent lamps, this would be quite an undertaking;

moreover, it is almost impossible to draft rules of this type so that they will be fair to all illuminants. There would, therefore, be considerable possibility of stirring up competitive recrimination which might seriously interfere with the general acceptance of the rule.

A definite measurement of the glare effect, independent of the particular equipment, seems to offer a more helpful solution. What is now needed is a simple, inexpensive and reasonably accurate instrument for measuring glare. With such a device available, the illuminating engineering features of the codes could readily be carried to definite conclusion.

Presented at the Boston Section Meeting of the American Institute of Electrical Engineers, Boston, Mass., May 7, 1919.

Copyright 1919. By A. I. E. E.

UTILIZING THE TIME CHARACTERISTICS OF ALTERNATING CURRENT

BY HENRY E. WARREN

ABSTRACT OF PAPER

By the invention of a very small, simple and reliable self-starting synchronous motor, in conjunction with convenient means for regulating the average frequency of an alternating circuit, there has been developed a new field of usefulness for electric power. It is now feasible to drive all kinds of timing devices such as clocks, graphic instrument movements, time recorders, etc., directly from the lighting circuits. Remarkable accuracy is obtained, and the amount of care is minimized by the elimination of winding and regulating.

MORE than a decade ago the possibility was recognized of increasing the accuracy of the alternations of current distributed for light and power to a degree where it might serve through synchronous motors to drive ordinary clocks. The system proposed, which was said to have been actually tried in Germany, was inadequate in using motors which required to be manually started and in the mechanism for enabling control at the central station.

Within a few years the author, impressed by the great opportunity to extend the use of alternating current into the field of clocks and various timing devices, recognized that the first requirement was a very small reliable self-starting synchronous motor. After many experiments the motor shown in phantom view in Fig. 1 was developed. This has a small bipolar field magnet, fitted with shading coils on its poles, and is excited by a winding *A* taking current from a single-phase circuit. Located in the field with a relatively large air gap is a disk rotor *B* with projecting pins *C*, the rotor and pins being made of hardened magnet steel. The rotor is mounted upon the shaft *D*, which is vertical to reduce wear of the bearings; and the relation between the rotor and the field is such that the weight of the former, which is only 1/20 of an ounce, is magnetically supported, so that when running it floats without appreciable end thrust. Motion from the rotor shaft is transmitted through the worm thread *E* to the gear *F* and thence by another similar reduction to the terminal shaft *G*. This gearing is enclosed in a nearly air-tight case *H* which contains an ample amount of lubricating oil. The worm thread on the rotor shaft forces this oil through the upper bearing and distributes it to all rubbing surfaces. The space around the rotor is guarded from entrance of dust and loss of oil by a tight cap of non-magnetic metal which is sealed into the case.

The rotor of this motor starts from rest and runs at synchronous speed because of the magnetic remanence in the hard steel. The initial starting torque is due to the fact that the poles induced in the disk by the rotating magnetic lines of the field persist locally during part of an alternation after the inducing magnetism has advanced, and by ordinary magnetic attraction tend to follow the field. The following reverse magnetic alternation overpowers these local poles in the rotor disk and sets up new ones, but the disk has moved meanwhile. At first there is a progressive shifting of the poles through the mass of the rotor. As the speed increases the driving torque continues strong until actual synchronism is reached, when the local poles in the rotor remain in step with the rotating field and no longer shift progressively through the mass of the rotor. In order for this action to take place, it is necessary to proportion and design the field, airgap, and rotor so as to maintain certain very definite relations among them, especially with regard to the distribution and intensity of the magnetic flux. Otherwise the magnetic poles of the rotor will shift at synchronous speed and the rotor will slip.

The rotor pins *C* considerably increase the synchronous torque of the rotor without interfering with its starting torque, but they must be properly proportioned and arranged to accomplish this result. Material such as is used for permanent magnets serves better than ordinary carbon steel for pins and rotors. Apparently the higher the remanence the greater the output.

The general appearance of the motor is shown in Fig. 2. The overall dimensions are 2 3/16 inches high, 2 11/16 inches wide and 2 5/8 inches deep. The weight is 15 oz. One of these motors consuming about two watts will deliver at its terminal shaft, making one rev. per min., slightly more than one-millionth of a horse power. Its efficiency is therefore one twenty-fifth of one per cent. Nevertheless, this minute amount of power is more than ample to drive clocks and various forms of timing devices. In fact it represents several times the output of a strong spring clock movement. These synchronous motors will start and run indefinitely at absolute synchronism when connected in circuit of the approximate frequency for which they are designed. Their synchronous operation is not affected by very considerable changes in voltage. For example, they will operate correctly when the voltage is 50 per cent above or below its normal value, but a motor designed for 60 cycles requires certain changes in its proportions in order to operate properly at 25 cycles. There is, however, no difficulty in supplying motors to operate at any standard frequency.

The starting torque of these motors is more than 50 per cent of their running torque. If overloaded beyond capacity they will generally come to rest and will not continue to run below synchronous speed. The period during which a motor will run without attention has not yet been determined, but some of them have been operating continuously 24 hours per day for more than 2½ years. It is evident that the lubricating oil which they contain should be renewed every year or two and if this is done, there seems to be no reason why the motors, when driving light loads such as clocks, should not run for many years. The oil can be easily renewed but the motor has its parts sealed together when it is built. The only place where the oil can escape is through the clearance space around the terminal shaft, but this is above the oil level when the motor is in operation. Owing however to variations in air pressure and temperature changes, a very slight amount of oil is occasionally forced out at this point.

The next requirement in order to improve existing distributing systems so that they might be used for the operation of clocks and other timing devices was an instrument which could be used in place of the present frequency meters to standardize the frequency at the generating station. The result achieved is shown in Fig. 3, which illustrates the type of master clock that has been installed in many of the principal power stations in the eastern part of the United States, and which has been adopted for controlling the frequency.

This clock contains two complete movements, one regulated by a pendulum and the other driven by one of the self-starting synchronous motors which have been described. On the large dial of the clock there are two hands, one of them black and the other gold. The black one represents the time as determined by the pendulum. This pendulum has a rod of invar and its rate is adjusted by two methods. One used in astronomical timepieces consists in adding or subtracting weights on a small platform carried by the pendulum rod. These weights are numbered to indicate the seconds per day which they will alter the rate. The other method is electromagnetic and consists in passing a very minute measured current through flat coils located below the pendulum bob. Within the pendulum bob there is a permanent magnet, the field of which cuts across the conductors of the coils. This combination of parts permits an exceedingly accurate adjustment of the rate of the clock. It is easy by this means to make the clock run a fraction of a second per day faster or slower.

The other hand which is mounted upon the large dial and is gold in color, is driven directly by gearing from one of the self-starting synchronous motors. The gear ratio between the hand and the motor is such that at normal frequency the gold hand will revolve at exactly the same rate as the black hand, and if they are started together they will remain so. These hands make a complete revolution in five minutes.

The operators at the switchboard are instructed to look at the clock occasionally and, if they find that the gold hand is gaining over the black hand, to lower very slightly the speed of the turbines. If the gold hand is running behind the black hand, the turbine speed is to be slightly increased. Small momentary variations in the frequency do not perceptibly affect the relative position of the gold and black hands, and since in general these fluctuations are equally likely to be positive and negative, there is shown only the error which is cumulative. Any real tendency of the speed to run either higher or lower than its normal value will gradually bring about a change in the relation of the two hands. This is really the only error in the frequency which should be corrected by the operator, because the governors of the turbines will automatically take care of the momentary fluctuations. The master clock may be considered as a device to maintain a true base line for the frequency.

The physical effort required to maintain this correct base line is very much less than has formerly been expended in some large systems in trying to keep the frequency correct by following the indications of an ordinary frequency meter. The reason for the difference is that the frequency meter shows only the instantaneous value and is consequently fluctuating above and below the average value. These fluctuations are only momentary as has been pointed out before, but there is no way by which the operator can tell whether the indication of the meter when he looks at it represents a high point in the frequency curve or a low point. Consequently, he is obliged to assume that it represents the average frequency, which it rarely does. Therefore when he adjusts the speed according to a reading of the frequency meter, he may make the average value worse than it was before. The next time he looks at the meter, the needle may happen to be on the same side of the average, in which case the operator assumes that he did the right thing previously. Or it may happen to be on the opposite side, when he concludes that it needs new adjustment in the reverse direction. The more frequently these adjustments are made, the more necessary they apparently become. The difference between determining frequency by the frequency meter method and that of the master clock may be made clearer by the following somewhat analogous illustration:

Suppose that a lake, the surface of which is covered with waves, is to have its depth measured. One method is to go out in a small boat and drop a sounding line overboard. The observer will be able to tell the depth of the water within a foot or two, but the readings which he makes will be constantly varying on account of the motion of the waves. This corresponds to the frequency meter method. A much more accurate means of determining the depth of water in the lake would be to enclose a considerable area of its surface in a structure such that the waves could not enter and the surface of the water would be smooth and level inside. Then a reading by a sounding line would instantly give the true depth. This corresponds to the master clock method.

With the master clock method, the only error that appears is a real one and is due to the tendency of the average frequency to run either high or low. This tendency in a large station may not become manifest for a considerable time interval, sometimes as long as 30 minutes or more. Therefore it is not necessary for the operator to make adjustments nearly as often as he did formerly where he depended upon the frequency meter. Perhaps it would be fair to estimate that one-half or one-third as many adjustments are needed where the master clock is used.

The only care which must be given the master clock is to compare it every few days with accurate time signals sent out from Washington or some astronomical laboratory. When it has been carefully regulated it should run within a very few seconds a week. If conditions are favorable one of these clocks will keep within one or two seconds per week. The winding of the movement which drives the pendulum is done automatically by the same motor which is geared to the gold hand on the large dial. As a safe-guard a second motor which drives a clock hand moving over a small dial is installed in the same case. In an emergency this hand can be used to regulate frequency by comparison with the black hand on the large dial. There should also be installed on the wall near the master clock a secondary clock with a second hand driven by one of the small synchronous motors so as to check at all times the frequency on the system. The master clock itself should stand upon a reasonably firm foundation so that it will not be subject to excessive vibration. Its motors, which require only four watts, may be connected to any nearby instrument transformer which can be depended upon to remain in circuit continuously. The effect of controlling the frequency by one of these master clocks may or may not be apparent on the frequency meters which have been in use. This will of course depend upon the error of these instruments. If however the average frequency is measured by more sensitive means, for example, by counting the total number of alternations during the day and determining the average mathematically, the effect of the master clock will at once be apparent. Any ordinary frequency meter can hardly be expected to keep the error in the average frequency below one or two per cent, while the master clock will easily, over a 24-hour period, keep the error in the average frequency below 1/100th of 1 per cent.

Ordinary frequency meters are subject to errors principally of the following kinds:

1. Calibration.
2. The effect of use and time.
3. Variations due to temperature.

Of the foregoing errors those due to calibration and use may be overcome by frequent recalibration, provided an accurate standard is available, which is not usually the case; but the errors due to temperature are very difficult indeed to eliminate

PLATE XXXV.
A. I. E. E.
VOL. XXXVIII, 1919

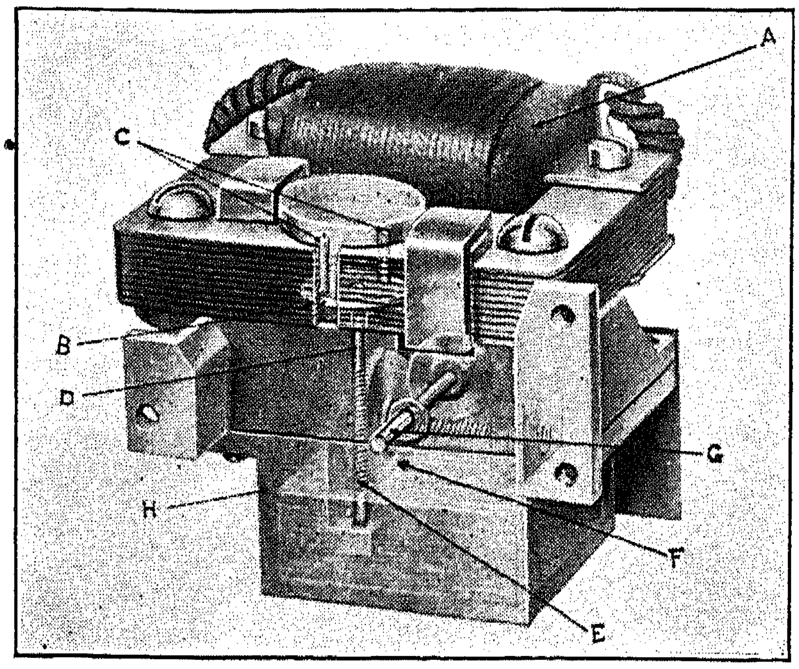


FIG. 1

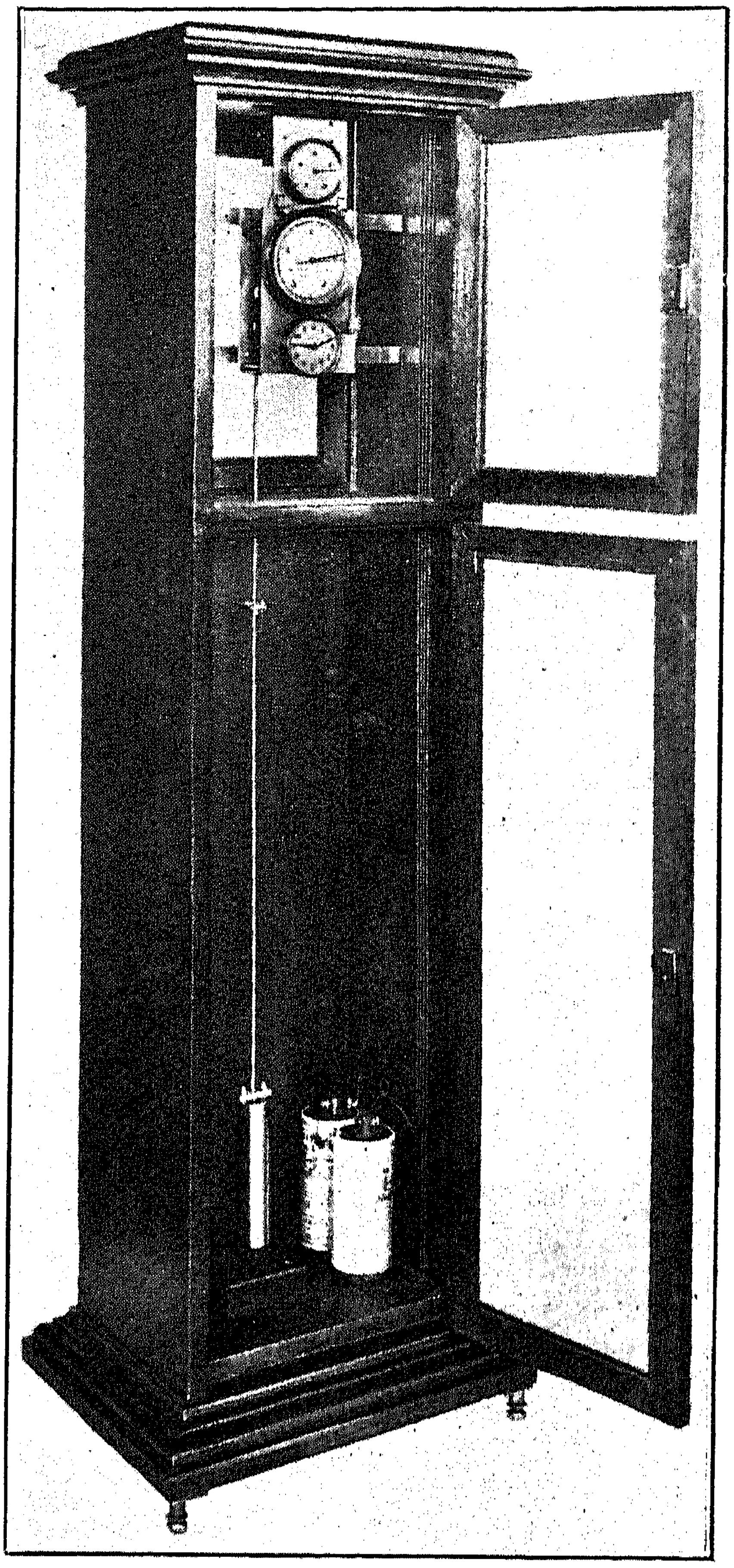


FIG. 3

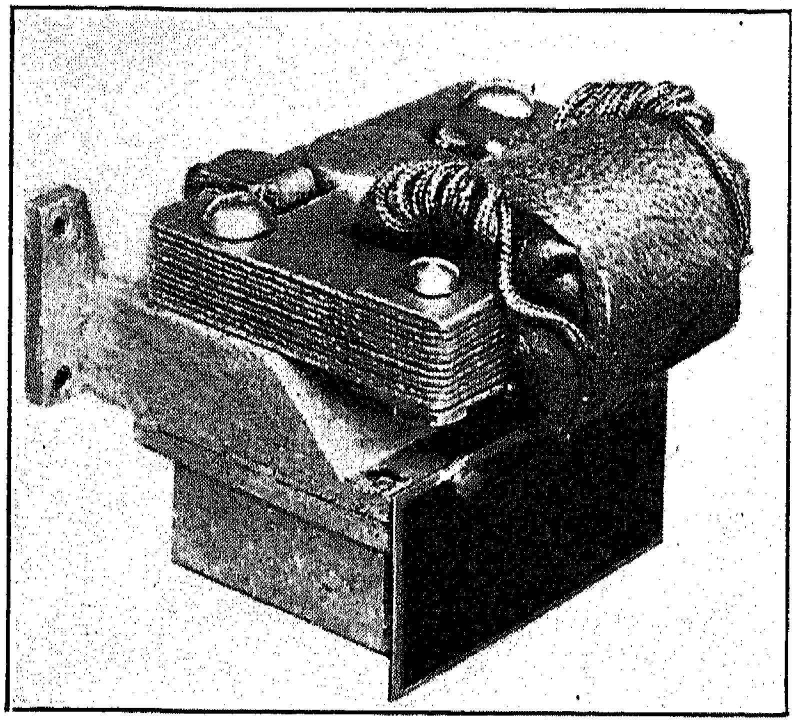


FIG. 2

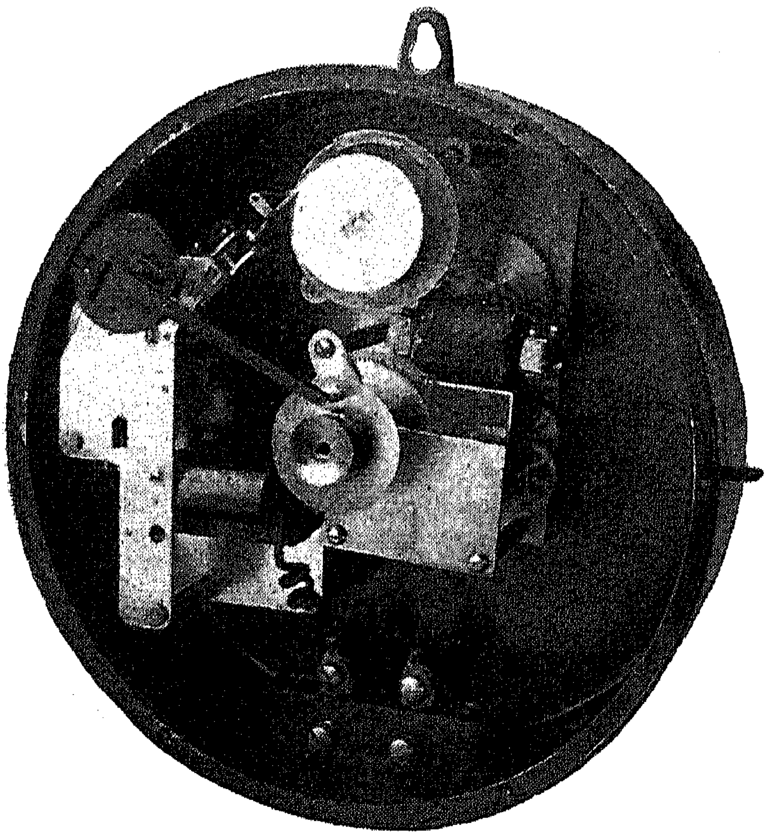


FIG. 5

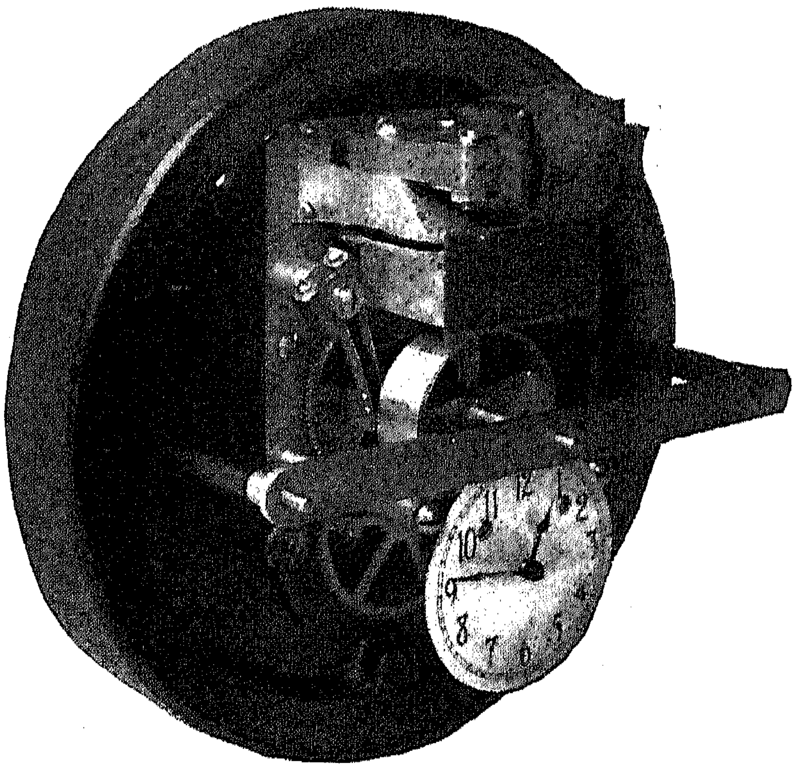


FIG. 6

[WARREN]

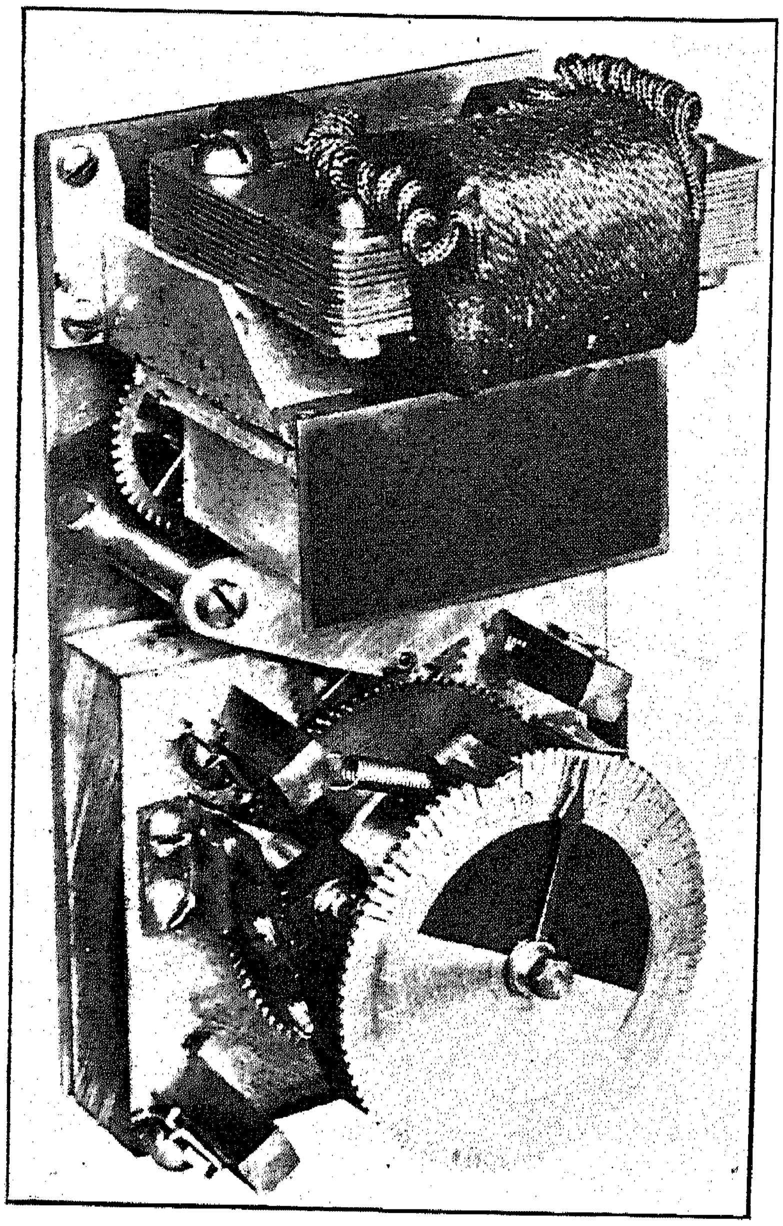


FIG. 7

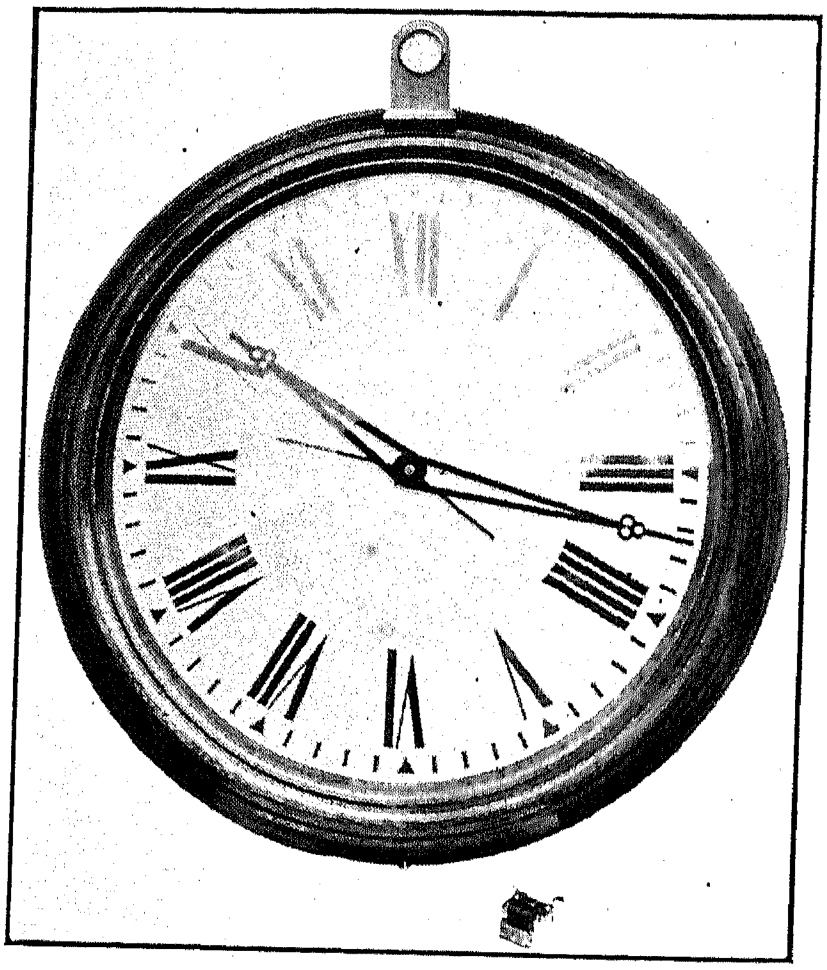


FIG. 8

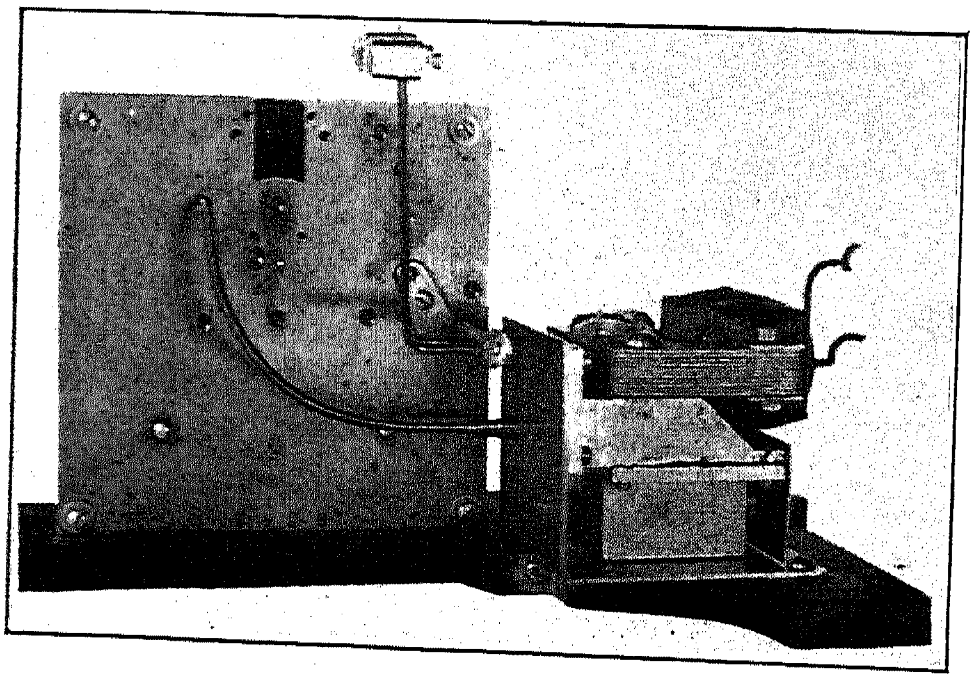


FIG. 10

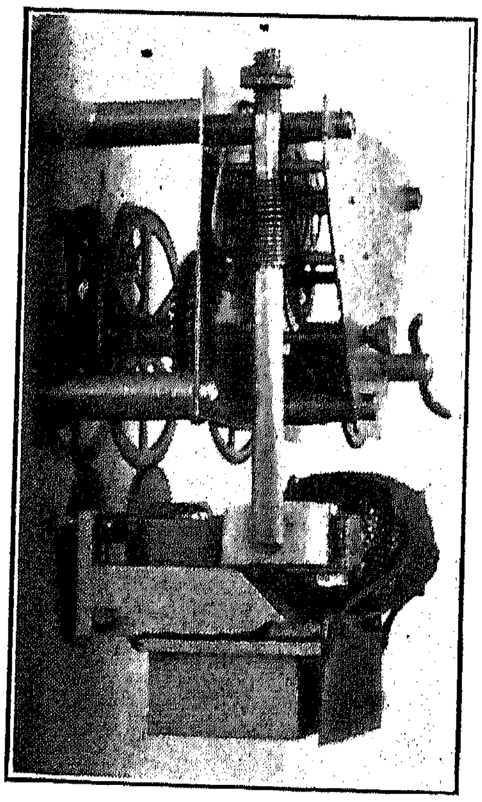
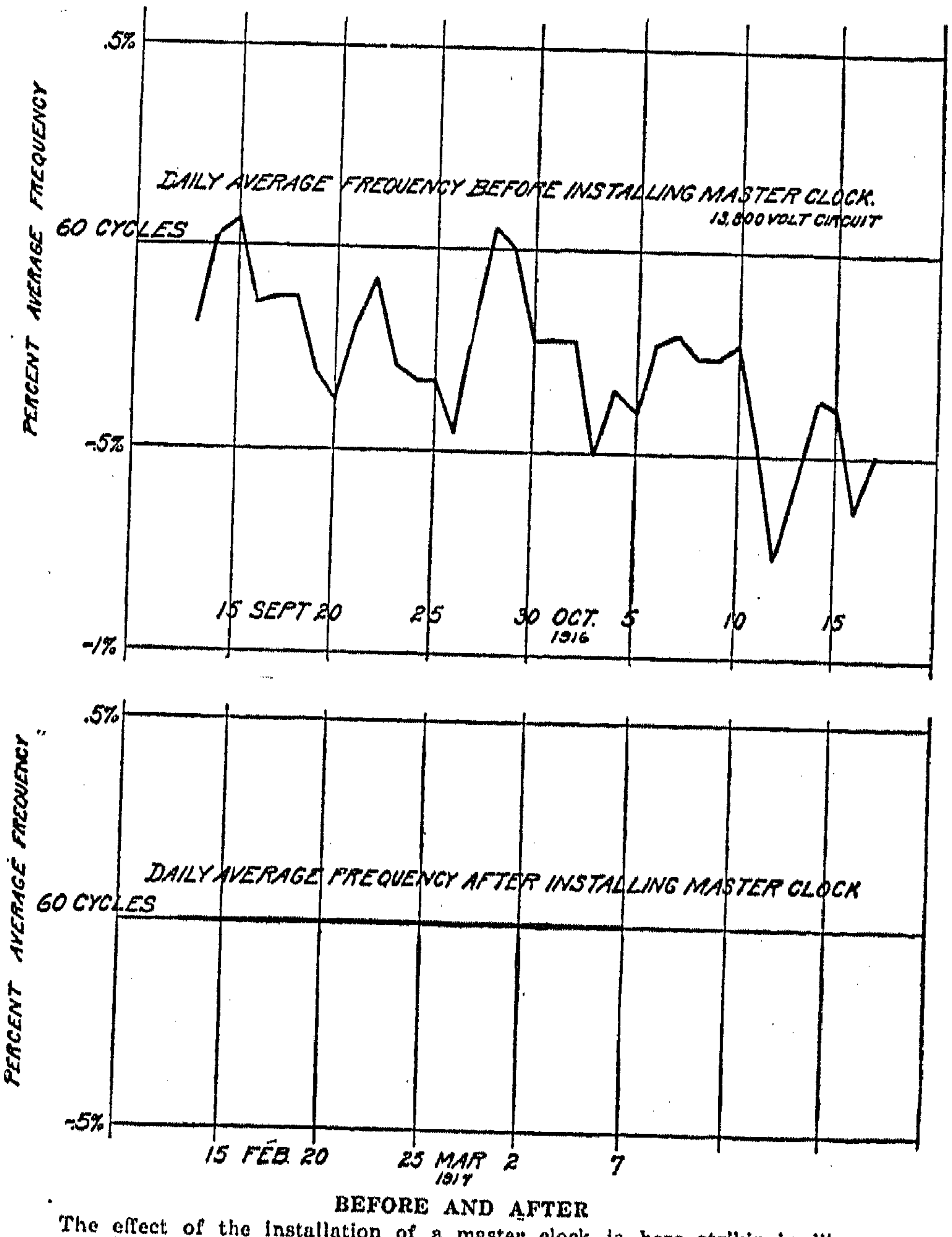


FIG. 9 [WARREN]

and these errors alone will ordinarily justify the master clock method. There is no reason why the frequency of a great system should be dependent upon the temperature at the switchboard, which has generally been true.

In Fig. 4 is shown an interesting comparison between the frequency obtained on a very large system by means of first class frequency meters which were often checked and conscientiously watched, and the frequency on the same system at a later period when the regulating was done by means of a master clock.



BEFORE AND AFTER

The effect of the installation of a master clock is here strikingly illustrated. Note the marked irregularities with the best regulation possible under the old system.

FIG. 4

The errors in the former case were principally due to variations in temperature, and these during the whole year covered a considerably wider range than shown in the curve, which covers only one month. These two curves do not represent all the variations in frequency during each day, but only the average value of the frequency as determined by dividing the total number of alternations during the day by the time in seconds. They may be said to form the base lines of the frequency. In one case the base line is very irregular and differs materially from its normal value, and in the other case it is a horizontal line having true normal value with an error too small to show on the plot. Experience has shown that where master clocks have been installed on various systems the average frequency before the master clock was started has usually been as much as one or two per cent above or below normal, although of course it was impossible to determine this until the clock began to operate.

The question will immediately arise whether there is any advantage in trying to hold the average frequency nearer than one or two per cent to its exact value, and superficially there might appear to be no good reason for doing so. Certainly the customers will be unable to tell the difference, because frequency meters which they may have are not better than those at the central power station, and there has been no other easy method of showing the error.

On careful analysis however, it will be apparent that there is a real advantage in keeping the base line of the frequency at its true normal value. In the first place, the machines connected with the system, including the turbines, are so designed as to run most efficiently at normal frequency. Some of these machines, for example hydraulic turbines, may have either their efficiency or their output materially affected by deviation from normal speed.

Perhaps the most obvious effect of errors in the frequency is upon those manufacturing establishments whose output comes from continuously running machines, particularly where textiles, paper, or flour are made. It is a well-known fact that textile machinery can be driven up to a definite speed with a constantly increasing output, but not beyond this without injury to the product. Now whether induction or synchronous motors are driving such machinery, the speed will depend directly upon the frequency. Consequently, if the average frequency is even one per cent lower than the most efficient speed for the machines, the output of the mill per day will be diminished by just that same percentage, and if the frequency is higher the product will suffer injury. The fact is of course recognized that it is necessary to allow a certain margin for safety between the average operating speed and the maximum speed to take care of temporary variations caused by load fluctuations or otherwise, but this same margin must exist in any case. It is clear that the output of a cotton mill connected to the system where the curve in Fig. 4 was made would be greater during the summer season when the frequency was nearly normal than later in the season when the frequency dropped, because of the temperature coefficient of the frequency meter. In a number of textile mills connected to a single system, the value of the lost output in dollars and cents due to this cause would soon exceed the trifling cost of the master clock equipment necessary to standardize the frequency. In paper mills the evenness and quality of the paper depend upon the accuracy and uniformity of the speed, so that there is a very obvious financial advantage in maintaining correct frequency. Doubtless many other classes of manufacturing are similarly affected, but the above examples are sufficient to show the financial desirability of the new method.

Beside the advantage to the user of electric power in maintaining the correct average frequency, there is a definite psychological advantage to the operating engineers and managers of power companies in being able to assure themselves and all of their customers that they are maintaining the frequency upon which they have agreed. A still greater gain from the standpoint of the power producer results from the ease with which systems having such standardized frequency may be connected in parallel without the slightest disturbance. Generally, where many systems are feeding into a large network, the individual variations in the frequency meters are such that it is necessary to make adjustments by raising or lowering the speed, sometimes to a very perceptible extent, before an individual station can come into synchronism with the network; but where master clocks are installed it is only necessary to wait until the machines have the right phase relation, because there will be no measurable difference in the average frequency of the independent station and the network when they are running independently. As at the present time the tendency is very clearly to increase the interconnection of big power producers towards the ultimate goal of a complete union extending all over the country, it is obvious that preliminary standardization of the frequency is a step in that direction.

Passing now from the advantages of master clock control which affect existing service, one comes to gains which open an entirely new field of service. Just as soon as the frequency of a system has been controlled by means of a master clock, it becomes feasible to connect anywhere in the line small self-starting synchronous motors like Fig. 2 and use them to measure time accurately. The most obvious use of such motors is to drive clocks, and this is being extensively done. Such clocks may ultimately supersede a large proportion of the spring- or weight-driven clocks that are in use today. Obviously the new clocks never need to be wound nor regulated and they will keep time in a manner which is most astonishing to one familiar with the poor time-keeping qualities of an ordinary clock. On a well regulated system these motor-driven clocks can be depended upon to remain correct within a few seconds, month after month. This requires that provision be made against interruptions of the service, which will be mentioned later.

To the electric light companies the great immediate advantage of the new kind of service is the ability to substitute these self-starting synchronous motors for the troublesome and unreliable clock movements that are used at present to drive graphic meters, maximum demand recorders, and similar devices. It is well known that these various kinds of meters require a large amount of attention to keep them wound, regulated and in repair. This is particularly true of the maximum demand recorders, which are distributed widely over large systems and thus require a great deal of traveling to keep them in operation. Substituting motors does not of course eliminate all attention, because it is still necessary to change the record sheets occasionally; but in many cases the recording time can be extended either by using larger paper rolls or a slower feed, and in all cases the attention for winding and frequent regulating is eliminated. It might be expected that these meters would, when motor-driven, give much more accurate records, and this has been found true. In the first place, the motors are far more powerful than the clocks and they are not affected by the varying friction of the paper feed. Therefore it has been found feasible to make better records by using more tension on the paper. Frequently such recording devices are used to analyze the cause of disturbances, and in such cases it is of great importance that the records should be synchronous. This is impossible where ordinary clock movements are used. Where there is a difference between two records, there is always more or less doubt whether this is due to an error of the clocks or not; but where synchronous motors are driving the recorders there need be no uncertainty in this respect. On account of the very small size of the motors shown in Fig. 2 it has been found easy to substitute them in all kinds of recorders where spring-driven clocks have been used. Fig. 5 shows a complete movement driven by one of these motors, suitable for a maximum demand recorder. There are certain instruments in which periodic contact making is required, and in these, this motor is equally well adapted to replace the clocks. Fig. 6 shows a printometer equipment whereby a circuit is closed for a fraction of a second every fifteen minutes. This interval is as exact as the regulation maintained by the master clock at the power station.

There is another class of instruments which have not come into as general use as they deserve, very largely because of the objectionable features of spring-wound clocks. These are time switches used to control sign lighting, street lighting, and for other purposes. Several forms of time-switch operated by motor have been designed which are in many ways more satisfactory than the ordinary types of time switches. Fig. 7 shows one of these switches with a capacity of 15 amperes designed for controlling street lights. The outside dimensions of this switch are as follows: height 4¾ inches width 2 7/8 inches depth 5 5/8 inches. It will be recognized that this is very much smaller than any kind of clock-driven time switch; it is in fact so compact that when encased it may be placed inside the iron pole of a street light. The advantages of such a switch are obviously the elimination of the winding and regulating, which have always involved a great deal of attention.

In the field of pure timing devices, the most important use of the motors is probably the driving of ordinary clocks. In some respects this is the easiest adaptation of the motor, because the power required is exceedingly slight and the necessary mechanism comparatively simple. As an illustration of the small power output needed, Fig. 8 shows two clocks, the one above with a 2½-in dial and the one below with a 36-in. dial, the latter a comparatively large clock for indoor use. Nevertheless the tiny motor which is also shown in the photograph is equally adapted for driving either of these clocks. Indeed, this small motor can drive still larger clocks. In order to indicate the almost inconceivably small power required to drive the hands of a clock through well-made gearing the following measurements were made:

Diameter clock dial	Hands	Horse power required with ample margin for safety
4 inches	unbalanced	0.0000000017
8 "	unbalanced	0.0000000022
12 "	unbalanced	0.0000000027
14 "	balanced	0.0000000014
Safe output of motor at synchronous speed		0.000001

These tests were made by measuring that torque which was more than ample to drive the train of gears and the hands in their most strongly resisting position. From these figures it is clear that any moderate size clock requires an amount of power which is negligible as compared with the output of one of these small motors.

In the driving of clocks it is necessary to make provision against interruptions in the supply of current, which would of course show as lost time on the dials, owing to the temporary stoppage of the motors. In a large modern power system, especially in the neighborhood of big cities, the interruptions are infrequent and generally of short duration, but they must be reckoned with. Two different devices are provided to safeguard these motor driven clocks against interruptions in the current. The simplest of these devices consists of an indicator which can have two positions. So long as the current flows continuously the indicator is not apparent on the face of the clock, but after an interruption lasting more than a few seconds this indicator appears on the dial and remains there even after the current is restored, thus showing and stating that the clock should be reset. For a brief interruption of the current the indicator, after appearing momentarily, returns to its hidden position. The device is magnetic and exceedingly simple and reliable.

The other means for guarding against interruptions consists in providing an auxiliary spring driven movement connected by differential gearing to the hands of the clock. This movement is at rest so long as the current flows through the motor, because its pendulum of magnetic material is prevented from vibrating by the field magnet of the motor. When an interruption occurs however, the pendulum is released so that the spring driven movement can drive the hands for the time being. Just as soon as the current is restored the pendulum is again stopped and the hands are driven through the motor. The spring movement is capable of driving the hands of the clock for 24 hours. It has not been deemed necessary to provide a very accurate auxiliary movement in these clocks, because it is not expected that the current will be interrupted for long intervals, and the error during a short interruption is very slight even if the auxiliary movement does not keep particularly accurate time. As shown by the illustration of the works for the auxiliary type in Fig. 9, this movement occupies very little room, and can be enclosed in any ordinary case. It is expected that as their use becomes widespread, these clocks will be available in all kinds of cases and at prices which compare favorably, considering their advantages, with common spring-driven clocks. After trying them people quickly realize their great advantage as compared with all other kinds. These advantages may be summed up briefly: Extreme simplicity; elimination of winding and of regulating; far greater precision in time-keeping; the ability to get time service from any lamp socket; low cost of maintenance; the avoidance of expensive and troublesome local master clocks and local independent wiring systems; the elimination of batteries, charging sets, relays, etc; and the avoidance of contact mechanism in the clocks themselves.

The service which these clocks give is so valuable in comparison with the cost of the current which they consume that their use can be extended far beyond the field that can be economically covered by existing systems for distributing time. Barring interruptions, the accuracy of time-keeping of the clocks is measured by the care given to the master clock at the power station. On the Boston Edison system where these clocks have been in use for about 2½ years, the service is so good that the clocks seldom vary more than 5 or 6 seconds from the true time. This is a degree of precision which is closer than can be read unless they are provided with second hands. Moreover, this remarkable quality of service really requires, as has been stated before, less attention from the station attendants than had previously been given where frequency meters were used for regulating purposes. The clocks are not absolutely noiseless but they are quieter than any ordinary clock which ticks. It is hoped that ultimately the very slight humming sound which they emit can be entirely suppressed, but this is to most people practically unobjectionable.

The clocks are not affected by temperature changes unless subjected to cold so intense as to thicken the lubricating oil. Different kinds of oil are used for different kinds of service. In some cases ordinary kerosene oil has been employed. This will enable the motor to run at very low temperature. The lubricating qualities of the oil do not seem to be of importance, because the friction of the moving parts is so exceedingly slight. Variations in voltage do not affect the operation of the motors in driving the clocks. A drop in potential of 50 per cent will not trouble the time-keeping qualities of the motor; but with auxiliary movements some difficulty has been experienced when the potential falls to a point where the pendulum of the auxiliary movement is released or permitted to vibrate while the motor also continues to run. In such a case the clock tends for the time being to gain. This could rarely happen in a first class distributing system. The objection does not apply to the indicating type of clock.

The field magnet winding for the motor is so well insulated layer by layer and from the ground that it will stand severe surges of potential due to lightning discharges. The windings are usually designed for 110 volts but potential above 2000 volts has been applied briefly in tests without causing damage. The fire risk of the clocks is evidently negligible and they are approved by the Underwriters' Bureau.

An important use for these clock motors appears to be the driving of time recorders, cost keeping devices, time stamps etc., for offices and factories. There are thousands of such appliances in manufacturing establishments which at the present time are operated by spring driven clocks or in some cases electrically by impulses from local master clocks. Spring driven clocks are objectionable because they require winding and regulating, and where a number are used in the same factory they cannot be depended upon to remain in synchronism. The electrically driven recorders installed hitherto are objectionable on account of the great expense of installation which requires wholly independent wiring and the trouble and cost of maintenance of batteries, charging devices, etc. Because of the ability to connect with the ordinary lighting wires of a factory, these time recording devices may, if driven by the new motors, be located almost anywhere without extra cost. The entire burden of winding and regulating and a large part of the cost of maintenance of the system are eliminated. It is in many cases feasible to attach the new motors to the old movements by a simple device such as a closely wound spiral spring serving as a coupling as shown in Fig. 10.

The indications are strong that as the district supplied by alternating current with accurately controlled frequency extends and this new service becomes ultimately available everywhere, there will develop many new and unforeseen uses for these small self-starting synchronous motors. Combining the functions of a wonderfully accurate clock with those of an exceptionally simple and reliable electric motor, a new instrumentality is available which can be adapted to cooperate with many kinds of existing devices and lead to the development of others for entirely new purposes.

Presented at the 35th Annual Convention of
The American Institute of Electrical Engineers,
Lake Placid Club, N. Y., June 24, 1919.

COOPERATION

PRESIDENT'S ADDRESS

BY C. A. ADAMS

INTRODUCTION

SINCE the last Annual Convention of this Institute the greatest war in the world's history has come to an end. Millions of lives have been sacrificed. Millions of men have been crippled in greater or less degree. Wealth to the extent of many billions of dollars has been destroyed. Most of the nations involved have been seriously impoverished. The whole social, economic and industrial order has received a severe shaking up and has in some countries been completely overturned. Widespread dissatisfaction with the old order prevails in nearly all of the countries involved in the war. Unrest is generally prevalent, and, however unreasonable the demands of the restless may seem, there probably *is* a reason. In any case "it is a condition not a theory that confronts us."

The old order has failed and can never wholly return. What will be the new order? What are we going to do about it? Are we as engineers, educated men, who pride ourselves on our breadth of vision, who complain because we are not properly recognized in large questions affecting the public welfare, going to crawl back into our little cabins and leave the steering of the ship to those we consider incompetent? Are we going to use our influence to keep the ship on the old course, in spite of the fact that it led to the rocks? Or are we going to ask the same question we ask when our engineering plans result in failure, namely, why? Are we going to scrutinize the ships' log to see why she ran on the rocks and then revise the sailing orders accordingly?

These are some of the questions, the consideration of which has urged me to deviate somewhat from the custom established by my predecessors, of discussing either Institute activities and policies or general engineering questions, in order to present a much broader question, of interest to us not only as engineers but also as citizens, a question which has repeatedly arisen in connection with my work for the Institute during the past two years.

Moreover this year the important question of Institute activities and policies is receiving a thorough discussion through our Development Committee before which we will all have an opportunity to express our views at this Convention.

The subject of *cooperation* to which I wish to call your attention has a decided bearing on several important Institute functions, and although it will be discussed largely in connection with these functions, it is the principle involved and its broad bearings on other questions as well, that I wish to emphasize.

My reason for selecting this subject is partly because of my recent experience in applying the principle, but more because the broad application of the principle seems to offer at least a partial answer to the questions propounded above, and also because this application is of peculiar interest to the engineer.

I am not proposing to cast aside the traditions of our fathers, nor the experience of the past, but rather to make use of that experience.

I do not propose any revolutionary program, but merely the hastening of the natural evolutionary development by the application of a little perspective and common sense. This does not involve altruism except insofar as a vision reaching somewhat beyond the end of ones nose may be considered altruistic. It is merely a step forward, a step of progress, which may prevent our being forced into some revolutionary stride which is doomed to failure and can mean only destruction and disaster as long as it endures.

COOPERATION IN GENERAL

The history of human society is the history of cooperation, at first between the individuals of a small group such as a family, then the cooperation of the small groups with each other to form a larger group such as a tribe, then of these larger groups to form a nation, then of several small nations into one larger nation, and now we are seriously considering a League of Nations.

The reason for these various increases in the size of the unit, is not an altruistic ideal, but the knowledge that each party to the cooperation will gain thereby, not only in strength, but also industrially and economically through a more extensive division of labor and an increase in the average productivity per unit of labor.

Every commercial transaction of the greatest or of the least magnitude, between individuals or groups of whatever size, is made on the assumption that it is profitable to both parties concerned, otherwise it would not ordinarily be made. Every such transaction involves a certain degree of cooperation, a certain degree of faith or confidence in each by the other. The more the mutual confidence and the cooperative spirit, the freer and the more efficient will be the interchange and the greater the mutual profit. This is merely the simplest possible illustration of cooperation to show its fundamental nature. Industrial cooperation from which my illustrations are drawn and with which engineers are mostly concerned, usually involves more than this simple and somewhat restricted cooperation between buyer and seller, it involves cooperation all along the line, internal and external, between producers, between consumers, and between producers *and* consumers.

The obstacles to cooperation, particularly between the larger units, are of two varieties, material, and human or intellectual. The chief material obstacles are space coupled with imperfect means of communication and transportation, differences in language, differences in coinage, weights and measures, and duties or tariffs. Some of these bear obviously only on international cooperation. The chief intellectual obstacles are tradition, custom, prejudice, suspicion, distrust, jealousy, narrow minded and short sighted selfishness or greed, or in general ignorance and lack of understanding one of the other. Most of this latter group bear upon minor or internal as well as upon international cooperation.

Most of the obstacles of both types may be looked upon as barriers which keep individuals or groups apart and thus prevent that mutual understanding which makes cooperation possible. It is my desire not only to point out the importance and value of cooperation, but also the part that we as engineers can play in helping to break down these barriers. My meaning can most easily be made clear by illustrations.

Standardization. Consider first the work of engineering and industrial standardization. The work of the Standard Committee of this Institute is second to none in its own particular field or in any other, either in this country or in any other country, in spite of the fact that the ground covered is a peculiarly difficult one. Why this success? Simply because of the *cooperative* spirit which has prevailed. Conflicting interests come together, pool their experience, share the work, place their cards on the table face up, and finally agree to abide by the resulting rules.

Does any one of these interests regret the results? Does any one feel that it has been a waste of time and money, that it has been chasing an illusory ideal, that it was visionary or impractical, that it was not a financially profitable investment in the long run?

But it is not only profitable to the producer, it is even a greater gain to the consumer by a reduction of cost, the elimination of at least some of the confusing and conflicting claims of competing salesmen, by rendering more definite the purchasers knowledge of the capacity of a machine of given rating and thus making possible a more suitable and accurate selection of a machine for a particular purpose. Thus it is an asset of no mean value to the industry and to the nation as well.

But this cooperative spirit does not always prevail, in fact rarely in the degree found in our Standards Committee. Until very recently the French National Committee of the International Electrotechnical Commission contained none of the designing or manufacturing engineers, it being assumed that the latter would be biased and partisan in their attitude. As a result the manufacturers formed a syndicate with a committee and standardization rules of its own, which were and even now are to some extent the rules generally employed in France although much inferior to our own. In some cases the American rules are specified. Even now the French rules originate in the Manufacturers Syndicate and are passed on to the French National Committee for their approval, with a consequent loss of time and efficiency.

As the work of our own Standards Committee expanded we found it overlapping the work of other Societies in adjacent fields, and cooperative relations were established. This failed from the lack of any suitable and generally acceptable cooperative machinery. A movement was then started by the Institute more than two years ago to organize a central body (the American Engineering Standards Committee) to coordinate all the work of engineering standardization being done by the various engineering organizations. The objects of this organization are to avoid overlapping, duplication of effort and the confusion of conflicting standards; to ensure the employment of an effective and sound procedure in the development of standards, to the end that a standard when promulgated may be not only workable but also acceptable to all concerned, and the only accepted standard of its kind; to stimulate desirable standardization and at the same time to shut off premature or ill-advised attempts at standardization.

The consummation of these objects will mean: an enormous saving of effort in the production of standards, a tremendous increase in the value and usefulness of standards, and, when the work is brought up to the needs of the time, an incalculable saving in the cost of production and the productivity of labor. International standardization will be a tremendous stimulus to foreign commerce; but such international standardization is in most cases exceedingly difficult without a central authoritative body with which similar foreign bodies may cooperate. Several instances of this difficulty which have actually arisen during the past two years could be cited, if time permitted. These have resulted in hopeless confusion, misunderstandings, and waste of time and money.

Great Britain has its Engineering Standards Association which, during its eighteen years of life, has demonstrated its reason for existence and its value to industry and to the Government. Its work in air craft standards alone, which it is doing on behalf of the Government, involves some fifty odd subcommittees.

France now has its Permanent Standards Commission a little more than a year old, and just getting under way.

Holland also has its Normalization Bureau for the same purpose, and Switzerland is just starting a movement in this direction.

With all these National Bodies our American Standards Committee has already established cordial cooperative relations.

The American Engineering Standards Committee, although organized last October is now in process of reorganization. But the success of any such movement is in direct proportion to the degree in which the cooperative spirit prevails, and the two years consumed before the first organization, and the present delay in the reorganization is due largely to some of the obstacles to cooperation mentioned in the second group above.

The conflict is chiefly and quite naturally between the desire to accomplish a highly desirable result and the desire on the part of each cooperating element to avoid paying the price, which may consist in a very slight loss in independence or prestige, although in the present case this will be negligible. It is the fear rather than the fact that causes the trouble. But even if the sacrifice were much more than it is, the price would be small in proportion to the result.

It is the generous cooperative spirit for which I am pleading, rather than the timid fearful spirit of him whose vision is of such short range that he dare not take a man's size step for fear of stubbing his toe, who holds the little present so close to his eye that it shuts out the whole landscape of the future, whose immediate self interest or small group interest is so dominant as to hide anything beyond.

Look around at the strikingly successful men of this country; rarely will you find one of the timid fearful type. He has vision even if sometimes of a very material nature; he sees a worthwhile goal some way ahead and starts for it; if he makes mistakes he tries again with added experience. The man who never makes a mistake never gets far. I am not advocating slap-dash methods, but rather more of the positive and less of the negative attitude, more constructive and less destructive criticism, more of the forward looking progressive spirit, and less of the visionless, timid, reactionary spirit.

To return for a moment to the American Engineering Standards Committee and its reorganization, may we not look to every member of this Institute to take part in a campaign of education and in the development of that wholehearted spirit of cooperation which will mean the support rather than the hindrance of many well meaning but timid objectors to this movement, which seems to promise so much in the development of our industries, in the increase in the productivity of labor, and in the breaking down of at least one of the barriers of international misunderstanding.

Research. Another fertile but as yet little cultivated field for cooperation is that of *industrial research.* Here we have a field where the obstacles of the second group are specially dominant, particularly short sighted self-interest and corporate interest, and the timid desire for secrecy.

That many of the most important and revolutionary developments of modern industry are dependent upon research needs no proof. It is not so obvious to many however that cooperation in industrial research is to any large extent feasible at the present time.

First let me point out that the line often drawn between scientific and industrial research is a very hazy one, namely between research the purpose of which is merely to extend the bounds of human knowledge and that intended primarily to meet an industrial need or to solve an industrial problem. In other words the difference is one of purpose rather than of method. But I think that, particularly since the war, most of our so called pure scientists have become greatly interested in the industrial value of their scientific work. Therefore when I speak of industrial research I include a very large part of the research work now being conducted in this country.

Consider for a moment the advantage of cooperative research Few individual corporations except the very large ones can afford either a suitable research laboratory or competent men to conduct the work. The result is that most of the research work is superficial and much of it misleading. The total cost is many times greater than of a comprehensive cooperative research led by the foremost experts in the particular field. Moreover some of the most vital and fundamental researches are either beyond the capacity of even the largest corporation laboratories or have to be postponed for those offering more chance of immediate returns.

I am fully aware of the difficulties arising from patentable developments, but these can readily be handled by suitable agreements between the parties to the cooperation.

After some experience in this field I am convinced that thorough going cooperation in industrial research would mean not only a reduction of the cost of research to a small fraction of its present figure, a much more rapid industrial development of the country as a whole and a material increase in the productivity of labor, but also an increase in the profits of every intelligent party to the cooperation.

Again I wish to point out that it is not so much a question of altruism as of a far sighted self-interest and patriotism.

Many other illustrations of cooperation in the engineering field could be cited, but those already briefly discussed will be sufficient for the present purpose.

Reverting now to the opening paragraphs of this address, let us consider some of the broader aspects of cooperation, particularly as they bear upon the great world problems which are insistent in their demand for solution.

Serious cooperation of any kind between widely separated groups is practically impossible without the mechanism of transportation and communication provided by the work of the engineer. Also with this mechanism developed even to its present degree of perfection, cooperation becomes not only possible but also desirable and necessary, even between comparatively widely separated groups. Moreover the problems which confront the world at this time are largely a direct result of this mechanism. Are we as engineers going to leave their solution to others? Are we going to be satisfied to play alone in our own little backyard and never look around us even after most of the fences have been removed by our own work. Are we going to fail to take an interest in our neighbor who has much that we need, from whom we have much to learn, and by cooperation with whom we can profit broadly? If we do offer cooperation is it to be with the provision that we gain all and sacrifice nothing? Are we going to be satisfied to sit back and criticize destructively the details of a cooperative agreement, priding ourselves on our critical faculty, however valuable that may be at times, when the real basis of the criticism is either intellectual pride or short sighted fear that we, as individuals or as a group, may lose a little of our independence or prestige; when the issue at stake is in some cases of such paramount importance (had we only the vision to see it) as to overshadow our very existence.

Let us rather make use of the logical habits of thought and freedom from bias which *should* at least be the result of our training, and take our part in the solution of the problems we have helped to create. Let us see the goal ahead and start for it without thought of fear or favor, as surefootedly as possible, but with that generous cooperative spirit which will surmount many an apparently impassible barrier, critical of details if so minded, but generously and constructively so.

Now just a word as to a specific and very serious problem, namely; that arising from the world-wide unrest in the labor group. That this condition exists must be admitted by the most optimistic. To ignore it is suicidal. It is difficult to characterise this condition in a few words as there are so many degrees of radicalism and of revolutionary spirit represented; but it may be safely stated then on the average, labor is unreasonable in its demands and seems to fail to realize that it cannot receive more wealth than it creates, or at best it has an exhorbitant idea as to the value of what it creates. For example in England labor is not only demanding higher wages but according to all reports is reducing the hourly output as well. There seems to be a prevailing notion in labor circles that the excess profits of the capitalist constitute a limitless supply which only needs the tapping; or to put it in another way, the labor group is trying to use its organized power as a monopoly, and to insist on "all the traffic will bear". The revolutionary element goes even farther.

It is very easy to criticise and condemn this attitude because of its unreasonableness; looking down from our superior height of intelligence. It is very easy to assume that it is merely a matter of showing the laborer the error of his ways and of forcing him into line. But would it not be wiser at this time to examine our own attitude with equal care, to make sure that we are setting an example which if followed by labor would solve the problem? Is demanding all the traffic will bear under the pressure of organized labor any worse than the same policy carried out by the capitalist whenever he has the monopoly grip. Unfortunately the two cases are not treated the same by our laws; labor monopoly is not subject to the same limitations and responsibilities as is capital monopoly; that disparity should be removed. But there are two forms of monopoly other than of labor which are not only not limited by but are actually fostered and protected by our laws, and in which the only limitation ever dreamed of by the owner is, "all the traffic will bear"—These are patents and titles to land.

Our patent system has a worthy motive, but who will claim that the commercial value of a patent bears any relation to the brains, industry and expenditure that entered into its development, or that in general the patent system works for justice to all those who share in the development of a patent. It is often a matter of pure accident as to which of two men who have conceived the same idea at about the same time reaps the reward, and it often goes to the less worthy. Often the *originator* of *an idea* makes a much less important contribution than the man who makes it a commercial product. But to point out the defects and the injustices of our patent system would take all day. This is another realm in which broad-gage cooperation would eliminate much of the unfair and gambling aspects of the patent system, and distribute the rewards in a much more equitable fashion.

Of all sources of monopoly our present land system is the most unfair, the most dangerous, the most far reaching in its influence. Under it a land owner is paid a premium for holding valuable land idle, for forcing a community to spend its money to build around his land and to take its time to travel around or past his land. The sight (or unimproved) value of land in this country constitutes about one-half of our national wealth, yet the landowning class as a group taken from the beginning of the settlement of the country, have been made a free gift of that wealth, which is the product of the industry of society as a whole. This is a statement which cannot be controverted.

Private ownership of land seems to me desirable, it has a wholesome stabilizing influence, and the injustice of our present system can be largely removed without sacrificing any of the real advantages of private ownership, by shifting taxation from individually created wealth to community created wealth which is the sight value of land *i. e.* by taking for the community what the community creates and leaving to the individual what he creates, by shifting taxation from industry to privilege. But this is a long story by itself and I touch upon it at this time only as one of a group of suggestions as to possible ways in which we can assist in progressive rational *evolution* of society rather than encouraging and inducing a *revolution* by a reactionary policy. I assure you that none of these suggestions can be called other than conservative at this time.

Are we sincere in our desire to be more recognized in our broader capacity as citizens and to be consulted in matters of broad public interest; if we are not sufficiently interested in those matters to make an intelligent study of them; if we are satisfied to accept without analysis the prejudices and propaganda of a biased group, however intellectually superior that group may consider itself; if we cease to utilize our logical habits of thought, our habit of analysis, of demanding the reason why, as soon as we step outside of the realm of those physical laws which force us to think straight? Are there no laws in this other realm of human relations which are just as inexorable as the physical laws with which we are so familiar? Is there no law of compensation which is the counterpart of our law of conservation of energy? Are we going to be allowed to go unpunished for crooked or careless thinking in this other realm any more than in the purely physical realm?

What can we expect of labor in the way of intelligent fairmindedness if we, with our superior advantages encourage the perpetuation of institutions which work for serious injustice to the many for the benefit of the few, or even if we allow them to persist because of our lack of intelligent fairmindedness.

My opinion as to the seriousness of the present world situation, as to the importance of the problem before us, is not mine alone but that of many thinking men in this country and in others. I have talked with many influential men in France and England and have found a striking unanimity on this point.

May we not as engineers utilize our training and experience, our method of thought of which we are so proud, to encourage clean, straight, unbiased and fairminded thinking, and to encourage that broad generous cooperative spirit which will mean so much in breaking down the barriers which separate classes within a nation, as well as the nations themselves, and which is essential to the solution of the world problem which is knocking so hard at our door?

Presented at the 35th Annual Convention of the American Institute of Electrical Engineers, Lake Placid Club, N. Y., June 24, 1919.

Copyright 1919. By A. I. E. E.

TRANSMISSION LINE RELAY PROTECTION

BY H. R. WOODROW, D. W. ROPER, O. C. TRAVER AND P. MACGAHAN

Foreword

As a result of the large development which has been made in the relay practises in the last few years, the Protective Devices Committee felt it was desirable to make an analysis of the transmission line protective relay situation throughout the country and prepare a summary to bring before the profession, experiences of the operating companies with this protective apparatus. With this in view questionnaires were sent out to sixty-one operating companies asking for their experience and present practise in regard to relay protection.

Replies were received from 32 of the operating companies and from these the following general analysis was made of the practise of relay protection for transmission lines.

The nomenclature used by the different operating and manufacturing companies for the various classes of relays is not in harmony and the Protective Devices Committee undertook the work of standardizing the nomenclature which it is hoped will be adopted by all parties so that any particular class of relays will be given a definite name. The following nomenclature has therefore been proposed by the Protective Devices Committee and this nomenclature is used throughout this paper.

Proposed Relay Nomenclature

Electric Protective Relay. An intermediate device by means of which one circuit is indirectly controlled by a change in conditions in the same or other circuits. The relay is ordinarily equipped with contacts to open or close an auxiliary circuit.

Directional Relay. Any relay which functions in conformance with direction of power or voltage or current or phase rotation, etc.

Power-Directional Relay. Any relay which functions in conformance with direction of power.

Note: This includes both uni-directional relays with single-throw contacts and duo-directional relays with double-throw contacts. The reason this name is preferred to "reverse power" is that the device is frequently used to function under normal direction of power. Furthermore, in some cases the normal condition of the system may permit power to flow in either direction. Relays for use in either alternating or direct-current circuits are to be classed as power-directional relays.

Polarity-Directional Relay. Any relay which functions by reason of a reversal of the normal direction of polarity.

Phase-Rotation Relay. Any relay which functions by reason of a reversal of the normal direction of phase rotation.

Current Relay. Any relay which functions at a predetermined value of the current. These may be either over-current relays or under-current relays.

Voltage Relay. Any relay which functions at a predetermined value of the voltage. These may be either over-voltage relays or under-voltage relays.

Watt Relay. Any relay which functions at a predetermined value of the watts. These may be either over-watt relays or under-watt relays.

Frequency Relay. Any relay which functions at a predetermined value of the frequency. These may be either over-frequency relays or under-frequency relays.

Temperature Relay. Any relay which functions at a predetermined temperature in the apparatus protected.

Open-Phase Relay. Any relay which functions by reason of the opening of one phase of a polyphase circuit.

Differential Relay. Any relay which functions by reason of the difference between two quantities such as current or voltage, etc.

NOTE: This term includes relays heretofore known as "ratio balance relays", "biased" and "percentage differential relays".

Locking Relay. Any relay which renders some other relay or other device inoperative under predetermined values of current or voltage, etc.

Trip-Free Relay. Any relay which prevents holding in an electrically operated device such as a circuit breaker while an abnormal condition exists on the circuit.

Auxiliary Relay. Any relay which assists another relay in the performance of its function and which operates in response to the opening or closing of its operating circuit.

Signal Relay. An auxiliary relay which operates an audible or a visible signal.

QUALIFYING TERMS AS APPLIED TO RELAYS

Notching. A qualifying term applied to any relay indicating that a number of separate impulses are required to complete operation.

Inverse Time. A qualifying term applied to any relay indicating that there is purposely introduced a delayed action, which delay decreases as the operating force increases.

Definite Time. A qualifying term applied to any relay indicating that there is purposely introduced a delayed action, which delay remains substantially constant regardless of the magnitude of the operating force. (For forces slightly above the minimum operating value the delay may be inverse).

Instantaneous. A qualifying term applied to any relay indicating that no delayed action is purposely introduced.

Where relays operate in response to changes in more than one condition, all functions should be mentioned.

PRESENT DAY PRACTISE OF TRANSMISSION AND TIE LINE RELAY PROTECTION

THERE was a time when the protection of an electrical transmission system with the single generating station and its radial feeders was not a matter of very grave importance. That time, however, has passed and the multiplicity of generating stations with tie lines and parallel feeders have greatly complicated the once simple problem. Furthermore the practise of inter-connecting large transmission systems has brought about conditions which require very serious consideration when laying out a protective relay scheme. An effective sectionalizing relay scheme is necessary in order to secure both continuity of service and maximum efficiency of transmission, and the most complicated system can be effectively relayed if all the conditions are thoroughly analyzed and proper relays, carefully set for both time and current values, are applied.

Most of the large systems of the present day have grown out of a comparatively small beginning, stations and lines being added as required, without taking into consideration the relay problem. As a consequence this problem has, in a great many cases, become rather difficult and it has become evident that the application of protective relays for sectionalizing trouble is one of the principal features to be considered in determining the design of a transmission system. This point should be borne in mind whether additions are being made to an existing system or whether an entirely new system is being designed. Numerous instances could be given, if space permitted, whereby a slightly different design in extending lines and adding stations would have given a much more simple and effective protective scheme.

The same argument applies to the design of an entirely new system, particularly if it is likely to become very complex, and in so far as practical, possible extension should be considered in the original layout so that various methods of protection can be studied and their advantages weighed against possible disadvantages of a design to which they can be applied. It will be found in practically every case, that one of the many sectionalizing schemes now in use can be combined with a proper layout of stations, transmission and tie lines to give proper selective action. Thus the many difficulties encountered when attempting to apply relays to a system laid out without regard for its protection may be avoided. It is therefore of paramount importance that the selective features of various types of relays be thoroughly analyzed before adapting them to any particular system.

The most perfect relay system cannot be expected to give the best results unless the man in charge of its installation and operation be thoroughly competent and familiar with the various precautions to be taken. It is absolutely essential that he know the characteristics and peculiarities of the relays as well as the scheme he is using. Results of experience have shown that an experienced man will secure more satisfactory results from an inferior type of relay than a less capable man can secure from the best equipment obtainable.

In the early days of the development of electrical transmission systems it was the rule to operate all lines radially so that a much simpler and less accurate relay was sufficient for the service required. But when an attempt was made to operate these lines in parallel it became apparent that a more reliable and accurate relay should be used. It was then that the phenomenal relay development which has taken place in the last few years was begun in order to meet the requirements of the operating companies with their various designs of systems. In order to secure the isolation of the proper feeder in case of trouble on these different systems, it has been necessary to bring out designs of several relays.

In the early days it was the practise in applying protective relays to set them in such a way as to prevent overloading the feeders. However, experience has shown that it is not practical to set relays in this way and expect them to give proper selectivity under heavy short-circuit conditions. For this reason the tendency now is to set the relays for short-circuit conditions to disconnect a defective feeder from the system, and depend on the operator to prevent overloading of feeders. Experience has shown that the overloading of a cable for a reasonable length of time is not as serious a matter as it was once considered to be, and it is sometimes desirable to load a cable beyond its rated capacity in order to prevent interruption of service to important customers.

One of the principal factors controlling the settings of relays is the amount of short-circuit current available under various conditions as outlined later in this paper.

The proper type of relays to be selected for protecting transmission lines is dependent upon whether the system is operated radially, in parallel by groups, or in loop, and the time allowable for holding a short circuit on the system. In general, however, the most simple form of relay protection where several parallel feeders are involved is the straight inverse-time current relays which are set to give times of operation which are progressively longer as the source of power is approached. In some cases, however, where an extended system is considered, the time requirements for the relays approaching the generating station are so long that it is considered dangerous or inadvisable to hold short-circuit currents on for this period. We believe a delay of two seconds to be the extreme time a dead short circuit should be allowed to remain on a system, except under very unusual conditions.

The introduction of directional relays at points where power is feeding into a station will materially reduce this maximum time setting as these relays can be given a short time setting as outlined later under the heading "Typical Applications of Current and Directional Relays."

Another method of shortening this time is by the use of the balanced system in which case current can only flow through the relay at time of short circuit as outlined later under the heading "Balanced Protection Schemes."

Another method has been developed for shortening this time by the combination of an under-voltage relay with the inverse-time-current relay as outlined later under "Combination of Under-voltage and Over-current Relays for Protecting Transmission Lines."

For relay protection on tie lines, between generating stations in general, the conditions are somewhat similar to the straight transmission line relay protection if the additional precautions are taken to prevent operation on heavy momentary surges of power that are likely to exist between two or more generating stations at time of synchronizing or momentary disturbances on the system. The relay desired here is dependent to a large extent upon the method of system operation, as in some cases an operating company will want the tie lines to open, in case of trouble on a portion of the system, whereas other companies want these tie lines held rigidly in service in all cases excepting in case of trouble on tie lines themselves. In the first case shorter time settings are given to the relays and the application of straight inverse time current relays with the possible application of directional relays, is all that is required. In the second case, however, application of some form of a balanced relay, as described later may become necessary. If the tie lines are not to stay in indefinitely, inverse-time current and directional relays with long time settings would give a very suitable operation and allow the tie lines to open in case of more serious trouble on either side of the system.

SAVINGS EFFECTED BY A PARALLEL OPERATION OF FEEDERS

The savings which can be effected by a parallel operation of feeders depends in a large degree upon the design of the transmission system and the ratio of the capacity of the line to the capacity of the individual synchronous converters or transformers, which are supplied by this feeder. In one system supplying converters varying from 500 kw. to 4000 kw. in size together with step-down transformers in substations of 1500 and 3000 kw. capacity and also industrial substations on the premises of customers ranging from about 500 kw. to 4000 kw. in capacity, it was estimated that if the feeders could be operated in parallel then a saving of 20 per cent could be made in the amount of investment. As the installation in question had a book value of about $5,000,000, there was a possible saving estimated at $1,000,000. This company has been operating feeders in multiple for about two years. During this period they have realized nearly 40 per cent of the possible saving in the investment in feeders, and this has been secured by an actual reduction in the number of feeders, notwithstanding a considerable increase in the maximum load.

The same company also reports in the four years preceding the installation of the relays permitting parallel operation of feeders that they averaged 27 burn-outs per year which could not be definitely ascribed to external causes, while in the two years since the feeders have been operated in parallel, the corresponding figure was 16 burn-outs. This would indicate a reduction of 40 per cent in the burn-outs of cables due to operation of the feeders in multiple, although the cables were actually more heavily loaded. While the time is rather too short to accept these figures as final and conclusive, it apparently indicates that the operation of feeders in parallel reduces the number of troubles due to internal causes.

TESTING AND SETTING RELAYS

In order to secure the best results from the relays, it is desirable to make a complete check on the current and time settings of each relay on the first installation and at least once every six months thereafter. Although the time for making these tests is determined to a large extent by the operating conditions, we feel that the best results will be obtained if a general plan is worked out for performing these tests at regular intervals.

If selective operation is to be obtained from the relays the greatest care and precautions must be taken in making these tests. The tests should be performed under conditions as nearly equal to the operating conditions as possible which makes it desirable to include the wiring and complete relay equipment in this test.

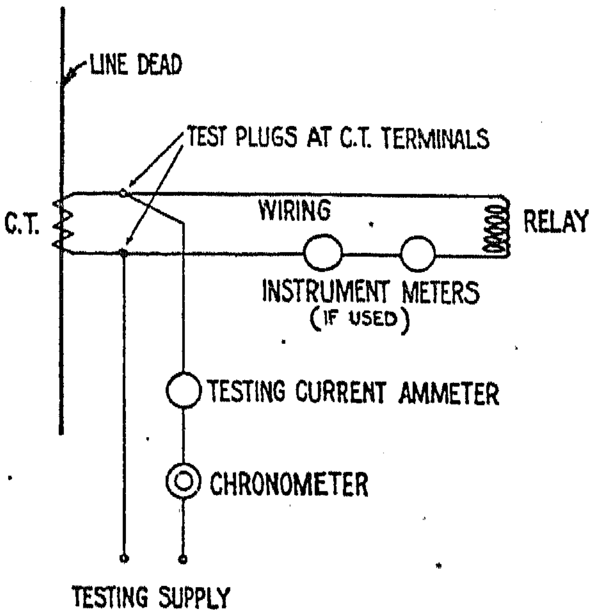


FIG. 1

In this test, wherever practicable, the test current should be applied to the primary of the current transformer. Where it is impracticable to do this the testing current may be supplied to the secondary terminals of the current transformer with the current transformer left in circuit as a shunt as shown in Fig. 1. The results obtained under these conditions very closely approximate those under short circuit and check the relay wiring and current transformer, as well as the relay. This will detect any short circuit in the current transformer or wiring and any open circuit in the wiring but open circuits in the transformer coils should be determined by a separate continuity test.

The degree of accuracy obtained when the relay is tested alone is dependent upon the type of transformer used and the secondary load. The error in this method is due to the high magnetizing current of transformers (especially with the low-ratio single-turn type on heavy secondary loading) on the heavy short-circuit conditions. As an example, a 40-volt ampere secondary load on a current transformer at full load imposes a 16-kv-a. load under short-circuit conditions of twenty-times full load. This, in many cases, is above the saturating point of the transformer and the ratio breaks down badly. The accompanying curves in Fig. 2 illustrate these conditions.

The time-current curve of a relay is affected by the nature of the secondary load or by the current transformer characteristics. It is therefore desirable to test each individual relay with current such as would be occasioned by short circuit. At least three points along the curve should be taken, one of which is approximately 150 per cent of the minimum value, one at as near maximum short-circuit current as the testing equipment will permit, and one intermediate point. The time should be taken by a chronometer or cycle counter which will eliminate the personal equation of the operator. The method of timing by a stop watch is entirely inadequate for close relay settings.

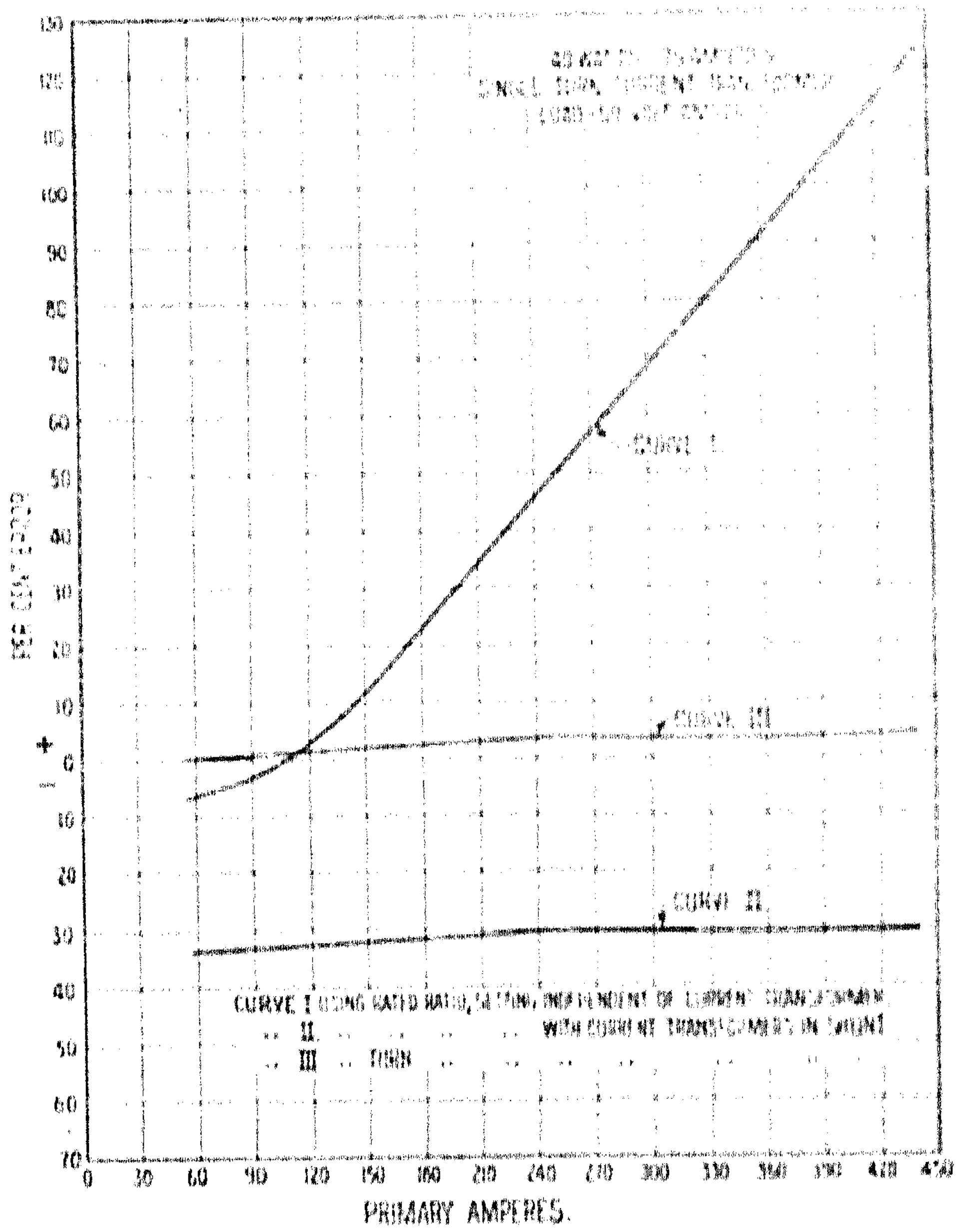


Fig. 2

An additional inspection should be made about every month for continuity of trip circuits (where this is not indicated by lamps) and for dirt or mechanical defects, and where conditions will permit, the relay should be operated to trip the circuit breaker. In case of bellows relays the bellows should be oiled with neatsfoot oil to prevent hardening.

Determination of Relay Settings

For selecting the settings on a relay, it is necessary to analyze carefully the fault currents which will be obtained on the circuits in question under various operating conditions. The limitations on these conditions are usually determined by the number of circuits in operation and the maximum and minimum number of generators connected to the system.

It is desirable to have the relays so set that selective action will be obtained under all probable conditions. It is therefore necessary to check the relay settings against the minimum conditions to be sure that they will operate under minimum connected generating capacity and to also check, the curve for maximum short-circuit conditions and allow a sufficient time between the operations of the relays, to permit the opening of the oil circuit breaker and to cover inaccuracies in the relays themselves.

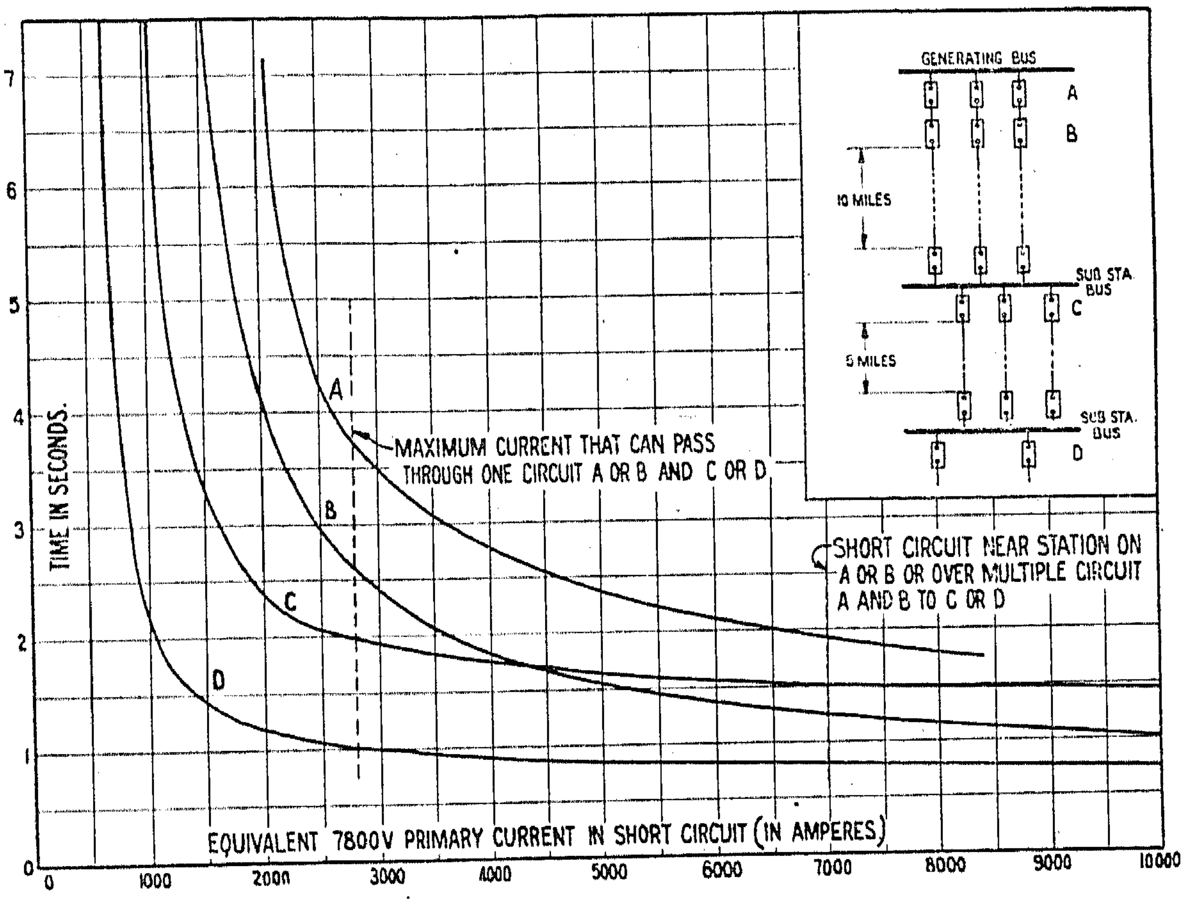


Fig. 3—Time-Current Curves of Inverse-Time-Current Relays

Where a large number of circuit breakers are in series and selectivity is desired between the several breakers, it is possible to apply, at the various points, relays having different characteristics in order to reduce the time that a short circuit is held on the system near the generating station. The time characteristic curves in Fig. 3 will illustrate a specific case where this method was adopted.

You will note from the curves that the maximum short-circuit current that can pass through relays *A*, *B*, *C* and *D*, is 2800 amperes and that the relays at this point are set selective with approximately ¾ of a second between each one successively. Greater currents may pass through circuits *A* and *B* with the short circuit on these feeders near the generating station, and the time characteristic curve being much steeper at this point will allow the relays to open the circuit very quickly. Although the currents through relays *C* and *D* may be much higher than 2800 amperes, these conditions represent times when more than one circuit (*A* and *B*) are operated in parallel and a proper selective action is therefore given with $^3/_4$ of a second between consecutive relays under any condition of operation.

The actual determination of short-circuit conditions is a rather difficult problem where a network is considered. Various formulas and calculating tables have been developed for simplifying this work which will give a sufficient accuracy for practical purposes.*

The relays in many systems have not been set according to this method but the settings have been determined to a large extent by the results of operation. For instance, if a relay appears to operate prematurely, the settings will be adjusted for a longer time and this will be worked back through the system. This method is very undesirable because either the final settings will be unwarrantedly high or unnecessary interruptions will occur. Moreover, in the meantime results will probably be sufficiently discouraging to cause a lack of interest in the matter. This does not mean that advantage is not to be taken of experience but rather that the basis should be more substantial. Experience should be harmonized with calculation before changes are adopted.

Some companies have made the time of their relay operation longer in order to permit the system voltage to drop in value, thereby reducing the rupturing strains on the oil switches. We point out, however, that considerable precaution should be taken when this is done as the prolonged current may do more damage than the greater rupturing arc in the switch.

METHOD OF TRIPPING

The use of circuit-opening relays for a-c. tripping has not given very satisfactory results, particularly where severe short circuits are possible. It is very desirable to have a definite uninterrupted supply of power for tripping. This has brought into use in many stations the storage battery for this purpose, which has proved to be reliable even in very severe system disturbances.

---

*G. E. Review, October 1916 and February 1919; *Elec. Journal*, July 1916.

Some companies have connected their control and tripping circuits to the excitation bus thereby obtaining the advantages of direct current, but sometimes this has led to considerable trouble due to the unstable conditions of this source, especially under a-c. short circuits. It has resulted occasionally in a complete disabling of the control system on account of abnormally high induced voltages on the excitation bus with single-phase short circuits on the a-c. generators. Great precaution should be taken against these conditions and for best results we feel that an auxiliary supply should be used if possible for obtaining at all times a constant power for the tripping circuits.

TYPICAL APPLICATION OF CURRENT AND DIRECTIONAL RELAYS

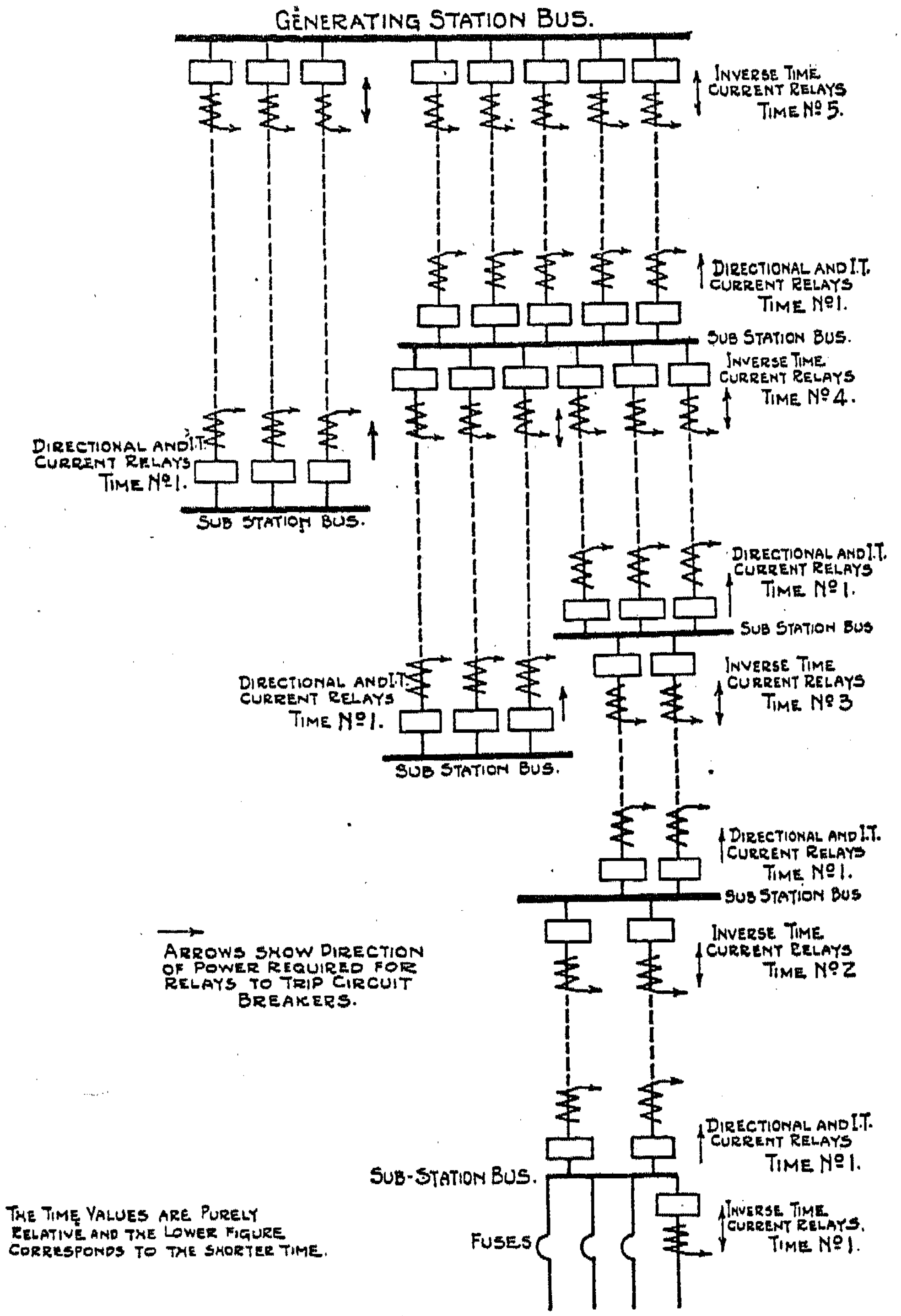
The combination of inverse-time-current relays with directional relays is becoming very extensively used in cases of parallel operation of lines or of feeders forming a ring system.

Some of the trouble which was experienced in the early days was mainly due to the difficulty in obtaining a directional relay which would operate correctly under all conditions of faults, especially where low voltage was encountered. The directional relays produced at the present time are apparently giving very satisfactory results, even under low voltage conditions down to 1 per cent of normal, and may be applied to loop or parallel group systems in series, in combination with inverse-time-current relays, if a careful analysis is made of the proposed installation.

For the protection of parallel groups in series, the relays are applied as shown in Fig. 4, with inverse-time-current relays placed at the transmitting end and directional relays (combined with an inverse-time-current element) at the receiving end. The directional relays are connected to close their contacts for power flowing from the bus to the line, and these contacts are placed in series with those of the inverse-time-current relays of the same circuit so that the circuit breaker is tripped only in case of power flowing in the direction away from the bus and of a magnitude exceeding a certain given amount for a certain time. Any time delay required is provided by the adjustments of the inverse-time-current relay element.

The settings should be made to suit each individual application and sufficient selective time leeway should be allowed to cover the time of operation of the oil circuit breaker, as well as the maximum probable error in the relay.

For protecting the loop system both the inverse-time-current and directional relays (with inverse-time-current element) are used as shown in Fig. 5. Each line, at each station, is provided with inverse-time-current relays. Where these inverse-time-current relays are given the lower time settings, directional relays are also used with the tripping contacts connected in series, so that both relays must operate before the circuit breaker is tripped.

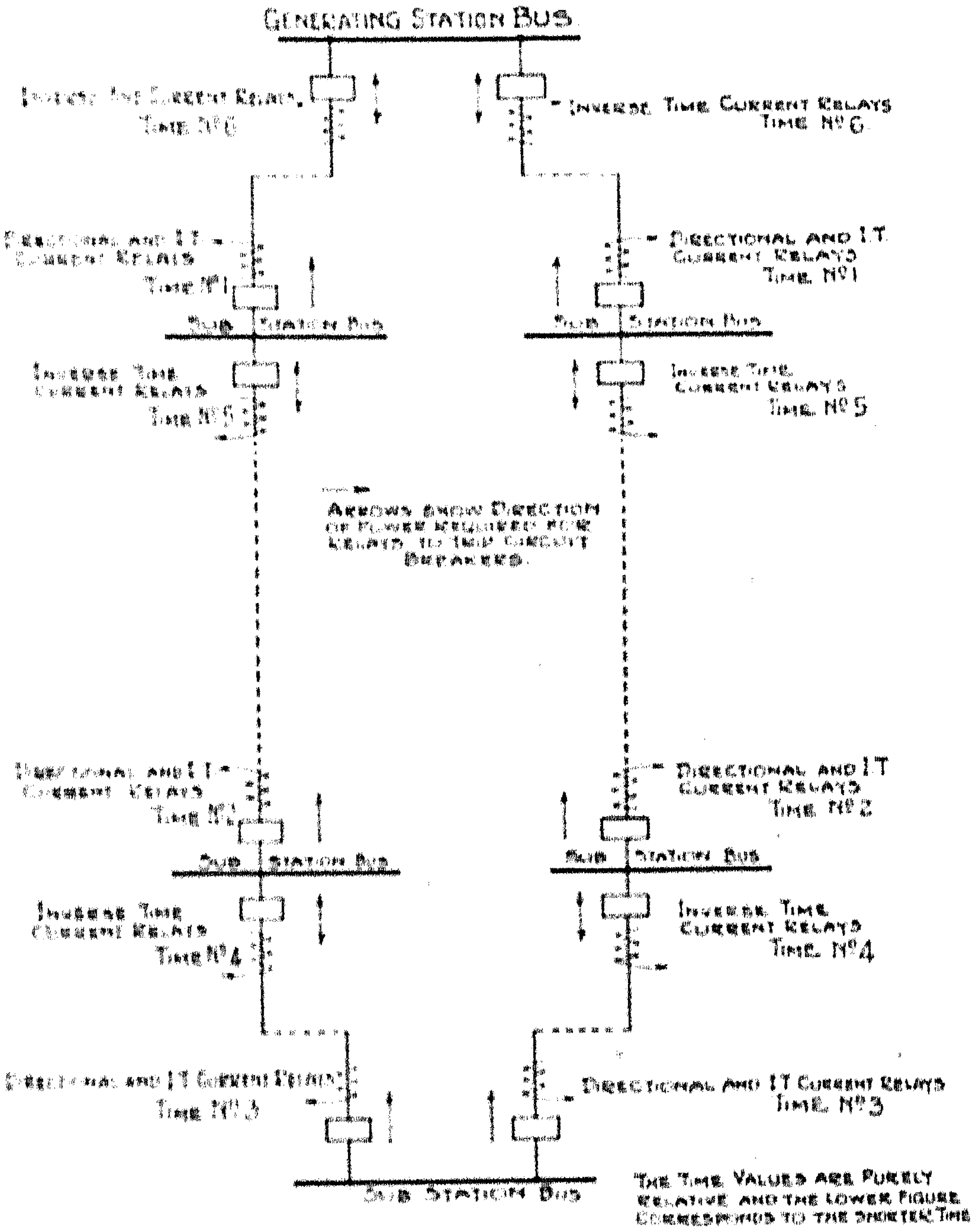

GENERATING STATION BUS.
INVERSE TIME CURRENT RELAYS TIME No 5.
DIRECTIONAL AND I.T. CURRENT RELAYS TIME No 1.
SUB STATION BUS.
INVERSE TIME CURRENT RELAYS TIME No 4.
DIRECTIONAL AND I.T. CURRENT RELAYS TIME No 1.
SUB STATION BUS.
DIRECTIONAL AND I.T. CURRENT RELAYS TIME No 1.
SUB STATION BUS
INVERSE TIME CURRENT RELAYS TIME No 3
DIRECTIONAL AND I.T. CURRENT RELAYS TIME No 1.
SUB STATION BUS.
DIRECTIONAL AND I.T. CURRENT RELAYS TIME No 1.
SUB STATION BUS
INVERSE TIME CURRENT RELAYS TIME No 2
ARROWS SHOW DIRECTION OF POWER REQUIRED FOR RELAYS TO TRIP CIRCUIT BREAKERS.
DIRECTIONAL AND I.T. CURRENT RELAYS TIME No 1.
SUB-STATION BUS.
INVERSE TIME CURRENT RELAYS. TIME No 1.
FUSES
THE TIME VALUES ARE PURELY RELATIVE AND THE LOWER FIGURE CORRESPONDS TO THE SHORTER TIME.

INVERSE TIME CURRENT AND DIRECTIONAL RELAYS AS APPLIED TO PARALLEL FEEDERS.

FIG. 4

Reference to the time settings given in the diagram (these are purely relative) will show that they are graded on the same basis as a single series of tandem connected substations, assuming that one end of the loop is open at the generator bus, and neglecting those breakers which would be incoming at the various substations. It should be noted that when the other end of the loop is considered open, the substation breakers which in the first case were incoming will now be outgoing and vice versa. The directional relays are also installed if the time of an incoming line is lower than an outgoing line further from the source. This must be checked separately for each end, in turn, considered opened at the generator station.

In order to secure the operation of the directional relays under bad power factor conditions, the connections are made so that the maximum torque is obtained with lagging power factor. This may be accomplished by several schemes of connections of the potential and current elements, which schemes may not be interchangeable with different types of directional relays. Some operators have found it advantageous when using directional relays with step-down transformers (particularly if not paralleled on the primary side) or reactors, to connect the voltage elements of the relays to the bus side of this apparatus in order to give a higher voltage in case of


Generating Station Bus
Inverse Time Current Relays
Time Nº 6
Inverse Time Current Relays
Time Nº 6
Directional and I.T. Current Relays
Time Nº 1
Directional and I.T. Current Relays
Time Nº 1
Sub Station Bus
Sub Station Bus
Inverse Time Current Relays
Time Nº 5
Inverse Time Current Relays
Time Nº 5
Arrows show Direction of Power Required for Relays to Trip Circuit Breakers.
Directional and I T Current Relays
Time Nº 2
Directional and I T Current Relays
Time Nº 2
Sub Station Bus
Sub Station Bus
Inverse Time Current Relays
Time Nº 4
Inverse Time Current Relays
Time Nº 4
Directional and I T Current Relays
Time Nº 3
Directional and I T Current Relays
Time Nº 3
Sub Station Bus
The Time Values are Purely Relative and the Lower Figure Corresponds to the Shorter Time

INVERSE TIME CURRENT AND DIRECTIONAL RELAYS AS APPLIED TO A RING SYSTEM.

FIG. 5

short circuit, thereby obtaining better operating conditions for the directional relays.

BALANCED PROTECTION SCHEMES FOR TRANSMISSION LINES

A great many operating companies have installed on a portion of their lines, protective equipment which is designed to be inoperative so long as the currents in different circuits or at different points in the same circuit are substantially equal. To this end, they have made use of (a) fully insulated lines in parallel between stations; (b) conductors in parallel having reduced separating insulation *i. e.*, split-conductor cable and (c) pilot wires of relatively small cross-section between the two ends of a line to sense the degree of balance of the currents at these points.

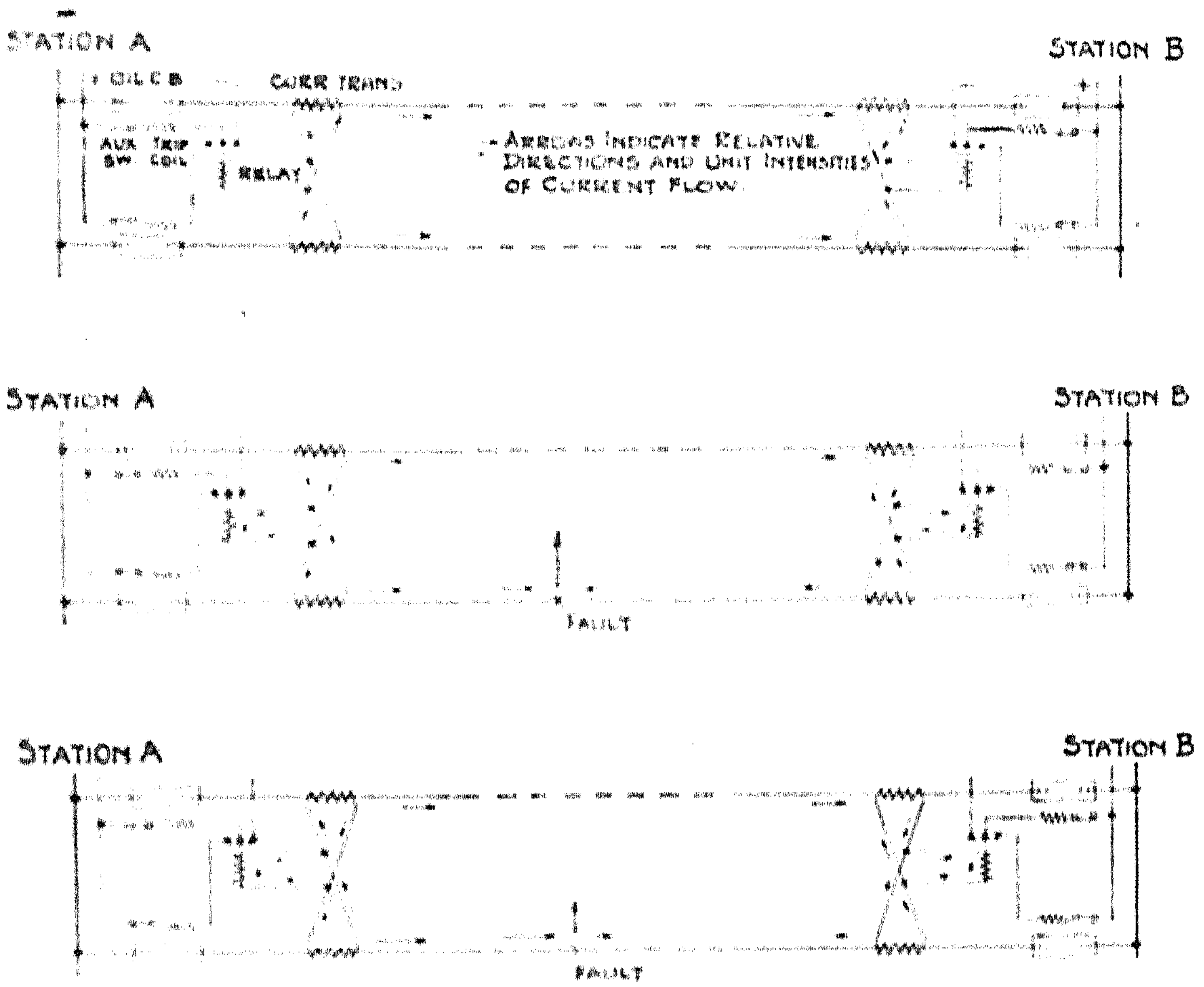
It is impractical to describe here the various modifications of these schemes in detail. However, the basic principles involved are discussed below, which principles are sufficiently flexible to allow application in various ways to the problem which may be in hand.

In the various diagrams accompanying the descriptions of these balances schemes, arrows are inserted to indicate the relative direction of current or of power. These arrows show power normally flowing from station A to station B. In each case, however, the equipment is equally suitable for power flowing in either direction.

In general, the over-current relays used are adjusted to operate instantaneously or nearly so. Some companies report that the opening of the breaker is so rapid that there have been cases where a cable has immediately rehealed, due to hot insulating material flowing into the opening made by the fault. Usually the point of breakdown has been actually located by subsequent inspection.

Balanced Protection of Two Parallel Lines (Not discriminating). An exceedingly simple scheme of balanced protection is illustrated in Fig. 6. This has given excellent results in a system which is so arranged that the lack of discrimination on the part of the relays between the sound and the injured line of a pair is not very vital. On the system referred to, each substation having such relay scheme, is provided with power over at least two different groups of lines. The complete, though temporary, interruption of one of these sources does not therefore kill the bus in question, and no hardship is imposed greater than the inconvenience of determining the good line preparatory to replacing it in service without its mate. This inconvenience is to a considerable extent compensated for by the freedom of the equipment from all alternating-current potential connections. While these potential connections, in present day relays, do not occasion the misgivings they were responsible for in older types, their elimination is a point not to be overlooked.

The balanced equipment as shown in Fig. 6 consists of "cross-connected" current transformers in the similar phases of the two lines, to the equi-potential points of the secondaries of which an over-current relay is connected. Reference to the arrows will show an assumed normal current flow, so long as the currents are equal and in the same direction in the two lines, there will be no current to flow through the relay coil, as demonstrated by the arrows on the secondary circuits. A through short circuit will not, therefore, improperly open these circuit breakers providing, of course, the line characteristics are such that a suitable balance is maintained.



FIGS. 6, 7 AND 8—BALANCED PROTECTION FOR TWO PARALLEL LINES (NOT DISCRIMINATING)

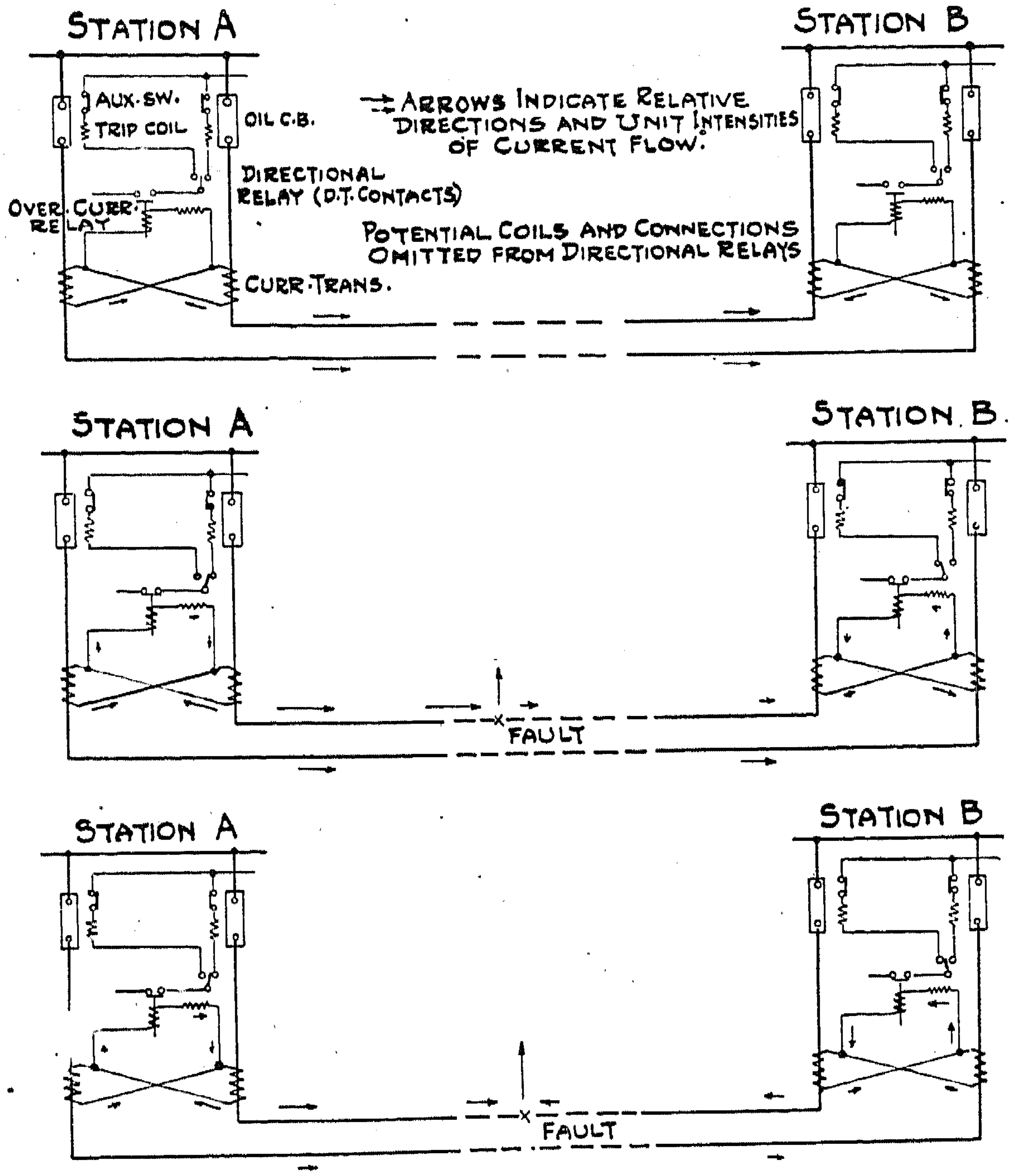
Assume a fault as shown in Fig. 7. In this case there is an unbalance of currents at substation A and a relative reversal of current at substation B. The differential of the secondary currents pass through the over-current relays, being an arithmetical difference at substation A and an arithmetical sum at substation B. Due to the differential current in the relay coils, the latter cause contacts to close, which trip both circuit breakers at each end of the two lines. Following such operation it is reported to be the practise to find the healthy line and connect it in service with time over-current protection (not illustrated in the diagrams) in place of the differential protection. If, at any time, the fault should occur so near station B that the currents at station A remained balanced, then the station A relay would not operate immediately but at B the differential current would be very great due to the reversal in the relative directions. Station B would, therefore, clear quickly after which there would be established a large differential at A. An open circuit in either of the lines will likewise unbalance the pair and result in its isolation.

In order to protect against short circuits on the substation bus itself, or any possibility which would not give a relative unbalance or relative reversal of current on a parallel group of lines, the generator ends of all feeders are provided with inverse-time-limit relays having comparatively high settings. It might be added here that with faults on the feeders between the generator station and substation, the substation relay clears the pair of lines from the substation, usually so quickly that the inverse-time-limit relay on the faulty line only at the generator end trips, thus leaving the good line of the pair connected at the generator substation.

Assume now a restricted fault in one of these lines, as in Fig. 8, such that the power direction remains throughout as formerly although the intensities have changed. It will be noted that the resulting difference in the currents in the two lines is reflected in the secondaries where it appears as a current through the relay coils and, causing the contacts to close, trips both circuit breakers at each end of the two sides.

Balanced Protection for Two Parallel Lines. (Discriminating). In order to obtain discriminating action in the relay equipment for short circuits in either of two parallel lines, direction relays are included in the scheme previously shown ln Fig. 6. With this modification the arrangement is as illustrated in Fig. 9. It should be noted that in case of an open circuit in one of these, selective isolation of the broken line is not obtained. The resulting unbalance would cause the opening of the sound feeder at one end and of the broken circuit at the opposite end, in case of sufficient current flow, as determined by setting of the time-current relay.

As in the previous case, so long as a balanced condition is maintained between the corresponding phases in the two lines, there will be no current in the differential or relay circuit. When a fault occurs, however, the vectorial unbalancing in the main circuits will be indicated in the relay coils as illustrated in Figs. 10 and 11. In these figures the arrows may also be considered as indicating direction of power flow. The moving contact of the directional relay will travel in the direction of the arrow.


STATION A
STATION B
AUX. SW.
TRIP COIL
OIL C.B.
ARROWS INDICATE RELATIVE DIRECTIONS AND UNIT INTENSITIES OF CURRENT FLOW.
DIRECTIONAL RELAY (D.T. CONTACTS)
OVER. CURR. RELAY
POTENTIAL COILS AND CONNECTIONS OMITTED FROM DIRECTIONAL RELAYS
CURR. TRANS.
STATION A
STATION B
FAULT
STATION A
STATION B
FAULT

FIGS. 9, 10 AND 11—BALANCED PROTECTION FOR TWO PARALLEL LINES (DISCRIMINATING)

Looking at the matter from another view point if we assume for the moment that only one line is in service, then the current

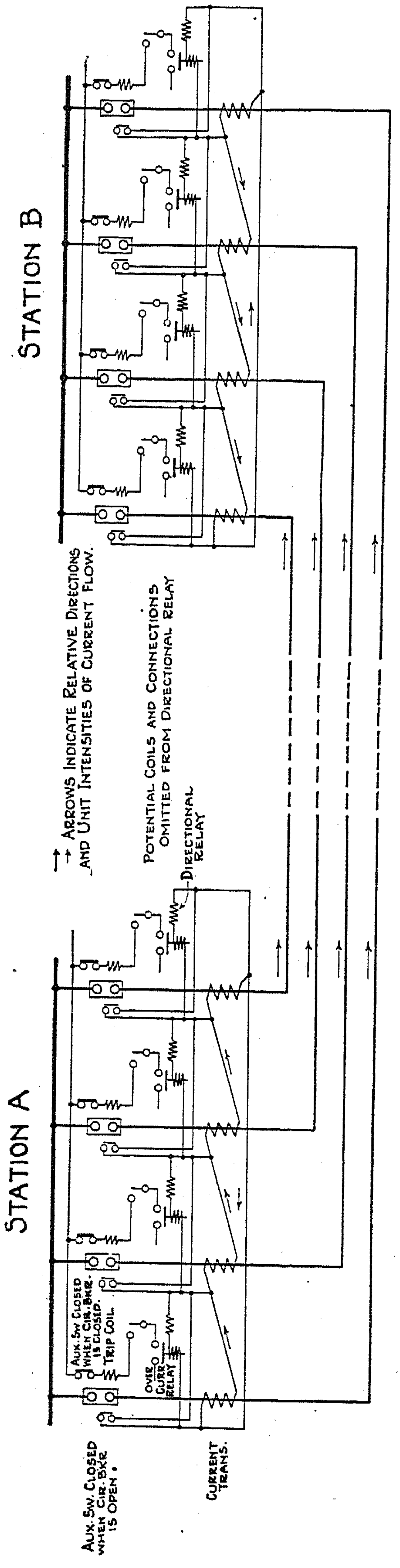

STATION B
STATION A
Arrows Indicate Relative Directions and Unit Intensities of Current Flow.
Potential Coils and Connections Omitted from Directional Relay
Directional Relay
Aux. Sw. Closed when Cir. Bkr. is Closed.
Trip Coil
Over Curr. Relay
Aux. Sw. Closed when Cir. Bkr is Open.
Current Trans.

Fig. 12

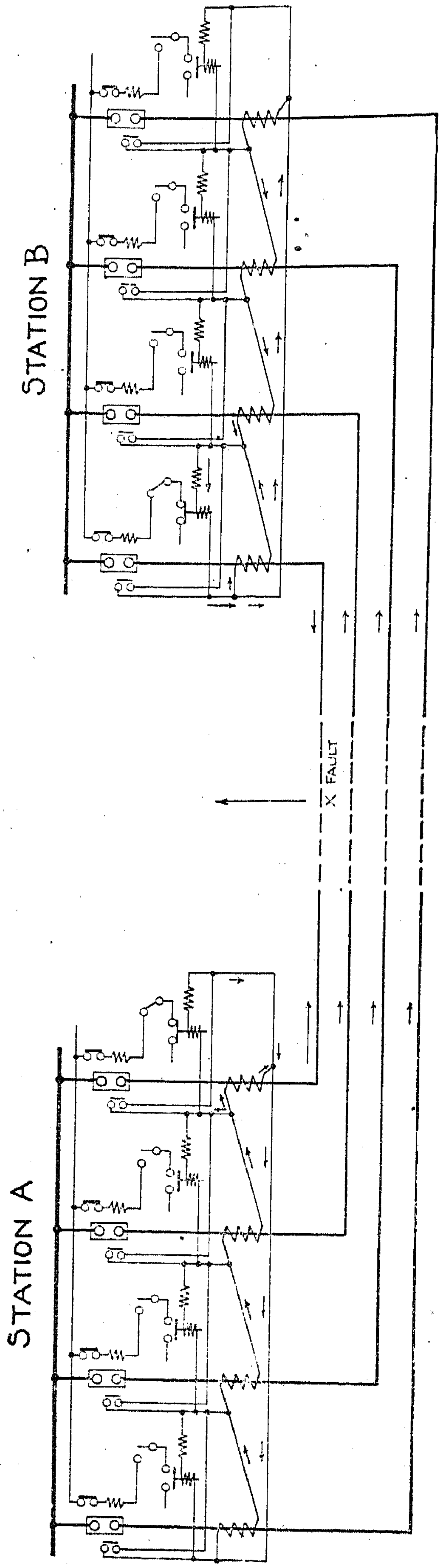


Fig. 13—Balanced Protection for Three or More Parallel Lines (Discriminating

and potential connections to the directional relay should be so made that for power flowing from the bus to the line the contacts would close on the side to trip the breaker of the line in service. When both lines are working, the circuit having the greater flow of power from the bus to the line will control the operation of the directional relay and, therefore, trip the breaker of the line in trouble which always carries the greater power from the bus to the line.

It should be borne in mind that power flowing from the line to the bus is of negative value, therefore, for cases such as shown in Fig. 10, where power flows from the line to station B bus over both circuits, the circuit having the greater flow from bus to line is the one having the least flow from line to bus.

In the event of one line being in service alone, it is the general practise automatically, by means of auxiliary switches on the circuit breakers, to introduce time-over-current relays in the protective equipment which will operate in conjunction with the directional relays in a manner similar to that described in connection with Fig. 4 or Fig. 5 (Tandem groups or loop circuits). This time delay also serves to prevent the opening of the sound line immediately following the tripping of the faulty one. In place of the single directional relay with double throw contact, it is also possible to obtain substantially the same results with two sets of relays with single throw contacts arranged in a manner similar to that described for the protection of three or more parallel lines and shown in Fig. 12.

Balanced Protection for Three or More Parallel Lines. (Discriminating). Where a number of parallel lines are involved the underlying feature is the same as described in connection with Fig. 9, although the treatment of the matter is necessarily different. Here, as shown in Fig. 12, the current transformer secondaries are connected all in series in a loop circuit so that, when the primary currents are all equal the secondary currents will also be equal and will circulate through the loop as one current of the magnitude of each, and practically none will pass through the coils of the over-current and directional relays which are connected across each current transformer secondary.

Fig. 13 illustrates a fault in one line such that power is reversed at station B in this line from the direction taken by the remaining sound lines. It will be observed that only in the case of the injured line is the power direction through the directional relays (as indicated by the arrows) such that the contacts will close. At station A this is because there is a preponderance of positive power flowing from the bus into this particular line. At station B the injured line is the only one having positive outward flowing power.

Inasmuch as the success of the scheme depends upon the relatively low impedance of the loop circuit as compared to the impedance of the coils, it will be apparent that some method should be provided to eliminate the useless impedance injected in the loop circuit by the relays of a dead feeder. This is usually done automatically by short-circuiting, by means of auxiliary switches on the circuit breakers or by auxiliary relays controlled by such auxiliary switches, the relay equipment of the line whose circuit breaker is open.

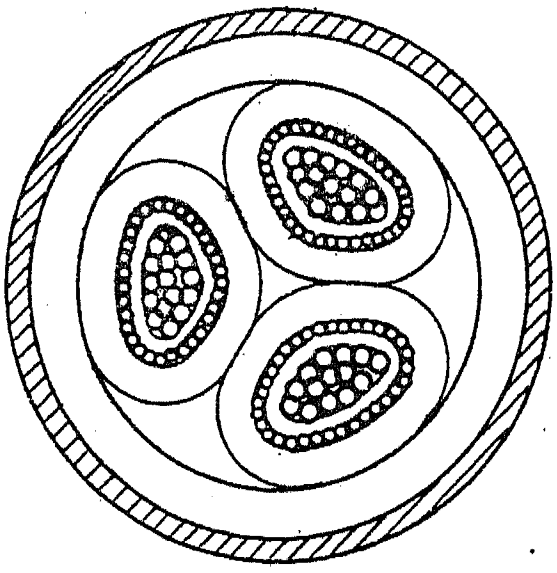


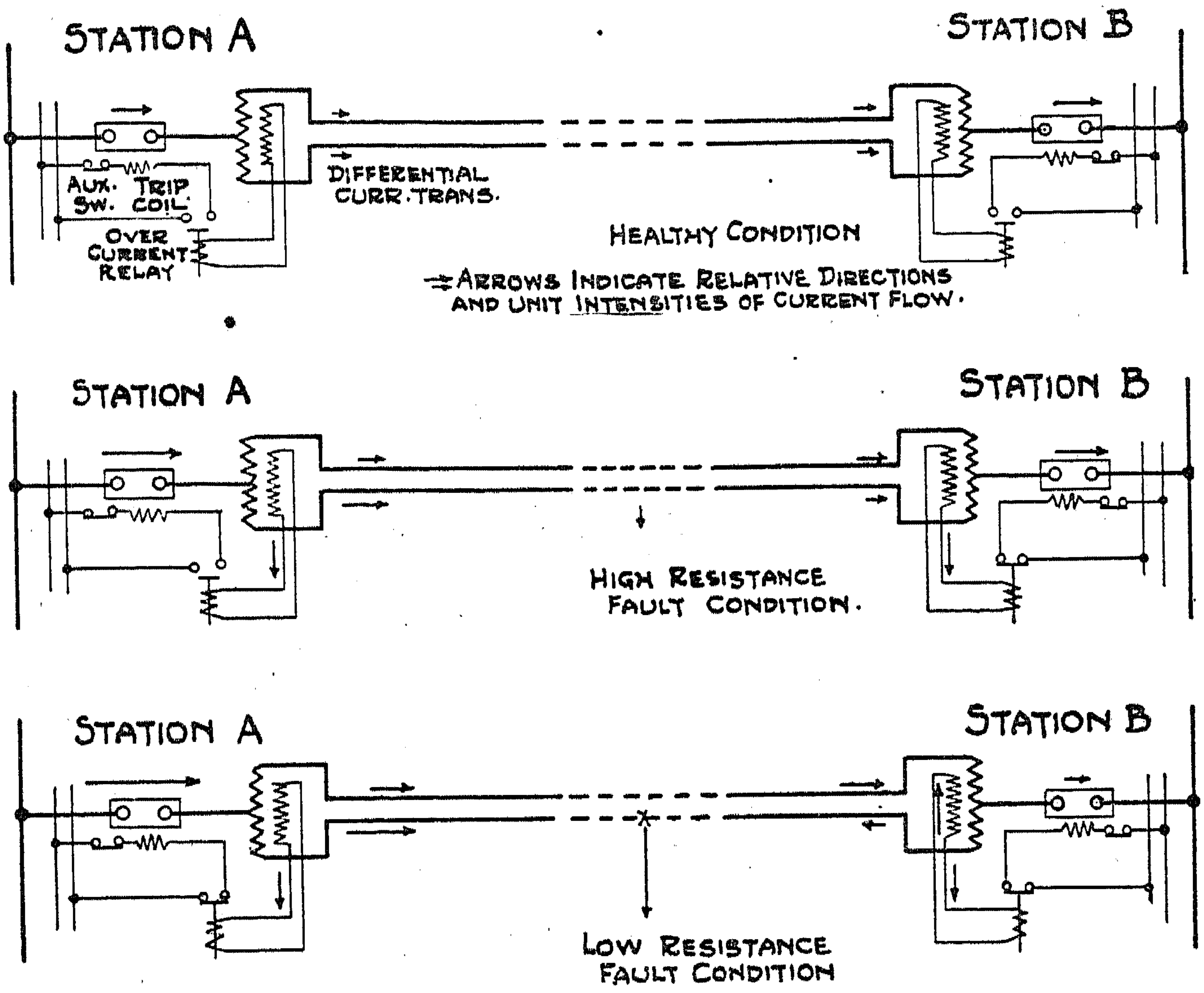
FIG. 14

It will be noted that when all but one line is out of service, the relay equipment of this last line will be short-circuited by the auxiliary switches or relays referred to in the preceding paragraph and accordingly the last line will be non-automatic unless some means is provided to open the loop circuit. This opening of the loop may be accomplished manually or it may be done automatically with considerable complication of auxiliary switches, etc., which complication is usually considered inadvisable. When the loop has been opened, each feeder will be left with over-current and directional protection at values determined by the settings of the over-current relays. Various other, also somewhat complicated, means may be used for inserting additional relays for the protection of this last line which on account of their variety will not be described here.

If an open circuit should occur in one of the conductors or if, when putting another line into service only the breaker at one end is closed, an unbalanced condition will result which may tend to open the good lines in use if the current flowing at the time, is sufficiently great. This danger becomes relatively smaller as the number of lines involved is increased because the secondary unbalancing will be inversely proportional to the number of lines in service. For instance, if the over-current relays are set to operate at the normal load of each feeder and if four lines are in, (3 continuous and one open at one point) an overload of three times normal on each feeder would be required before any trouble could be encountered.

One installation of this scheme is described in a paper by Mr. Philip Torchio, "Relays for High-Tension Lines," TRANS. A. I. E. E., Vol. XXXVI, page 361, March 1917.

Split-Conductor Protection. Split-conductor cables are being tried out in this country by two operating companies who report good results to date. The cable is comprised of six conductors, each of the three phases being "split" into two halves. A section of the cable used is shown in Fig. 14.


STATION A
STATION B
AUX. TRIP SW. COIL
OVER CURRENT RELAY
DIFFERENTIAL CURR. TRANS.
HEALTHY CONDITION
ARROWS INDICATE RELATIVE DIRECTIONS AND UNIT INTENSITIES OF CURRENT FLOW.
STATION A
STATION B
HIGH RESISTANCE FAULT CONDITION.
STATION A
STATION B
LOW RESISTANCE FAULT CONDITION

FIGS. 15, 16 AND 17—SPLIT-CONDUCTOR SYSTEM PROTECTION

The healthy and faulty conditions are illustrated in Figs. 15, 16 and 17. The action of the relays is similar to that described in connection with the two-parallel-line equipment shown in Figs. 7 and 8, excepting that under end fault conditions, additional devices (such as are described in the paper referred to in the next paragraph) are required to provide the necessary unbalancing. Repetition here will therefore be unnecessary.

The above greatly curtailed description is intended to cover in a simple way the underlying principle of split-conductor protection. For more specific information the reader is referred to the paper by Mr. W. H. Cole, "Split-Conductor

Cable—Balanced Protection," TRANS. A. I. E. E., Vol. I. 1918, page 757.

Pilot Wire System with Balanced Pressures. Adaptations of the Merz-Price protective system have been used by two operating companies. One of these reports very satisfactory results, the other is not so favorable due to an inherent difficulty in obtaining the proper degree of balance.

Fig. 18 illustrates the scheme of connections. The current transformers in each end of the circuit have their secondaries connected in opposition so that when current flows through the feeder, a potential is produced across each current transformer secondary. As these pressures, at the two ends of the line, are produced by equal currents passing through similar transformers they also will be equal and, being in opposition, no current will flow.

If, however, the input to the cable exceeds its output the corresponding potentials across the transformers will become unbalanced and a current will pass through the pilot wires and, at the same time, through the relays connected in series at each end. The operation of these relays serves to trip the circuit breakers.

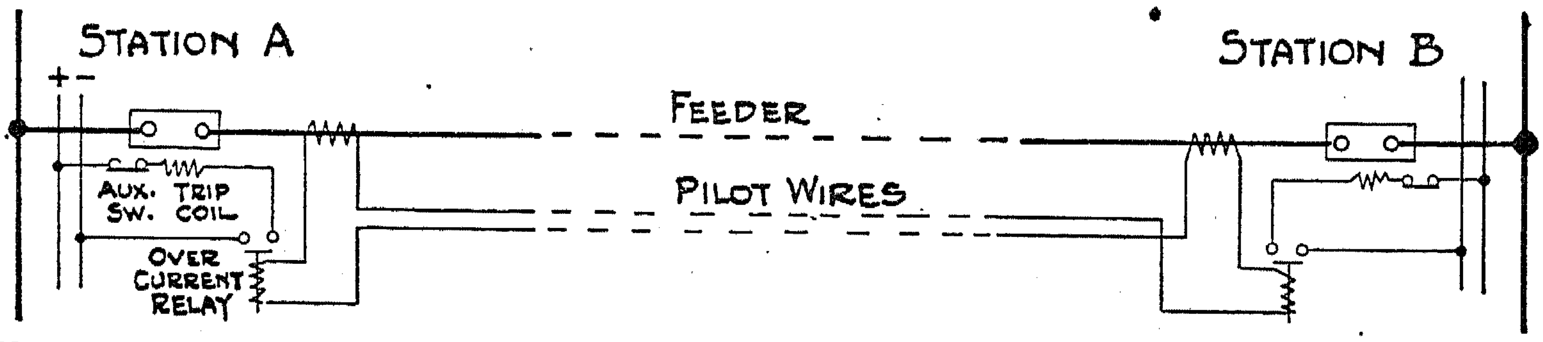


FIG. 18—PILOT WIRE SYSTEM WITH BALANCED PRESSURES—MERZ-PRICE SYSTEM

A modification of this method of protection is illustrated in the paper by R. F. Schuchardt: "Protective Relays" TRANS. A. I. E. E., Vol. XXXVI, page 383, 1917.

COMBINATION OF UNDER-VOLTAGE AND OVER-CURRENT RELAYS FOR PROTECTING TRANSMISSION LINES

One of the companies reported an application of a combination of under-voltage and over-current relays for their system which has given them very satisfactory results.

In this scheme all the circuit breakers in any section are mechanically locked in the closed position by means of a magnetically operated lock. It is controlled by a voltage relay which releases the lock when the voltage falls below a predetermined value (approximately 70 per cent). Upon the occurrence of a short circuit the voltage drops and when it has fallen below the predetermined point the voltage relays act to unlock the circuit breakers which are then free to be opened by the over-current relay or other over-current device. Thus, in most cases, only the breakers in the affected section can open on short circuit. It sometimes happens, however, that when two stations are close together, breakers in an unaffected section may be unlocked but usually the over-current device will act to open the proper ones only and clear the trouble before the others can operate.

Referring to the diagram Fig. 19 the under-voltage relays are connected to the secondaries of potential transformers on

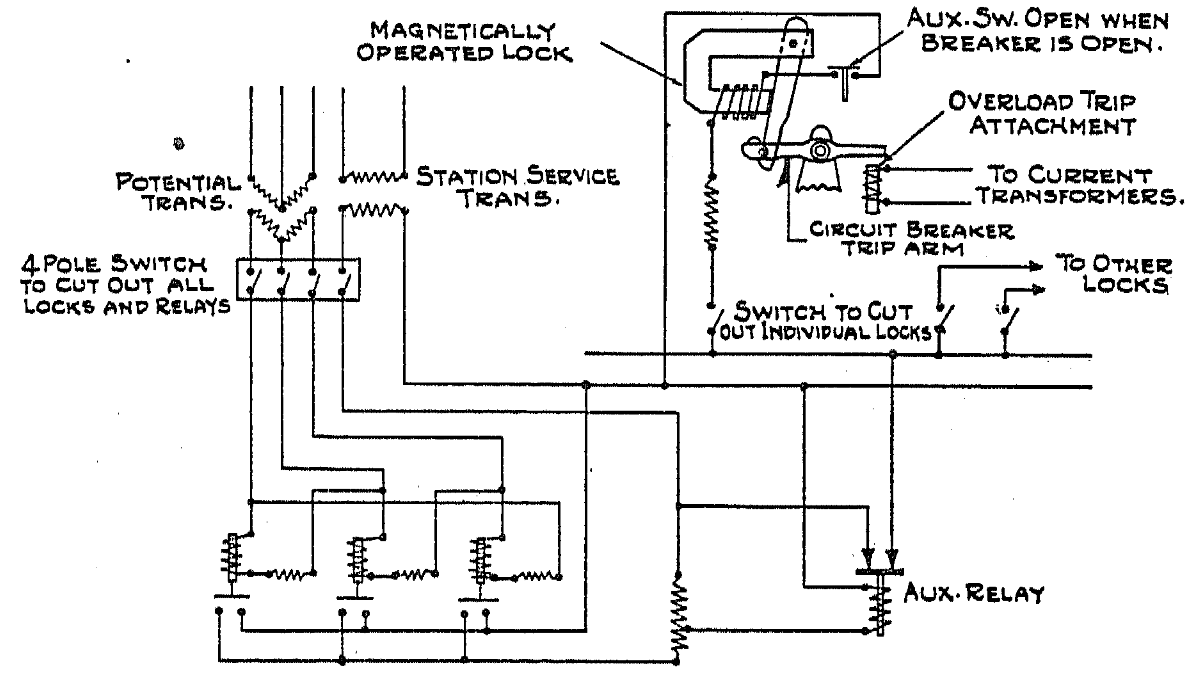


FIG. 19—UNDERVOLTAGE AND OVERCURRENT RELAY—SCHEME OF CONNECTIONS

the line buses. An auxiliary relay is actuated from the station service transformer and is controlled by the under-voltage relays. In its turn this auxiliary relay controls the locks on the circuit breakers. One set of under-voltage relays, with the auxiliary relay, may control a number of locks on a number of circuit breakers. The operation is as follows:

When the line potential drops below a predetermined value, one or more of the under-voltage relays drop and short-circuit the auxiliary relay through a resistance which causes this auxiliary relay to open. This, in turn, releases the latch and leaves all the circuit breakers under its control free to operate in case of over-current. Thus, so long as the potential remains above the predetermined value, no breaker can be opened by its automatic over-current feature.

This scheme, illustrated in Fig. 19, is reported as giving successful results on an over-head system covering 4000 square miles where there is a large number of stations located on an average of approximately 10 miles apart. As the lowest voltage occurs nearest the short circuit, it is probable that only the circuit breakers in the immediate vicinity will be unlocked and allow the circuit breakers to be automatically opened if the current is of sufficient magnitude.

APPENDIX

In reply to a general questionnaire which was sent out to sixty-one operating companies, information was received from thirty-two giving analysis of their relay protective schemes, from which the following summary was made.

Of the companies reporting, about 60 per cent cover supply systems for the larger cities and the remaining 40 per cent cover widely distributed territories of a number of smaller cities in which cases there are in general a number of water power stations.

Ques. 1. *a.—How often do you check current and time settings on your relays?*

Ans.—From the general reports submitted from central stations it seems to be the general practise to make a periodic check on the current and time settings of the relays.

Of the companies reporting, 70 per cent make this periodic check and the average complete check for each relay is once every three months; 27 per cent make no periodic check but most of them report making checks after cases of trouble and at intervals depending on operating conditions; 3 per cent report that they make no check whatever on the relay system.

Ques. 1 *b.—Have you a regular routine for making tests and inspection; What is it?*

Ans.—Approximately 70 per cent of the operating companies reporting have a regular routine of making tests.

Although the reports are not very definite, some of the larger companies have found it desirable to disconnect the circuit from service and test the relays by applying the testing circuit to the current transformer secondary terminals with the complete relay circuit left intact so that current transformer is left in shunt with the relay circuit under test. This method is found to give more accurate test results under varying conditions of various loads, relay circuits and different types of current transformers.

A few of the companies make their tests on the relays with the feeder alive which necessitates disconnecting the trip circuits and for the period of test the feeder circuit breakers are left non-automatic. This test may be made at relay terminals or at current transformer terminals in which case a check is made of the wiring.

Ques. 1. *c.—Do you calibrate with a stop watch, or do you use some other instrument? What is it?*

Ans.—For good relay calibration it is apparently becoming the practise to use some form of an automatic chronometer or cycle-counter in which the personal equation of the tester is eliminated. Of the companies reporting, 63 per cent have adopted this method of calibration where 37 per cent still hold to the old form of the stop watch.

Ques. 1. *d.—Do you make a more frequent inspection for dust and dirt and mechanical defects? At what intervals?*

Ans.—Of the reporting companies 65 per cent make a more frequent inspection of the relays for dust, dirt and mechanical defects and these additional inspections vary from periods of one week to one month.

Ques. 2. *a.—Do you test and adjust relays with the current according to the setting or to the current that will probably flow at the time of short circuit?*

Ans.—There is a growing tendency to test the relays with currents approximating the short-circuit conditions as borne out by the fact that 50 per cent of the operating companies reporting are now testing under these conditions. The results obtained by these companies show that this method is a marked improvement over the earlier method of small currents.

Ques. 2. *b.—In adjusting relays for selective operation how do you determine the current and time settings of the several relays?*

Ans.—With companies, where the testing is made approximating short-circuit conditions, the settings are arrived at in general from the determinations of the short-circuit conditions and the relays on which selective operation is desired are set to give selectivity under these conditions with allowances made in time settings for the actual operation of circuit breakers, the minimum time being determined by the estimated requirements for preventing operation on system load and surges.

In some companies the settings are determined by a "trial process" which is the result of the actual operations obtained under trouble conditions. Others use either the calculating table (See *G E Review*, October 1916), or Mathematical Calculations—(See *G E Review*, June 1916 and *Electrical Journal*, July 1916) for determining these conditions while others report the advisability of installing recording instruments for determining these conditions in a purely practical manner.

Ques. 2. *c.—In determining the current settings of your relays do you consider the probable short-circuit current with all generators in operation or with reduced generating capacity such as corresponds to the average or minimum load?*

Ans.—Of the operating companies reporting their settings determined by short-circuit conditions; 62 per cent determine their settings with the minimum generating capacity and in addition some of these consider also the maximum generating conditions, and 25 per cent consider only the maximum generating conditions, 13 per cent consider the average generating conditions.

Ques. 3. *a & b.—Do you use a-c. or d-c. for tripping oil circuit breakers on transmission lines? Is it satisfactory?*

Ans.—Of the replying companies 80 per cent use d-c. tripping current and most of them find it satisfactory and preferable and of the 20 per cent that use a-c. current only in one case did they state that they believed the a-c. to be preferable.

Storage batteries, and in some cases primary batteries, have been used for giving direct current for tripping.

Some of the central stations have had trouble with their circuit-breaker control circuits when connected to the d-c. excitation bus, due to the surges produced on the excitation system in cases of heavy short circuits on the high-tension system.

Ques. 4—*Do you connect watthour meters or indicating wattmeters to the same current transformers as your relays?*

Ans.—In 30 per cent of the cases neither watthour nor indicating wattmeters are connected to the same current transformers as the relays.

In 35 per cent of the systems the relays are connected to the meters in cases where a high degree of meter accuracy is not required.

In 35 per cent of the systems, the relays are connected with the meters with no restrictions mentioned.

Ques. 5—*Do you attempt to operate underground or overhead transmission lines with a ground on one phase, and if so, for how long before disconnecting?*

Ans.—Of the systems answering the questionnaire 67 per cent are grounded and the ground line is automatically disconnected.

Of the remaining systems, which are ungrounded, 25 per cent operate for short intervals (no longer than necessary and in no case longer than 30 minutes) with a ground on one line and 75 per cent disconnect the grounded line immediately.

Ques. 6.—*What is the longest time (in seconds) as determined by your relay settings and switch constants, that a short circuit is permitted to exist on your transmission lines before the switch is opened automatically?*

Ans.—The average maximum time as reported from the operating companies on which a short circuit is kept on the system is $3^1/_3$ seconds. This time varies with different companies from ¼ of a second to as high as 15 seconds.

We wish to point out that the above is not the time on which the heaviest short circuit is held on the system which cases are undoubtedly much shorter than given above. In some cases where the longer time is permitted, there has been trouble experienced due to synchronous apparatus falling out of step. Two seconds is apparently a good limit for heavy short circuits.

Ques. 7.—*About what percentage of your transmission line troubles are ground and what percentage are short circuits between phases?*

Ans.—Of the systems reporting, an average of 70 per cent of the troubles are started from ground. There is considerable variation in the reports; for example, some give as high as 90 per cent to ground while others give as low as 25 per cent to ground. We feel this discrepancy is probably due to the fact that some of the operating companies have no records to show what portion of the trouble did actually start from the ground.

Ques. 8.—*Do you ordinarily keep all transmission lines in service or do you keep some lines dead and in reserve?*

Ans.—Of the operating companies reporting 93 per cent keep all of their good transmission lines in service without any held in reserve with the exception of two cases where with overhead high-voltage transmission lines one of two parallel lines is taken out of service for reserve during lightning disturbances and in one of these cases the line is grounded at this time.

Ques. 9.—*Where you have several schemes of relay protection that accomplish practically the same purpose, will you kindly state your preference and your reasons?*

Ans.—Of the companies reporting 13 per cent prefer the combination of time-current and directional relay schemes, 13 per cent the balanced pair, 6 per cent split conductor, 6 per cent straight inverse-time-current protection, and 3 per cent balanced relay with pilot wires.

And of the remaining systems there was only one relay scheme in operation from which no comparison could be drawn.

Ques. 10.—*Have you any scheme of relay protection which in your opinion is particularly effective for your conditions? If so will you kindly state in some detail the desirable features of this system?*

Ans.—Of the companies reporting, 32 per cent feel that some form of a balanced relay protection is well adapted for their conditions. Of this 32 per cent, 6½ per cent prefer the split-conductor scheme, 6½ per cent, two lines operated as a unit with balanced overload protection: 16 per cent, two lines in multiple with the balanced directional and current relays for disconnecting a single defective line only; and 3 per cent use pilot wires.

22 per cent find that the current-directional relay combination gives particularly good results for their conditions and some of these find that the connection of the potential circuits of the directional relay to the substation bus side of transformers, where not in parallel on high-tension side, give preferable operating conditions, as a higher voltage is maintained under short-circuit conditions.

3 per cent find special ground relays desirable (Refer to A. I. E. E. TRANSACTIONS p. 361 vol. 26).

6 per cent prefer definite time-limit relays as they have given very satisfactory service.

3 per cent prefer no automatic protection on very high-tension lines.

3 per cent find that a combination current and under-voltage relay system has solved, for their conditions, difficulties that they have not been able to overcome with any other scheme.

Balance report no special conditions.

Ques. 11—*Have you some installations or schemes of operation for which you find difficulty in securing a satisfactory scheme of relay protection? If so, please describe?*

Ans.—Several companies reported that they have had difficulty in finding a proper relay protection in cases where they have taps on the tie lines between stations. This is particularly true where the protection of these lines would have been of the balanced-relay type were it not for the taps.

Other companies at present using the balanced-current relay protection, which automatically disconnects both feeders in case of trouble on either, on parallel lines would like to find a simple relay scheme which would disconnect only the damaged line of the pair in case of trouble on one of the lines.

Some difficulty has also been experienced in finding the proper relay protection for tie lines between stations where very heavy momentary surges may exist with the flow of power in either direction without any actual trouble on the tie lines themselves.

Large variations in generating equipment, in operation with water power systems, caused the operating companies difficulty in finding a relay arrangement which would provide protection under all these different conditions.

Ques. 12.—*Do you attempt to operate all of your transmission lines in parallel by groups? Or in parallel?*

Ans.—Of the companies reporting 60 per cent operate with the major part of their lines in parallel and 40 per cent operate with some of their lines in parallel by groups and in some other cases the lines are strictly radial.

Ques. 13—*Have you in operation a balanced-relay system with pilot wires?*

94 per cent of the companies reporting use no balanced-relay system with pilot wires. In one instance it was reported that pilot wire scheme had been tried out but its use discontinued.

Ques. 14 *a. b. & c.—Do you use a balanced-relay scheme consisting of split-conductor cable or lines in multiple?*

When trouble occurs on one line of two in multiple is the good line again placed in service while repairing the other line?

What protective scheme do you have on the single line during the repairs on the line in trouble?

Ans.—72 per cent of the reports indicate that no balanced scheme of any kind is used.

6 per cent use split-conductor protection of the six-conductor cable type.

3 per cent use a similar scheme excepting that the six conductors comprise two separate three-phase circuits. For this scheme it was reported that after the two lines had been disconnected, due to a fault in one, the good line was found by test and put back in service as quickly as possible with inverse-time-limit protection at each end.

19 per cent report other schemes of balanced protection with directional relays in which selective isolation of the injured line is attempted.

Ques. 15 *a. b. & c.—What are the principal difficulties that you have experienced with relay protective system?*

Have you experienced any false operations or failures of relays to operate?

Have you been able to attribute such difficulties to defects in the relay, the design of the relay protective system, the difficulties in making proper connections, or to the irregular operation of oil circuit breakers? Please answer in some detail?

Ans.—Practically all the companies report trouble with the operation of their protective relay system. Of these troubles there are 46 per cent reported due to the difficulties in the relay, 20 per cent to improper application of the relays, 15 per cent to improper settings of the relays, 9 per cent to defective wiring, and 10 per cent to defective operation of the oil circuit breakers.

Several of the companies reporting trouble with the relays find that they were due to the chattering of the bellows type of relay in which cases some of the trouble had been eliminated by replacing this relay with other time-current types and in some cases it was eliminated by the installation of a saturating reactor in shunt with the relays. The shunt reactor eliminated the trouble and gave a desirable characteristic time curve at a considerably lower cost than required to replace the relays.

Some defective operations of directional relays are reported, in most cases of which the relays were of the old type.

Some companies report a change in calibration with time and although these cases are not very explicit, the trouble could probably be avoided by a more frequent check and inspection of the relay.

Of the troubles due to improper wiring, some of these are attributed to a wrong connection of the potential element of directional relays and some to a wrong connection of the balanced or differentially connected relays and the remainder is due to poor workmanship.

Ques. 16—*What is the maximum current that can flow into a short circuit on a transmission line just outside the generating station?*

a. The maximum instantaneous current?

b. The maximum sustained current?

Ans.—Report covers the experience from thirty-one operating companies with systems varying from 25,000 to 455,000 kv-a. whose short-circuit currents, at any point, vary in value from 100,000 to 1,650,000 kv-a. and sustained short-circuit kv-a. values vary from 20,000 to 340,000 kv-a.

GENERATING STATION DATA

Of the systems reporting, 67 per cent operate with the grounded neutral. Of these 40 per cent are grounded without resistance, 23 per cent grounded through low resistance and 4 per cent are grounded through a high resistance where special ground relays are required for selective operation on grounds.

TIE LINE RELAY PROTECTION

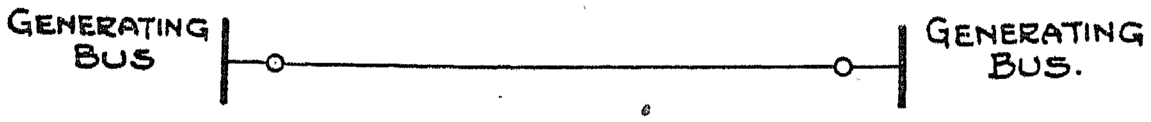


DIAGRAM A

About one-third of the companies reporting had tie lines that could be classified under this connection. Of these—

55 per cent of the lines were protected by means of inverse-time-current relays only.

5 per cent use balanced protection with pilot wires.

5 per cent use split-conductor installations.

35 per cent were provided with directional and current relay protection.

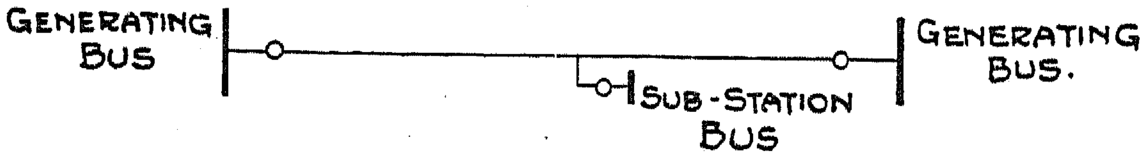


DIAGRAM B

About one-fifth of the companies reporting had tie lines that could be classified under this connection. Of these—

85 per centof the lines reported were provided with inverse-time-current relays only.

15 per cent were provided with current and directional protection.

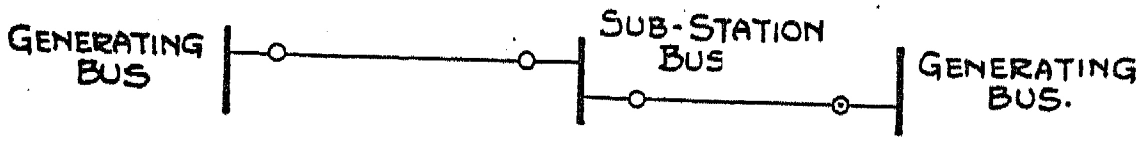


DIAGRAM C

About one-fourth of the companies reporting had tie lines that were classified under this connection. Of these—

50 per cent of the lines reported were protected by inverse-time-current relays only.

25 per cent by current and directional relays.

10 per cent were provided with balanced protection using pilot wires.

15 per cent were provided with balanced pair protection.

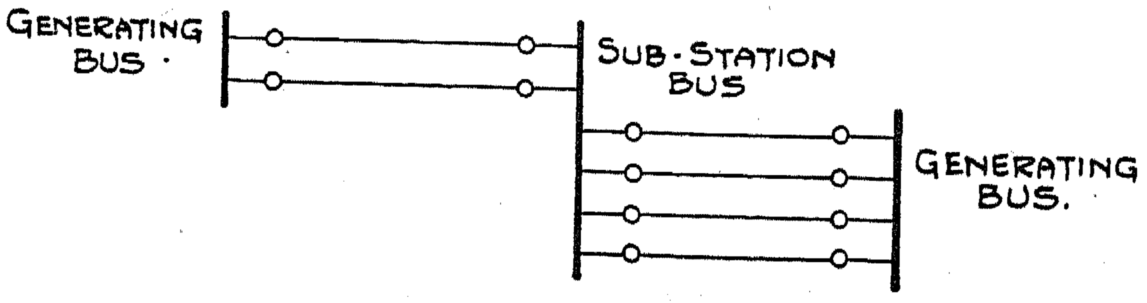


DIAGRAM D

About one-tenth of the companies reporting had tie lines that were classified under this connection. Of these—

65 per cent of the lines reported were protected by means of inverse-time-current relays.

35 per cent were equipped with current and directional relays.

TRANSMISSION LINE RELAY PROTECTION

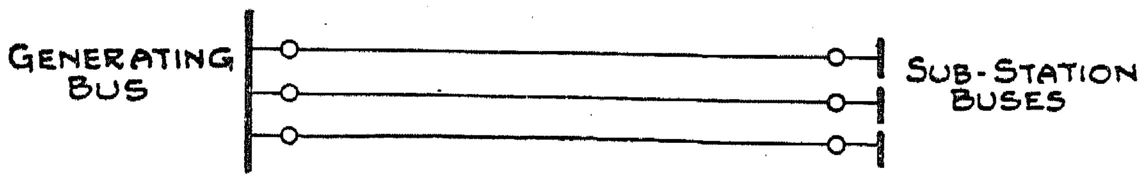


DIAGRAM E

About one-eighth of the reporting companies have lines that were classified under this type. Of these—

75 per cent of the lines were protected by inverse-time-current relays.

25 per cent of the lines were protected by directional relays and current relays.

(Even on a radial system the secondary connections of the step-down transformers may be in parallel and cause a feed back in case of trouble which makes a directional relay desirable.)

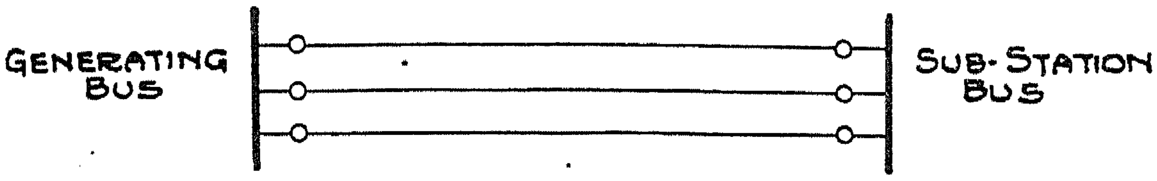


DIAGRAM F

About one-half of the companies have lines in this classification. Of these—

40 per cent of the lines were reported to have current and directional protection.

55 per cent were equipped with inverse-time-current relays only.

5 per cent were protected by balanced relays of the interconnected directional type.

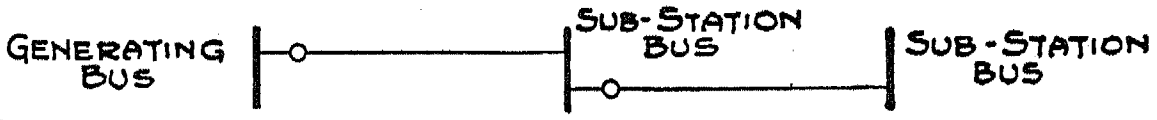


DIAGRAM G

About one-tenth of the companies have lines under this classification of which one-half are protected by inverse-time-current relays and one-half by balanced connections.

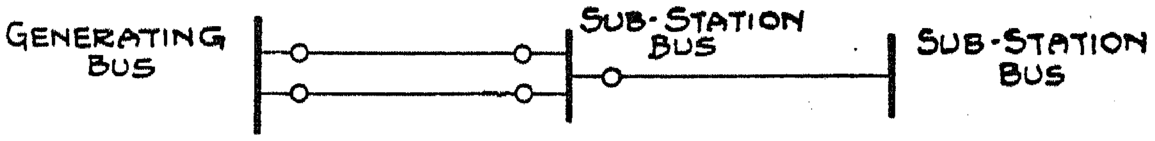


DIAGRAM H

About two-fifths of the companies have lines under this classification. Of these—

55 per cent were reported to have inverse-time-current relay protection.

30 per cent were equipped with directional and current protection.

7½ per cent were equipped with balanced relay protection with pilot wires.

7½ per cent were equipped with balanced relay protection scheme using directional relays.

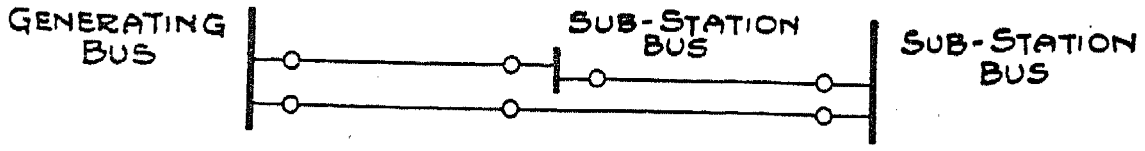


DIAGRAM I

About one-eighth of the companies have line classifications in this list. Of these—

75 per cent were reported to be protected by means of current relays and directional relays.

25 per cent by means of inverse-time-current relays only.

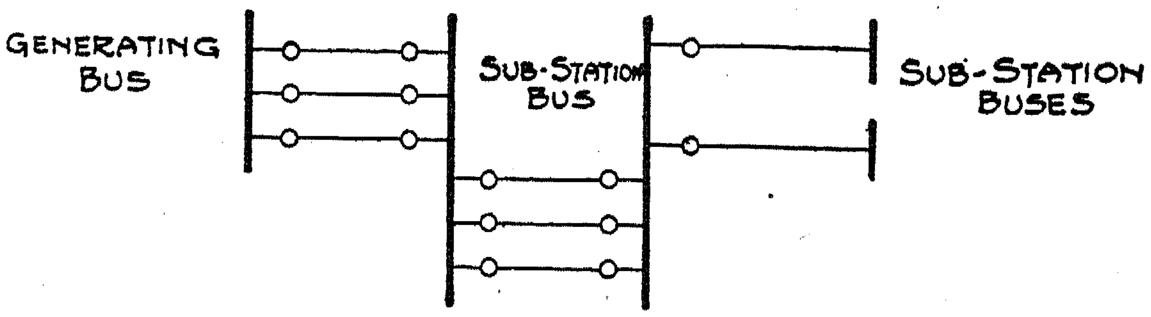


DIAGRAM J

Two companies reported lines in this classification and in both cases straight inverse-time-current relay protection was used.

Presented at the 35*th Annual Convention of The American Institute of Electrical Engineers, Lake Placid Club, N. Y., June* 24, 1919.

Copyright 1919. By A. I. E. E.

GROUNDING THE NEUTRAL OF GENERATING AND TRANSMISSION SYSTEMS

BY H. R. WOODROW

Abstract of Paper

Four different high-tension transmission systems in the New York district have been grounded at different times during the past four years, all of which are operating with the grounded neutral at the present time. These systems were all grounded in a different manner and the results obtained from the effect of grounding showed that the troubles have been reduced.

In addition to the grounding, special ground relays have been installed on three of the systems which disconnected the feeder very quickly on a ground and the results show that the addition of this quick acting relay further reduces the stresses on a system.

6600-Volt 25-Cycle System

It had been the policy of The New York Edison Company to operate its high-tension feeders radially with the system neutral ungrounded, but with selective ground indicating relays on all feeders to locate a ground on any feeder. This method of operation gave the operators time to disconnect defective grounded feeders from the system before developing into short circuits, and as a result 70 to 80 per cent of the system ground failures were cleared in this way.

During the later years, however, it became desirable to study means of paralleling feeders at the substations and of reducing the time limit for disconnecting the defective feeder to avoid short circuits following a ground on account of the increased capacity current of the greatly larger mileage of cables on the system.

After a careful analysis it was decided to establish a definite ground current by means of grounding transformers connected to the system bus and connect the selective ground relays to trip the oil circuit breakers automatically.

The method of grounding through zig zag-connected transformers was adopted as the latest generators installed are delta-connected, from which a neutral cannot be brought out. The limiting resistances were connected in the primary side of the grounding transformer and of a value to limit the total ground current with both generating stations operating in parallel to 400 amperes. The selective ground relays, which are operated from sheath type current transformers, are adjusted for operation on 140 amperes minimum and on 300 amperes in one second.

The above described system was placed in operation in the early part of 1918 and the accompanying Fig. 1 will show the results which have been obtained. Although the time that this system has been grounded is too limited to draw any definite conclusion, the total system troubles have slightly decreased and of these total troubles the number which have developed into short circuits is reduced to 60 per cent of those before grounding.

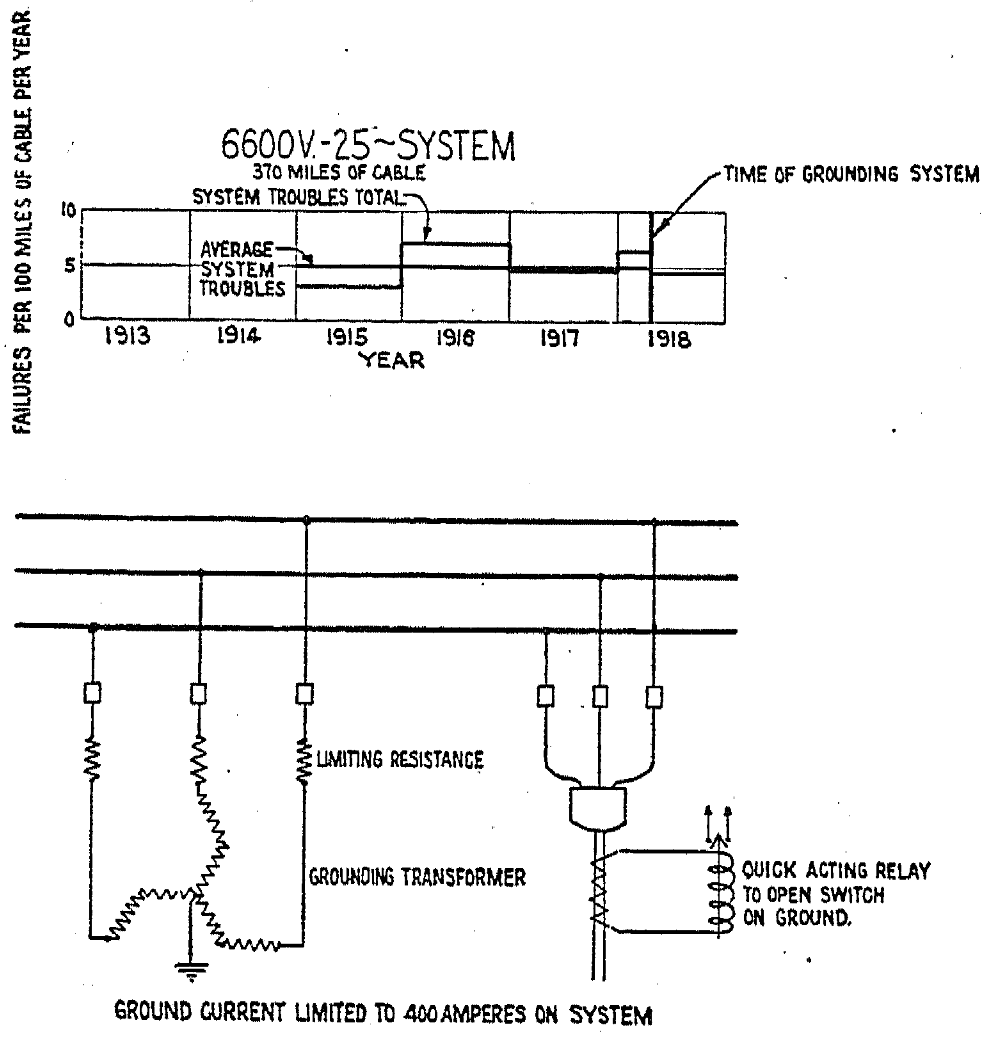


Fig. 1

7800-Volt 60-Cycle System

The new 201st Street station of the United Electric Light & Power Company was placed in service with the same general scheme of operating with an ungrounded neutral with selective ground indicating relays. In the first few years of operation, however, several generator failures were experienced and at the recommendation of the manufacturer the system was grounded by means of the generator neutral through a two-ohm resistance, which would allow a ground current of approximately 2000 amperes to flow.

The results of the system troubles before and after grounding are shown in Fig. 2, and although no general system disturbances were experienced before the grounding, there has been an apparent reduction in the number of system troubles since grounding.

16,000-VOLT SYSTEM

When the 16,000-volt system was adopted for supplying the northern territory of the 25- and 60-cycle systems, step-up and step-down transformers were provided on each feeder, thereby isolating the high-tension side of each feeder. During the first two years of operation the number of cable failures was very large, most of which occurred in the cable splices.

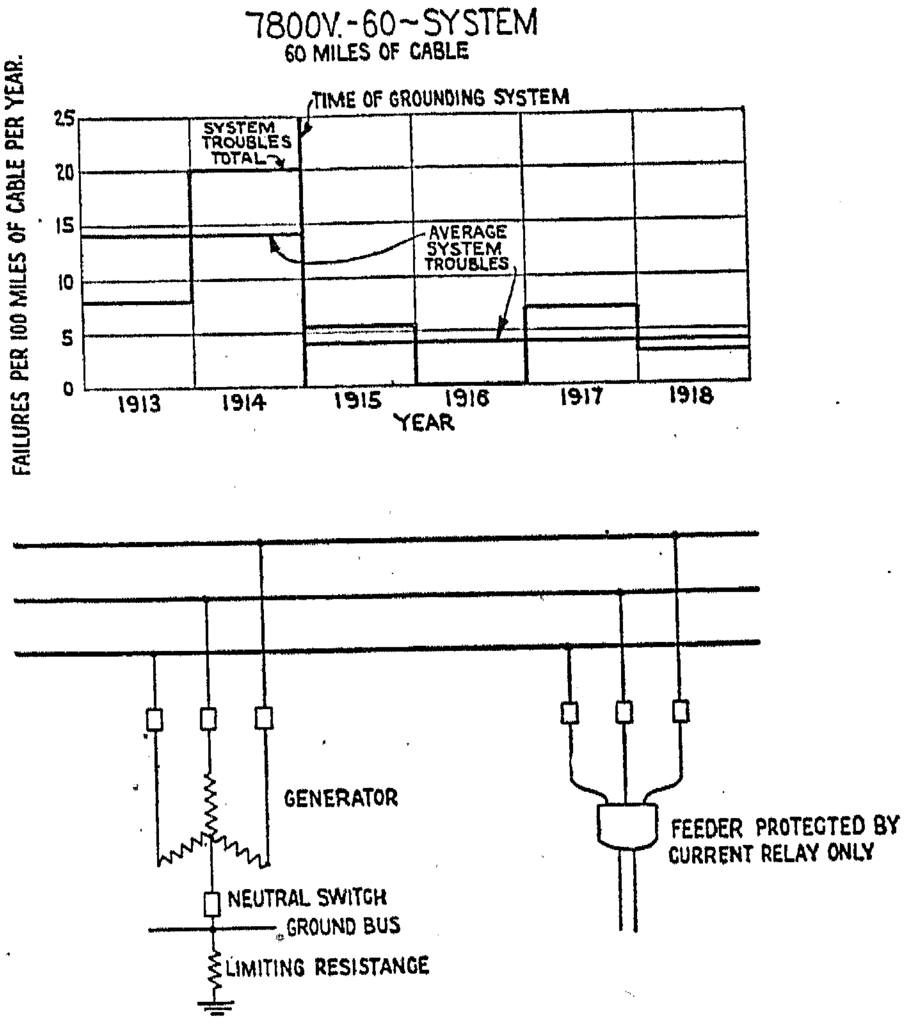


FIG. 2

To remedy this trouble a number of splices on the 60-cycle system were changed and the transformer connections were changed on both the 25-cycle and 60-cycle systems to make a solid ground on the high-tension side. The systems were operated in this manner for a period of approximately two years and no material change in the number of failures occurred. During this time, every ground on the system was a short circuit due to the solidly connected neutral, thereby materially increasing the stresses on the transformers and cables. It was decided to change the method of grounding and provide quick-acting relays to disconnect a feeder practically instantaneously on a ground.

The method adopted for grounding these feeders as shown in Fig. 3 followed the suggestion of Dr. Creighton of using potential transformers with the neutral grounded and the secondaries connected in delta with a quick-acting relay in the loop.

In the year and a half that this system has been in operation, the results have been encouraging although the number of system troubles, as shown in Fig. 3, was only reduced by approximately 28 per cent; the number of short circuits have been reduced by 82 per cent. In 73 per cent of the cases in the present method of operation the failures are disconnected from the system before short circuit occurs.

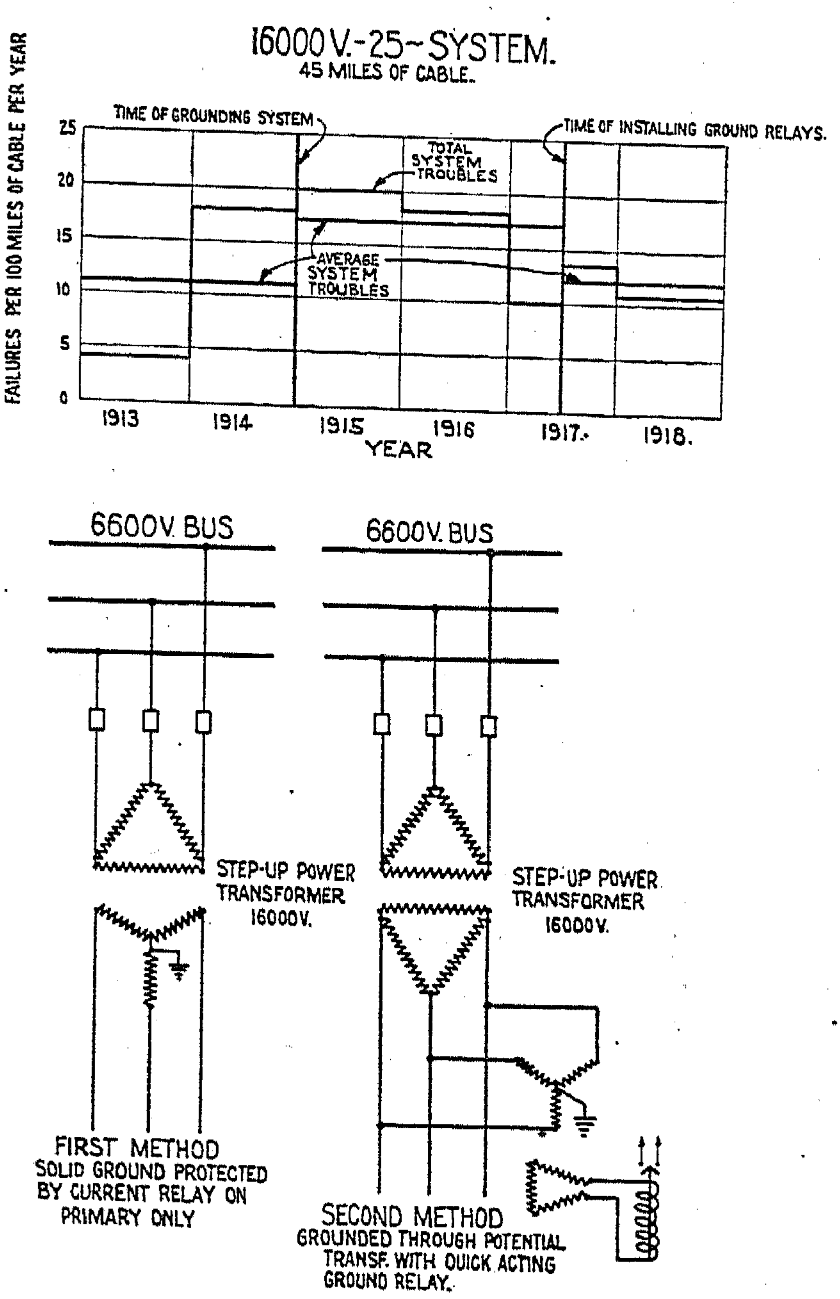

16000 V.-25~SYSTEM.
45 MILES OF CABLE.
FAILURES PER 100 MILES OF CABLE PER YEAR
TIME OF GROUNDING SYSTEM
TOTAL SYSTEM TROUBLES
TIME OF INSTALLING GROUND RELAYS.
AVERAGE SYSTEM TROUBLES
25
20
15
10
5
0
1913
1914
1915
1916
1917.
1918.
YEAR
6600V. BUS
6600V. BUS
STEP-UP POWER TRANSFORMER 16000V.
STEP-UP POWER TRANSFORMER 16000V.
FIRST METHOD
SOLID GROUND PROTECTED BY CURRENT RELAY ON PRIMARY ONLY
SECOND METHOD
GROUNDED THROUGH POTENTIAL TRANSF. WITH QUICK ACTING GROUND RELAY.

FIG. 3

These figures are the results on the 25-cycle system only, as a number of changes in the cable joints have been made on the 60-cycle cables and the results would be confused as it would be impossible to determine what portion was due to betterment of insulation and that due to the method of grounding.

25,000-VOLT 25-CYCLE SYSTEM

In the installation of 25,000-volt 25-cycle cables, the neutral of the transformers at the generator end was solidly grounded.

The type of joint used in this cable was of the oil-filled type as described by Mr. Torchio before the Institute, and practically no failures have resulted in the splices. In the first fifteen months of operation, however, a number of cable failures occurred, some of which could be traced to defective manufacture of cable. To relieve the stresses on these cables it was decided to install grounding relays connected to a sheath transformer for disconnecting the cable very quickly on a ground, as the method of operation of these feeders necessitated a long time setting on the normal current relays.

These cables have been in operation with this method of protection for three years and, as shown in Fig. 4, the number of failures has decreased until no failures have occurred during the past year.

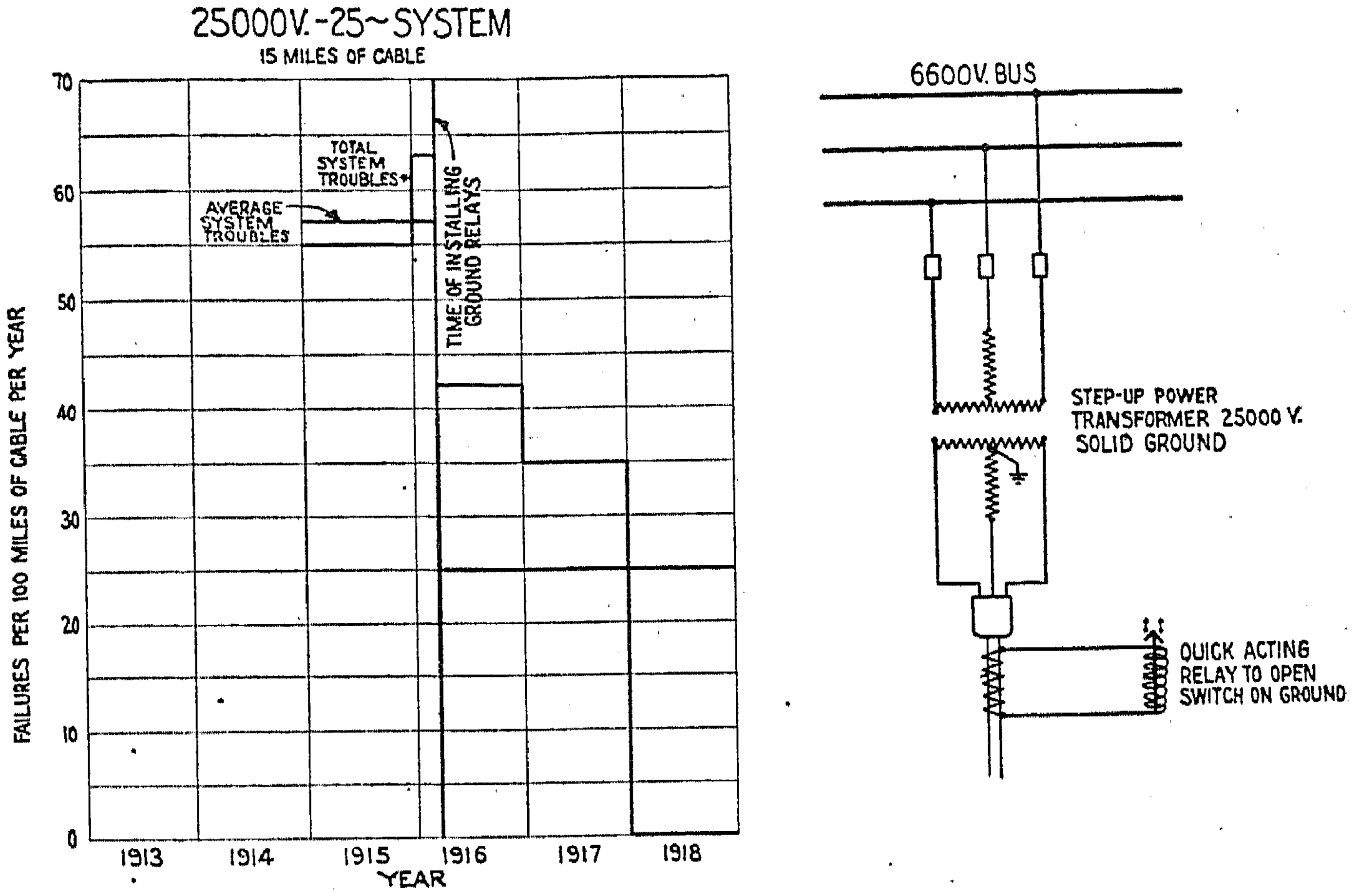


Fig. 4

Conclusion

Our experience has shown unquestionably that the stresses on the cables are relieved materially by grounding, but we further believe that it is desirable to limit this grounding current as much as possible, thereby reducing the stresses on the system and particularly the burning of lead sheath and grounding connections. Also the installation of grounding relays, which can be set to disconnect a feeder more quickly than the straight normal current relays, will further greatly relieve the potential stresses due to a prolonged grounding current which occurs even with a grounded system.

Presented at the Thirty-fifth Annual Convention of the American Institute of Electrical Engineers, Lake Placid Club, N. Y., June 24, 1919.

Copyright 1919. By A. I. E. E.

GROUNDED NEUTRAL TRANSMISSION LINE

BY W. E. RICHARDS

Abstract of Paper

The paper describes the conditions on the system in Toledo which was originally delta-connected. Serious trouble was experienced when a short circuit occurred especially with synchronous apparatus connected to the line which invariably failed due to the voltage drop occasioned by the severe short circuits on the delta system. This trouble was overcome by changing the transmission to a Y system with the neutral grounded. The effect of a short circuit with the new connection has been decidedly minimized.

THE subject will be treated with especial reference to the operation of a grounded-neutral transmission line having several elements not always desirable in handling high-tension voltage within city limits, namely, extremely high and long-span construction connected directly with an extensive underground transmission with its various connections to conversion apparatus.

The system in Toledo was built more on the line of developments following existing industrial requirements than a prearranged system where future requirements were anticipated.

In this case, an industry, by reason of circumstances, was located some four miles from the generating station. Conditions were such that it was not desirable for the customer to generate his own supply. His requirements started with a demand of from two to four thousand kilowatts, with the ultimate expected demand of eight or ten thousand. As the industry has exceeded their expected increase, the demand on this system will shortly run to sixteen thousand kilowatts, and possibly greater.

Geographical conditions and city ordinances determined that this transmission should be in part purely underground. There was fortunately located on the route selected, several substations which were badly in need of additional energy and such other developments that the company had in mind, that it was decided to install a 23,000-volt 25-cycle system, it being felt that this was the limit of successful high-voltage underground operation. Certain features presented themselves at this time which made it desirable to have the transformers delta connected. This feature was thoroughly discussed and at that time no serious objections presented themselves and it was put into operation the early part of the year of 1915.

Some trouble developed due to weak spots in cable and joints but these faults were surprisingly low, considering the quantity of cables that were put into service at this time.

The system rapidly settled down to a successful continued operation and no faults developed that did not have a reasonable explanation, such as fault in material and joints, and joints not properly made.

Since correcting this condition the system has operated successfully with reference to the general plan, but a serious condition arose when a short circuit occurred, particularly disastrous to synchronous apparatus connected with the system, which would invariably fail by reason of the voltage drop occasioned by the severity of a short circuit on the delta system. The cables suffered severely, and in fact, in some cases the covers of the manholes blew off, taking on more the nature of an explosion if the short circuit occurred in the manhole.

It was then decided to change this transmission to a Y system with the grounded neutral. The transformers being constructed for delta operation, it was necessary to take the core out of the case, cut the coils apart and parallel them for 23,000 volts Y. A five ohm-resistance was inserted, which afterwards was removed, thereby operating the neutral directly to ground. The results of a cable failure now has a decidedly different effect.

In some cases no superficial evidence is present at the point of failure, and when such evidence is present, it is not of a severe nature to destroy adjacent cables and in no way presents the disastrous appearance as when trouble would occur with a system operating delta.

It is true that by experience a better understanding of protective apparatus resulted in changes to such apparatus which were beneficial, but it was felt that the successful operation of this system was largely due to the change from delta to Y grounded.

Referring to the Figs. 1, 2 and 3, the system is graphically shown, and presents some unique features as before mentioned.

Fig. 1 is the general scheme of the transmission, carrying

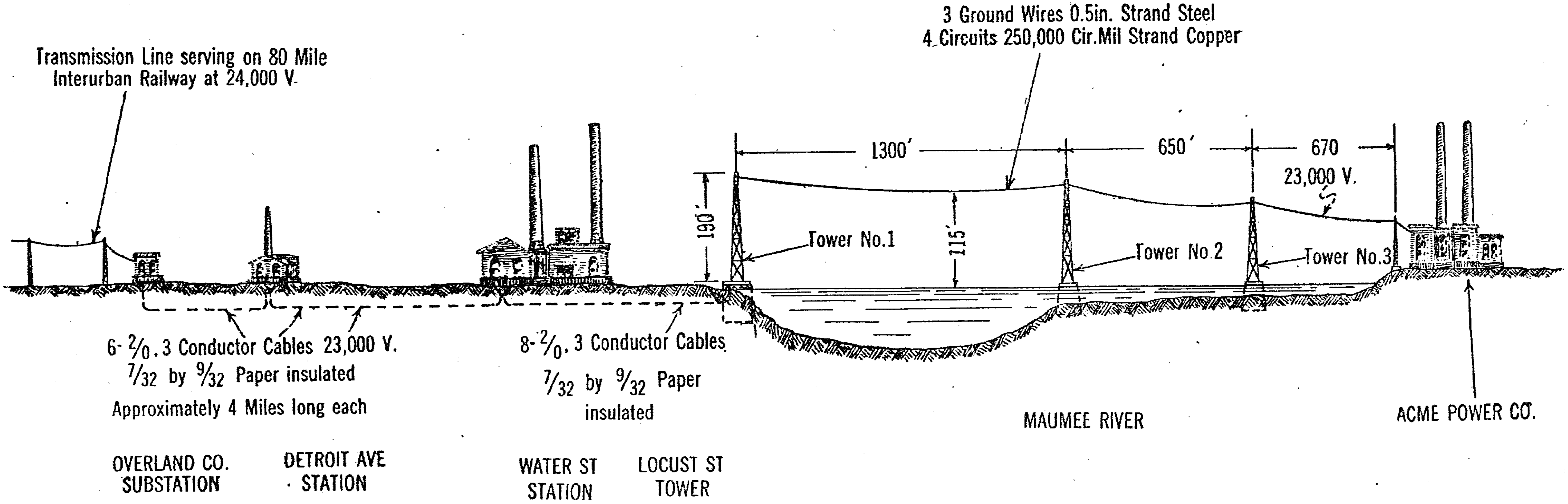

3 Ground Wires 0.5in. Strand Steel
4 Circuits 250,000 Cir.Mil Strand Copper
Transmission Line serving on 80 Mile
Interurban Railway at 24,000 V.
1300′
650′
670
23,000 V.
190′
115′
Tower No.1
Tower No.2
Tower No.3
6-2/0.3 Conductor Cables 23,000 V.
7/32 by 9/32 Paper insulated
Approximately 4 Miles long each
8-2/0.3 Conductor Cables
7/32 by 9/32 Paper
insulated
MAUMEE RIVER
ACME POWER CO.
OVERLAND CO.
SUBSTATION
DETROIT AVE
STATION
WATER ST
STATION
LOCUST ST
TOWER

Fig. 1—General Scheme of Transmission

such information as will give the physical characteristics of the system.

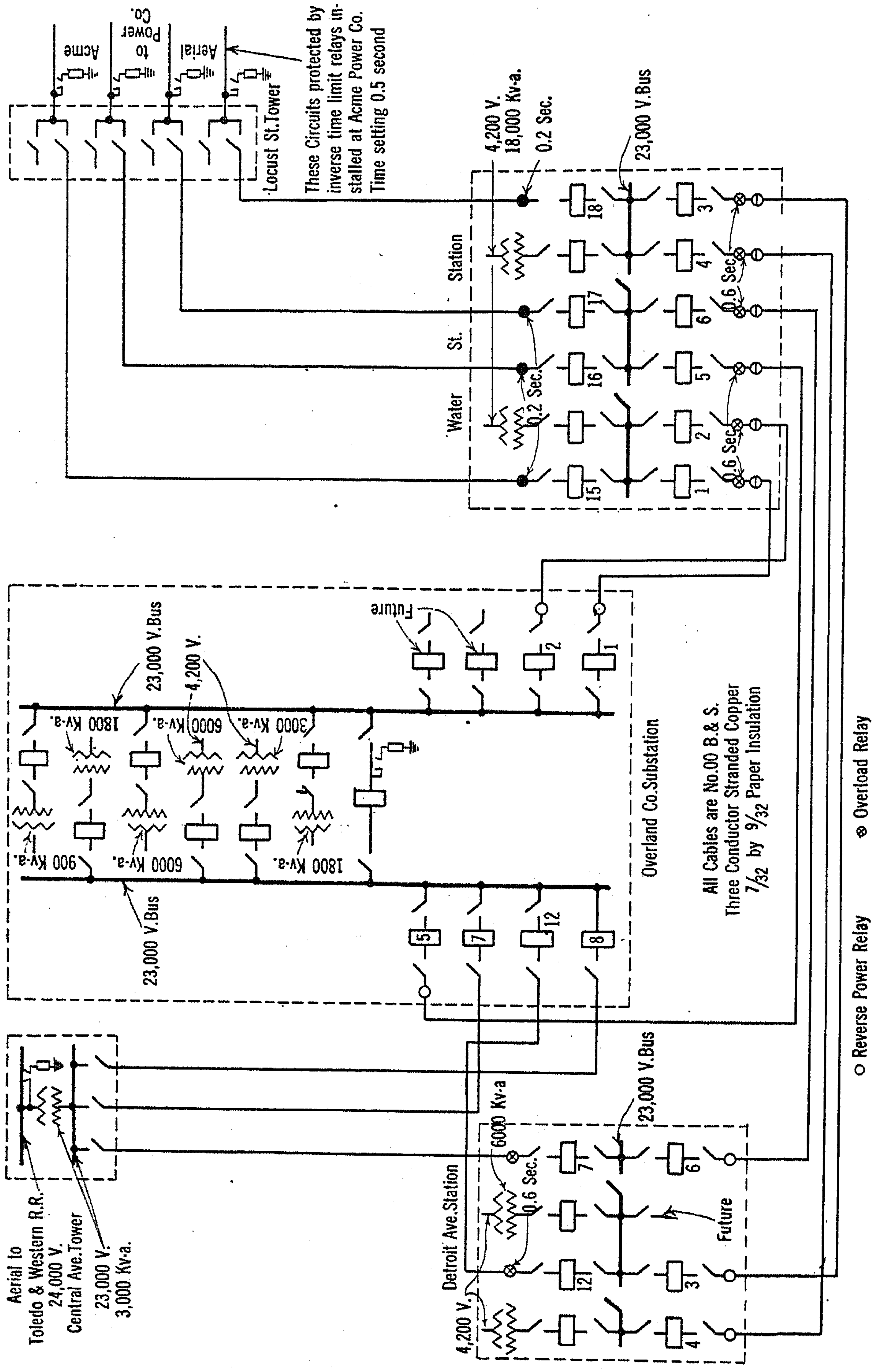


FIG 2—HIGH-TENSION SYSTEM OF THE TOLEDO RAILWAYS & LIGHT CO.

Fig. 2 is a one-line diagram of the system, showing protective apparatus.

Fig. 3 is a detail of the landing tower of the river crossing.

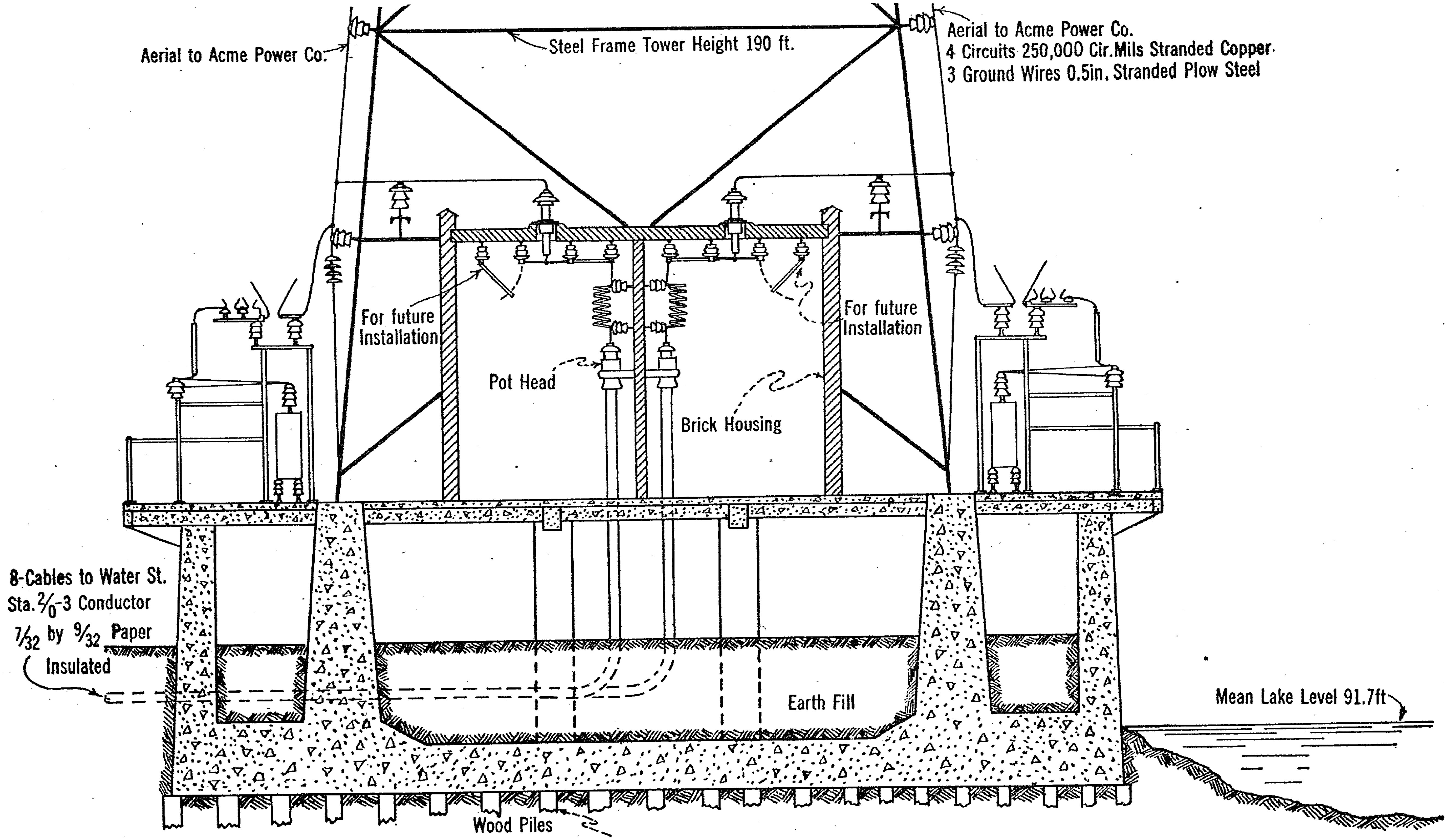

Aerial to Acme Power Co.
Steel Frame Tower Height 190 ft.
Aerial to Acme Power Co.
4 Circuits 250,000 Cir.Mils Stranded Copper.
3 Ground Wires 0.5in. Stranded Plow Steel
For future Installation
For future Installation
Pot Head
Brick Housing
8-Cables to Water St. Sta. 2/0-3 Conductor 7/32 by 9/32 Paper Insulated
Earth Fill
Mean Lake Level 91.7ft
Wood Piles

FIG. 3—SECTION AT BASE OF TOWER NO. 1

With reference to this crossing, it might be interesting to note that no trouble has been experienced by insulator failures due to lightning or other causes. This part of the system has passed through some severe lightning storms, but has shown no sign of weakness, due largely to the fact that the insulator values are about 200 per cent above working voltage, the disk type being used on the short spans, and the well known oil-insulated wood strains for the long spans.

The system has been in successful operation now some two years and is considered by this company as being successful in all points of construction.

DISCUSSION ON "TRANSMISSION LINE RELAY PROTECTION" (WOODROW, ROPER, TRAVER, MACGAHAN), "GROUNDING THE NEUTRAL OF GENERATING AND TRANSMISSION SYSTEMS" (WOODROW), AND "GROUNDED NEUTRAL TRANSMISSION LINE" (RICHARDS), LAKE PLACID, N. Y., JUNE 24, 1919.

PROPOSED RELAY NOMENCLATURE

J. R. Craighead: The terminology expressed in this paper is open to an objection in the heading of the first definition. The heading reads, "Electric Protective Relay," and the definition that follows is so worded as to include other than protective devices. For instance, a telegraph relay is included in the definition according to its wording.

While there is a distinction between protective relays and relays for other operating purposes, that distinction does not belong in the definition itself, but rather in a classification of relays which is subsidiary to the general definition. The word "protective," therefore, should be omitted from the heading.

J. A. Johnson: It is not quite clear to me why an exception is made in the use of the generic term in the case of the "watt relay." Why not "power relay" in conformity with "current relay," "voltage relay," etc., and also with "power directional relay"? I would suggest that the designation of "power relay" be chosen.

A. H. Lawton: I would like to suggest that under qualifying terms as applied to relays there be added definitions of circuit closing and circuit opening. There has always been more or less confusion as to whether a relay should be called a closing circuit relay or a circuit opening relay, and I think the subject needs definition as to the action which the relay has upon its tripping circuit.

E. E. F. Creighton: The question of terminology is very interesting to every one who sat in the Committee. We hope that these terms will be continued and used, and thus avoid a great deal of confusion. The discussion on each term was long, and although objections can be made to each one, we finally, in spite of those objections, accepted the term as the best one possible.

One comes to me now—the word "instantaneous." We are now working along lines where the word "instantaneous" might mean a tenth of a second, a sixtieth of a second, or even one-two hundredth of a second. Instantaneous is a misnomer.

H. T. Conover: Mr. Creighton deplored the use of the word "instantaneous" to describe the relay operation, and some time ago when this report was being discussed at the Bureau of Standards, the same feeling was expressed there, and at that time the word "immediate" was suggested as a substitute for the word "instantaneous." I would like to offer that to the Committee as a suggestion. While the word "immediate" leaves something to be desired, it certainly has not the rigor and the absolute characteristics that the word "instantaneous" has and would be more liberal and allow for an operation that is rapid but at the same time does not take place in zero time.

O. C. Traver: The term "Electric Protective Relay" was proposed in an effort to restrict the field to be covered by the other terms, which followed it. Possibly, some other preferable means of accomplishing this purpose will be substituted at a later time.

There seems to be but slight preference between the generic terms "Watt Relay" and Power Relay." Even if "Power" should be the better when used alone, the combined term "Over Power" is considered very objectionable on account of previous usage.

Circuit closing and circuit opening contacts might well be defined.

Although objections may be raised to the term "Instantaneous," it must be remembered that in the physical world there is no such thing as instantaneous action in the absolute sense. Practically, universal usage sanctions "Instantaneous" as applied to relays, and as no confusion results there seems to be no reason for changing to some other term, possibly no more accurate.

PRESENT DAY PRACTISE OF TRANSMISSION AND TIE LINE RELAY PROTECTION

J. R. Craighead: This paper emphasizes the fact that we cannot specialize too narrowly on one branch of engineering without inviting trouble in other allied branches. The transmission line engineers have for a long time chiefly given their attention to the questions of efficient transmission, safety and protection in other senses than relay protection, and have consequently allowed relay protection to be developed as a separate art which must be adapted to transmission lines constructed without consideration of relay protection.

J. A. Johnson: As networks become larger by the interconnection of systems, the necessity for better means of isolating faults with a minimum of disturbance becomes greater. The delay of seconds after the occurrence of a fault and the use of reactance are, after all, but make-shifts adopted on account of the limitations of the circuit breaker and ought not to be, and, in my opinion, will not be permanently tolerated. Some other means must be found to accomplish the purpose, and I believe that we must look for it in improvements in the circuit breaker itself. It may well be that some entirely new and revolutionary principle of operation will be the means of accomplishing the result, but in any event I am convinced that a big advance in the effective isolation of faults by more efective circuit breaking devices is on the books for the near future.

I would invite the attention of inventors to this problem. We have accomplished much, but we must guard against the self-satisfied attitude, keep in mind the ideal to be attained, and strive forward to the day when the size limitations of transmission networks will disappear and the isolation of faults take place instantaneously and entirely without disturbance to operation of unfaulted circuits.

I believe that the future system should be the reverse of the present practise in that the time of operation of relays should become shorter as the generating station is approached, as the disturbance to the system is greater with short circuits near the generating station, and consequently such trouble should be cleared from the system in the shortest possible time.

P. Torchio: Referring to Mr. Johnson's remarks, I fully appreciate, and I think most of us appreciate, the work of development of the Protective Devices Committee. We have with us at present only two solutions to cope with the requirements of handling large powers, and the two are either the balanced system, including split conductor or pilot wire, or the other system developed by American engineers in the last few years, depending on selectivity by timing relays.

Mr. Johnson gave a knock-out to the development of this system of protection because necessarily if a system of that kind depends on having the longest time of selectivity on the relay near the station, that must be the last to go. Mr. Johnson is probably right in aiming at an ideal goal, that ideal goal being the development of a circuit breaker that will open any kind of short circuit, and any amount of power. He does hope that such development will be made, but to my knowledge there is not now on the market a circuit breaker that will cope with the stresses imposed by opening a circuit as met in a central station of large size.

Now, with the failure of the large circuit breaker, we have followed another line, to reduce the amount of power that will go into a short circuit. If we reduce the amount of power that will go into a short circuit, and that is the only practical way that we now have available, then we can use the selective relay that these engineers are developing.

So I do not want to discourage this American development because I believe that at present it is the right line to follow. We have no other alternative from an economical standpoint, and I believe that from an engineering standpoint it can be made successful.

J. A. Johnson: I had no intention whatever of casting aspersions on the present relays; on the other hand, I have every admiration for and desire to express appreciation of the work that has been done. I believe that under present conditions, and with the apparatus that is at present developed, the lines that we are following are right. They are the best that we can do.

The ideal purpose will perhaps be accomplished in cable systems by means of the differential principle, which in a great many cases, enables faults to be disconnected before they develop into short circuits but on the other hand in high-tension overhead lines, I firmly believe that some new development of circuit breakers, carrying with it a new development of relays, has got to be made and I simply want to encourage progress in that direction, not at all to disparage the progress that has been made up to the present time.

D. W. Roper: I would like to add a word about the maximum delay of relay operation and its effect on the synchronous apparatus.

Some years ago, in the course of switch experiments by the Commonwealth Edison Company, some large generators were made with the reactance coils in series. Experiments were made to determine the effects on synchronous apparatus in case of short circuits at the bus and at some distance electrically from the bus. The experiments showed, and subsequent experiments have verified, that a short circuit on the bus will disconnect the synchronous apparatus, regardless of how quickly you open that short circuit.

The types of breakers that we have available apparently would not open the short circuit quick enough to prevent disconnecting the synchronous apparatus in the substation. If, however, the short circuit, instead of being on the bus is a comparatively short distance away—a hundred or two feet on a cable—so that you get a perceptible amount of resistance in series in the arc, then the synchronous apparatus will not be disconnected if the breaker on the transmission line operates properly and in the correct time.

There are, however, other effects that have to be safeguarded and one of them is the effect of the burnout on other lines in adjacent conduits. Our experience indicated that in the days when we had the instantaneous relays, the ordinary burnout resulted in a small hole in the lead sheath of the cable, perhaps half an inch or an inch or two in diameter, depending upon the location with respect to the station.

As we introduced a time element into the operation of the breakers, we got larger burns on our cable so that with these delayed actions of two seconds or thereabouts, instead of having a small hole in the cable we now very generally involve all three failures in the cable burnout before the breaker opens, and sometimes there is perhaps two or three feet missing. That is one of the many important things that we now have to guard against in these burnouts.

If you have an eight-second limit, as one man has suggested, I venture to state that on the system of the Commonwealth

Edison Company, we would in nearly every case involve other cables in the conduit by burns through the conduit system into adjacent ducts. Our experience has indicated that to be the case with very much less than eight seconds, where the relay not operating properly had delayed the action to perhaps three seconds.

P. H. Adams: We are using a maximum setting of two seconds on the lines of the Public Service Electric Company of New Jersey, under short-circuit conditions. These relays, on overloads as high as 100 per cent, operate at from six to ten seconds. I believe that the Committee intends that the two seconds value should apply to short-circuit conditions and not to overload conditions.

R. N. Conwell (by letter): Many operating companies are undoubtedly making trial installations of newly developed relays or relay schemes to fulfill certain conditions in their systems. I would suggest that complete reports to the Protective Devices Committee on the operation of these trial installations would prove of great value if compiled and published annually, for in this way, wasteful duplication would be eliminated and real progress obtained. The Protective Devices Committee should also maintain a list of trial installations so as to afford opportunity for simultaneous tests of several different types of relays or relay schemes for the same conditions.

The co-ordination of transmission, station and substation design, and the application of protective relays is necessary in all systems, but as systems increase in size, the matter increases in importance. A new line loses much of its commercial value if it is so connected into the system that trouble cannot be properly sectionalized. Tie bus or section reactances do not completely fulfill their functions if the transmission system is so interconnected that these reactances are shunted by lines, thus increasing the requirements for discrimination on the part of the relays, in addition to subjecting much of the transmission system to needless strains under short-circuit conditions. Careful grouping of lines from the standpoint of relay protection as well as operation will undoubtedly result in the minimum cases of trouble, minimum short-circuit current, minimum discrimination required of the relays, and minimum interruptions to service.

Careful analysis of many systems will reveal the fact that more effective protection may be obtained by changing operating schedules to conform with relays already installed. The possible slight reduction in efficiency in transmission will be more than compensated for by the reduction in the number of interruptions.

The effectiveness of any application of relay protection is dependent upon complete cooperation between those having charge of operation and those having charge of the protection of the system. The most carefully planned application of relays for transmission line protection, and the most carefully calculated settings will fall far short of the requirements if the system is not normally operated in such a way that the relays may function as designed and set.

A. W. Copley (by letter): In this paper the authors have laid emphasis upon the desirability and importance of taking into account the relay system to be employed when a transmission system is being planned. At times, however, departure from standard relay practise may be warranted, because of certain advantages in a type of system for which relay protection has not been entirely developed. The recurrence of similar cases finally brings the new development into the standard class.

C. P. Osborne (by letter): The average operating engineer is prone to let well enough alone, especially during such times as we have been passing through in the last few years when it was difficult to secure financial assistance for betterments.

The company with which the writer is connected cannot boast of its relay protection. However, in a country where there is a small amount of lightning and where weather conditions are more or less uniform, I feel that protective devices are not as important as in a more severe climate. My experience is that it has been only in recent years that manufacturers have put out protective relays which are a real protection to a transmission system. The Bellows type relay has proven one of the most satisfactory in the writer's experience, but it must be properly taken care of and tested periodically and even then its protection is not of the best. The experience of a great many companies certainly shows that the a-c. relay does not give the protection to transmission systems or tie lines that the d-c. controlled relay does. One of our greatest troubles with high-tension oil switches, where a-c. trips are used in series with the line for protection, is the inaccuracy of the relay. The writer has found it necessary in some cases to change the mechanism to get a relay to properly trip, and our opinion is that the manufacturers have not as yet gotten an a-c. relay for high-tension transmission lines which is accurate enough to give the operating engineer assurance that his lines are protected and his circuit breakers will open when line trouble is experienced. Where there are four or five high-tension lines generating from stations feeding into a network at the main distributing point where all tie lines are in parallel, it is too often the case that trouble on one high-tension line will cause disturbances which will affect the whole system. Especially is this true if one line should have a swinging ground and we have frequently found this to be the case where insulators are breaking down. I am a firm believer in running grounded neutrals on high-tension lines where it is possible.

The authors of the paper bring out a point in the first paragraph of the fourth page of the paper which seems to the writer is putting the reliability of the relays up to the operator instead of making them more or less fool proof and automatic. I do not believe the operator should in any way be responsible for the action of the relays, except that the relays must be kept in good working condition.

It is difficult for operating companies to keep up to date on relay installations as any installation which is two or more years old is more or less out of date. The advances in recent years make so many different changes and different kinds of relays for protective apparatus that the operating companies are often at sea to know just what to order for giving the best protection to their lines.

P. M. Lincoln: I think the work that this Committee is doing is one of the most important that we have before our Institute. The thing which our transmission systems and supply systems must have above everything else is reliability. Service must be reliable and as systems increase in size we have two elements that are working against that quality. In the first place, the very fact that the systems increase in size and extent of itself makes them more vulnerable because of the greater amount of apparatus and equipment and lines which are exposed to the possibility of trouble spreading. In the second place, the need of greater reliability becomes greater as the system increases in size. If the larger systems cannot give service without the possibility of continual interruptions, the service loses its value. I certainly hope that the Committee will continue its work and give us in the future, as they have in this report, resumés of the advances that are made from time to time in protection of systems.

C. P. Steinmetz: Broadly and bluntly speaking, the problem of relay protection essentially consists in producing a simple, reliable and automatic device to disconnect an abnormal circuit without interfering with the operation of any normal line. *This problem has not yet been solved.* The formidable difficulty in the way of solution of the problem is that the abnormal currents or voltages exist not only in the abnormal circuit element but also in many or most of the normal circuit elements. And therefore, any device depending on the existence of abnormality of current and voltage would not inherently discriminate between the normal and abnormal circuit element, and cannot solve the problem.

The attempt of solution thus has been made discriminating by the degree of abnormality, which means time element. This is the present solution which naturally is limited, as you all know. Therefore, the problem on the whole is to discriminate between normal and abnormal circuit elements, and it still stands unsolved as the most formidable and most important problem before you.

E. T. Moore: I presume the discussion of this paper from an industrial man's standpoint is rather different from that of a distributing company. However, the functioning of a relay is common to both viewpoints and I therefore venture a question regarding the protection of a feeder line carrying energy to an electric furnace. Are the present types of relays capable of giving necessary protection? Certainly no type except the induction type, and even these will not always function in the same time and in the same way in each instance of overload.

N. L. Pollard: I would like to emphasize the necessity for each system to have some one man who is thoroughly familiar with the relay situation to have charge of the relays. It is certainly a waste of money to install expensive relays and then neglect to properly maintain them. The man in charge of the relays should be properly equipped with suitable timing and calibrating instruments.

R. F. Schuchardt: It seems very strange how long it took for us to appreciate the marked effect of these relays on the investment. We were all apparently too willing to wait for the coming of perfection instead of using the best that was available, and having the development take place by experience rather than expecting too much from the manufacturing engineer.

Perhaps five or six years ago, we in Chicago came to the conclusion that we must realize some of that saving in investment which is possible by fairly complete interconnection of the transmission system. We tried what was best at that time. We built an artificial line by using some of the existing lines in part and adding artificial sections in a substation and tried out what was then available. As a result, certain improvements were made and we put the improved devices on our system.

What is now found on the Chicago system is the result of that experiment and because of the extent of that system with its 800 odd miles of underground transmission lines, the experience may be of interest since sooner or later other companies will grow and will have to meet the same problems that we have had to solve.

During the war we were forced to look for more opportunities to save on our investments, so we put into effect more interconnections requiring relays, even though we knew they were not perfect. A description of the system used in Chicago was contained in a paper before the Institute some time ago. Some of you may be familiar with it, and the main elements are contained in this report. The system is a combination of time-element selective relays, unidirectional relays and balanced relays.

It is a rare case now when the relay system fails. We do have times, occasionally, when more than the faulty line is disconnected but those are not by any means a very large proportion of the total failures.

Now, I want to emphasize particularly one statement made in the report with reference to the ability of the men employed to maintain the relays because it is only by having the proper talent applied at that end and learning from their experience that the manufacturers will be able to know wherein to improve.

I would like to see the Protective Devices Committee serve throughout the year as a clearing house. Such a scheme was recently put into effect by the Electrical Apparatus Committee of the N.E.L.A. in which the member companies send to that Committee, descriptions of all cases of apparatus operation that they think might help to indicate in which direction the development should be.

J. C. Parker: I would like to emphasize the remarks of Mr. Schuchardt. The matter of relay protection is one, of course, of distinct importance in the engineering of our power stations, primarily of importance only because it involves insurance of continuity of service, not because of its protection of the apparatus itself, protection of the apparatus is of importance only insofar as it involves the continuity of service.

If we keep that thoroughly in mind, our question as to the relative speed of operation of relays close to a fault or close to the generating apparatus will automatically solve itself.

Mr. Schuchardt called attention to the fact that we are a little tardy in availing ourselves of the best that has been developed to date, possibly through a spirit of false economy or possibly through the desire to wait for the ultimate ideal of perfection in our apparatus. I rather wonder whether that is not due to the fact that as engineers we are too little inclined to recognize the ultimate object in all of our large power stations which, as in all engineering enterprises, is an economic one. There is no factor that is more significant in the capacity of a central station organization to hold and retain business at a satisfactory earning rate than continuity of service.

I have in mind the instance of one corporation which a few years ago was forced to accept a reduction in its receipts for power service from two large customers which, on a five-year contract, aggregated $50,000, a condition which will doubtless continue through the rest of the history of that concern. The reduction in earnings was due absolutely to lack of continuity of service. Obviously, such a concern might have spent easily a half million dollars, if necessary, in securing continuity of operation.

I know at the present time of another very large and very important concern which faces the loss of several important consumers because of its failure to isolate faults through adequate relay protection. If we keep in mind the commercial significance of continuity of service and keep in mind the actual dollars involved in that sort of thing, I believe we will not hesitate to spend fairly large sums of money in using the at present confessedly imperfect protection devices rather than waiting for absolute protection.

H. C. Don Carlos: There is one point brought out in the paper which I wish to question and that is the limit of two seconds which was set as a maximum time for relays. In the Ontario Power Company's plant there has been maintained a time setting of eight seconds on the generators, and on the cables supplying the hydro system from the Ontario Power Company we have maintained a time setting of not less than four seconds. We have never yet had a case of cable failure on account of this long time setting, so our experience would not agree with the recommendation in the paper for short time setting. As a matter of fact on an extensive transmission system it is impracticable, I believe, to maintain such a short time setting of relays, as you will readily see that we could not graduate the relays on long feeders and get selective action with the difference in time that we would have to give to the different relays at the different stations. It has been our experience that we could not maintain a setting of less than three-quarters of a second difference in time between relays and get any results. I do not believe that with present type of relays and circuit breakers they can be depended upon to duplicate their action. The relays are not accurate enough in their time relay and the circuit breakers will take longer to operate one time than another. I would say that this difference in time between the different successive relay settings would be necessary.

D. A. McKenzie: I would like to know how the value of two seconds was arrived at as an ultimate limit. The synchronous apparatus would be off the system before that time had elapsed and the extension of the time limit to allow a relay further out on the system to operate would not increase the damage due to heating very much.

F. L. Hunt: Regarding the matter of maximum time setting on relays, I feel quite sure that circumstances under which we have to operate will control that. We use three and four second settings right along.

F. C. Hanker: The paper prepared by the Protective Devices Committee will go a long way toward establishing uniform practise in relay protection and brings out very clearly and definitely the importance of giving full consideration to this phase of the problem when laying out transmission systems.

The one big advance that is evident is the development of a relay with a dependable time element that maintains its accuracy. As late as 1907 the very lack of such a dependable time element forced the adoption of a control system on the New Haven electrification, involving the installation of control wires over the electrified section. The additional cost was probably on the order of $40,000 to $50,000 and could have been avoided if we had had the modern types available.

The importance of the information that has been submitted is recognized by everybody and it is suggested that the Committee for the ensuing year continue the work and include other applications such as balanced protection where currents are out of phase, as in two-phase three-phase transformer banks. The usual schemes for these involve more or less complicated wiring and special arrangements should be investigated. Similarly, cases where parallel transmission lines are involved, having tap connections at sub-stations, should be carefully analyzed.

O. C. Traver: Regarding Mr. Johnson's comments, it would be very nice to have a system which would reduce the time of operation as you approach the generators, but at the present time in the art there are no relays which will perform this function excepting so far as inverse time delay will accomplish the result. We agree with him entirely on the development of breakers for giving us quicker time. It certainly is something that needs much attention.

While it is true, as Mr. Johnson suggests, that selective relaying must sometimes be handicapped on account of circuit breakers of insufficient interrupting capacity, still the question of cost will always be a deciding factor, the loss in revenue due to poor service and of equipment due to delayed action, being balanced against the increased capital outlay. The evidence submitted by Prof. Parker, though relating primarily to relay equipment, indicates how one can misinterpret the effect of these details on dividends.

On the other hand, the use of current limiting reactors, we consider in general, an aid to good relaying in that they tend to insure ample potential for directional relays and provide a more nearly definite amount of fault current for all conditions of operation.

Dr. Steinmetz stated that the relay problem still stands unsolved. That is true much the same as if the same remark were made concerning any other matter of engineering, relay perfection does not exist. This is also the case with the greater portion of power station equipment in about the same degree. Relays of a reasonable accuracy are available. The question has resolved itself into a matter of application more than of manufacture.

A number have spoken of the maximum interval of two seconds. There has been discussion on both sides, to increase it and to decrease it. The maximum interval is largely a function of the system and the conditions which have to be met.

A two-second delay maximum is considered advisable; first, on account of possible damage to equipment, particularly in the larger metropolitan systems, and second; on account of the greater liability of synchronous apparatus falling out of step.

The damage to equipment may involve overhead lines as well as cables. Arc over of an insulator, if cleared promptly, frequently leaves that insulator uninjured, whereas an additional delay might cause the complete loss of the line at an embarrassing time. The effect of sustained shorts circuits on a cable was brought out in the discussion by Mr. Roper.

SAVINGS EFFECTED BY PARALLEL OPERATION OF FEEDERS

H. C. Albrecht: I do not understand what relation reduction of 40 per cent of the burnout of cables on one particular system has to parallel operation. To me this seems to be but a coincidence. "After parallel operation was adopted, the number of burnouts was reduced by 40 per cent."

E. A. Hester: Regarding Mr. Albrecht's statement with reference to the case cited in which a saving had apparently been effected by operating feeders in parallel it may seem coincident since only one incident was given. However, it has been called to our attention by at least two other companies that as soon as they began to operate their transmission lines in parallel they noticed a very decided reduction of transmission line trouble and consequently a reduction in maintenance cost. The committee made inquiries of several operating companies regarding their experience along this line but most of them replied that they had never made the change over from radial to parallel operation with relays, one method or the other having been used from the beginning. That the same should be experienced by three companies who had made such a change and at a time when their transmission systems were rapidly growing and becoming more complex would indicate that a reduction in maintenance cost at the same time that parallel operation was adopted is more than a coincidence.

TESTING AND SETTING RELAYS

H. C. Albrecht: I question whether or not it is practicable to test on the primary of the current transformer. It is expensive and takes considerable time. I would like to have more information as to where this is being done and in what manner. I am somewhat astounded at the curves in Fig. 2 showing the error in current transformer testing under various conditions. I notice that this curve refers to a single-turn current transformer. It was my impression that one of the standard lines of these transformers could be loaded up to 400 per cent load with only 2 per cent error, while this particular curve shows 70 per cent error for four-times load. I also question whether this transformer was not a bus type transformer, and whether we should not differentiate between single-turn and bus-type transformers, and it seems to me that this is a very poor record for even the bus-type transformer.

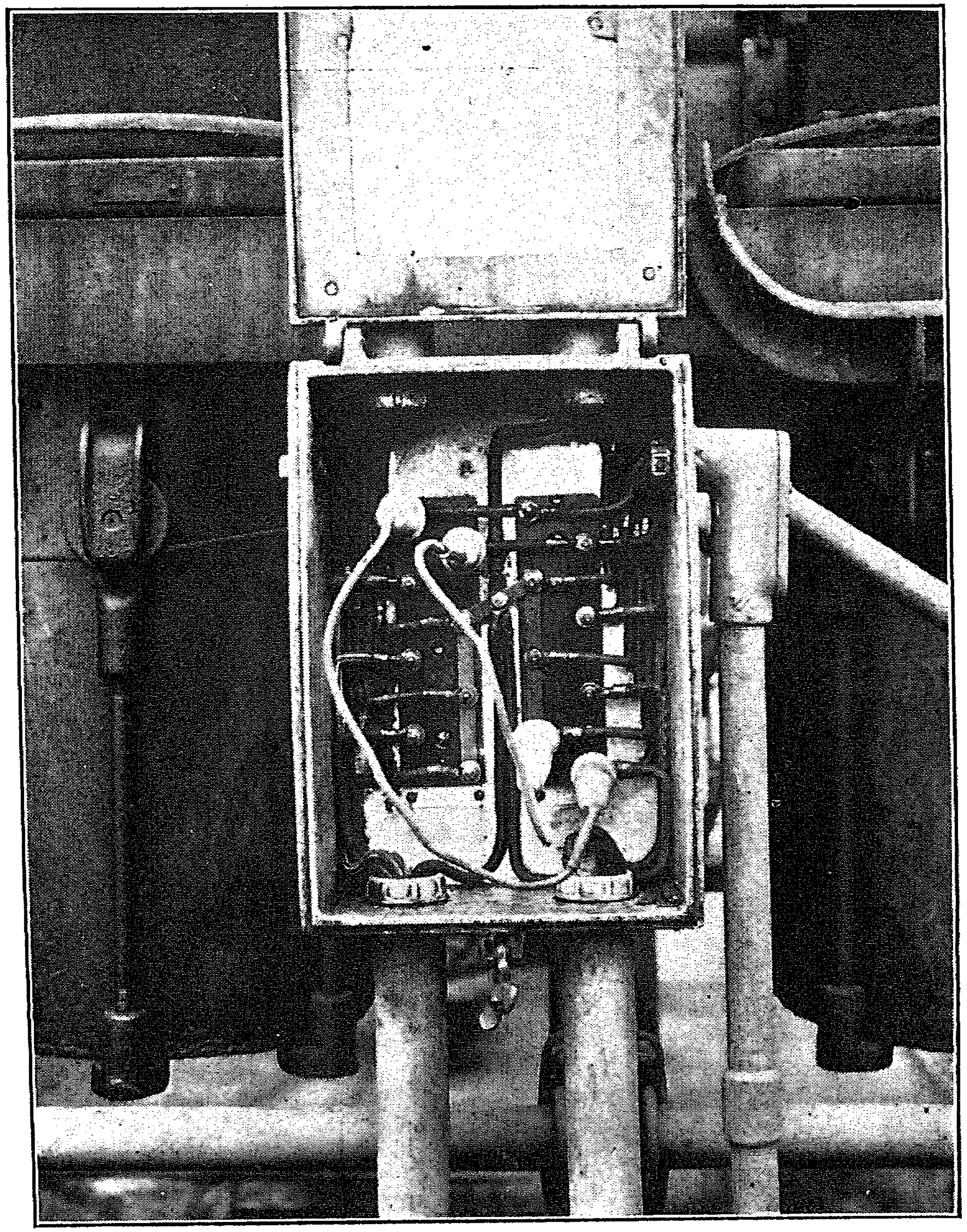
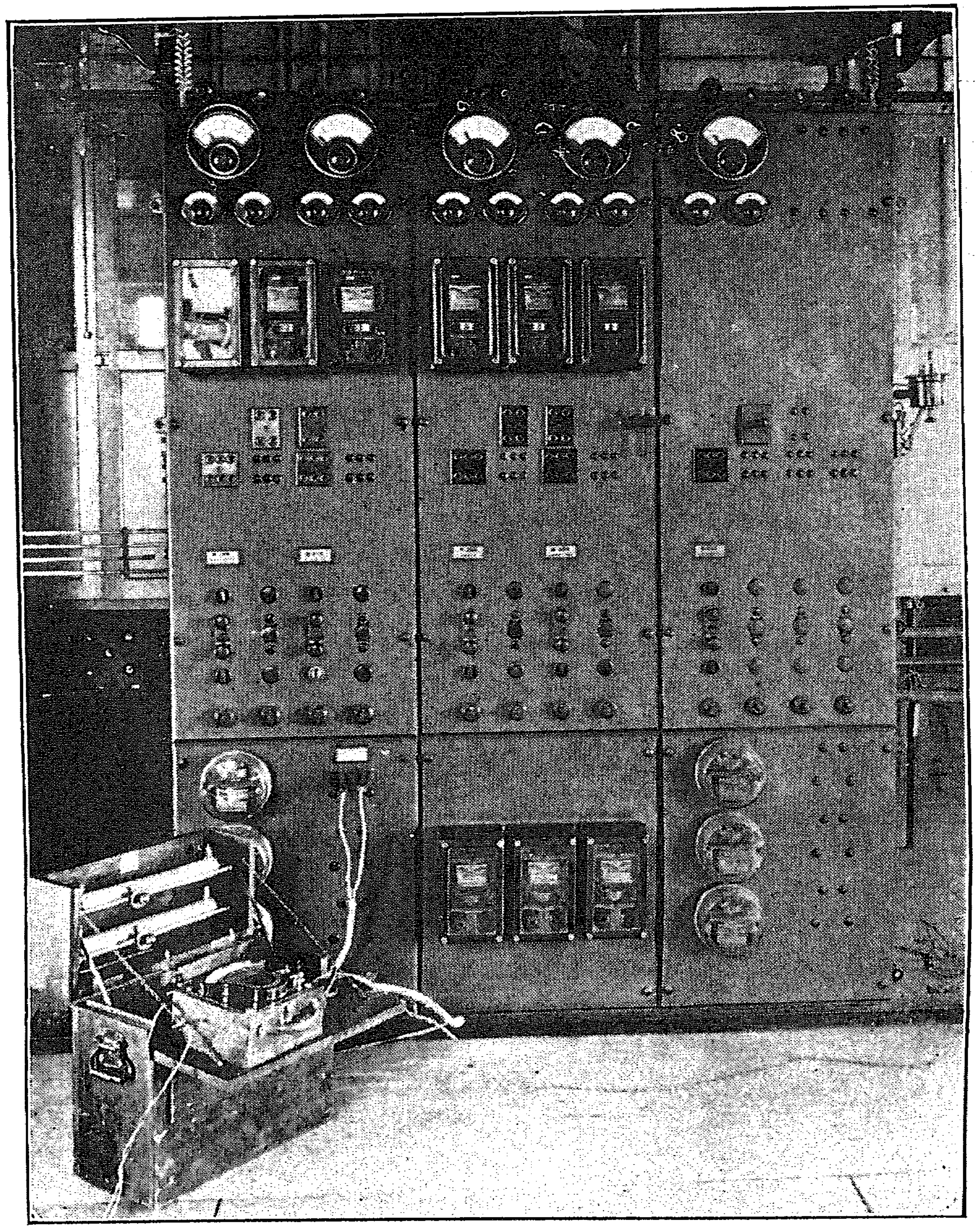


Fig. 1—Pull Box on Oil Switch Showing Test Leads Connected in Shunt to Relay Leads at Current Transformer Terminals



[CONWELL]

Fig. 2—Front of Line Panel Showing Relay Testing Set Connected to Test Leads

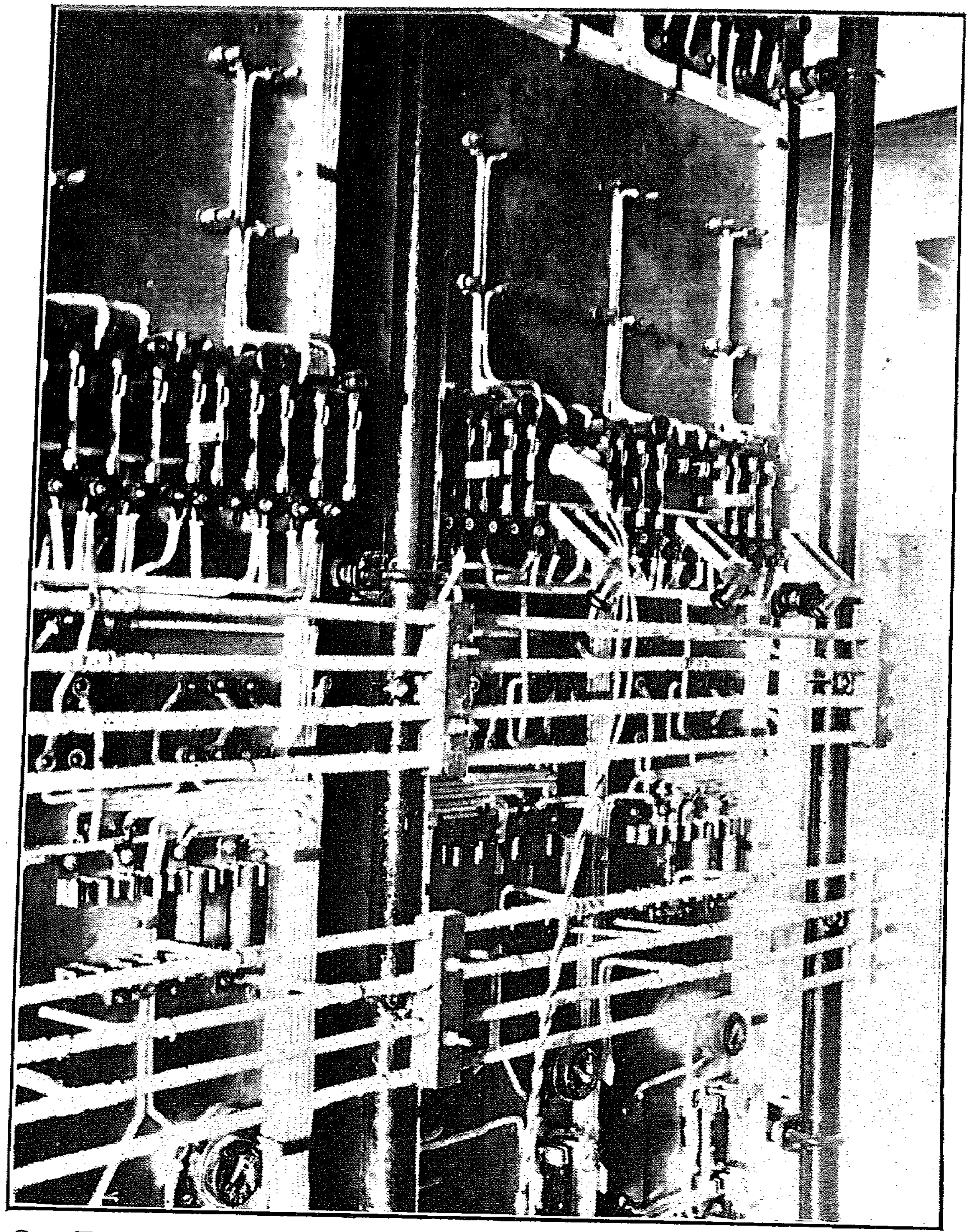
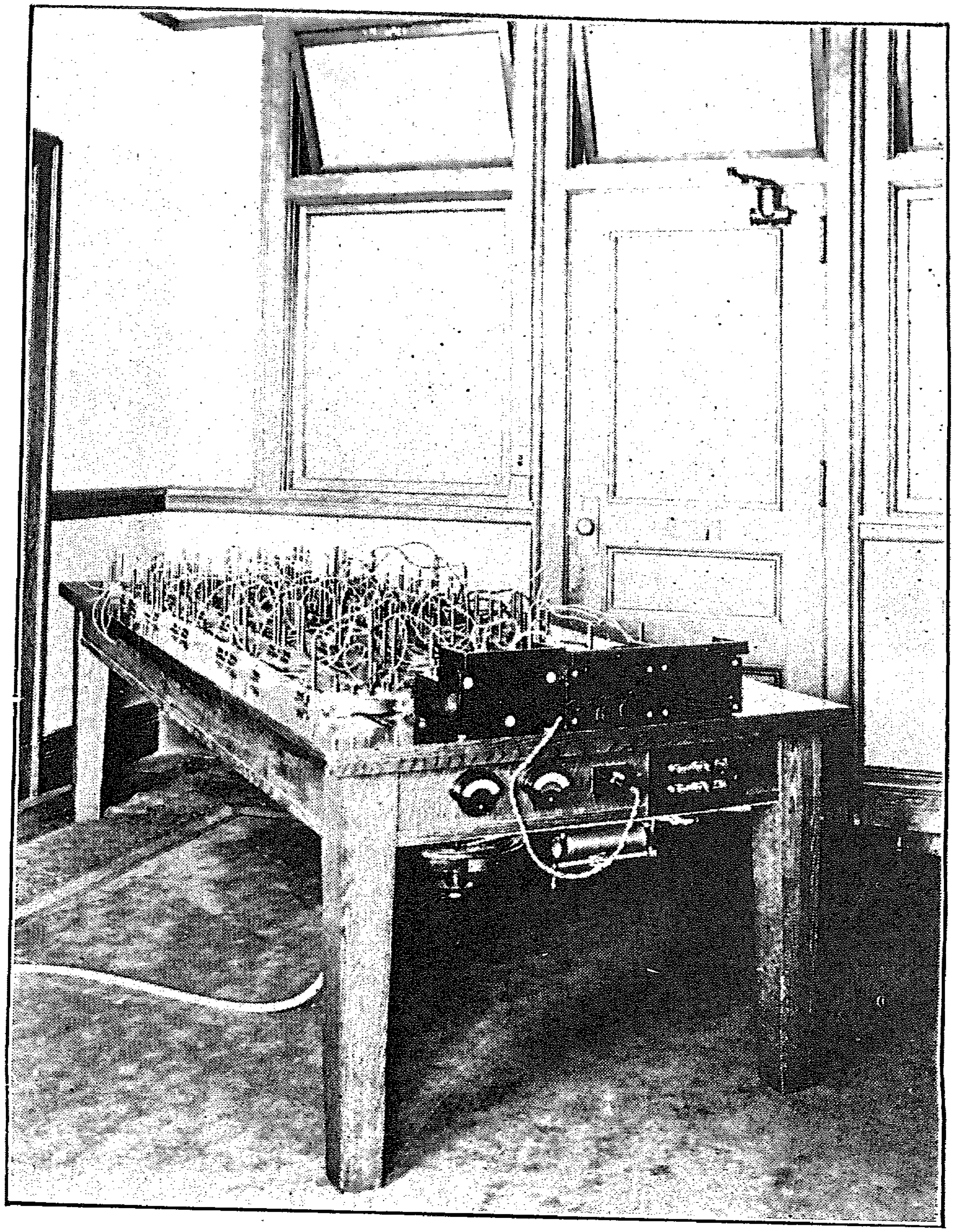


FIG. 3—BACK OF LINE PANEL, SHOWING LEADS TO CYCLE COUNTER CONNECTED TO TRIP CIRCUIT OF RELAY—SWITCH IN CURRENT LEADS CLOSED



[CONWELL]

FIG. 4—CALCULATING TABLE USED FOR THE DETERMINATION OF SHORT-CIRCUIT CURRENT VALUES

E. G. Merrick: It is not necessary to reduce the time settings of the relays near the source of power below two seconds for protection to the generators and transformer equipment, as a modern generator and transformer will withstand short-circuit current at the terminals for several seconds without difficulty.

The improved relay situation of the Montana Power Company, as mentioned by Mr. Lincoln, is the result of a recommendation of a Committee as taken from test table data and the present operation shows 90 per cent perfect operation. The present main difficulty is due to high resistance grounds, which do not give sufficient current to trip out the relays, and this feature will probably have to receive further consideration in the future.

R. N. Conwell (by letter): The test schedule which should be adopted for relays will probably be best determined by testing the relays very frequently at the start and keeping a record of all tests. An inspection of the test record will clearly indicate the time interval which should be allowed between tests, the interval varying with types of relays, location and duty.

The test method described on pages 801 and 802 has been used on the Public Service Electric Company's system with considerable success on relays connected to high-turn ratio bushing-type current transformers, used in conjunction with power-directional and current relays on 26,400-volt transmission lines. Fig. 1 shows one of the pull boxes mounted on the side of each outdoor oil circuit breaker, the terminals of the bushing-type current transformers being brought to a terminal board to which the relay leads are connected and a pair of test leads running from binding posts on the switchboard, through each pull box is connected to two studs on the same terminal board in each pull box. The relay may then be tested in shunt with the current transformer by connecting the testing supply with necessary control apparatus and meters to the binding posts at the switchboard See (Figs. 2 and 3) and at the oil switch, connecting with jumpers, the test leads to the studs, to which the current transformer leads and relay leads are connected.

Frequently, relays are installed and cut into service without any final check, other than inspection, being made of the wiring. Experience has shown that even when a standard color scheme is used for the secondary leads, in fully fifty per cent of the new installations, the relays will be found to be cross-phased, a ground connection improperly located, or some other mistake made which will result in faulty operation. With the complicated wiring necessary for the more recent applications of balanced protection, the problem of phasing out becomes difficult.

A single-phase, four-quadrant power factor meter, reading from zero to unity, lag and lead, has been used for work of this character, the results obtained being such as to warrant the general adoption of this test method. If currents are to be phased, the potential coil is energized by connection to any convenient 110-volt a-c. source of the required frequency. The current coil is then connected successively in each circuit to be phased, direct and reversed readings being taken from which the relationship of the currents may be determined. Currents in phase will give the same deflections on the power factor meter. If check readings are not obtained the vector diagrams may be drawn from the readings and the error in wiring determined. Although the meter used has the usual cosine scale, a scale marked in degrees would eliminate the necessity of converting cosines to the equivalent angles.

For the phasing of potentials, the current coil of the meter is energized with an artificial load, test current being obtained as above, the potential coil being connected to the circuits to be phased.

Two conditions must be fulfilled to obtain satisfactory results: (1) the excitation of the meter coil used for reference must remain constant in direction and value throughout any series of tests and (2) sufficient torque obtained to give correct indications on the meter. Generally, half load on the meter with full potential is sufficient.

C. P. Osborne (by letter): There is not enough attention paid to checking and time-setting of relays by most operating companies and unless this is followed up by one certain engineer they are very apt to be neglected. The regular routine testing is, in my opinion, the only way to keep relays in a workable condition. The inspection of relays should be made by every operator on shift every day. This is just as important as it is for the operator coming on shift to look over the minutes and discuss with the operator going off shift any difficulties which he may have had or which may be on at the time of the change in shift. Relays should be tested under operating conditions wherever possible, thereby not only checking the apparatus under load conditions, but also giving the operator a chance to get familiar with the workings of the apparatus. The writer does not believe that watt-hour meters should be connected in series with relays where it can be avoided.

N. A. Carle: I think the method outlined will be of assistance in eliminating a few more interruptions due to inaccuracies in current transformers.

D. A. McKenzie: One gentleman has questioned the accuracy of the curve in Fig. 2 with reference to the operation of current transformers. I do not get the distinction between the single-turn current transformer and the bus-bar type but would think that both of these transformers would have similar characteristics. In my opinion the 75-ampere current transformer of a single-turn type would not give satisfactory results and should not be used in any relay protective scheme regardless of voltage rating or secondary load, unless possibly all the current transformers in the series were of the same type and carried the same secondary load.

I would like to ask if any company has made an investigation of the standard instrument-type current transformer under short-circuit conditions to determine the comparative shapes of the wave form on the secondary current as compared to the primary and also to determine the phase relation between these two. With a current transformer carrying a secondary load of 40 volt-amperes at five amperes secondary current and operating at twenty times full load and attempting to deliver 16 kv-a. the iron in the transformer would have been saturated long before this limit was reached and the wave form considerably distorted. This would have some effect on the use of directional relays containing potential elements. It would appear to me that wave distortion would affect their operation.

I do not believe it is practicable to set relays from the primary side of the current transformer and it should not be necessary to subject all your equipment to short-circuit strains for periodic testing.

I think that the current transformer may be tested at relatively low values of current and ammeters placed in series with the secondary winding will bring an indication of the normal operation of the current transformer and continuity of secondary wiring. It might be desirable to test the current transformer each time after testing the relay, and I believe that a relay should be tested immediately after each operation regardless of the time of the routine test.

The maximum indicating ammeter as mentioned by Mr. Craighead is a desirable piece of apparatus for obtaining operating conditions under short circuit.

H. R. Woodrow: Some of our most severe system disturbances have been due to failures in current transformers and we have therefore adopted the single-turn type of transformer for our generating stations where very heavy short-circuit currents are to be expected.

The record shown in Fig. 2 of the paper is for a standard 75-ampere single-turn current transformer without any special precautions being taken for accuracy and although this is an unusually low current for a single-turn transformer it is a type which is to be found on the system. For standard 400-ampere single-turn type of transformer the error would be in the order of four to five per cent. The accuracy is largely dependent upon the amount of secondary loading and, as a matter of fact, the length of the secondary wiring changes the characteristics of the relay settings even where identical relays are used. We have even found conditions, where the secondary loading is so high on a low-rated single-turn current transformer, that the relay was practically inoperative as insufficient current could be obtained due to the saturation of the current transformer.

As pointed out by Mr. McKenzie, a series ammeter in the secondary circuit of the current transformer will indicate continuity of service but it will not show the change in the numerous contacts in the circuit which may come loose and thereby require abnormal secondary loading of the current transformer which will entirely change the characteristics of the relay setting.

We have made investigations determining the wave shape of the secondary current of a current transformer up to 200 amperes (secondary current) and found that the wave became very peaked but with the installation of directional relays, if proper potential connections are made, this wave distortion will not change selectivity of direction, which is all that is required of a directional relay

The single-turn and bus-type transformers are in the same class giving considerably greater mechanical strength but a tendency to poorer accuracy especially in the lower current types.

I agree with Mr. Merrick that most generators and transformers are capable of withstanding short-circuit currents for a period of two seconds as regards heating but many are not capable of withstanding the mechanical force produced under short-circuit currents.

As pointed out by Mr. Conwell considerable care should be taken in the connections of directional and balanced relays as an improper connection will invariably result in failure of proper selectivity.

O. C. Traver: I might say concerning the bushing transformers that the single turn is considered just the same whether it is directly around the core or whether it goes around by a circuit of several miles; the difference is so slight that it is scarcely noticeable.

I would like to make a plea in favor of using a higher turn ratio on bushing transformers. From the manufacturers' standpoint, time after time we have to ask the operating company if we cannot use a higher tripping point for the relays. He may think that we are trying to put something over on him, but almost invariably it is for this same reason which has been brought out several times this morning—namely, that the accuracy of a bushing transformer with about 70 amperes turns (that is, 70 amperes with a single primary turn) is exceedingly poor, especially when used with a time relay. If instantaneous action is involved, it does not make much difference. So, in general, if you can start at 150 amperes for your minimum operating point when using your relays with a bushing transformer, you will begin to get reasonably good results.

If you can, as Mr. Woodrow has just mentioned, use 300 amperes instead, you will begin to get accuracy equal to the best accuracy you can get in an instrument transformer, and sometimes much better on account of the much greater amount of iron that is used in the single-turn transformer.

Determination of Relay Settings

H. B. Vincent: The paper speaks of the successful relay settings being three-quarters of a second. I would like to ask if this is sufficient in all cases. Is it understood that the Consolidated Gas, Electric Light & Power Company of Baltimore, and others use a shorter value very successfully, and I also raise the question—"Does this entail anything special in the relay or in the testing of the relay and breaker?"

J. R. Craighead: The improvement that has been made within the last few years in the accuracy and speed of operation of oil circuit breakers has rendered the shorter times spoken of by Mr. Vincent practicable in a number of cases, and in order to obtain the benefit of these shorter times a corresponding improvement in relays is required. This improvement has already made progress but there is still room for further advance along that line in order to keep the oil circuit breaker, and the relay in such relation that we will not waste time in either. I believe it is quite practicable to cut the differential of time somewhat below the three-quarters of a second mentioned with satisfactory results.

A thorough understanding of conditions is the most necessary thing in the application of relays, and the difficulty of getting that understanding has been mentioned in the paper. The best way of getting the data is the practical method of obtaining actual measurements when possible. Until recently, this has been impracticable under operating conditions. By the development of a new instrument called an "Impulse Ammeter," records are now made of the approximate highest effective value reached during a short circuit on an operating line. This device is an indicating ammeter with a current scale high enough to cover the short-circuit current value. Under the pointer is a record paper which under short circuit, is punctured by a spark discharge from the pointer, thus recording the highest reading of the instrument during the short circuit.

H. C. Albrecht: The Philadelphia Electric Company adopted a short time ago the use of the cycle counter in setting relays, and since that time on our 60-cycle relays we have been using a time interval of twenty-five cycles or five-twelfths of a second. That is based on allowing fifteen cycles or a quarter of a second to the oil circuit breaker operation, and ten cycles in addition for inaccuracies in relays, etc.

I would like to question whether too much attention is not paid to reduction in bus voltage on short circuits, as our experience has shown that the sustained short-circuit value is not very much reduced from the instantaneous value.

Regarding the question of whether to set relays for short-circuit conditions, I question whether it is necessary to test at currents approximating maximum short circuits, as most relay curves flatten very much before this point, and it is the practise of our company, for instance, to set the relays at the point where the curve starts to flatten.

N. L. Pollard: Mr. Don Carlos questioned the minimum setting of relays on lines in tandem. The Public Service Electric Company maintain a difference of four-tenths of a second between the settings of successive relays and get very satisfactory results. It would seem to me that one-half of a second would be sufficient for almost any type of relay that we have on our system.

Will Mr. Don Carlos tell us what type of relay he referred to? The relays that we are using are of the induction type, and we have no difficulty at all in operating with a difference in the time setting of four-tenths of a second.

R. N. Conwell: The Public Service Electric Company is using a calculating table (See Fig. 4) similar to that referred to on page 804. Such a table is necessary if successful relay operation is to be obtained on large interconnected networks. In many of these systems the mathematical calculation of the short-circuit value of current is practically impossible, or at least, very tedious. The table quickly supplies information relative to the effects of changes in operation or design. The work of making a complete analysis of a system is not much greater than that necessary for relay settings so that the rupturing duty required of circuit breakers and other valuable data may be obtained with the same set up and at the same time.

R. F. Schuchardt: In connection with the time-element relays used in Chicago, our cable breakdowns now are more severe than they were in earlier days before we had so much power or so much ability to get energy into a fault. That may be due to the increase in power or to the increase in time setting or perhaps both.

D. A. McKenzie: Referring to Fig. 3 of the paper, it is interesting to note that selectivity between breakers *B* and *C* changes at about 4200 amperes and I would like to inquire if the breaker capacity was the determining feature in this. Should it not be necessary to have those curves so adjusted that they do not cross.

I was interested to see that three-quarters of a second was considered a permissible interval between relays. That appears rather short on account of the different types of circuit breakers which would be used, some of which may operate faster at one time than another, depending upon adjustment since last operated and to mechanical condition.

I believe that the relay problem for protection of electric steel furnaces is an important one and can be solved if the wave form of a primary current and the distortion in current transformers under short-circuit conditions are considered.

H. R. Woodrow: Several have questioned the advisability of allowing three-quarters of a second time interval between consecutive relays for selective operation as shown in curves of Fig. 3. There are a number of conditions which effect the allowable time interval. First, the type of relay, its characteristics and reliability; second, time of operation of the oil switch (time from closing of relay contacts until switch ruptures the arc); and third, the care with which the relays are set and kept in operating condition.

As pointed out, several companies have found a shorter time allowance desirable but I believe the conditions require a very careful analysis, especially where a time allowance below three-quarters of a second is considered.

It is interesting to note from Mr. Craighead's discussion that a maximum indicating ammeter will soon be placed upon the market which will give the operating companies a more accurate indication of the currents experienced under short-circuit conditions.

I would consider that if the relays are calibrated out to the flat point of the curve, as mentioned by Mr. Albrecht, it would give conditions approximating those under short circuit. We do not feel however that proper selectivity can be obtained unless the calibration point is carried out to this value.

Referring to Mr. McKenzie's questions regarding the changes in characteristics of the different circuits, shown on Fig. 3, would say that these characteristic curves were made in this manner to reduce the time which a short circuit would be held on the system near the generating station and still give selectivity with the lower values of current when a short circuit occurs at the end of the line. The dotted line in curve 3 shows the current and selectivity when the short circuit occurs on the end feeder.

METHOD OF TRIPPING

A. H. Lawton: In connection with the statement that the use of circuit-opening relays for alternating-current tripping has not given very satisfactory results, I beg to inquire what has been the experience with automobile storage batteries for tripping purposes? Has any contact difficulty been experienced due to low voltage, or has the system been entirely satisfactory?

N. A. Carle: The previous speaker asked about the use of the automobile storage battery. We have been using automobile storage batteries in a 1000-kv-a. substation for about two years and have had very good satisfaction.

H. C. Don Carlos: In regard to the use of automobile storage batteries for a source of tripping current, we have used these rather extensively for the last two years and we have had very good results with them and we have recently used the ordinary dry cell for tripping current very successfully and the indications are that the maintenance cost on the dry cell will be very much lower than with the storage battery and will give equally as good results.

I might say that we have used both the lead cell and the Edison battery for tripping current and I don't know that we are in a position to say just at the present time whether one would give better results than the other or how the maintenance and operation charges would compare with the two.

D. A. McKenzie: I believe that the lead cell is going to be more economical in maintenance than the Edison cell, but as far as my experience goes the dry cell will be quite satisfactory for the purpose of tripping switches in isolated stations. I believe that there are some companies that are using dry cells for tripping even switches with 125-volt direct-current control with, as far as I know, perfectly satisfactory results.

H. B. Vincent: No mention is made in the paper of using circuit-closing relays with alternating current by introducing an auxiliary relay.

E. A. Hester: Those who have expressed themselves on the subject are apparently of the same opinion as regards the use of the automobile storage battery as a source of tripping current. And in so far as we have been able to determine from cases which have come under our observation the service obtained has been very satisfactory.

Referring to Mr. Lawton's question as to whether difficulty is experienced with contacts at such low voltage, we have known of but little trouble from this cause and when such trouble has appeared it has been overcome by means of a small auxiliary switch so constructed that a very positive and high contact pressure was obtained. This solved the difficulty and yet allowed the use of sensitive relays for selective action.

As regards the maintenance and operation of lead cells and Edison batteries we do not have comparative costs, though as stated by Mr. McKenzie, it is very likely the lead cell will be more economical. It is also very interesting to note the statements made by Mr. Don Carlos and Mr. McKenzie that dry cells have actually been used for tripping purposes and have given satisfaction even on 125-volt control circuits.

It is stated by Mr. Vincent that no mention was made in the paper of using circuit-closing relays with alternating current by introducing an auxiliary relay. From this we understand that he refers to tripping directly from current transformers on the power circuit. The chief reason for not including this was that a complete description of the relays used and of direct-trip relay schemes would have been necessary in order to make such a discussion of any value. Therefore since the Protective Devices Committee hopes to conduct some investications on special problems, among which is that of direct tripping, it was decided not to try to incorporate this in the paper. However we may say that while several satisfactory methods and schemes have been worked out, direct tripping is not yet on the same footing with the use of a separate source. In our opinion it is possible to devise some means of utilizing the energy from current transformers which will be satisfactory for tripping. The circuit-opening scheme has never proven very satisfactory, more because of contact trouble than any thing else, so there is evidently only the alternative of using a circuit closing relay with auxiliary device.

One scheme which has been used with a certain amount of success is that of taking the tripping current from a voltage transformer connected to the station bus. The disadvantage is that when trouble occurs the voltage is likely to fall so low that sufficient energy can not be supplied by the transformer to trip the circuit breaker. It would therefore appear that since such a scheme is likely to fail on the worst cases of trouble, just when it is most needed, its use would never be justifiable.

TYPICAL APPLICATION OF CURRENT DIRECTIONAL RELAYS

D. A. KcKenzie: The study of this paper indicates that there is apparently no field for the definite-time relay, even for protection of parallel groups of lines in tandem or for the ring system, which cases would appear to me to be the most logical places to use this type of relay where progressive time settings are required. I am of the opinion that the definite-time relay should not be used in transmission line protection but it is a question in my mind whether the definite-time relays should not be used for generator protection or for protection against bus-bar troubles. The current setting would be very large under these circumstances and would be determined by the sustained short circuit obtainable from the generator.

E. A. Hester: Mr. McKenzie states that in his opinion definite-time relays should not be used for transmission line protection. We understand this statement to refer to relays having a strictly definite-time element and agree with Mr. McKenzie from this point of view. However it must be remembered that practically all relays now manufactured are made with a minimum definite-time element and in most cases this is adjustable. This feature has proven itself to be very valuable and while not a definite-time relay in a stricter sense of the word it is often applied as one, current setting being made at such values that operation is on the definite part of the curve. The progressive time settings are obtained by adjusting the definite-time element.

As regards the use of definite-time relays for protection against generator or bus bar troubles we believe it best to use some form of a definite-time relay though the time interval allowed should not be very long. As stated by Mr. McKenzie the current setting will necessarily be very high and for this reason an adjustable minimum definite inverse-time-limit relay can be used to an advantage as the operation will come on the definite part of the curve.

BALANCED PROTECTION SCHEME FOR TRANSMISSION LINES

H. C. Albrecht: On balanced protection in order to obtain continuity of service for two parallel lines, without the discriminating method as shown in Figs. 6, 7 and 8 of the paper, it would be necessary to use four lines between stations, which I do not believe is feasible in many cases.

The discriminating scheme shown in Figs. 9, 10 and 11 for the operation of two or more parallel lines, seems to be full of merit, particularly where the feeders are used between generating stations.

I would like to hear something in the discussion regarding experience with split-conductor cables (which seem to be rather expensive) being used by only two systems. I would appreciate having first-hand information as to this experience up to date.

D. A. McKenzie: The mechanically opposed type of differential relay has not been mentioned and I know of one instance where this type of protection is operating very satisfactorily. This type of relay is not complicated with any potential elements and therefore should have a considerable field of application.

The balanced type of relay used with the split-conductor cable system seems to give quite satisfactory results, although it is handicapped by considerable increase in the cost of cable as the company using it is extending this system considerably.

N. L. Pollard: Someone raised the question as to what results had been obtained with split-conductor cables. We have ten split-conductor cables in service at the present time, and the relay operation to date has been perfect. Since these cables were put in service with proper relays, several insulation failures have occurred and the operation of the relays was perfect in each case. These failures occurred not only between the outer conductor and sheath but also between the inner and outer conductor of a phase.

R. N. Conwell (by letter): It is to be noted that the trend of relay development is toward the adoption of the balanced principle as affording the best method of sectionalizing trouble and cutting the faulty feeder out in minimum time. At the present time, however, most of these schemes are open to one or more serious objections and the limitations of each scheme should be carefully considered before application is made. Some of these objectionable or limiting features are as follows:

1. There seems to be a sharp line of demarkation between protective schemes applicable to paired lines and those applicable to three or more lines, requiring an entirely new installa-

tion of relay protection for three lines when the number of lines in parallel is increased from two to three.

2. The balanced principle applied to parallel feeders will not, as a rule, clear bus or other faults originating within stations or substations, since the fault current under these conditions generally divides equally between the lines, thus maintaining a balance. Additional over-current protection must be provided if faults of this character are to be cleared automatically.

3. When applied to paired radial feeders, some balanced schemes fail with the occurrence of a fault at the substation end of a feeder, due to equal division of fault current between feeders.

4. When practical application is made of seemingly simple balanced relay schemes, the resulting secondary wiring frequently becomes complicated, introducing other weak links in the chain of protective apparatus. Certainly the tendency should be towards simplicity in apparatus and wiring, if ruggedness and uniform performance is to be obtained.

We have under construction or in operation on the Public Service Electric Company of New Jersey system four complete installations of relays, with several modifications in two cases, which may be considered as on trial. It is hoped that sufficient data will be available after a year's operation to make an accurate comparison of the relative merits of the installations. Such comparison is not possible at the present time, owing to the difficulty in obtaining correct settings in one installation, and lack of operation in two others, the fourth being under construction.

A. W. Copley (by letter): A type of transmission system not covered by the authors, but which is frequently met with, has involved the development of a protective layout possessing certain features which can be advantageously applied to the more usual type of system involving parallel feeders.

On certain transmission lines, particularly those for the transmission of power to railway substations, located rather close together, it is desirable to operate the substation bus on taps from the transmission lines, rather than to sectionalize the lines. This introduces a problem, which is not taken care of by the schemes mentioned in the paper. It is necessary in this case to disconnect one transmission line, in case of trouble due either to ground or short circuit, from the bus at every substation, as well as at the ends of the lines. Power directional relays used in connection with breakers between the lines and bus will adequately care for short-circuit conditions, as a short circuit on one line will draw power from the other line through the substation bus, and power flow through the breaker between the affected line and bus will be in the reversed direction from normal.

For the solution of the ground protection problem, balanced protection between the two lines is unreliable on account of the possibility of the normal currents in the two lines being unbalanced. The desired protection may be obtained by the use of power directional relays, connected differentially between the conductors from bus to line and with potential coils supplied by the voltage between an artificial neutral and ground.

With this connection, true directional action takes place on the occurrence of a ground, as the direction of the potential applied to the relay is always dependent upon the particular conductor which is grounded. For instance, on a single-phase system, if a ground occurs on one conductor current in the differentially connected relay will be in one direction, and if the ground occurs on the other wire it will flow in the opposite direction. The potential coil, when connected from neutral to ground will have its potential reversed in a corresponding manner under such conditions. The arrangement for a single-phase circuit is shown in Fig. 5. Three-phase protection will be similar to it. The scheme has been used very successfully on the electrifications of the P. R. R. between Philadelphia and Paoli, and between Philadelphia and Chestnut Hill.

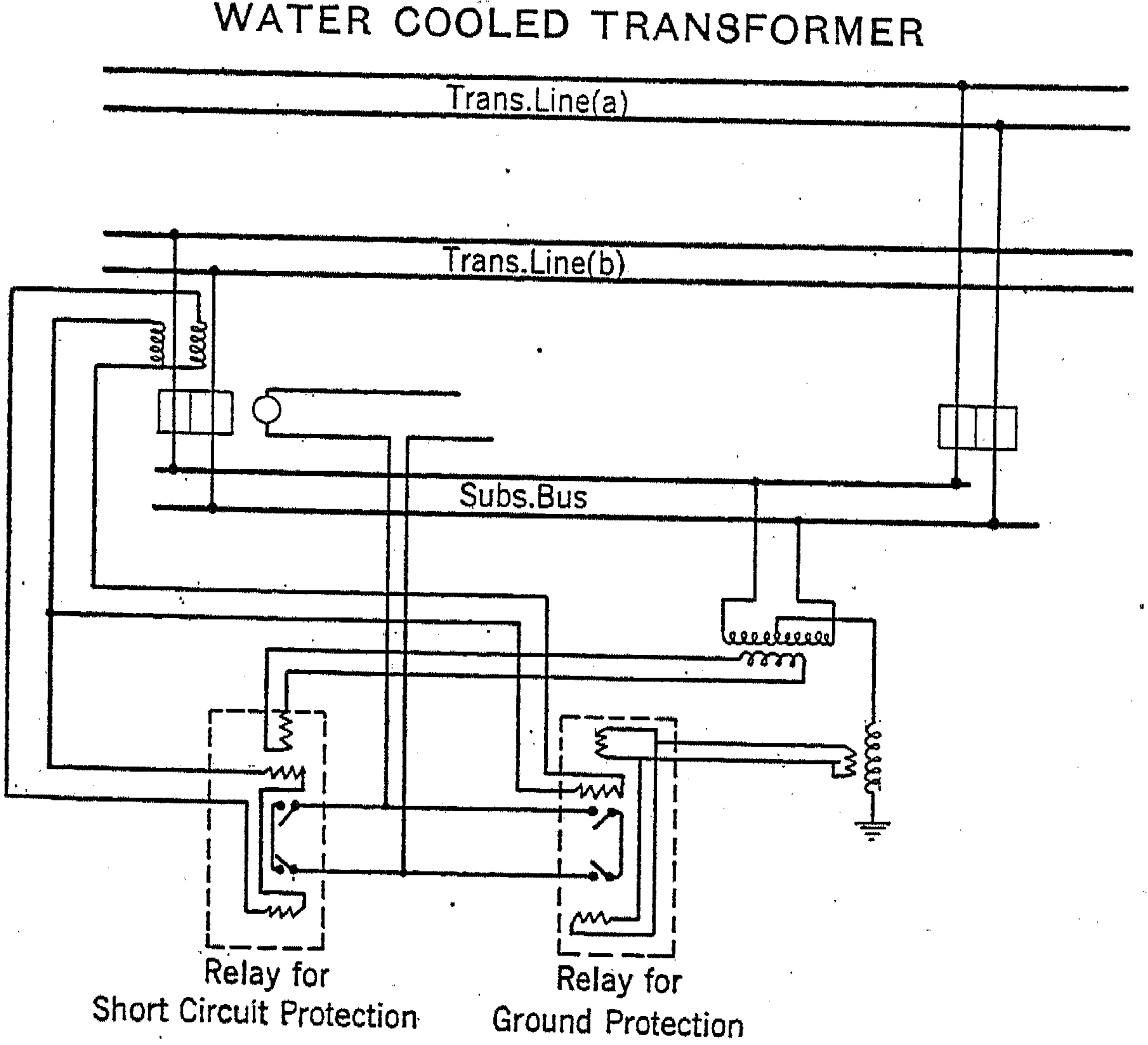


FIG. 5

In order to apply the scheme it is only necessary to establish an artificial neutral at the substation by the use of potential transformers or resistances and to connect a potential transformer from this point to ground, making the connections as indicated. It will be noted particularly that there is neither current nor potential applied to the relay when the lines are clear of ground, but when a ground occurs both current and potential are simultaneously applied in the proper direction to cause selective action.

The scheme is also applicable to the protection of ordinary parallel transmission lines when the neutral of the system is grounded through high resistance, and consequently the current flow incident to the occurrence of a ground may be so low that it is of the order of the unbalanced current which might be normally expected in the parallel lines. In such a case the protection, as shown by the authors in Fig. 9, 10 and 11, will not be satisfactory, but the scheme which has been described will give full protection.

O. C. Traver: With reference to Mr. Albrecht's remarks concerning the balanced scheme shown in Fig. 6 it might be mentioned that this arrangement is applicable where the four lines are installed for other reasons. In the instance cited in the paper the substations were in loop formation and each bus was connected by a pair of lines to each of two different stations from which power was normally available. All the lines were primarily needed for service.

Mr. Conwell states that for balanced schemes an entirely new installation of relays is required when increasing the number of lines from two to three. This is true of some schemes but not of all. Space did not permit of a description of the various possibilities in detail in the paper although in some cases references were made to results which would be obtained with varying numbers of lines in service. For example the scheme illustrated in Figs. 12 and 13.

Regarding Mr. Conwell's objection to complication in balanced schemes, it should be borne in mind that when transmission systems become more complicated, successful protective equipment must also involve more complication. This may be in the primary connections, the secondary connection or in the contact and trip coil connections. The degree of this complication and the least objectionable variety must be decided in each case on its own merits.

The scheme described by Mr. Copley was not reported in the replies to the questionaire. As the intention of the paper was to catalogue actual experience any scheme which did not have some operating company to stand sponsor was not included. We are greatly indebted for this addition which otherwise could not have appeared in the record. The mechanically balanced relays referred to by Mr. McKenzie were not mentioned for the same reason.

COMBINATION OF UNDER-VOLTAGE AND OVER-CURRENT RELAYS FOR PROTECTING TRANSMISSION LINES

J. A. Johnson: In connection with the combination of undervoltage and over-current relays, I would like to suggest a modification which appears to me to be simpler and less expensive in application. Instead of having the under-voltage relay operate locks on each individual circuit breaker, let it simply control the supply of current to the tripping bus. The circuit breakers are then left free to be operated by the control switch, or by hand, but cannot trip automatically unless the voltage relay has operated to connect the tripping bus to the source of tripping current.

This plan, in its essential features, is in use by one of the power companies at Niagara Falls in connection with its ground relays. The system is a radial one, each feeder having a relay connected in the neutral of its star-connected current transformers. This relay is selective on grounds, the generator neutral being grounded through resistance but the relay may also operate in case of single-phase switching between two feeders at the receiving end.

To obviate this, a relay was connected in the generator neutral, arranged to control the supply of current to the tripping bus. No feeder switch can be tripped by its ground relay unless current is actually flowing in the generator neutral.

It seems to me that this system of combining the effects of under-voltage and over-current for relay operation has great possibilities, and is worthy of more general application than it has yet had.

D. A. McKenzie: As far as I know the application of a master relay for similar purpose at the Ontario Power Company's plant at Niagara Falls has been very satisfactory.

E. A. Hester: We are much interested in Mr. Johnson's proposed modification of this under-voltage and over-current scheme and can see no reason why it will not give the same results as the one described in the paper. If anything the modification is more desirable since it leaves the circuit breaker free for manual operation which is certainly one thing in its favor.

It is interesting to note that this scheme of controlling the trip circuit by means of a relay has actually been successfully used on another application. We are familiar with the ground relay scheme although we have never heard of any trouble due to single-phase switching. However it is easy to see that the installation of a relay in the generator neutral to close the control circuit would prevent tripping from this cause.

We agree with Mr. Johnston in his opinion that the possibilities of combination under-voltage and over-current protective relays are great and believe the next few years will bring forth developments which will overcome its present difficulty and make its application more general.

GROUNDING THE NEUTRAL

P. M. Lincoln: I note in this report that apparently the efficacy of the grounded neutral has been fairly well demonstrated. There are two cases which have been cited in this report in which testimony is given that grounding of the neutrals has given better protection and better operation. The two cases cited today are those of the New York Edison Company and the company out in Toledo.

This has been a thing which I have studied for many years and have advocated, that is, the grounding of the neutral. It is my opinion that reliability of service can be very considerably increased if the neutrals of the systems be grounded and the larger the system the more desirable such a move becomes.

I happened some years ago to personally come into contact to considerable extent with the system of the Montana Power Company. That system has been growing rapidly. It is the system, as you all know, which supplies power to the Chicago, Milwaukee & St. Paul for their electrification across western Montana. Years ago they operated delta-connected transformers and ungrounded. At first they had very little trouble, when the system was small, but as the system increased in size and capacity they began to have considerable difficulty. A ground or a short circuit at one point would apparently cause a blow-up of apparatus maybe hundreds of miles away and after studying the matter, the engineers of that company decided to adopt the grounded neutral for their system. They did this about 18 months or two years ago and the testimony from the engineers operating that system has been very definitely to the effect that the troubles which they have had have been very materially decreased on account of the presence of that grounded neutral.

F. D. Newbury: I wish to say a word in favor of the use of the grounded neutral from the standpoint of generator protection. In advocating the grounded neutral in order to protect apparatus, we have often encountered the feeling on the part of some operators that this increased protection was only secured at the expense of more frequent interruptions to service. That was an argument that a generator designer could not very well combat, so I am particularly pleased to see in both of these papers evidence that grounding neutrals is an advantage, even from the standpoint of service. With that argument removed, I hope that there will be a continued growth in the practise of grounding neutrals.

N. A. Carle: In regard to grounding the neutral, I would like to ask if anyone has had experience on moderate sized systems entirely overhead, operating at 13,000 volts with the grounded neutral, and if such systems have been using the ground wire on their lines or not.

R. F. Schuchardt: This paper by Mr. Woodrow reminds me very much of the vote by states on an amendment to the Federal Constitution. The system in Chicago was, I think, the first state to ratify the grounding of the neutral. Back in 1901 and 1902 certain investigations made by George Eastman, at that time head of our testing department, brought out very conclusively to our minds that grounding the neutrals of the system was the only proper way to operate. One by one the other states have ratified and even New York is now in the fold.

It might be worth while to state that the system in Chicago is sectionalized, that is, we limit the total capacity which we connect to a single section, separating the sections through reactors. That limit at the beginning was 50,000 kw. and now is at least doubled and may grow as the system grows. On each section we ground only one of the units in operation. We connect each neutral to a neutral bus which is connected to a ground through approximately two and one-half ohms resistance.

Originally we grounded solid but our experience soon indicated that with the increased capacity, the destruction and general disturbance resulting from a fault became very undesirable and we have found from perhaps fifteen years' experience that that amount of resistance in the neutral, combined with the relay system which we now use, is quite satisfactory.

E. E. F. Creighton: In regard to the grounded neutral, going back to history, the following may be of some interest to those who advocated a non-grounded neutral under certain reservations. The arcing ground suppressor which was available a good many years ago made it possible to operate a non-grounded neutral system with greater continuity of service because of the fact that you could operate the system for any length of time if that neutral was solidly grounded by a switch. In that development, which was aided by the Public Service Electric Company of New Jersey more than any other company, the radial system was in use and extremely good results were reported from the use of the suppressor.

For reasons of economy, as have been pointed out in the papers today and as all the operating engineers know, lines are being connected together at both ends and in loops, and no one has yet devised a way of localizing where the arcing ground is taking place, unless a radial system is used.

As Dr. Steinmetz pointed out, there is nothing in the current anywhere that would indicate which one of the many feeders was grounded. Therefore, the method of protecting by the use of the arcing ground suppressor fails because it is impossible to open the circuit that is at fault. Since all systems are going more and more to closed loops in one form and another, the arcing ground suppressor has not been found a permanent solution.

Naturally, with a grounding of the neutral the practise has gone to the suppression of the arc by the use of a fuse in series, following the same principle and practise as was first outlined in the arcing ground suppressor. In that development, however, it has been found impossible to use a fuse, since the fuse takes a time to operate depending upon the amount of power or the amount of current sent through it. When the system is heavily loaded and all the power is available, the fuse will blow in a very short time. That is the time when the fuse should hold the longest in order to extinguish the greatest volume of arc. On the other hand, at some odd hour of the night, or of the week, the power is at a minimum, the same fuse will hold for an abnormally long time; in fact, long enough to trip the circuit breaker so it has been found that the fuse is undesirable.

F. L. Hunt: About a year ago we grounded the neutrals on our system and we feel quite certain that the operation of the relays is much more definite and positive than it was before.

N. L. Pollard: It might be interesting to some of you to know that we are going to ground the neutral of the Public Service Electric Company's system at Newark, N. J. We came to a decision about three months ago that by grounding the neutral, we would eliminate many of our troubles.

About seven or eight years ago we seriously considered grounding the neutral, but since our system was operating differently at that time, and since we would have been obliged to establish several artificial neutrals. We finally decided against grounding. At this time, we installed an arcing ground suppressor, which apparently eliminated some of our troubles for the time being, at least. As our system grew larger, our troubles naturally increased until we decided recently that grounding the neutral was absolutely necessary.

We have two hundred and fifty miles of cable and four hundred miles of overhead lines in the Northern Division, which is the part of the 60-cycle system which we anticipate grounding.

A. E. Silver: I want to add a word of testimony to the benefits of the grounded neutral system. In my work, I have been associated with, at least three systems that were initially constructed and operated isolated from the ground which have been changed over to grounded neutral. In one case, with a Y connected system it could be done very easily; in the other two cases it was necessary to put on artificial neutrals. However, in each case the results have been very good indeed. We have gotten much better service, fewer punctures of insulation, fewer breakdowns of bushings and so on, so that we feel very thoroughly convinced from our own practise that the grounded neutral is practically the only system for high-voltage transmission. I might say that these systems that I have mentioned were each of 60,000 volts or higher.

F. C. Hanker: The paper analyzing the operation of the New York systems and giving the actual results for periods prior to and immediately following the installation of neutral grounding resistances is of particular value as confirming the contention of engineers relative to the advantages of neutral-grounding. The relay committee has presented a paper outlining the more general relay problems and it is suggested that they extend their activities to include a survey of the different systems that are operating with the grounded neutral.

There has always been a hesitancy among operating engineers to grounding the neutral due to the feeling that continuity of service would be seriously affected. That is one feature that is easily understood, whereas the contention that the operation of an ungrounded system with a partial ground was dangerous and likely to have more serious results than the interruption of service on individual feeders was not so self evident.

The information given in the paper prepared by the Committee on "Relays and Protective Devices" would indicate that the operators are recognizing the condition that exists. We believe that a review of the experiences of these various companies as suggested, will be of advantage in convincing operators on systems that are ungrounded of the advantages of this method of operation.

Four different methods of grounding the neutral are described. The first scheme using zigzag transformer and limiting resistances operates on relatively small currents and is applicable to systems where sheath-type transformers can be used. A modification of this arrangement using three current transformers connected in star and arranged with a low-current relay in the neutral connection will give similar operating results.

The second method outlined by Mr. Woodrow is probably one that is more generally used, the resistance varying from zero, corresponding to a solid ground, to six to eight ohms.

Probably the reason this method is the one most generally used is that it can be applied to existing systems with minimum change. It involves, of course, the use of current transformers in all three phases of each feeder in order to insure protection of all conductors.

Where the feeders are of relatively small capacity the adoption of this scheme does not require excessively heavy ground currents and avoids unnecessary damage to apparatus and cables in cases where grounds occur. A combination using this method on the small current feeders and adopting the relay connected in the neutral of the current transformers will probably meet the needs of the majority of installations and still avoid radical changes in the lay-out.

H. R. Woodrow: Mr. Hanker states that the three current transformers connected in star, with a low-current relay in the neutral, will give results equal to the special grounding transformers and I would like to point out that extra precautions are necessary as the third harmonics, produced in the three current transformers star-connected will cause considerable current to flow in the neutral, and although this arrangement is practical, special precautions in regard to this must be taken.

On a high-resistance ground where they have had trouble with the standard relay protection, I feel that a special ground relay should be given very serious consideration. This is very simple to do and we have adopted it throughout our system, with very good results. The grounded feeders are disconnected from the system before short circuit develops in 85 per cent of the cases.

The method of grounding is dependent to a large extent upon the system. We have shown four different methods in which we have grounded various parts of our system, and in each case the method of grounding was adopted because of the special condition under which that system must operate. It might not be clear that in some of these cases the method will operate only on a radial system.

In Fig. 3 the grounding there by means of potential transformers can only be accomplished where the feeder is isolated at both ends by power transformers. The method as shown in Fig. 1 necessitates the use of a ground relay in the substation, where everything is in parallel operation.

ANNUAL REPORTS OF TECHNICAL COMMITTEES

ANNUAL REPORT OF THE POWER STATIONS COMMITTEE

To the Board of Directors,

IN answer to the request of the Board's Committee on Technical Activities that suggestions be offered for improving the technical work of the Institute, we incorporate our suggestions in this Committee's report for the year 1918–1919:

(A) AS TO PLAN OF WORKING OF TECHNICAL COMMITTEES

Can improvements be made in the co-ordination of the work of the authors of Institute papers and that of the Technical Committees?

In the past the great majority of papers in our PROCEEDINGS were secured either from direct offers of authors or through the solicitation of Committees.

Lately it has been the custom to refer every paper, before acceptance, to the Meetings and Papers Committee, which practically consists of a Chairman and a Secretary, with the Chairmen of the Technical Committees as members. Unsolicited papers are referred by the Meetings and Papers Committee to one of its members for endorsement or criticism, the member to report being the Chairman of the Technical Committee most interested in the subject of the paper in question. Papers solicited by a Technical Committee are accepted upon the recommendation of its Chairman, without previous submission to the members of his Committee, except by title.

In practically all cases, therefore, papers are not submitted and discussed by any Committee in advance of presentation before the Institute.

Hence, in the past, with few exceptions, the main work of the Technical Committees has been confined to secure especially qualified authors to write papers on subjects in which the Committee was interested. While the papers are nominally submitted under the auspices of this or that Committee, in fact the Committee itself never has an opportunity of acquainting itself with them. It follows that the discussion in the open meetings is greatly made up by comments and criticisms of

---

NOTE:—No reports made by Traction and Transportation Committee and Electrical Machinery Committee.

members of the same Committee under whose auspices the paper is presented.

The suggestion arises: *Could not material advantages be secured by the Committee discussing the paper before it is presented at the general meeting of the Institute?*

In fact, should not the Committee share in the responsibility with the author or write, as an introduction, a brief statement of the opinions and conclusions of the Committee, either endorsing the author's paper in full or qualifying any point of divergence, giving its reasons and evidence to the contrary views?

If this course were followed, the papers and the discussions would improve, because the author would gladly revise, condense or modify his draft, if that would make his paper more clear and complete, eliminating small inaccuracies, superflous matter or statements susceptible to misconstruction, and adding information of value to corroborate the conclusions.

Also when views fundamentally and radically at variance are prevailing on a given subject, the Committee should secure different papers, carefully discussed and prepared to convey comprehensive, clear and substantiated statements of the facts from the different angles.

The suggestion is not to curtail the freedom of the writers, but only to assist them with whatever constructive criticism the Committee can give.

The result would be that the Institute PROCEEDINGS would become more condensed, valuable and readable than under the present method, and the importance of the Committee's work immeasurably enhanced.

The suggestion is in the direction of creating an organic administration of the technical work of the Institute, which will automatically expand to reach out for, and divulge in crystallized form, the information and experience of more and more members. This work would grow and widen from year to year along lines of greater benefit to, and closer co-operation with, each member of the Institute.

In the interest of effectiveness, this suggestion possibly would displace the newly organized Committee on Technical Activities. The respective Chairmen of Technical Committees as members of the Meetings and Papers Committee should, through its Chairman or individually, report to the Board of Directors in matters pertaining to papers, reports, co-operation with sister societies or institutions, etc.

(B) AS TO SUBSTANCE OF STUDIES OF COMMITTEES WITH SPECIAL REFERENCE TO THE POWER STATIONS COMMITTEE

There exists considerable indefiniteness as to what field a Committee, like the Power Stations, should cover. A power station presents a great complexity of features, like those of design, mechanical and electrical equipment, operation, efficiency of generation, production costs, etc. Many of these subjects, for instance, are strictly mechanical, though perhaps of preeminent importance only in electrical power stations.

Take, as an illustration, the steam turbine. Its design and construction are carried out by mechanical engineers and frequently discussed by the Society of Mechanical Engineers. Similarly, the features of operation, efficiency of generation, and production costs are mainly objects of study by committees of associations of operating companies most vitally interested in the subject. Features of design of electrical apparatus would, under the present organization of committees, come under the jurisdiction of specific committees, like Electrical Machinery, Measuring Instruments, Protective Devices.

Without extending further this analysis, one readily sees that, if we are bound too closely by these limits of existing specialization, we lose the opportunity of doing constructive work in a broad sense. We must, therefore, overlap in other fields under the limitation of not conflicting with other activities in so far as our work would be mere duplication of efforts already satisfactorily covered by others.

To illustrate this principle, let us outline one of the studies which this Committee has had under consideration for the last two years and which will shortly be presented to the Institute. The subject is "Limits of speed and power of single-shaft steam turbo generator units." Three papers are being prepared, two covering the steam end and one the electrical end. It is hoped that these papers will give an idea of limits of power and efficiencies of machines of existing types, *i.e.* a single-shaft steam turbine or a single-shaft generator for steam turbines. Since the study was first considered, its realization having been retarded by the war, there occurred three or four serious mechanical failures of steam turbine wheels which have created considerable concern to the users and aroused a widespread interest in one phase of this theoretical subject. The details of such occurrences can, for obvious reasons, only be discussed by the manufacturers and the users. This has been done but, to a large extent, the findings are properly considered confidential matters, often having a legal aspect, which cannot be made a subject of investigation and report by this Committee. One the other hand, the users would greatly benefit by any work of this Committee that, regardless of specific experiences, would outline the possibilities and limitations of power capacity of single-shaft turbo generators for different speeds permissible by the given frequency. The erroneous remark is often heard that it is unsafe to go beyond a certain limit in the capacity of units for central stations. In reality, units of twice or three times the power of the largest existing units can be manufactured to have the same or even greater factor of safety. Possibly such units may be more costly, or less efficient, or too heavy, or of too large dimensions, or be undesirable in situations where they would concentrate all the power of a station in two or three units instead of five or six or more, but we should not dismiss the study of the possibility of such machines simply from the wrong assumption that they cannot be made as safe as smaller capacity units, because in some situations the larger units may yet prove to be of advantage.

In any event, if they are undesirable, we should know why they are so. The papers under preparation will discuss these features and the Committee believes that such study will be of benefit to manufacturers and users if the study is intelligently carried out.

As we stated before, this departure of invading some of the fields of other technical societies or associations must be done with the understanding that the Committee will not attempt to duplicate the work of others, like, for instance, the Prime Movers Committee of the N. E. L. A. The latter, as a committee of an industry, can obviously fully investigate every operating feature or defect of apparatus and secure information to which this Committee cannot properly have access. On the other hand, features of engineering and design of machines can only constructively be discussed with the co-operation of users and the principal assistance of engineers thoroughly familiar with calculation and design of machinery.

It would appear to us, therefore, that there is a clear line of demarcation for the field of activities of this Committee, which, if directed along the plan first suggested, offers the promise of giving material aid to all the interests concerned.

From this illustration we submit that, as to substance, the work of the Committees must in certain cases be broadened even to the extent, sometimes, of invading the field of other technical societies or committees of the Institute where this is necessary to make a comprehensive study of the subject, under the condition of not duplicating work already satisfactorily and more properly done by others.

(C) PROGRAM OF FUTURE ACTIVITIES

Other proposed work of this Committee for the ensuing year consists of:

A paper outlining the statistical items which are essential in reporting efficiencies of power plants—this to the end of standardizing and making possible comparisons between different plants, which are not possible at present on the basis of the information given in all the published reports of United States Bureaus, State Commissions or other Governmental bodies.

A comprehensive review of the relative merits of shunt and compound-wound exciters, steam and motor-driven exciters, etc. has been considered and is recommended for the incoming Committee.

While the war continued it was found impracticable to make progress along the study of a national policy for the generation and utilization of power, as referred to in last year's report, but some of the members have further studied and investigated the broad subject and will submit suggestions and recommendations to the Committee or the Board as soon as practicable.

PHILIP TORCHIO, *Chairman*

ANNUAL REPORT OF THE TRANSMISSION AND DISTRIBUTION COMMITTEE

To the Board of Directors,

THE Committee on Transmission and Distribution submits the following report for the year 1918–1919:

The need for further reducing the membership of the Committee, as pointed out by the preceding chairman was again brought out very forcibly during the past year.

The large and scattered membership made it impossible to call meetings and it was found necessary to conduct the work entirely by correspondence. This method is generally unsatisfactory, but was particularly so in this case because many of those appointed seemed to consider their connection with the Committee as purely honorary, involving no obligation on their part to enter actively into the work. Several of the members made no reply to any of the communications sent out during the year.

The number of papers presented during 1918–1919 was somewhat smaller than usual because activity during the war period was directed toward intensified production and after the armistice to general readjustment.

However, the papers which were presented dealt with subjects of timely interest and constituted a valuable addition to the literature on overhead and underground transmission.

At the November meeting, held in Toronto, Mr. S. Svenningson presented a paper dealing with the construction of a 110,000-volt transmission line crossing the St. Lawrence River. The preliminary investigation, leading to the adoption of a single 4800-ft. span with 350-ft. towers, is outlined, together with a general description of the design and construction of the towers, insulators and cables. The provisions for protection from ice and method of calculating sag is also included.

At this Annual Convention a session is devoted to the presentation of papers covering both overhead and underground transmission problems.

Messrs. W. S. Clark and G. B. Shanklin, in a paper on high-tension single-conductor cables for polyphase systems, set 25,000 or 30,000 volts as the limit for three-conductor cables and point out the adaptability of single-conductor cables for voltages up to 55,000. The paper compares the dielectric, inductive and general line characteristics of single-conductor and three-conductor cables, discusses the advantages and disadvantages of each type and states that the induced sheath voltage or current is the only factor in single-conductor operation which requires serious consideration.

Mr. R. W. Atkinson presents a paper on the dielectric field in an electric power cable. The data given cover, particularly, the field of three-conductor, three-phase cables, and is primarily the solution by physical measurements, of some of the geometric problems of this type of cable. Methods are outlined by which it is possible to determine from electrical measurements certain specific quantities such as the permittivity and resistivity of the dielectric of three-conductor cables.

The potential stress distribution in three-conductor cables is given, most of the data being based on measurements with electrodes in an electrically conducting liquid, thus approximating closely a homogeneous dielectric. Some additional potential measurements were made with exploring electrodes built into a three-conductor cable having commercial insulating material.

Mr. A. E. Silver in a paper on the problems of 220-kv. power transmission suggests 220-kv. as the logical voltage for the high-capacity, long-distance transmission of energy from distant coal fields and water power plants. The author discusses very fully the important problems introduced by large concentration of energy at high voltage and covers in detail the economic and technical considerations underlying the design of a 220-kv. system. The paper indicates that while further investigation and research are necessary for a complete solution, established principles of design and present types of equipment can, with minor changes, be adopted for 220-kv. transmission.

The points brought out in the paper will aid in working out this advance in the field of high-voltage transmission by promoting constructive discussion and directing research along the lines indicated.

Mr. F. W. Peek, Jr. has given us the results of his investigation of the operating and protective characteristics of various types of spark gaps. The paper shows that there is a marked difference in the relative lightning spark-over voltages of different gaps, as well as a great difference in the settings imposed by operating conditions. Both of these factors must be considered in comparing relative protective values.

It is pointed out that the combination sphere-horn having electrodes of points, horns and spheres gives very good protection over the whole range of frequency or wave front. The spheres discharge the very steep waves and the horns the moderate waves, the points taking care of continuous high-frequency, static, and waves of very slanting front.

The comparative protective values of various types of gap are shown graphically and some fundamental points in the design of bushings and insulators are pointed out.

RECENT DEVELOPMENTS

High-Tension Insulators. There have been no radical changes in the design of high-tension insulators during the past year, and efforts along this line have been toward perfection of details and improvements in reliability.

A great deal has been written on the subjects of insulator troubles, but the difficulties experienced have been no greater than might have been expected in high-voltage work. It is believed that a proper appreciation of the factor of depreciation of insulators and the inspection and replacement of faulty material will go far toward improving operating results.

Practise has clearly demonstrated that a line which is liberally designed as regards insulators will operate satisfactorily in spite of a considerable amount of depreciation. As a result there is little doubt of the tendency to go to considerably higher voltages, wherever there is sufficient load to warrant a conductor large enough in diameter to keep down the corona losses.

The past year has seen the installation across the Tennessee River of the longest span in the world, the very heavy stress occasioned by this span being carried on a single insulator of comparatively small size.

Underground Cables. The material progress which has been made in the last few years in the field of underground cables may be traced to the growth in power requirements incident to the war, as well as to the more exacting demands of cable users.

The recent practical developments are a natural outcome of many investigations dating back to the papers of Gorman in 1901, Raynor in 1912 and Klein in 1913, all of whom clearly demonstrated the start of corona in paper insulated cables at stresses of between 25,000 and 30,000 volts. Developments in the United States have been closely paralleled in foreign countries.

Great general interest is shown regarding the subject of dielectric losses. This matter has become of great commercial importance in the last few years due to the increased loading of the conduit systems. A great variety of opinions exists regarding the actual importance of losses and particularly as to the temperatures at which these are most important. Standardization of ideas on these questions is desirable.

Several methods, with numerous variations, are being used for making dielectric loss measurements, the results checking very closely. However, on account of the magnitude and nature of the quantities sought, all of the methods require more care and skill than do the more usual measurements on electrical apparatus.

Many investigators have studied the problems connected with the stresses in dielectrics. Since there are inherent limitations to the development of the usual three-conductor cable, proposals have been made to avoid these. One plan involves the use of single-conductor cables and another the use of the electrical equivalent of three single-conductor cables but under one sheath.

One direction of present day development is toward the standardization by operating companies of larger sizes of transmission cables. One company is planning to install a large amount of 11,000-volt cable with conductors of 600,000 cir. mils, and 22,000-volt with 350,000 cir. mils. While this is not larger cable than has been made in individual instances, it is larger than has been standard practise for any company in the past. Another company is making a practise of operating two cables as large as 4/0, 26-kv. as a single unit.

There has been an increase in the use of split-conductor cable for transmission line protection. Two large operating companies seem to have partly standardized on this type of cable for future installations.

Cable users are now fairly well informed as regards the operating temperatures of their cables. By comparatively simple means it is possible to obtain an accurate knowledge of conditions of an existing system. A large amount of data is in the possession of operating companies as regards temperature rises of conduit systems under different conditions of installation, and it would be of extreme value to the industry as a whole for such data to be made generally available. Existing systems which have poor cooling characteristics might be improved in some cases and the design of new systems would be greatly facilitated.

Operating companies understand much better than formerly the properties of cables which have to do with successful operation and there is an increasing tendency toward the elimination of non-essential features of specifications and for co-operation with manufacturers so that the latter will not be hampered in securing to the highest degree the essential properties some of which cannot yet be covered by specifications.

SUGGESTIONS FOR THE FUTURE

There has been considerable discussion during the past two years on the subject of dielectric losses in underground cables. Very little real progress appears to have been made, although several large cable users are planning to incorporate in their cable specifications a clause relating to dielectric losses.

Your Committee has been advised that cable manufacturers have been working on this problem with a view to standardizing equipment and methods of measurements, and it is recommended that succeeding committees give this matter their attention.

Allied very closely with the subject of dielectric losses, is the question of safe operating temperature of cables. Mr. R. C. Powell presented a paper which appeared in the PROCEEDINGS for August 1916, but very little additional information has been made available.

There is a growing tendency on the part of those operating high-tension transmission cables to base their current ratings on the heat dissipating characteristics of the individual duct line and there is no doubt that some progress has been made along these lines. It is very desirable that the Committee secure one or more papers covering this phase of underground transmission.

As regards insulators, it is very desirable that the results of tests conducted by Professor Ryan on the aging of insulators be made available, as this question is of vital interest in the operation and maintenance of high-voltage lines.

Referring again to the reduction in the membership of the Committee, your Chairman feels that the work would be greatly facilitated and the results of much greater value if the personnel were limited to not more than ten members.

E. B. MEYER, *Chairman.*

June 2, 1919.

ANNUAL REPORT OF THE INDUSTRIAL AND DOMESTIC POWER COMMITTEE

To the Board of Directors,

FOR two terms back, the Industrial and Domestic Power Committee has studied the general idea of investigating the application of electrical power to industry. During this term, the idea has been reduced to a definite plan, and the plan put definitely in motion.

It is impossible in this report to more than indicate the general features of the plan, its organization and personnel. Fourteen men, *i. e.*, the committee as appointed, started work in September, 1918. May 1919 finds the committee increased to 43, not including men who are helping in the work without committee recognition.

It was decided early in the term to actively prosecute the study of application of power in industries by the formation of sub-committees. Each sub-committee was to concentrate on its branch of the work, subdivide its activities as seemed wise and go right ahead. As it was questionable as to the correct final procedure, it was decided that the investigations would be based upon.

a. The application of electric power to specific processes,

b. The application of electric power to specific electrically driven machinery,

and that part of the sub-committees would work to one thought; the remainder to the other.

Nine sub-committees were formed, and have been under operation this term as follows:

1. Sub-committee on motors with particular reference to speed torque characteristics.

A. M. Dudley, Chairman.
R. H. Tillman A. C. Lanier
Wilfred Sykes John C. Parker

2. Sub-committee on Domestic Power Applications.

H. Weichsel, Chairman
James Dixon C. L. Kennedy
A. F. Welch A. M. Harrelson
Bernard Lester

3. Sub-committee on Applications in Printing Industry.
W. C. Kalb, Chairman
W. E. Date
John D. Nies
J. C. Lincoln
Carl F. Scott

4. Sub-committee on Applications to Cranes and Hoists.
W. L. Merrill, Chairman
H. W. Eastwood
E. Friedlaender
R. H. McLain

5. Sub-committee on Application to Machine Tools.
H. D. James, Chairman
T. E. Barnum
W. T. Snyder
R. H. Goodwillie
W. C. Yates

6. Sub-committee on Application to Passenger and Freight Elevators.
R. H. Goodwillie, Chairman
H. D. James
H. P. Reed
David L. Lindquist
Charles H. Roth

7. Sub-committee on Application in Textile Industry.
H. W. Cope, Chairman.
E. T. Foote
S. B. Paine
C. T. Guilford
D. H. Sadler

8. Sub-committee on Application in Cement Industry.
R. B. Williamson, Chairman
A. M. Dudley
C. A. Kelsey
H. Weichsel
Arthur Simon

9. Sub-committee on Application in Woodworking Industry.
L. E. Underwood, Chairman
Truman Hibbard
S. A. Staege

Under the plan, other committees will be formed and put under way as the work develops. Plans are also under way in the Office of the Secretary of the Institute to care for records from the sub-committee activities as they are turned in. The activities of the sub-committees are further interchanged through the Chairman of the Industrial and Domestic Power Committee, acting as an advisory personal clearinghouse.

The actual records of sub-committee work for the term are not numerous. Much time has been spent in planning the sub-committee work, determining its personnel and securing appointment thereof. Certain of the sub-committees have not advanced beyond this stage. All have submitted reports of activities for this term indicating full co-operative support from their associates and definite sustained interest in the work. In order to bring out more clearly the work in hand, excerpts from several of the reports follow.

From the sub-committee on the printing industry, Chairman Kalb reports the following divisions:

1. Press operation on raised surface or typographical printing, this to cover motor and control application for typical job presses, magazine and newspaper presses.
2. Contributory device operation relating to raised surface printing. This includes linotype, monotype, stereotype, electrotyping, conveyors or carriers, hoists, folders, paper cutters, tension devices and electric heating as applied to metal melting pots and bookbinding tools.
3. Types of presses used in surface and sub-surface printing, this to cover lithographic, planographic, rotogravure and embossing. This division will also include power application on bronzing machines and wood printing.
4. Contributory devices used in connection with surface and sub-surface printing and embossing.

Chairman Goodwillie indicates the work of the sub-committe on the elevator industry as follows:

Scope—The committee has undertaken the study of the electric elevator from the standpoint of its use, *i. e.*, fulfilling its purpose as a mechanism for raising and lowering vertically, passengers and freight in a hatchway.

The subject is being treated by considering comprehensively the requirements of elevator service and leading to the fulfillment of these requirements by the application of electric power to the motor and controlling devices.

The sub-committee on machine tool application has given much thought to its subject. Chairman James in part indicates his committee's plan as follows:

1. Control Equipment: This term will include the complete control apparatus, including disconnecting means and overload protection which is used for the control of the machine tool motor. The complete equipment may or may not form a single unit. Note: It has been common to speak of the machine tool controller as that part of the control equipment used for starting, stopping, reversing, or regulating the speed of the motor. In many cases, this controller did not include the disconnecting means and overload protection. It was, therefore, thought desirable to select a term which would include all of these items.

2. Application Requirements: It was agreed to divide the control equipment into two classes in reference to the features required for particular applications. These divisions are as follows:

    1. General Application Requirements: These requirements are common to all machine tool applications and it is recommended that they be incorporated in all machine tool control equipments. They are as follows:
        1. 1 Overload Protection.
        1. 2 Low-Voltage Protection.
        1. 3 Enclosure or Equivalent Protection.
        1. 4 Disconnecting means.
    2. Additional Application Requirements.
        2. 1 Armature Speed Control.
        2. 2 Field adjustment for Speed Control.

2. 3 Reversing or Non-reversing.
2. 4 Manual or Automatic.
2. 5 Dynamic Brake.
2. 6 Drift.

The next item on the program will be to compile a list of machine tool applications which comprise only the machines more commonly used. This list will then be analyzed and the additional requirements needed for each application specified.

The sub-committee on the cement industry through its Chairman, Mr. Williamson presented the results of earlier activities in an admirable paper read to the Institute at its regular meeting November 8th, 1918. This meeting was held under the auspices of the Industrial and Domestic Power Committee. Chairman Williamson reports that his committee is collecting such additional data as will supplement this paper and in course of time it may be advisable to publish either a revision of it, or an appendix. As an example, we hope before long to secure some information regarding the application of synchronous motors in this industry. In course of time this additional information should add considerably to the value of the paper.

As a committee we feel that our work, and its plan is not only possible but practicable but that two perhaps three years are necessary to determine this. We feel that this plan should be continued another year and recommend this. We also call attention that it takes much time to get together a working personnel and so far as possible, and where not inconsistent with other plans, it is recommended that this existing personnel be kept intact, the recommendation extending to the sub-committees' makeup.

We must call attention to our belief that a certain amount of definite interest in the Institute is being generated by this plan of extending work among a large group. We believe Institute men are anxious to contribute service if a method be indicated. I believe that our sub-committee chairmen are helping materially in this, and their activities have resulted already in bringing several desirable electrical engineers on to our roster.

The committee particularly appreciates the prompt help and attentions which our executive offices in New York through the instrumentality of Mr. F. L. Hutchinson, have constantly rendered.

H. G. PIERCE, *Chairman*

ANNUAL REPORT OF THE LIGHTING AND ILLUMINATION COMMITTEE

To the Board of Directors,

I beg to submit on behalf of the Lighting and Illumination Committee the following report for the year 1918–19.

The activities of the Committee for the current year were composed largely of the plans for a session of the Institute for a discussion of Industrial Lighting Codes, this subject being one of those suggested by last year's Committee, but which, due to the abnormal conditions which prevailed at that time, could not be arranged. As the outcome of these plans a paper was presented by G. H. Stickney at a joint meeting of the Institute and of the Illuminating Engineering Society on April 11, 1919 in New York under the auspices of this Committee of the Institute. At this meeting Mr. Stickney presented an outline of the general scope and reasons for these codes. The session was highly successful and promoted valuable discussion on the part of representatives of the lamp companies, the state labor departments and manufacturers in general.

Another feature of the Committee's work during the present year has been the suggestion which the Chairman was authorized to make to all the Institute sections throughout the country calling attention to the advantage to be gained by devoting one of the Section meetings at each center during the current year to some aspect of Illumination, preferably as a joint session with the local branch of the Illuminating Engineernig Society where such a local branch existed. This suggestion met with a widespread response and a large number of the Institute sections devoted one meeting of the year to some particular division of the lighting and illumination fields and a number of valuable papers and discussions on this topic were presented before the local sections. In some cases the Committee cooperated with the local sections by suggestions as to possible speakers for illumination subjects.

At one of its later meetings, the Committee, following the suggestion of the Board's Committee on Technical Activities, decided upon steps to be taken in plans for presenting one comprehensive paper before the Institute during the coming year on "An Analysis of Daylight Saving," and a symposium on "Systems of Electrical Distribution for Street Lighting" at a later date.

According to the custom of the Committee during the past few years, and also acting upon the suggestion of the Committee on Technical Activities, the following brief summary of progress in electric illumination during the past year, is herewith presented.

PROGRESS IN ELECTRIC ILLUMINATION

Street Lighting. One of the effects of the war was to curtail street lighting installations throughout the country, existing systems being extended only when absolutely necessary, and work on new installations being practically at a standstill during the war. With the signing of the armistice, renewed activity has become evident in this field, and present prospects for extensive developments appear bright.

To illustrate the effect of the war on curtailment of installation work, the construction work on extensive street lighting in Los Angeles and San Francisco were suspended during the war, but the work on these new systems was resumed almost immediately after the signing of the armistice. It may be added that the two installations here mentioned constitute, in themselves, notable achievements in the street lighting field. Other recent developments in street lighting include, among other cities, portions of Chicago, Denver and Dallas.

In the fixtures to be used with incandescent lamps in street lighting, there is an increased tendency to apply some form of prismatic refractor. New lanterns have been developed of both upright and ornamental bracket types, using dome shaped prismatic refractors with an outer globe, which has the effect of breaking up the concentrated rays without loss of the redirective value of the refractor. These units exhibit a bright, sparkling appearance which is considered desirable in street lighting if undue glare is avoided. They direct an unusually high proportion of the light to the street surface.

In recently designed street lighting systems there is an increased tendency to take advantage of the wide range in the incandescent lamps available for operation on circuits of the same kind, thus making it possible to select the lamp sizes according to the strictly engineering requirements of each given class of street and thus affording a far more effective illumination result for a given outlay than is possible with a single fixture and one given size of unit of either large or small size as the case may be.

Studies tending toward still greater flexibility in installation and extension of street lighting systems, and toward a reduction in the disproportionate investment charges which are necessary in the common form of series street lighting system, have led to the development of new plans and devices for current supply. A new automatic movable-coil constant-current regulator, in sizes up to ten kilowatts for pole mounting, and operation from a time clock or remote-control relay switch, is especially useful for the extension of street lighting at distances from substations.

Another promising development of street lighting is the operation of the street lamps from existing secondary multiple or house lighting circuits rather than from series circuits. This form of installation avoids the separate high-tension circuits, special transforming and regulating equipment and the cost of the high-voltage insulation for series systems. Small synchronous motor clock switches on the lighting circuits are available for controlling the lamps. Another proposed method is the use of simple remote control switches operated by a so-called pilot wire.

A great deal of interest has also been displayed in highway lighting, and there is a possibility that some of the states in placing contracts for road work will include the erection of suitable poles for the lighting of the state highways, on a larger scale than has been considered practical in the past.

The tendency for sharing the expense of ornamental street lighting between the city and property owners and merchants is illustrated by the action of the property owners and merchants to pay one-half of the total annual cost for the ornamental street lighting in the Triangle District of San Francisco. A defense of proportionate sharing of this expense on a 50 per cent basis for the city and 50 per cent basis for the merchants and property owners, based on Syracuse, N. Y. conditions, has recently been discussed in one of the leading engineering weeklies.

Under war time street lighting economies, one interesting statement has been advanced that the average streets in this country require an increase of illumination of 70 per cent, but that a considerable reduction of white-way lighting could be afforded under extreme conditions like those during the war. The small percentage which the fuel used for street lighting bears to the total fuel used in this country has also been a matter of discussion, brought on by war conditions.[1]

Street lighting curtailment under war conditions, proved unpopular in a number of localities and cases have been cited where the popular demand for regular lighting conditions in the streets of a city or town was so great during the period of curtailment, that the regular lighting conditions were restored under pressure of public sentiment. In certain cases where curtailment of street lighting was carried out, interesting problems of rebates during the curtailment period were created.

Saving coal by efficient street lighting was an alternative to curtailment and one case which has been reported in a part of Chicago's street lighting system indicates an annual saving of about 3000 tons of coal by the substitution of type C Mazda lamps for arc lamps.[2] The magnitude of the street lighting in the larger cities may be realized from the published figures in New York City, where there are 2800 miles of lighted streets at an annual cost of about $700,000. to the city.[3]

To the foregoing, there might be added a list of the new street lighting units and standards which have been developing recently and which have made available a great variety of effective equipment for use in this field.

Protective Lighting. While the ideas of flood-lighting the fronts of buildings and yard surfaces with light from projectors, is not new, its use during the war has received increased attention as a vital measure of plant protection. This has resulted in an unusual appreciation of light as a factor of usefulness for industrial plants and undoubtedly has had some effect in convincing manufacturers of the necessity of higher intensities of interior illumination.

Factory Lighting. One of the most notable influences of the war upon the lighting field has been the widespread acceptance of the fundamental value of good lighting to factory production. The pioneer tests of Mr. Wm. A. Durgin, of the Commonwealth Edison Company in Chicago, on the relations of higher illumination intensities to factory production rates, has brought out forcibly the interesting fact that illumination is a factor of primary importance up to an intensity which makes vision possible, but even beyond this, an increase in the intensities to much higher values than those required for vision itself, bring about rather surprising results from what seems to be a stimulating effect of highly cheerful surroundings upon the employees, thus increasing production by percentages which are very large in comparison with the costs for the increased illumination.

---

[1] J. R. Cravath in *American City*, October 1918, p 303.

[2] G. T. Dunklin in *American City*, June 1918, p 517.

[3] D. F. Atkins in *Modern City*, February 1918, p 13.

The importance of factory lighting in war work was also emphasized by the appointment of a National Committee on Lighting to act as a sub-committee of the Advisory Commission, Council for National Defense, referred to by last year's Committee in its report. The publication of a Code of Lighting by this Committee on Labor, with a suggestion that the Code be put in effect in every state in the country, has resulted in bringing these regulations before the various state authorities from a new standpoint. The paper by Mr. Stickney, found in another issue of the Institute PROCEEDINGS, contains an excellent summary of the developments in these codes.

Motion Picture Lamps. Considerable development work has been directed to the production of a satisfactory motion picture lamp. The use of the incandescent lamp for motion picture work will undoubtedly become quite general.

Searchlights for War Purposes. The searchlight has been of the utmost importance in the experiences of the recent war. The 60-inch searchlights first used by the Allies together with the automobile power plant and the truck, brought up the weight of each to eleven tons. Specifications which were prepared by American engineers during the progress of the war were directed to a reduction of weight to make the equipment more practical for army work, to the use of a 60-inch metal mirror, and to the greatest possible simplicity.

The so-called Lynn searchlight is now said to be the most powerful searchlight in the world, giving a beam of an intensity of over three times that of the most powerful searchlights in use at the beginning of the war. Moreover, the spectrum of the light from these units approaches a daylight character so nearly as to make them very useful in revealing aeroplanes at great distances above the earth's surface.

The importance of the searchlight is indicated by the general conclusion of army engineers that the only satisfactory method of meeting an aeroplane bombing operation is by means of searchlight defense. In fact it has been stated that the city of Bruges was never bombarded at night by allied planes due to the protection afforded by seventeen enormously powerful searchlights.

Automobile Headlighting. The two 1918 reports of the Committee on Automobile Headlighting Specifications of the Illuminating Engineering Society and the corresponding discussion as published in the TRANSACTIONS of that Society, March 20, 1919, constitute a valuable fund of information on this subject. The Committee on Lighting Legislation of the Illuminating Engineering Society has recently formulated proposed regulations for incorporation in a model law in regard to headlamps on motor vehicles. The Specifications for Headlight Tests of New York State are also of interest in this connection.

C. E. CLEWELL, *Chairman*

ANNUAL REPORT OF COMMITTEE ON ECONOMICS OF ELECTRIC SERVICE

To the Board of Directors,

DURING the first half of this Institute year your chairman and three other members of the Committee were in military service. It will be readily appreciated by anyone who was in active service in engineering and construction lines that there was consequently no opportunity to carry forward the work of the Committee. This should not be considered as an index of the importance of a Committee on Economics of Electric Service to the Institute. During the past year the normal conditions in connection with electric service were so disarranged that there was little such a Committee might have done except record the effects of war conditions on the service, which might have become of historic value but which could not have had any considerable immediate usefulness. Last year the Committee took charge of a New York meeting of the Institute, presenting a paper on The Effects of War Conditions on the Cost and Quality of Electric Service which served its end in bringing people to a lively appreciation of the almost incredible effects the war conditions had on electric service and the great difficulties the managers of electric properties were forced to face in continuing effective service at reasonable prices during the war. The preceding year the Committee took charge of a meeting considering, in five papers, the matter of valuations of properties, and the production of effective inventories both for use in valuations in the regular operations of the properties.

Now that the war is over electric service will gradually become normal, both in the matters of cost and character, but it is not to be expected that conditions will return to those of pre-war times. It is therefore important that the Committee on Economics of Electric Service should keep careful account of the matters under its jurisdiction and by timely reports and papers enable the Institute to assist in bringing out from the rather chaotic war conditions sound and reasonable bases for the establishment of electric service under the new conditions. It is important also that this Committee of the Institute should work in cooperation with committees of like scope of other organizations, including those of a more or less commercial character, so that these economic questions both as to cost of service, quality of service and value of properties may be solved for the future from as broad a point of view as possible. The Institute's place in this work should be an important one as the recognized leader amongst purely technical societies having particular regard for electrical matters.

WM. B. JACKSON, *Chairman.*

ANNUAL REPORT OF THE PROTECTIVE DEVICES COMMITTEE

To the Board of Directors.

SHORTLY after the signing of the armistice last November, there were indications that the members were rapidly discontinuing their various war activities, and upon making an attempt to resume the work of the committee, which had been practically dormant during the war, it was found that the members of the committee and the members of the Institute with whom they were in correspondence, were in general, ready, willing and anxious to co-operate in the work of the committee. Work on the Relay Questionnaire was, therefore, pushed rather aggressively and was discussed in several meetings of a subcommittee and later in meetings of the Protective Devices Committee. The result of this questionnaire and of the committee work appears in the form of papers which are to occupy one of the technical sessions of the Annual Convention.

The future activities of the committee were discussed at some length at one of its meetings. The questionnaire had, in the opinion of the members of the committee, brought forth information so interesting and valuable that it was their opinion that similar investigations should be undertaken by the committee and that for this purpose the number of members on the committee should be increased so as to permit the investigation of more than one subject at a time. The subjects for investigation recommended by the committee, in the order of preference, are as follows:

1. Schemes of Relay Protection.
2. Oil Circuit Breakers.
3. Grounding of the Neutral.
4. Lightning Arresters.
5. Current Transformers.
6. Potential Transformers.

In connection with the work on the Relay Questionnaire it was brought out by several members of the committee and by several of the members of the Institute with whom we corresponded during the work of the questionnaire, that in a number of cases the operating companies were experimenting with various types of relays and schemes of relay protection, and that there were no facilities for co-operation or for comparing experiences so that each of the parties interested in such matters could get the benefit of the experience of all. In some cases it was found out that one company was at some expense investigating a particular type of relay for a particular purpose, and that another company a year or two earlier had made the same experiment under practically identical conditions and had found that the relay was not adapted for the purpose. It is, therefore, recommended that the Protective Devices Committee undertake to devise some scheme of facilitating co-operative research on subjects within the scope of the committee and that the scheme include some method of communicating, at regular intervals, to the engineers who are co-operating in this manner, such information as has been developed by all of the contributing members.

D. W. Roper, *Chairman.*

ANNUAL REPORT OF ELECTROCHEMISTRY AND ELECTROMETALLURGY COMMITTEE

To the Board of Directors,

TECHNICAL papers on electrochemistry and electrometallurgy, involving chiefly chemistry, physical-chemistry and metallurgy, are naturally discussed more intelligently, more critically and more effectively, in our sister society, The American Electrochemical Society, which includes in its membership most of the chemists, physical-chemists and metallurgists who are interested in such subjects. Hence papers on these subjects naturally drift to and belong to that society.

In connection with many electrochemical and electrometallurgical plants there are however features of a purely electrical character which chemists, physical-chemists and metallurgists are not able to discuss critically and intelligently, such as regulation, power factor, load factor, balancing of loads, different kinds of currents, long-distance transmission, relative advantages and costs of water power *vs.* steam power, etc. Papers on the latter subjects ought to go to the A. I. E. E., or perhaps better yet, be presented at joint meetings of these two societies. Although the Chairman of the present committee has endeavored to procure papers of this kind, he has not yet been successful.

The Chairman recommends that the above plan be adhered to, and that there should be no rivalry but rather a mutually beneficial cooperation between these two national societies. As joint meetings are difficult to arrange, it might be advisable that in case of a particularly good paper on an overlapping subject, arrangements be made that such a paper might become the joint property of both societies, be read and discussed at the meetings of both societies, and be published in both TRANSACTIONS.

CARL HERING, *Chairman.*

ANNUAL REPORT OF THE ELECTROPHYSICS COMMITTEE

To the Board of Directors,

IT has been the endeavor of this committee for the last few years:

1. To encourage original papers of high technical standard, marking advances in electrophysics.

2. To have each year a broad, interesting, general lecture, free from mathematics, dealing with modern physics.

3. To promote a more complete cooperation and mutual understanding between the engineer and the physicist.

This general policy has been endorsed by the present committee.

The following are the activities of the committee for the past year:

LECTURES

"Lectures on Research". Drs. Whitney and Millikan. October 1918. Philadelphia. Arranged by the Committee of 1917–1918.

"An Evening with the New Astronomy". Dr. John A. Brashear. Mid-winter Convention, 1919. New York. Arranged by Meetings and Papers Committee.

TECHNICAL PAPERS

A technical session devoted to electrophysics was held at the Mid-winter Convention. The following papers were read:

"Ionization of Occluded Gases in High-Tension Insulation", G. B. Shanklin and J. J. Matson.

"The Dielectric Strength of Air Films Extrapped in Solid Insulations". F. Dubsky.

"Abnormal Voltages in Transformers". L. F. Blume and A. Boyajian.

It is difficult for the engineer to keep up with the rapid evolution and progress in modern physics, as most of the work is clothed in mathematics and appears in philosophical magazines never read by engineers. The general lectures were arranged to overcome this, and have proved very successful and *popular*.

It is hoped that arrangements for a joint meeting with the American Physical Society can be made soon.

F. W. PEEK, JR., *Chairman*

ANNUAL REPORT OF THE TELEGRAPHY AND TELEPHONY COMMITTEE

To the Board of Directors,

A NOTEWORTHY development during the past year has been the drawing together of the arts of telegraphy, telephony and radio signaling into one subject of Electric Communication. War conditions have very materially aided this tendency. The enormous increase in traffic, due to the war, made it imperative that each method of rapid, long distance communication be developed to its utmost capacity.

With the entry of our country into the war, the requirements of the military and naval authorities imposed a certain amount of concentration in the direction of communication facilities, and, when on July 31, the Postmaster General was empowered to take control of all telegraph and telephone lines in this country, unity of direction was further advanced.

On December 1, the facilities of the two large commercial telegraph companies were consolidated, and on December 4, the marine cable systems of the various cable companies were taken over by the government.

From an engineering standpoint these developments had a retarding effect upon the plans of the private companies for apparatus and construction improvements; but several Boards appointed by the government were given opportunity to experiment with suggested technical improvements, and to distribute traffic loads where they might be moved in the most expeditious manner possible over the lines of the combined systems.

STANDARDIZATION

During the year a considerable amount of contributory work has been done along the line of standardizing telephone, telegraph and radio terms, and symbols.

RAILROAD COMMUNICATION

The Association of Railway Telegraph Superintendents was, during the year, absorbed by the American Railroad Association; but the engineering work carried on by the Association has not been neglected by the railroad telegraph and telephone officials. Indeed, the manager of the Telegraph Section of the Railroad Administration has been enabled to coordinate technical effort to the end that the railroads will undoubtedly benefit from the work done by technical committees.

LIGHTNING PROTECTION

During the year considerable improvement has been made in the design of lightning protective apparatus used to safeguard telegraph and telephone lines from lightning and accidental contact with foreign wires carrying higher potentials than the signaling circuits. The vacuum type of arrester has been improved in effectiveness and reliability.

PRINTING TELEGRAPHS

On the lines of the Western Union Telegraph Company the use of the multiplex printer system has been considerably extended. About 80 per cent of this company's wire traffic is now handled by means of the Multiplex, and the Morkrum printer duplex systems. The Postal Telegraph-Cable Company, on the other hand, which has experimented during the past twelve years with three different printing telegraph systems, on April 1, 1919 discontinued the use of printing telegraphs on all of its lines, reverting exclusively to the Morse manual system.

SUBMARINE CABLE SIGNALING

During the past year considerable improvement has been made in the traffic carrying capacity of submarine telegraph cables. The application of the cable relays and amplifiers invented by Heurtley, Dixon, Brown, Gulstad, and Bruce, to cable operation, has made longer direct-working possible, and has resulted in increased speeds in words per cable in unit time. On the cables operated by the Western Union Telegraph Company between Hearts Content, Newfoundland, and Valentia, Ireland, the printing telegraph system has been applied with satisfactory results.

[The following contributions to this report are submitted by the members of the Committee whose names appear under the various subheads.]

TELEGRAPH LINE CONSTRUCTION

R. E. CHETWOOD

No radical changes in the design or type of telegraph construction have been made during the past year. There has, however, been a decided tendency towards stronger construction, both for the pole line and for the wires. Stronger pole line construction has been obtained by using more poles per mile of line, by using stronger poles and by the use of as short poles as right-of-way conditions will permit.

Developments in the telegraph art which lessen the harmful effect on telegraph transmission of the high electrostatic capacity of conductors in cables have permitted the use of aerial and underground cables to a somewhat larger extent than formerly. Stronger and more reliable construction has thus been obtained in and near cities where, on account of right-of-way conditions, it is often necessary to carry the wires of two or more pole lines on one line.

As iron wire is a satisfactory conductor for many classes of telegraph circuits, both with manual or with automatic operation, a considerable increased strength in line wires has been obtained by using iron wires of large size, such as No. 4 and No. 6 B. w. g.

INTERFERENCE TO COMMUNICATION CIRCUITS FROM NEIGHBORING POWER LINES

A. W. COPLEY

A mutual interchange of information and co-operative work between power companies and communication companies toward the solution of inductive interference problems in particular cases, has resulted in improved conditions, and at the same time, caused the maintenance of friendly relations between the parties.

Conditions obtaining on the power circuits during regular operation are given the greatest attention working out the solution, as serious interference under such conditions is intolerable. Abnormal transient conditions, such as short circuits on the power lines, are of comparatively infrequent occurrence and short duration, and if protection of the communication circuits against interference at such times were to be generally attempted, the cost and inconvenience of the arrangements would be out of all proportion to the benefits gained. It is, therefore, considered reasonable to protect only against physical damage to the property or personnel of the communication company under such abnormal conditions.

For the prevention of interference in telephone circuits under operating conditions, co-ordinated transpositions of the power and telephone circuits give a solution in most cases. Wave-form correction in the power circuits by resonant shunts or other means, may sometime be resorted to, especially when transpositions are impracticable. There has been no standard in the past by which the tendency of a machine to cause telephone interference could be measured. The new telephone interference factor now defined in the Standardization Rules provides a measure of this tendency and its use will assist in the adjustment of interference difficulties.

Telegraph interference from transposed power circuits is so small as to be negligible, except during abnormal conditions. In cases of interference from power lines which cannot be transposed, such as trolley circuits, the use of resonant shunts in the telegraph circuits has given very good corrective results. Much has been accomplished in the prevention of interference from such circuits by a careful layout of the power system, particular regard to substation locations being given, so as to minimize the inductive interference in the communication lines.

LONG DISTANCE TELEPHONE TRANSMISSION

O. B. BLACKWELL

The features of perhaps greatest general interest at this time in long distance telephone transmission are telephone repeaters, loading, and multiplex-telephony.

The vacuum tube amplifier has been highly developed for use in telephone repeaters and has become the preferred form of amplifier for such use. The transmission gains which may be obtained with such an amplifier in two-way operation depend in each case on the electrical conditions of the line in which used. This has had a large effect on cable and line construction.

In the loading art it has required the development of coils having especially great magnetic stability and uniformity. Such electrical stability is also particularly important on long circuits which are composited for Morse operation to prevent the telegraph interfering with the telephone transmission.

The repeater and loading developments have considerably extended the use of telephone cables for long distance transmission. They have, however, made it imperative to keep open wire circuits as free as possible from cable in order to prevent such electrical irregularity from reacting on repeater operation.

A multiplex telephone circuit has been put into commercial use between Baltimore and Pittsburgh, giving four circuits over a single pair of wires in addition to the ordinary telephone circuit over the same pair. This multiplexing depends on the use of "carrier currents." In order, however, to be effectively transmitted over telephone circuits, these carrier currents are of comparatively low frequency and the ratio between them and the telephone frequencies being transmitted is, therefore, low. This brings in peculiar problems in the handling and separating of these different frequencies on the one pair of wires.

The vacuum tube plays an important part in this system, not only as an amplifier but also as a modulator and demodulator. The transmission of these frequencies over telephone circuits makes more severe requirements on the circuits than do ordinary telephone frequencies in view of their larger attenuations, the greater effect on them of lengths of cable and their increased tendency to crosstalk. The multiplex system promises to have a considerable field in comparatively long distance telephone service.

TELEPHONE OUTSIDE PLANT PRACTISES

W. R. McGOVERN

At the present time the best practise in outside plant construction is represented by an all cable and twisted pair distribution in the medium and large exchange districts. In the small exchanges where estimates of future growth do not show the need along any given route for more than from 10 to 15 circuits, it is more economical to use crossarm construction with galvanized wire. Even in these exchanges it is desirable to place a small amount of cable in the immediate vicinity of the exchanges.

In those exchanges where cable and twisted pair wire are used, the cable portion of the plant should be so designed, and supplied with distributing terminals, that the amount of twisted pair wire is held at a minimum. In thickly populated areas the twisted pair wire should appear only as the connecting link between the pole route and the subscribers premises.

The dividing line between the underground and the aerial portion of the cable plant is a variable determined by local conditions, but in general the circuits should be placed underground, if estimates of growth for several years in the future show that they will exceed a few hundred in number.

Among the tendencies of telephone construction is the more extended use of fine-wire cable, it being theoretically practicable to use 24-gage, or smaller circuits in the zones near the exchanges. The use of small-gage wire in the aerial cable plant will result in economy; and a more extended use of fine-wire cable in the underground plant will conserve conduit space as well.

Another tendency is towards the use of cable terminals of such a type that potheads are unnecessary. The so-called sealing chamber type of terminal allows paper cable to be taken directly into the terminal, and its use will become more extended in the future.

STATUS OF AUTOMATIC TELEPHONY

ARTHUR BESSEY SMITH

There is at the present time a general recognition that automatic telephony has arrived. It has been established that automatic switching is very flexible, offering a number of solutions for every traffic problem.

The switching apparatus has become fairly well standardized as to mechanism, electrical circuits, processes of manufacture and maintenance. The recent trend of development has been along the line of increasing the factor of safety, making more definite the standards of adjustment, and increasing the life of parts subject to wear.

Automatic switching is a recognized efficiency device for many classes of service. The private automatic exchange is widely used in commercial institutions, either isolated or with trunks to the public system. It is maintained by ordinary electrical employees.

The rural automatic telephone has become firmly established. It is served by party lines from a city central office or from a small switchboard at the center of the rural area. Common battery is successfully used for voice transmission as well as for dialling.

Single office and multi-office exchanges up to 70,000 lines are in use and have proved their adequacy. The application of automatic switching to metropolitan areas is a recognized possibility.

The automatic switching of toll lines is established. It is possible to dial directly from the originating toll board to the called party's telephone. This has increased the time-efficiency of toll lines.

ARMY SIGNALING

E. B. CRAFT

The telephone has become an essential part of the modern war mechanism. Because of the vast territory and enormous masses of men involved in military operations of the past four years, the military telegraph has assumed an entirely new character.

We now find networks of wires and cables reaching from all points of the fighting zone back to bases of supply often hundreds of miles away. These networks make use of the most modern developments of the communication art, such as high speed printing telegraphs and telephone repeaters for long distance circuits.

While necessarily of a temporary type of construction, these wires systems with their exchange centers, compare in magnitude and complexity with those of our large industrial centers.

Probably the most important development and one which will have the most far reaching effect upon the art in the future, relates to the development and application of thermo-ionic devices. The design and manufacture of these devices commonly known as vacuum tubes, has been placed upon a basis which probably could not have been reached within a decade under normal peace time conditions.

Some of the more important applications comprise the following:

Radio telephone for communication to and between airplanes while in flight.

Determination of direction by radio compass.

Signaling through the ground by means of leakage currents.

Overhearing of enemy operations through the use of energy amplifying devices.

As a result of the intensive development of the past two years there has been made available to the engineer, a device which will in the future undoubtedly play an important role not only in the communication but in other branches of the engineering world.

RADIO SIGNALING

JOHN STONE STONE

Radio engineering has continued to feel the powerful stimulus of the World War.

Secrecy has necessarily veiled a considerable amount of the original discoveries and inventions made in this field as well as the complete statistics which would disclose the enormous expansion of the manufacturing industry in this line during the past year, because all the development and manufacture were necessarily conducted for the military and naval services and under strict governmental observation and regulation.

The general tendency has been towards a more systematic and sound engineering development of radio communications. Sustained-wave-train radio telegraphy has, to a great extent, replaced the older and less satisfactory damped-wave-train, or spark system, while radio telephony has been developed into a most practical form for air plane service as well as for land and ship stations.

These advances have largely been the fruit of careful quantitative studies of the audion, resulting in the possibility of accurately predetermining its design when it is to be used either as an oscillating generator, detector, amplifier or modulator.

Advances have been made in minimizing "interference" and the amount of radiated power which can be effectively "modulated" by the voice has been increased with a considerable resulting increase in the range of radio telephony.

DONALD MCNICOL, *Chairman*

ANNUAL REPORT OF THE MARINE COMMITTEE

To the Board of Directors,

THE status of the application of electricity to merchant vessels was such that at the beginning of this season's Institute work it was deemed necessary to greatly enlarge the membership of this Committee. In previous years the shipbuilding industry comprised about five or six fairly large size steel shipbuilding plants. The representatives of these companies as well as the representatives of the few manufacturing concerns which supplied the shipyards comprised the Marine Committee. These gentlemen were fully informed as regards the methods of installation and the various requirements of the classification societies, ship owners, naval architects, etc., so that standard requirements in the nature of governing rules were hardly necessary. The war brought about a great expansion in the shipbuilding industry with the outcome that a great many men were brought into the newly organized plants to perform the work of electrical applications. It seemed fitting to bring these new men into contact with the more experienced members of the Institute and to prepare for all concerned some guiding regulations to make for the proper development of electrical installations on board merchant vessels.

Seven meetings of the committee were held. The first few meetings were used for the purpose of organizing both the committee itself and the work to be done. To this end sub-committees on the two main questions pertaining to the work were appointed. These main committees were called the Ship Installation Committee and the Electric Ship Propulsion Committee. The Ship Installation Committee was further subdivided into a Power Apparatus Committee, a Lighting Committee, and a Distribution Committee.

The sub-committees of the Ship Installation Committee prepared the necessary details pertaining to the recommendations and reported in conference to the Ship Installation Committee. In turn the Ship Installation Committee laid the results of its work before the Marine Committee proper. I, therefore, have the pleasure of submitting for the consideration of the Board of Directors the report of the Merchant Ship Installation Committee which establishes the Rules and Requirements as approved by the Marine Committee.

This report speaks for itself as regards the activity of the Ship Installation Committee. It may be stated that the work of the Ship Propulsion Committee has shown a similar amount of work but the nature of its investigations preclude the incorporation of the data that have been made, as the status of this application does not admit of the definite formulation of Rules and Requirements. I am attaching, however, as an addenda to this report a considerable amount of exceedingly interesting and important material which cannot at this time be put into print but will remain in the files of the Institute for the work of the Committee next year.

The undertaking of this work as is always to be expected grew with the investigations made by the sub-committees so that the work of the Ship Installation Committee is by no means complete. There are five items which though acted upon by the Power Apparatus Committee and the Lighting Committee were not considered by the main committee. There are seven items still to be completely considered by the sub-committees. In general, these subjects include such questions as the following:

(completed by sub-committee)
Control equipments.
Alternating-current apparatus control.
Radio installations.
Running lights, etc.
Fire-alarm systems.

(not completed by sub-committee)
Tabulation of generating sets.
Storage battery installation.
Gyro-compass installation.
Gyro-stabilizing installation.
Tabulation of wires and conductors.
Appliances in vicinity of compass.
Inspection report to Classification Societies.

The data pertaining to these questions are not to be printed but are attached as an addendum to this report for the benefit of the committee next year.

The attendance at the meetings of this committee have all been large and representative. The committee membership as a whole is to be congratulated upon the work accomplished. The chairmen of the main committees as well as the sub-committees have worked earnestly and faithfully. It is to be expected that next year, by following the work here begun, that the Institute will be enabled to lay down the first requirements for marine installations that have ever been made. It is suggested that next year the concluding work of the committee should be done in conference with the Classification Societies and the Bureau of Standards.

H. A. HORNOR, *Chairman*

ANNUAL REPORT OF COMMITTEE ON USE OF ELECTRICITY IN MINES

To the Board of Directors,

I BEG to submit the following report regarding the activities for the past year of the Committee on the Use of Electricity in Mines.

The Committee provided papers for one session of the Philadelphia Meeting at which time a paper on "The Use of Electric Power in the Mining of Anthracite Coal", by J. B. Crane, Vice President, Lehigh Navigation Company, and a paper Drum Shapes as Affecting the Mine Hoist Duty Cycle and Motor Rating," by F. L. Stone were presented.

Having been Chairman of the Schenectady Section during the past year has prevented me from devoting as much time to the Committee work as I would liked to have given to it. However, I have given considerable thought to the question of making the activities of this Committee felt among mining men, and while I have no definite plans to recommend for doing so, I do believe that closer cooperation between the various technical societies, both national and local branches, will prove very beneficial. As the Committee on Development has this whole matter in hand at the present time I have taken no action with reference to it other than a talk with our local representative on this Committee concerning the question.

K. A. PAULY, *Chairman.*

ANNUAL REPORT OF THE IRON AND STEEL INDUSTRY COMMITTEE

To the Board of Directors,

THE increase in the use of electrical energy in the Iron and Steel Industry in the last few years has been remarkable and partly responsible for the prominent position the United States has taken in furnishing steel products to the entire world.

Over 500 motor equipments are in use in this country, driving main rolls with a maximum horse power of over 850,000, while the total horse power of auxiliary motors exceeds 1,350,000, making a total of 2,200,000 h. p., which requires about 650,000 kw. in generator capacity. In addition, electric furnaces require approximately 250,000 kw. in generator capacity, or a grand total of 900,000 kw. generator capacity in the iron and steel industry.

This is by far the largest amount of electric energy required in any one industry, and from all indications will continue to increase considerably in the future.

The activity of the Iron and Steel Industry Committee has mainly been confined to arranging joint meetings with the Association of Iron and Steel Electrical Engineers for the purpose of presenting and discussing technical papers of interest to members of both societies.

Practically all members of the A. I. E. E. especially interested in the steel industry are members of the A. I. & S. E. E., and attend the latter's technical monthly meetings and yearly conventions.

While members of the A. I. E. E. are primarily interested in the design and development of electrical apparatus, the membership of the A. I. & S. E. E. consists mainly of men having responsible charge of electrical machinery in the steel industry and is primarily interested in the installation and operation of electrical apparatus.

It is suggested that both societies closely co-operate, keeping in touch with the latest and best practise and progress in the art, and avoid duplication of effort as much as possible.

One of the main subjects to be taken up jointly by the two societies is "Standardization." In general, the Standardization Rules of the A. I. E. E. have been adopted by the A. I. & S.E.E., however they cover primarily standardization of terms and conditions which characterize the rating and behavior of electrical apparatus without going into details of construction.

The Standardization Committee of the A. I. & S. E. E. feels that for its work, standardization rules should also refer to such details and dimensions as will reduce cost and complexity and not retard progress in design and development. In its *Proceedings* of 1917, Mr. H. D. James, then Chairman of "Co-operative Standardization," expresses himself very ably in regard to advantages of Standardization as follows:

> Those interested in the application and manufacture of electrical apparatus realize the many advantages in standardizing equipment. Not all, however, stop to analyze these advantages, and in order to bring these to your attention, the Standards Committee wish to present the following advantages of standardization:
>
> 1. The best average equipment is obtained when a considerable number of qualified persons is willing to agree upon a particular arrangement. Such an agreement is based upon the available facts and when the personal bias of individuals is eliminated.
>
> 2. The adoption of a standard by an association tends to establish engineering and commercial practise in that particular field. The first approximation towards standards crystallizes criticism and suggestion, so that the tentative standard may be improved.
>
> 3. A standard equipment enables operating men to interchange parts of an equipment between different machines and reduces the number of spare parts required.
>
> 4. A standard equipment enables the manufacturer to carry this equipment in stock, so that prompt delivery can be made. It also reduces the misunderstandings which sometimes arise regarding the details of such an equipment.
>
> 5. A standard equipment permits the manufacturer to organize his work so that a better product can be obtained at a reduced cost.
>
> 6. A standard equipment saves the time of an executive. It enables him to delegate his authority to a subordinate where the choice of the equipment is limited to recognized standards.
>
> 7. Standard equipment simplifies the work, of the purchasing departments. Where standards have been agreed upon, price forms can be issued and a great deal of delay incidental to the purchase of this apparatus will be avoided.
>
> 8. In making plans for the application of electrical equipment to the various types of machinery required, the draftsmen will have available standard outline drawings, together with weights and dimensions, so that he may proceed expeditiously with his work.
>
> 9. Where equipment is agreed upon, the question of the mechanical interchangeability of parts can be more readily considered; for instance, the mechanical dimensions of a grid resistor frame.

10. Standard equipment will concentrate the attention, and increase the number of research investigations to improve this standard equipment.

11. The adoption of standards indicates the stability of any art, showing that it has passed through its initial stages and has become a recognized factor in the industry. This results in placing engineering upon a higher plane and commands the respect for those engaged in this particular art.

12. It is important that the electrical engineers of the Iron and Steel Industry take an active part in national standardization, so that the requirements of their industry may receive proper consideration in parallel with the standardization work of other organizations.

Standardization forms the basis of a great deal of our work. We are accustomed to take for granted many of our standards, such as weights and measures, screw-threads, pipe fittings, standard time, standard railroad gage, etc. In every direction we see evidences of standardization. If it were not for these every-day standards, it would be very difficult for us to do business or to design apparatus. Standardization of motors, controllers, cranes, etc., is the carrying out of this general idea, which has existed for generations among civilized people.

Manufacturers of electrical apparatus should get together for real co-operation, to a much greater extent than they have done in the past, and not stop in the standardization of characteristics, voltages, frequencies, ratings, torque, etc., but begin to standardize mechanical dimensions, shafts, gears, pulleys, etc. They are generally in sympathy with the subject of standardization, but think it is the duty of the user of electrical apparatus to first decide on the proper standards and then co-operate with them, so as not to interfere with the progress of the art.

The problem then before us is to find the proper standards; the first step towards solving this problem should be to make general requirements for the installation and operation of electrical apparatus in the steel industry as uniform as possible.

It is suggested that the Iron and Steel Industry Committee take up this work jointly with the A. I. & S. E.E., and formulate a plan by which sufficient practical and theoretical data can be collected through the members of both societies, and finally draft standard rules for the installation and operation of electrical apparatus in the iron and steel industry. A great amount of detail work and time will be required to carry out this plan, and the closest co-operation of all members of both societies interested in this subject will be required.

It is recommended that the "Board of Directors" of the A. I. E. E. confer with the Executive Committee of the A. I. & S. E. E. in regard to this subject.

E. FRIEDLAENDER, *Chairman.*

ANNUAL REPORT OF THE INSTRUMENTS AND MEASUREMENTS COMMITTEE

To the Board of Directors,

THE Committee during the past year has had the difficulties and handicaps incidental to the war period to contend with, but these were borne by all similar committees and associations so that nothing is gained by amplifying them in this report.

Several meetings were held and in addition an effort was made to transact as large a volume of the business of the Committee as possible by correspondence. The Committee has been responsible for the preparation of several papers on subjects relating to the functions of the Committee and while some of these papers have been completed, others are still in preparation. It has not been possible, however, to present any of these papers due to the crowded calendar of meetings. It is strongly recommended, however, that a session be set aside, possibly at the next Mid-winter Convention, for a group of papers on the subjects within the scope of this Committee. Justification for such a session is based on the success of the similar session held during the initial year of this Committee's activity.

The Committee through its membership was active with the Bureau of Standards working with the War Department to standardize purchase specifications for electrical instruments and accessories for overseas service, and joined in other governmental activities within the functions of the Committee.

The demands of the Government for standard instruments, for special designs of the standard types and for instruments to meet requirements new to the art, required a development of far reaching magnitude. The Institute was largely represented in the personnel engaged in this work and early in 1918 the output met the needs of the Government in all its special fields and met as well the unusual demands of the country.

The appeal by the Government to manufacturers heretofore specializing on a particular line of instrument or accessory to undertake the manufacture of instruments or accessories foreign to their regular production, resulted not only in increased production but the enlisting of resources and individuals new to a given problem. Not only did this produce the increased quantity desired by the Government but also resulted in a greatly improved product from a technical standpoint.

Space does not permit other than a passing reference to these developments, some of which are briefly summarized herewith:

HIGH SENSITIVITY MEASURING DEVICES

Prior to the war thermoelectric couples had been employed for measuring low values of alternating current. These couples were almost exclusively of German manufacture and not extremely accurate nor sensitive. There have since been developed and put on the market, thermoelectric couples of American manufacture which are superior in every way and are now available for the accurate measurement of alternating currents of any frequency. Their sensitivity is such that a current of one milliampere will produce a deflection of 3 millivolts in a direct-current voltmeter.

The recent perfection of the thermo-ionic tube has put into the hands of the physicist and engineer, a new device for use in measurements of precision. By means of thermoelectric amplifiers, very feeble currents may be greatly amplified and their properties studied.

A peak voltmeter has been developed which makes use of the unilateral properties of the thermo-ionic tube. With this device it is possible to measure the instantaneous potential between any two points in an electrical circuit, without conuming any part of the power in the circuit and without disturbing its electrical constants.

Vacuum gages of high precision have been developed, making use of these tubes. Pressures from 0.001 millimeter down can be directly measured.

Quantity production of hot-wire ammeters.

Bureau of standards Type D decremeters.

Wavemeters.

Consideration of the foregoing and the uses to which they were put in Government service, will make it apparent that some of the very significant developments, while valuable material for the Institute, could not properly during the major portion of the year, be made the subject of papers or reports.

The widespread use of maximum demand apparatus requiring accurate timing and synchronous operation renders very interesting the development of the Warren timing element and system.

The Warren self-starting synchronous motor constitutes a very accurate time-keeping element in the space of 2⅛ by 2⅝ by $2\frac{9}{16}$ inches capable of delivering more power than many of the present spring driven clocks, and this without the usual weekly or monthly winding. It is, therefore, particularly adapted for use in graphic and demand meters, time switches, etc. where the advantages of a time-keeping element with such characteristics are obvious. An even more important asset, for certain applications, than any of those mentioned, is that it is possible to synchronize the records of graphic or demand meters—a feature which heretofore has been very difficult to accomplish.

The small timing element can be installed in any device connected to the alternating-current mains of a company where constant speed and synchronous operation are desired. The element itself, when combined with an accurate clock, is used as a master clock at the central station both for providing uniform time throughout the system and for regulating frequency.

S. G. Rhodes, *Chairman*

ANNUAL REPORT OF EDUCATIONAL COMMITTEE

To the Board of Directors,

FROM September 1918 to January 1919, all the principal colleges in the country were under a military rule in connection with the Student's Army Training Corps. During that time the activities of the institution of learning and their policies were governed entirely by the War Department and its Educational Committee. Before the armistice it looked as if such an arrangement would continue throughout the whole academic year, and for this reason, your committee did not make any effort to plan its work, merely keeping itself in readiness for an emergency.

After the disbandment of the Student's Army Training Corps the colleges found themselves facing many difficult problems on account of extremely confused scholastic records of many students, mixed schedules, lack of instructors, and the necessity of finishing the academic year in such a way that a large number of students, including those discharged from the regular army, could start the next academic year with a clean slate. The committee did not feel, therefore, that the time was opportune for discussing any general educational policies, or sending out questionnaires, until conditions became more normal.

At one time during the war there was considerable talk about radical changes in technical education, as part of the general plan of reconstruction after the war. It seems, however, that the colleges of engineering are essentially going back to the pre-war standards, methods, and curricula. An investigation recently made by one of the leading technical journals, among the principal schools of engineering in the country, has shown an overwhelming majority of the deans and professors to be in favor of no radical changes from the ante-bellum methods.

Educational reconstruction, if any, will have to follow and not precede the general economic and industrial reconstruction. The country as a whole first has to decide what is technically and economically right, what is feasible to accomplish and what kind of men and women will be needed as leaders under the new regime. The first steps of this reconstruction will have to be carried on by the present college graduates, and the coming generation will have to be educated in accordance with the newly laid principles of justice and brotherhood, and with sound principles of scientific engineering.

One important direct contribution of the war to the problem of education has been the introduction of elaborate psychological tests for analyzing a person's intelligence, fitness and caliber for a particular kind of work. Some leading universities have either introduced this system to test their students, or are planning to use it in addition to regular examinations. This is only a beginning towards a great new principle, namely that of fitting education to the individual, instead of trying to fit every individual to the same stereotyped courses of instruction.

If there is a recommendation that the retiring committee cares to make to its successors, it is that the new committee should add its influence to the general acceptance of the sound principle that the principal purposes of education are self-realization, inspiration for co-operative service, and the development of intelligence and moral sense, rather than an indiscriminate cramming with undigestable information. Psychological tests made before, during, and after the college course are among the first steps in this direction, and the new committee might well devote its effort to an adaptation of such tests for the needs of electrical and mechanical engineering.

V. KARAPETOFF, *Chairman.*

Presented at the 35th Annual Convention of the American Institute of Electrical Engineers, Lake Placid Club, N. Y., June 24, 1919.

Copyright 1919. By A. I. E. E.

HIGH-TENSION SINGLE-CONDUCTOR CABLE FOR POLYPHASE SYSTEMS

BY W. S. CLARK AND G. B. SHANKLIN

ABSTRACT OF PAPER

Three-conductor underground cable is limited to a line potential of about 25,000 or 30,000 volts. The transmission and distribution of large amounts of power at this voltage involves heavy investment in copper.

The only factor that limits the voltage rating of single-conductor sheathed cable is over-all diameter. Line voltages of 44,000 and 55,000 volts are perfectly practical and mean a saving in copper over three-conductor cable of as much as 70 per cent. Induced sheath voltage or current, depending upon whether sheath is open- or short-circuited, is the only factor in the operation of single-conductor cable that needs serious consideration.

In this paper the dielectric, inductive and general line characteristics of three-conductor and single-conductor cable are compared. The advantages and disadvantages of each type of cable are brought out in a way that will aid in deciding the merits of individual problems.

INTRODUCTION

SINGLE-CONDUCTOR sheathed cable has been used only to a limited extent for conducting polyphase energy. This has been but a natural result of conditions. Multi-conductor cable has generally met underground service conditions satisfactorily and there has been a sufficient overlap with overhead high-tension service to make one of the two meet all but a very few exceptional cases.

Single-conductor cable is more economical than multi-conductor only when there are large quantities of high-tension power to be handled. It is only within recent years that steadily increasing units of power have reached figures that make the substitution of single-conductor cable for multi-conductor an attractive proposition.

Another feature favorable (although not welcome from an economical standpoint) to a more extended use of high-tension single-conductor cable is the tendency towards abolishment of overhead lines in densely populated districts.

COMPARISON OF MULTI-CONDUCTOR AND SINGLE-CONDUCTOR CABLE

The use of multi-conductor cable for transmitting and distributing large amounts of power economically is limited in two ways; by the maximum allowable voltage, and conductor cross section, or current carrying capacity.

The highest operating voltage in ducts is about 30 kv. This voltage will probably never be safely exceeded to much of an extent in ordinary three-conductor cable, because of the tangential stresses first pointed out by Höchstadter.[1]

Conductor cross section is limited to about 250,000 or 300,000 circular mils. Larger conductors than this would make the cable unwieldy, difficult to manufacture, transport and install. The radius to which it could be safely bent would not be small enough for installation purposes. For a rise of 30 deg. cent. a 300,000-cir. mil, 30-kv., three-conductor cable will carry approximately 230 amperes per conductor under favorable conditions. This represents 12,000 kv-a. per cable. The copper loss will be 6.54 watts per duct-ft. per cable at 55 deg. cent. With 12/32-in. by 6/32-in. thickness of insulation and 9/64-in. lead sheath the outer diameter is 3.63 in. Since this is the highest voltage and largest cross section that it is practical to use we may regard this design as having the least amount of copper and lowest I^2R losses for the given amount of power to be transmitted.

Now, let us compare this limiting size of three-conductor cable with different designs of single-conductor cable. Comparison is best made by reference to the design data in Table I. Cable (a) is the three-conductor cable discussed above. Cable (b) is single-conductor cable, has the same voltage to neutral as (a) and the same total copper cross section, also the same current density. The kv-a. capacity and copper loss per duct-ft. are therefore the same. The two must be distinguished between however, as (a) is a separate three-phase circuit, while (b) is only one leg of a three-phase circuit. It is assumed that the total amount of power to be transmitted will require three cables or some multiple of three, of either type and the cables will each be in a separate duct. It is then easy to see how closely the conditions coincide.

---

[1] "Three-Core Cables" E. T. Z. Heft. 47, 1915. Abstract in *London Electrician*, 1916, p. 209.

TABLE I

COMPARISON OF THREE-CONDUCTOR AND SINGLE-CONDUCTOR CABLE THREE-PHASE Y CONNECTION

Cable	Total copper cross section cm.	Core diam. in.	Cond. diam. in.	Thick. insul. in.	Lead sheath in	Overall diam. in.	3 ϕ volt to neut. Kv.	Current per cable	Kv-a. capacity per cable	$I^2 R$ loss per ft. per cable watts	Ratio a/b	Kv/.cm. G_{max}
						Three-conductor cable.						
(a)	3 x 300,000	0	0.631	12/32x6/32	9/64	3.63	$30/\sqrt{3}$	3 x 231	12,000	3 x 2.18	......	......
						Singl-eeconductor cable.						
(b)	900,000	0	1.09	17/32	1/8	2.40	$30/\sqrt{3}$	693	12,000	6.54	0.505	18.3
(c)	275,000	1.16	1.30	1 3/32	9/64	3.76	$55/\sqrt{3}$	378	12,000	6.32	0.370	19.5
(d)	550,000	1.00	1.30	1 3/32	9/64	3.76	$55/\sqrt{3}$	378	12,000	3.00	0.370	19.5

An advantage of (b) over (a) is in its smaller over-all diameter. The cost of installation would be less, especially as regards simplicity of joints, and the cable itself would be cheaper because of less insulation and lead. Another advantage is that (b) can be more safely overloaded with less danger of deterioration developing, as there are no tangential stresses.

The disadvantages of single-conductor cable depend upon individual cases and involve rather complicated considerations such as: transposition at manholes, necessity of breaking the continuity of lead sheath or accepting the additional loss introduced by short-circuited sheath. This will be dealt with later in a way that will aid in deciding the merits of each individual case.

We would not especially recommend the substitution of cable (b) for (a) although there probably are cases, such as short tie-ins, where it would be advantageous. The real advantage of single-conductor cable lies in the opportunity for much higher operating voltage with consequent reduction in copper cross section and increase in line efficiency.

Cable (c) is designed to transmit at $55/\sqrt{3}$ kv. the same amount of power as (a) and (b) with the same loss per duct ft. There is, therefore, no gain in line efficiency, but the amount of copper has been reduced to less than one-third of that needed in (b).

If the limiting feature is line loss rather than investment in copper the copper cross section can be increased until the proper blaance is reached. In cable (d) the copper cross section is doubled and the I^2R loss thereby even more than halved because of the lower operating temperature. Cable (c) is designed to operate at 55 deg. cent. and (d) at about 37 deg. cent., 12,000 kv-a.

Liberal copper cross section is a good investment in any cable and particularly so in this type, for the additional cost involved is solely that of the bulk copper, which can be realized upon when the cable is scrapped. Reference to the design data of (c) and (d) will show what is meant by this. They both have exactly the same conductor diameter, which is fixed by the voltage rating and not by the amount of copper. The conductor is filled out to this diameter by wrapping the strands over a core, usually made of hemp rope. The only dimension that is different in (c) and (d) is the diameter of this core. Obviously, (d) is good for 41 per cent overload in an emergency.

Cable (d) with only 550,000 cir. mils of copper can transmit 41 per cent more load than (b), which has 900,000 cir. mils copper, for the same temperature rise.

DESIGN OF SINGLE-CONDUCTOR CABLE

The most important point to be considered in designing a high-tension cable is the maximum voltage stress. In a recent paper[2] a method of determining the maximum allowable stress is worked out in detail. For paper cable impregnated with mineral hydro-carbon base compound the maximum working stress is placed at between 19 and 20 kv-eff. per cm. (48.3 and 50.8 volts per mil.) The formula for calculating the maximum stress in single-conductor cable is

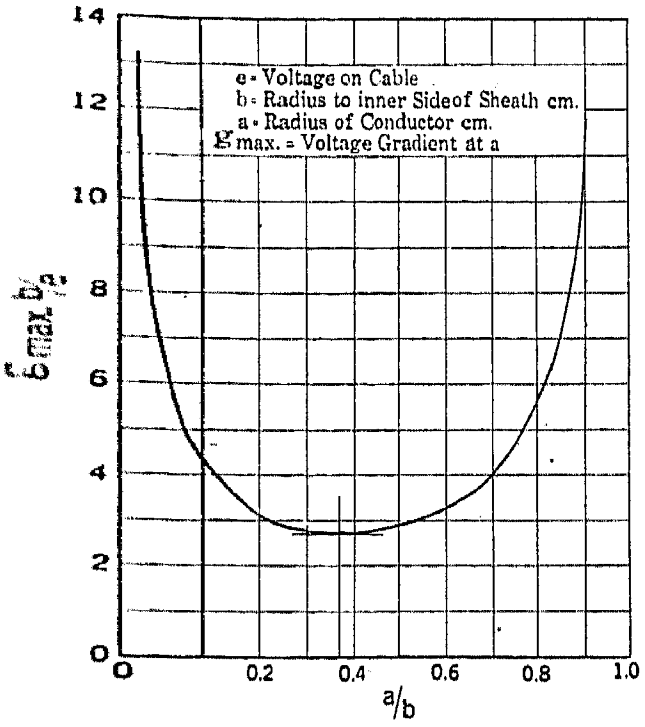


FIG. 1—SHOWING THE MOST EFFICIENT CROSS SECTION OF SINGLE-CONDUCTOR CABLE

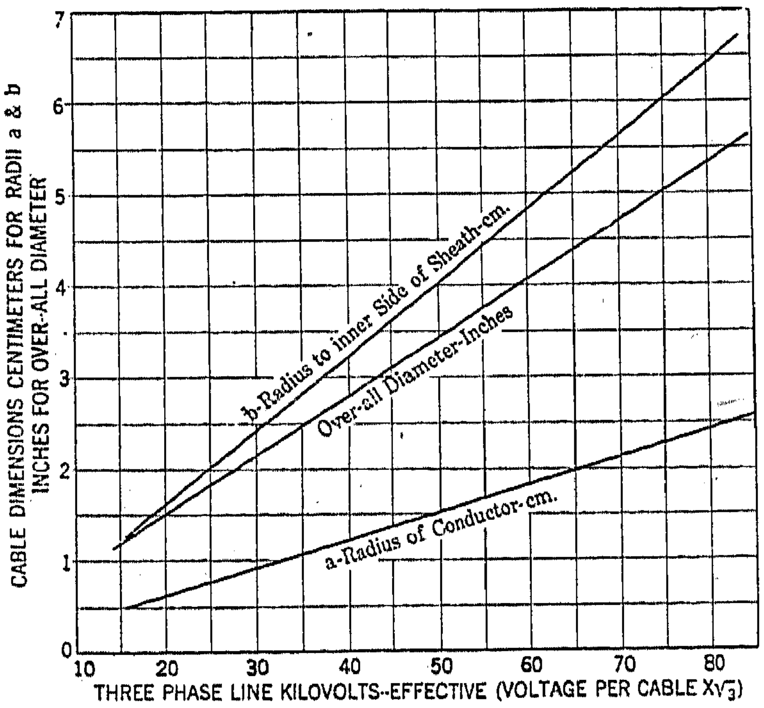


FIG. 2—SINGLE-CONDUCTOR CABLE—SMALLEST OVER-ALL DIAMETER FOR GIVEN VALUE OF G_{max}

$$G_{max} = 19.5 \text{ kv./cm.}$$

$$b/a = \epsilon = 2.718$$

$$g_{max} = \frac{e}{a \log_{\epsilon} b/a}$$

where

e = impressed voltage.

a = radius of conductor in cm.

b = radius to inner side of sheath in cm.

A curve is given in Fig. 1, calculated with the above formula and assuming different values of the ratio a/b. The most efficient cross section of cable, that is, the smallest cross section for any given conditions, is determined from the minimum ordinate of this curve. Ratio a/b found in this way has a value of 0.370, or b/a is exactly equal to the naperian logarithm base, $\epsilon = 2.718$.

---

[2] "Ionization of Occluded Gases in High-Tension Insulation," by G. B. Shanklin and J. J. Matson. TRANS. A. I. E. E., 1919, p. 489.

A safe working stress is selected at 19.5 kv. per cm. (49.5 volts/mil.) With $a/b = 0.370$ the most efficient dimensions of cable are calculated at various voltage ratings and are given as curves in Fig. 2.

Cable (b) in Table I, has a conductor diameter that cannot be reduced to the most efficient value because of the large cross section of copper involved. The "skin effect" is rather high,—7 per cent at 60 cycles. If this were a low-voltage cable it would be well to use a cored conductor to reduce "skin effect." There is an interesting relation between "skin effect" loss and dielectric loss in different cross sections of high-voltage cable, whereby, one is so nearly neutralized by the other as to allow considerable lee-way in designs. Space will not allow us to deal with this feature. It is mentioned here simply as an explanation of why the more desirable coreless conductor is used in cable (b).

With one thirty-second in. thickness of impregnated paper, $g_{max} = 18.3$ kv. per cm. and ratio $a/b = 0.505$. Reference to Fig. 1 shows this ratio to be high. The overall diameter is 2.4 in., whereas the minimum diameter for the same voltage rating in Fig. 2 is 2.15 in. However, the over-all diameter of 2.4 in. is sufficiently small to be easily handled and any further reduction would not be of great importance. When feasible, it is better not to stress a cable up to the maximum limit of 19.5 kv. per cm. That is why cable (b) is stressed to only 18.3 kv. per cm. The factor of safety is not only greater but due to lower dielectric loss, the cable can safely operate at a higher temperature.

With higher-voltage cable, over-all diameter is of greater importance, in fact, it is the only thing that limits the voltage rating. The practical limit of over-all diameter is about 3.9 in. (9.9 cm.) at the very most. From Fig. 2 the corresponding voltage rating is $57/\sqrt{3}$ kv. Cables (c) and (d) in Table I are rated at $55/\sqrt{3}$ kv., the nearest standard voltage to this. It would not be practical to use cable rated at, say, $66/\sqrt{3}$ kv. The over-all diameter would then be 4.46 in. (11.33 cm.) With special machinery for manufacture and special means of transportation and installation it would be possible to use such cable, but the extra expense involved would hardly warrant the benefit derived.

GRADING OF SINGLE-CONDUCTOR CABLE

It, no doubt, has occurred to the reader that since the over-all diameter is limited, further gain in voltage rating could be accomplished by grading the insulation. Osborne[3] and Beaver[4] demonstrated the advantages, from a theoretical standpoint, of graded cables. Actually, however, from a practical standpoint grading has few possibilities.

If all other factors are ignored grading is of benefit only if:

(a) there are high grade insulating materials of reasonable cost to select from, whose permittivities cover a fairly wide range, and individually have consistent values of permittivity that do not vary in different batches, or with ageing, etc.

(b) these materials have the same allowable working stress, regardless of their individual permittivities. If the allowable stress varied directly with permittivity it is apparent that even better results could be obtained by grading.

Neither of these two conditions actually hold. As far as (a) is concerned there are only three materials that can be considered, and of these three, one, rubber, introduces doubtful features. Aside from its high cost there are questions of its stability and length of life when used in composition with other materials, especially in such extremely high-voltage cable. The truth is, rubber has never been given a thorough try out in high-voltage cable because of its prohibitive cost.

The three materials mentioned and the range of their 60-cycle, relative permittivities at 55 deg. cent. are listed below:

Rubber, 4.6 to 6.0
Varnished cambric, 3.8 to 5.2
Impregnated paper, 2.8 to 4.0

A better representation of permittivity characteristics can be obtained from the dielectric loss data which will be given in this paper and those in two papers read before the Institute in 1917[5] and 1919.[2]

It is apparent from the above values that rough grading could be accomplished by using these three materials, but, condition (b) upsets this combination. In the paper, "Ionization of Occluded Gases, etc.,"[2] the safe maximum working stress for impregnated paper is placed at 19.5 kv. per cm. (49.5 volts per mil) while that for varnished cambric is only 16.0 kv. per cm. (40.6 volts per mil) because of its higher permittivity. Rubber insulation is not dealt with in that paper. We have made tests on rubber insulated cable, however, and found that its corresponding stress is a little less than that for varnished cambric, 15 kv. per cm. (38.1 volts per mil).

---

[3] "Potential Stresses in Dielectrics" by H. S. Osborne, TRANS. A. I. E. E., Vol. XXIX, p. 1553.

[4] "Cables" by C. J. Beaver, *Journ. I. E. E.*, Vol. 53, 1915.

[5] "Insulation Characteristics of High-Voltage Cables" by Clark and Shanklin, TRANS. A. I. E. E., 1917, p. 447.

It follows from all this, that practically no advantage is gained by an attempt to grade with these materials, because their maximum allowable working stress is inversely proportional to their relative permittivities. It is much better to have the insulation a homogeneous, compact mass of impregnated paper and aim towards a uniformly high grade product with low temperature coefficient and dielectric losses.

Beaver, in his paper on "Cables,"[4] advocates the metallic inter-sheath method of grading as furnishing means of overcoming the almost universal objections to the variable permittivity method. He proposes connecting these inter-sheaths to transformer taps and holding them at some definite, predetermined potential. As far as we know, this has never been tried out in practise, and we doubt if any operating company would be willing to assume the risk involved. The chief objections to this method are:

a. The inter-sheath must necessarily be of flimsy construction and there is danger of damaging it or even breaking its continuity during transportation and installation. High local potentials would concentrate at this point, resulting in failure.

b. The length of line would be very limited, as the inter-sheath could not carry the heavy charging current in long lines.

c. This method is based on the supposition that it is desirable to equally stress the whole insulation cross section. Actually, it is not desirable, because of the high dielectric loss involved.

d. Weak spots in the different layers would be lined up over the whole length. Failure of one layer would throw full potential on the others, also the short-circuit current might fuse the inter-sheath at some point far distant from the original point of failure.

The only other way of increasing voltage rating without prohibitive increase in over-all diameter is by stressing the cable above the safe limit we have assumed of 19.5 kv. per cm. This value is just about a maximum of what popular experience has shown to be conservative practise. It is more than a coincidence that recent laboratory investigations[2] check it very closely. The only radical departure from this practise we know of has been tried in Germany. In several instances underground cables have been worked at stresses much higher than 19.5 kv. per cm.[6] Just how successfully these experiments worked out over a reasonable period of time we do not know.

The single-conductor cable referred to in Litchtenstein's paper operated at a stress of 42 kv. per cm. At the time his paper was published it had operated intermittently for twenty months, however, the frequency was only 16 $^2/_3$ cycles and the current only 2.4 per cent of full-load current.

DIELECTRIC LOSSES

When estimating the watts loss in a conduit line the dielectric loss is often ignored. This is a great mistake. Even in the highest grade cable this loss is an appreciable part of the total, although, as a rule, a small part. The higher the voltage rating and operating temperature the more nearly it approaches the copper loss. In poor grade cable this loss may be even more than the copper loss. Consequently, the very highest grade of impregnated paper is absolutely necessary for high-tension cable.

The amount of space that can be given to dielectric energy loss characteristics is necessarily limited in this paper for the inductive characteristics of single-conductor sheathed cable and the general line characteristics must also be dealt with. We will limit the data presented to three designs of single-conductor cable and then show an empirical method of calculating the dielectric characteristics for any cross section of cable either single- or three-conductor.

The loss measurements were taken by the "Compensated Dynamometer Wattmeter Method," which is described in detail in an article published in 1916.[7] The energy loss data given

---

[6] "Underground High-Tension (60-kv.) Cables" by Leon Litchtenstein E. T. Z. Jan. 1913, Abstract in *Elec. Rev.* March 7, 1913.

"30,000 Volt, Three-Phase Cable System, Berlin" by W. Pfaunkuch, E. T. Z., April, 1916.

[7] "Compensated Dynamometer Wattmeter Method of Measuring Dielectric Energy Loss", by G. B. Shanklin, *General Electric Review*, Oct. 1916.

TABLE II.
CABLE SAMPLE NO. I.

Dia. over bare copper strands 1.186 in. (3.012 cm.).

Insulation—26/32 in. (2.064 cm.) thickness paper filled with mineral hydro-carbon base compound.

Temp.	Freq.	e kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-10}$	$\cos\theta_d$ per cent
23 deg. cent	60	12.01	0.276	0.0219	148.3	3.11	0.610	0.660
		18.01	0.411	0.0497	147.2	3.09	0.607	0.673
		21.00	0.489	0.0671	148.0	3.14	0.616	0.654
		23.99	0.559	0.0878	147.2	3.14	0.616	0.657
		26.98	0.630	0.1118	146.8	3.15	0.619	0.657
		29.97	0.700	0.1372	147.2	3.16	0.621	0.655
		32.96	0.772	0.1959	125.0	3.17	0.623	0.758
		35.95	0.840	0.2960	98.3	3.16	0.621	0.980
		41.93	1.018	0.6790	58.3	3.28	0.645	1.592
		47.90	1.200	1.3320	38.7	3.39	0.666	2.320
61.5 deg. cent	60	12.01	0.292	0.404	8.05	3.26	0.640	11.50
		18.01	0.440	0.920	7.95	3.28	0.643	11.61
		21.00	0.513	1.258	7.90	3.28	0.643	11.70
		23.99	0.589	1.637	7.90	3.28	0.645	11.61
		26.98	0.660	2.090	7.82	3.28	0.644	11.72
		29.97	0.732	2.560	7.89	3.28	0.644	11.68
		32.96	0.807	3.100	7.82	3.28	0.645	12.01
		35.95	0.888	3.890	7.49	3.31	0.650	12.20
		41.93	1.053	5.680	6.96	3.38	0.663	12.88
		47.90	1.246	7.750	6.64	3.49	0.686	13.01
60 deg. cent	25	11.99	0.136	0.304	10.60	3.60	0.707	18.68
		17.98	0.206	0.689	10.55	3.64	0.717	18.62
		20.97	0.240	0.940	10.52	3.64	0.715	18.68
		23.97	0.273	1.211	10.64	3.64	0.714	18.50
		26.96	0.310	1.538	10.68	3.66	0.720	18.40
		29.95	0.343	1.901	10.60	3.64	0.715	18.50
		32.94	0.380	2.300	10.58	3.67	0.722	18.40
		35.92	0.414	2.790	10.42	3.66	0.720	18.80
		41.91	0.494	3.980	9.92	3.76	0.738	19.28
		47.90	0.591	5.640	9.14	3.93	0.772	19.90
100 deg. cent	60	12.01	0.333	1.300	2.50	3.56	0.700	32.45
		18.01	0.501	2.730	2.65	3.58	0.703	30.32
		21.00	0.588	3.780	2.60	3.60	0.707	30.60
		23.99	0.670	4.970	2.60	3.60	0.706	31.00
		26.98	0.769	6.12	2.65	3.67	0.721	29.55
		29.97	0.849	7.750	2.60	3.64	0.716	30.50
		32.96	0.940	9.440	2.56	3.66	0.719	30.50
		35.95	1.037	11.890	2.45	3.70	0.728	31.90
		41.93	1.250	17.650	2.24	3.80	0.747	33.80
		47.90	1.482	23.500	2.19	3.94	0.775	33.10

TABLE II. (*continued*)

Temp.	Freq.	e kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-10}$	$\cos\theta_d$ per cent
100 deg. cent	25	11.99	0.169	0.827	3.91	4.16	0.816	40.90
		17.98	0.254	1.880	3.89	4.17	0.820	41.30
		20.97	0.296	2.550	3.88	4.17	0.820	41.10
		23.97	0.339	3.330	3.88	4.17	0.820	41.00
		26.96	0.380	4.280	3.82	4.16	0.818	41.90
		29.95	0.422	5.190	3.88	4.17	0.820	41.00
		32.94	0.469	6.290	3.88	4.21	0.827	40.70
		35.92	0.519	7.680	3.78	4.25	0.836	41.20
		41.91	0.630	10.970	3.60	4.43	0.870	41.60
		47.90	0.753	14.980	3.44	4.65	0.911	41.50
60 deg. cent	60	12.01	0.283	0.288	11.25	3.18	0.623	8.45
		18.01	0.431	0.644	11.35	3.22	0.634	8.29
		21.00	0.509	0.875	11.35	3.27	0.642	8.20
		23.99	0.579	1.134	11.40	3.25	0.639	8.15
		26.98	0.651	1.339	11.37	3.25	0.639	8.20
		29.97	0.722	1.780	11.35	3.25	0.639	8.24
		32.96	0.801	2.245	10.89	3.28	0.645	8.52
		35.95	0.877	2.840	10.25	3.30	0.649	9.01
		41.93	1.036	4.270	9.25	3.36	0.660	9.86
		47.90	1.232	6.070	8.48	3.45	0.677	10.30
60 deg. cent	25	11.99	0.130	0.253	12.82	3.47	0.683	16.22
		17.98	0.195	0.573	12.70	3.46	0.680	16.39
		20.97	0.230	0.775	12.76	3.52	0.691	16.10
		23.97	0.263	1.011	12.76	3.51	0.691	16.05
		26.96	0.298	1.262	12.80	3.52	0.691	15.78
		29.95	0.331	1.580	12.76	3.53	0.694	15.92
		32.94	0.366	1.931	12.65	3.55	0.697	16.02
		35.92	0.402	2.330	12.45	3.58	0.704	16.12
		41.91	0.486	3.400	11.61	3.71	0.730	16.69
		47.90	0.574	4.960	10.38	3.82	0.750	18.10
23 deg. cent	60	12.01	0.281	0.0262	124.0	3.16	0.621	0.777
		18.01	0.424	0.0595	123.0	3.17	0.623	0.782
		21.00	0.495	0.0796	124.5	3.18	0.626	0.767
		23.99	0.565	0.1038	124.5	3.18	0.626	0.767
		26.98	0.640	0.1318	124.0	3.20	0.629	0.762
		29.97	0.710	0.1684	120.0	3.20	0.629	0.794
		32.96	0.780	0.2760	88.6	3.20	0.629	1.072
		35.95	0.854	0.4130	70.5	3.20	0.629	1.347
		41.93	0.995	0.8290	47.7	3.20	0.629	1.990
		47.90	1.179	1.4590	35.3	3.32	0.653	2.580

in this paper can be considered as supplemental to those in the two papers previously referred to [2] and [5] and are dealt with in the same way.

In Table II, 25- and 60-cycle data are given for a high-tension single-conductor cable (No. 1) having the following specifications.

Conductor. 58 strands, each 0.104 in. diameter in two layers, on a core 0.770 in. (1.956 cm.) diameter. Total conductor diameter 1.186 in. (3.012 cm.) Cross section 625,000 cir. mils

Insulation. 26/32 in. (2.064 cm.) thickness paper filled with mineral hydro-carbon base compound.

Sheath. 0.101 in. (.257 cm.) thickness of lead.

Over-all Diam. 3.02 in. (7.67 cm.)

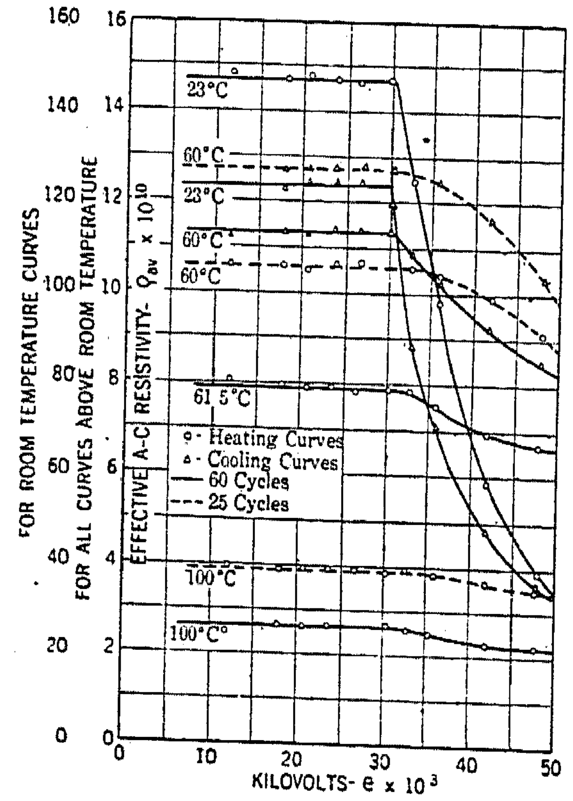


Fig. 3—Cable No. 1—Single-Conductor (See Table II)

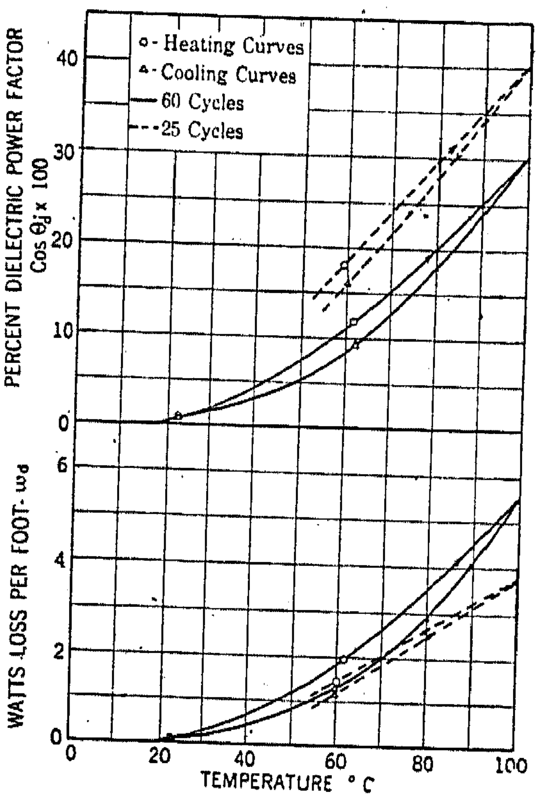


Fig. 4—Cable No. 1—Single-Conductor (See Table II) Curves Plotted at 25.4 kv.

The measurements in Table II are given consecutively in the same order in which they were taken in the laboratory. The nomenclature and the method of calculating the different values can be found in Appendix A. In Figs. 3 and 4 curves are given of effective a-c. resistivity vs. impressed voltage, watts loss per ft. vs. temperature, and dielectric power factor vs. temperature. These curves show the more important characteristics. The other characteristics may be obtained from Table II. Space does not allow the presentation of them all in the form of curves.

Referring to Fig. 3 the resistivity (ρ_{av}) is constant up to the impressed voltage at which internal ionization starts, in other words, the dielectric loss varies as the square of the voltage

TABLE III

CABLE SAMPLE NO. 2

Dia. over smooth conductor surface 1.311 in. (3.33 cm.)

Insulation—24/32 in. (1.905 cm.) thickness paper filled with mineral hydro-carbon base compound.

Temp.	Freq.	e Kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-10}$	$\cos\theta_d$ per cent
24 deg. cent.	60	12.01	0.309	0.0196	185.2	3.06	0.679	0.527
		18.01	0.468	0.0442	184.6	3.10	0.689	0.525
		21.00	0.549	0.0603	184.0	3.11	0.692	0.524
		23.99	0.628	0.0789	184.0	3.13	0.695	0.524
		26.98	0.707	0.0996	184.0	3.14	0.697	0.523
		29.97	0.785	0.1420	159.0	3.13	0.695	0.604
		32.96	0.864	0.2218	123.2	3.13	0.695	0.779
		35.95	0.940	0.3550	91.6	3.11	0.692	1.050
		41.93	1.125	0.8900	49.7	3.21	0.714	1.890
		47.90	1.327	1.8500	31.2	3.19	0.735	2.920
60 deg. cent.	60	12.01	0.328	0.298	12.15	3.26	0.724	7.55
		18.01	0.492	0.674	12.08	3.26	0.724	7.60
		21.00	0.580	0.925	12.00	3.29	0.730	7.60
		23.99	0.660	1.211	11.95	2.28	0.730	7.64
		26.98	0.744	1.529	12.00	3.28	0.730	7.63
		29.97	0.829	1.911	12.00	3.29	0.732	7.71
		32.96	0.910	2.410	11.90	3.28	0.730	8.03
		35.95	0.990	2.940	11.05	3.27	0.726	8.26
		41.93	1.190	4.650	9.51	3.38	0.750	9.34
		47.90	1.400	6.910	8.34	3.47	0.772	10.32
60 deg. cent.	25	11.99	0.150	0.251	14.40	3.56	0.790	13.95
		17.98	0.227	0.559	14.51	3.59	0.796	13.72
		20.97	0.269	0.768	14.40	3.65	0.810	13.62
		23.97	0.304	1.006	14.35	3.60	0.798	13.96
		26.96	0.345	1.260	14.40	3.63	0.805	13.68
		29.95	0.387	1.565	14.40	3.69	0.814	13.50
		32.94	0.423	1.906	14.35	3.65	0.809	13.68
		35.92	0.464	2.304	14.09	3.66	0.814	13.90
		41.91	0.551	3.480	12.69	3.75	0.831	15.05
		47.90	0.655	5.350	10.77	3.87	0.858	17.09
100 deg. cent.	60	12.01	0.372	1.16	3.13	3.57	0.793	25.90
		18.01	0.561	2.56	3.20	3.60	0.799	25.25
		21.00	0.656	3.64	3.06	3.61	0.803	26.30
		23.99	0.746	4.63	3.13	3.60	0.799	25.80
		26.98	0.850	5.85	3.13	3.64	0.804	25.60
		29.97	0.950	7.10	3.18	3.66	0.813	25.00
		32.96	1.032	9.55	3.08	3.68	0.816	26.95
		35.95	1.173	11.79	2.81	3.74	0.831	27.90
		41.93	1.410	17.70	2.50	3.83	0.851	30.00
		47.90	1.665	25.50	2.26	3.94	0.875	32.00
100 deg. cent.	25	11.99	0.188	0.87	4.20	4.17	0.926	38.20
		17.98	0.280	1.92	4.24	4.14	0.917	38.20
		20.97	0.330	2.57	4.32	4.20	0.931	37.00
		23.97	0.379	3.37	4.28	4.21	0.935	37.20
		26.96	0.426	4.26	4.28	4.21	0.935	37.20
		29.95	0.472	5.26	4.28	4.20	0.931	37.30
		32.94	0.529	6.61	4.32	4.26	0.947	37.90
		35.92	0.579	8.09	4.01	4.25	0.944	38.90
		41.91	0.699	11.98	3.68	4.37	0.971	40.90
		47.90	0.836	16.72	3.45	4.57	1.013	41.70

TABLE III (*continued*)

Temp.	Freq.	e Kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-10}$	cos θ_d per cent
60 deg. cent.	60	12.01	0.318	0.219	16.60	3.15	0.700	5.72
		18.01	0.478	0.490	16.64	3.16	0.700	5.69
		21.00	0.555	0.665	16.70	3.15	0.700	5.71
		23.99	0.631	0.870	16.64	3.15	0.700	5.75
		26.98	0.719	1.101	16.64	3.17	0.704	5.70
		29.97	0.800	1.381	16.64	3.18	0.706	5.76
		32.96	0.882	1.759	16.37	3.19	0.708	6.05
		35.95	0.964	2.222	14.65	3.20	0.711	6.43
		41.93	1.156	3.690	12.00	3.29	0.729	7.64
		47.90	1.370	6.040	9.56	3.40	0.755	9.22
60 deg. cent.	25	11.99	0.147	0.190	19.00	3.49	0.774	10.81
		17.98	0.220	0.430	18.85	3.49	0.774	10.89
		20.97	0.253	0.590	18.75	3.45	0.765	11.11
		23.97	0.290	0.769	18.80	3.45	0.765	11.08
		26.96	0.330	0.973	18.80	3.50	0.775	10.93
		29.95	0.363	1.199	18.80	3.45	0.765	11.02
		32.94	0.402	1.452	18.70	3.49	0.774	10.97
		35.92	0.446	1.811	17.90	3.55	0.789	11.30
		41.91	0.532	2.879	15.36	3.63	0.804	12.90
		47.90	0.634	4.810	12.00	3.75	0.833	15.90
29 deg. cent.	60	12.01	0.301	0.0241	150.0	2.99	0.665	0.670
		18.01	0.460	0.0538	151.0	3.05	0.676	0.649
		21.00	0.539	0.0731	151.5	3.06	0.679	0.648
		23.99	0.612	0.0959	151.0	3.05	0.676	0.652
		26.98	0.701	0.1220	151.0	3.00	0.666	0.645
		29.97	0.774	0.1548	146.0	3.08	0.684	0.668
		32.96	0.859	0.2190	124.8	3.12	0.691	0.775
		35.95	0.935	0.3310	98.2	3.11	0.689	0.985
		41.93	1.118	0.7360	60.0	3.18	0.707	1.579
		47.90	1.323	1.7750	32.5	3.31	0.734	2.800
29 deg. cent.	25	11.99	0.133	0.0213	170.0	3.35	0.744	1.340
		17.98	0.209	0.0476	170.5	3.34	0.741	1.269
		20.97	0.245	0.0654	169.0	3.35	0.744	1.278
		23.97	0.274	0.0846	170.5	3.28	0.728	1.290
		26.96	0.311	0.1074	170.0	3.32	0.736	1.284
		29.95	0.350	0.1348	167.2	3.36	0.744	1.285
		32.94	0.386	0.1782	153.1	3.36	0.744	1.405
		35.92	0.416	0.2425	134.0	3.33	0.739	1.620
		41.91	0.496	0.5740	77.0	3.41	0.756	2.760
		47.90	0.590	1.2440	45.4	3.54	0.786	4.410

up to this point. From here on ionization introduces additional loss and ρ_{av} drops rapidly. The minimum voltage at which ionization starts is 30 kv. and the voltage stress at the conductor is

$$g_{max} = \frac{30}{a \log_\epsilon b/a} = 23.1 \text{ kv. per cm.}$$

Since this cable is designed to operate at $44/\sqrt{3} = 25.4$ kv., or at a voltage stress, $g_{max} = 19.5$ kv. per cm., it is apparent that there is no danger of internal ionization during operation. The divergence between the 25- and 60-cycle watts loss increases with temperature (Fig. 4). The power factor, of course, is higher at 25 cycles. No 25-cycle measurements at room temperature are given but will be for the other two cables.

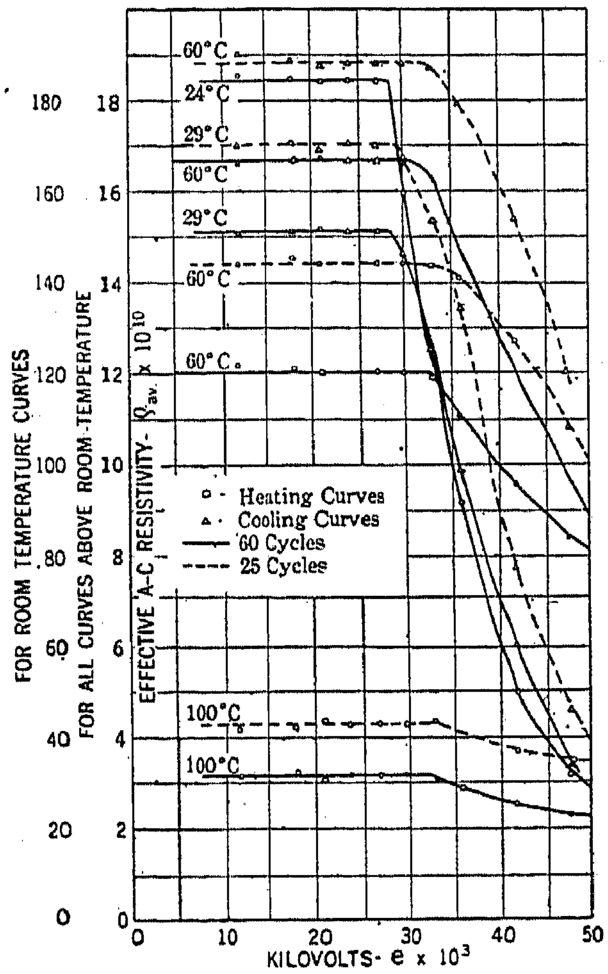


FIG. 5—CABLE No. 2—SINGLE-CONDUCTOR (SEE TABLE III)

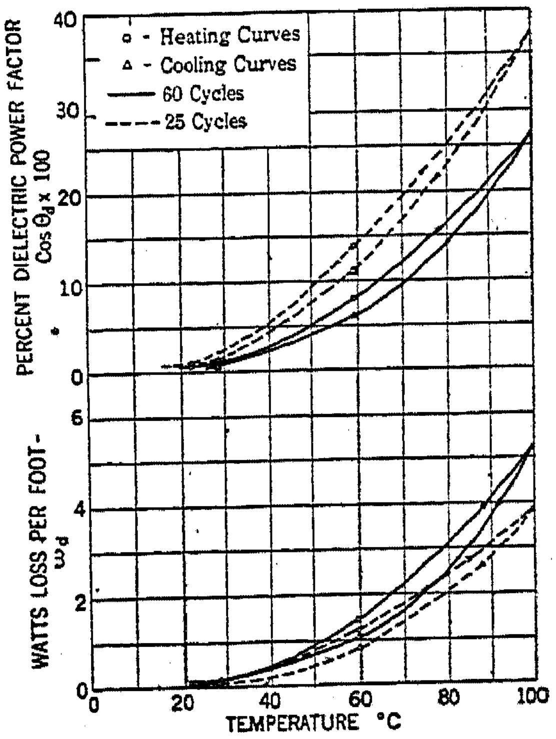


FIG. 6—CABLE No. 2—SINGLE-CONDUCTOR (SEE TABLE III) CURVES PLOTTED AT 25.4 kv.

In Table III measurements are given for cable No. 2, having the following specifications:

Conductor. Same as that for cable No. 1 except that a 1/16 in. (0.1578 cm.) lead jacket was placed over the bare strands to give a smooth surface. Conductor diameter 1.186 + 0.125 = 1.311 in. (3.33 cm.)

Insulation. 24/32 in. (1.905 cm.) thickness paper filled with mineral hydro-carbon base compound.

Sheath. Same as cable No. 1.

Over-all Diameter. Same as cable No. 1.

The test on this cable was made to determine by comparison with No. 1, the benefit derived from a smooth surfaced conductor. The ρ_{av} vs. E curves in Fig. 5 give a minimum ionization voltage of 28.5 kv., which corresponds to a voltage stress at the conductor, g_{max} = 22.5 kv. per cm. This is even less than the voltage stress in cable No. 1 and would indicate that there is practically no difference between stranded and smooth surfaced conductors. The watts and power factor curves in Fig. 6 show No. 2 to have slightly less loss than No. 1. This is only a variation that often occurs in different batches and has nothing to do with the type of conductor. It is due to slight differences of the compound, thoroughness of filling, and compactness of the paper wrappings.

There are two factors involved in high-tension cable that are of equally vital importance. One, high dielectric losses, has already been emphasized. The other, loosely wrapped paper and poor filling is best brought out by the measurements on a special length of cable (No. 3) given in Table IV. This cable had the following specifications:

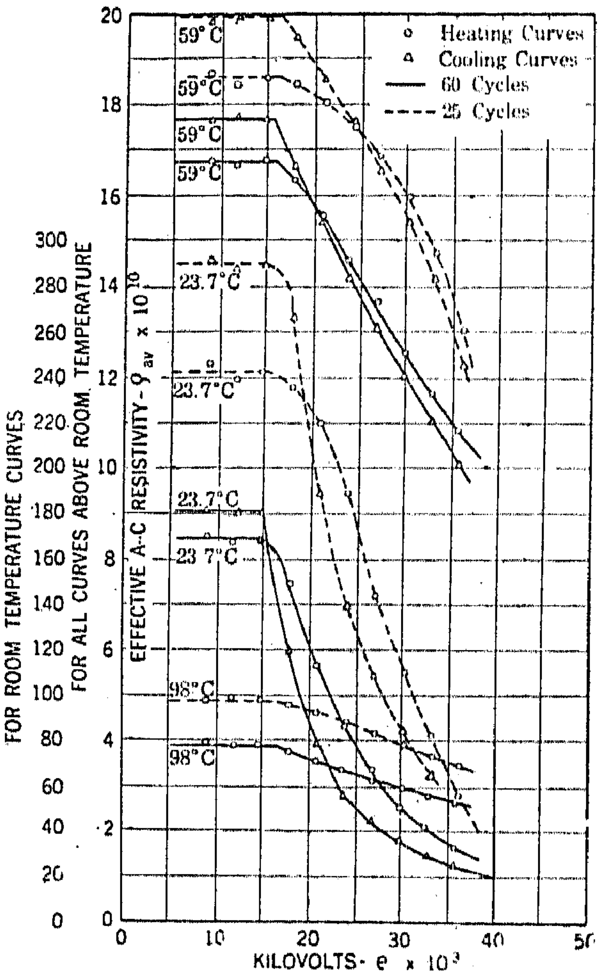


Fig. 8—Loose Wrappings (See Table IV)

Conductor. Same as cable No. 1.

Insulation. 22/32 in. (1.746 cm.) wrapped on loosely and poorly filled with mineral hydrocarbon base compound.

Sheath. 9/64 in. (0.357 cm.) thickness of lead.

The first few wrappings of paper were loosely applied with the remaining wrappings applied in the usual way. The length was then vacuum treated and filled with compound, but not left in the compounding tank for the usual period of time. Consequently, the spaces between the loose wrinkled wrappings were not well filled. These wrinkles are plainly shown in Fig. 7, the photograph being taken after all but the first two paper wrappings were removed.

The ρ_{av} vs. E curves in Fig. 8 show decidedly more ionization than occurred in Cables Nos. 1 and 2. Ionization starts at

PLATE XXXIX.
A. I. E. E.
VOL. XXXVIII, 1919

[CLARK AND SHANKLIN]

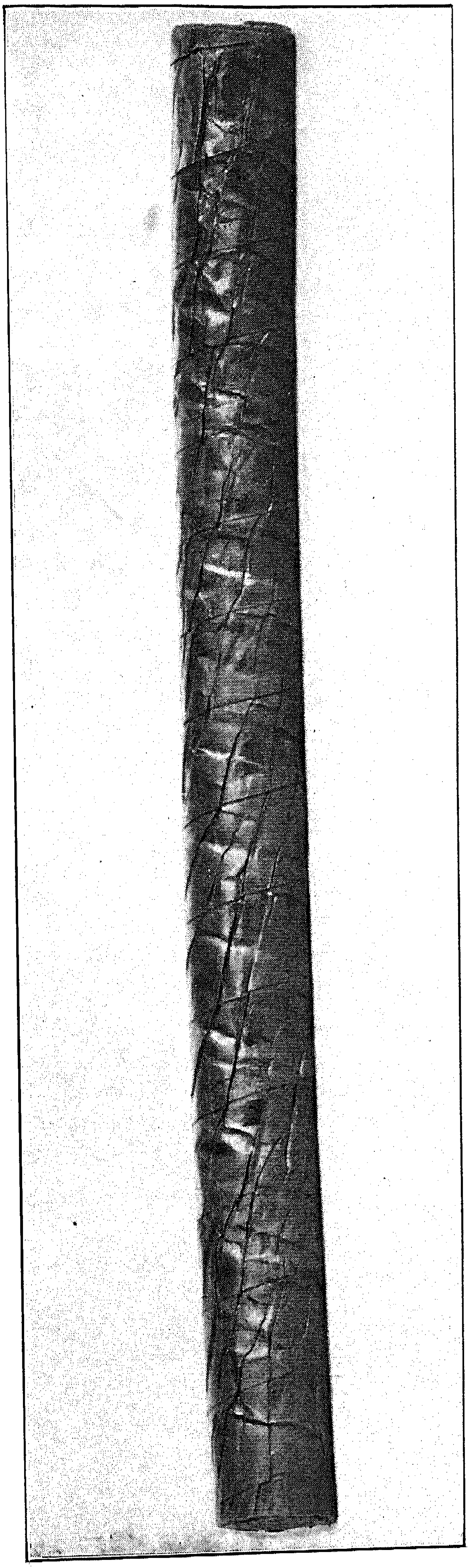


FIG. 7—CABLE NO. 3—WRINKLING OF IMPREGNATED PAPER WRAPPING NEAR CONDUCTOR (TWO WRAPPINGS ONLY REMAINING)

TABLE IV.

CABLE SAMPLE NO. 3

Dia. over stranded conductor surface same as Cable No. 1.

Insulation—22/32 in. (1.746 cm.) thickness paper loosely wrapped and partly filled with mineral hydro-carbon base compound.

Temp.	Freq.	e kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-10}$	$\cos\theta_d$ per cent
23.7 deg. cent	60	9.00	0.221	0.0118	170.0	2.96	0.651	0.595
		11.99	0.296	0.0214	168.0	2.97	0.653	0.601
		14.99	0.374	0.0331	169.0	3.01	0.660	0.590
		17.98	0.449	0.0540	148.9	3.02	0.663	0.669
		20.97	0.525	0.0976	112.0	3.02	0.663	0.885
		23.95	0.599	0.1651	86.2	3.02	0.663	1.153
		26.94	0.679	0.2730	66.2	3.04	0.668	1.495
		29.93	0.760	0.4440	50.2	3.07	0.675	1.952
		32.91	0.844	0.6510	41.3	3.10	0.681	2.345
		35.90	0.925	0.9710	33.0	3.12	0.685	2.920
23.7 deg. cent	25	8.98	0.094	0.0082	246.0	3.12	0.664	0.971
		11.97	0.126	0.0148	240.0	3.16	0.671	0.981
		14.97	0.158	0.0230	242.0	3.16	0.671	0.976
		17.98	0.191	0.0342	235.0	3.18	0.675	0.998
		20.99	0.222	0.0499	220.0	3.16	0.671	1.074
		24.01	0.257	0.0761	188.2	3.21	0.682	1.233
		27.05	0.296	0.1270	143.2	3.28	0.697	1.585
		30.11	0.342	0.2030	111.0	3.40	0.723	1.978
		33.21	0.401	0.3340	82.0	3.62	0.770	2.505
		36.34	0.498	0.5870	56.0	4.11	0.874	3.240
59 deg. cent	60	9.00	0.228	0.120	16.72	3.05	0.670	5.86
		11.99	0.306	0.216	16.60	3.08	0.676	5.86
		14.99	0.385	0.334	16.72	3.09	0.679	5.80
		17.98	0.461	0.492	16.32	3.09	0.679	5.94
		20.97	0.540	0.705	15.51	3.10	0.681	6.22
		23.95	0.620	0.979	14.56	3.12	0.685	6.58
		26.94	0.702	1.328	13.62	3.14	0.690	7.01
		29.93	0.785	1.770	12.56	3.16	0.695	7.54
		32.91	0.871	2.307	11.64	3.18	0.700	8.03
		35.90	0.960	2.958	10.83	3.22	0.709	8.57
59 deg. cent	25	8.98	0.098	0.107	18.65	3.26	0.693	12.20
		11.97	0.132	0.193	18.45	3.29	0.700	12.10
		14.97	0.165	0.301	18.56	3.28	0.696	12.20
		17.98	0.199	0.436	18.42	3.29	0.700	12.21
		20.99	0.234	0.609	18.05	3.30	0.704	12.42
		24.01	0.271	0.815	17.54	3.36	0.714	12.50
		27.05	0.313	1.078	16.88	3.43	0.730	12.72
		30.11	0.363	1.416	15.91	3.58	0.763	12.95
		33.21	0.421	1.859	14.74	3.76	0.799	13.28
		36.34	0.529	2.518	13.05	4.20	0.894	13.10
98 deg. cent	60	9.00	0.240	0.514	3.92	3.12	0.686	23.80
		11.99	0.321	0.924	3.87	3.13	0.688	24.00
		14.99	0.405	1.441	3.87	3.16	0.695	23.60
		17.98	0.489	2.150	3.74	3.19	0.700	24.45
		20.97	0.575	3.110	3.52	3.20	0.704	25.60
		23.95	0.662	4.260	3.34	3.22	0.707	26.90
		26.94	0.754	5.710	3.17	3.24	0.711	28.16
		29.93	0.852	7.530	2.96	3.29	0.724	29.50
		32.91	0.952	9.710	2.77	3.32	0.729	31.00
		35.90	1.047	12.340	2.62	3.32	0.729	32.80

TABLE IV. (*continued*)

Temp.	Freq.	e kv.	i_d $\times 10^{-3}$	w_d	ρ_{av} $\times 10^{10}$	K_{av}	C $\times 10^{-10}$	$\cos\theta_d$ per cent
98 deg. cent	25	8.98	0.118	0.415	4.84	3.62	0.770	39.20
		11.97	0.160	0.722	4.93	3.70	0.786	37.60
		14.97	0.199	1.149	4.84	3.66	0.780	38.50
		17.98	0.242	1.684	4.77	3.73	0.794	38.70
		20.99	0.289	2.410	4.56	3.79	0.805	39.80
		24.01	0.334	3.270	4.37	3.81	0.809	40.75
		27.05	0.386	4.440	4.10	3.87	0.823	42.50
		30.11	0.445	5.770	3.90	3.99	0.849	43.10
		33.21	0.514	7.440	3.68	4.17	0.887	43.60
		36.34	0.623	9.650	3.41	4.65	0.987	42.60
59 deg. cent	60	9.00	0.226	0.115	17.60	3.02	0.664	5.64
		11.99	0.299	0.202	17.76	3.01	0.661	5.63
		14.99	0.376	0.317	17.67	3.02	0.664	5.64
		17.98	0.454	0.484	16.61	3.04	0.668	5.94
		20.97	0.529	0.711	15.40	3.04	0.668	6.44
		23.95	0.609	1.008	14.14	3.06	0.674	6.91
		26.94	0.690	1.380	13.10	3.09	0.679	7.43
		29.93	0.773	1.851	12.03	3.11	0.684	8.00
		32.91	0.859	2.430	11.05	3.15	0.692	8.60
		35.90	0.951	3.180	10.10	3.19	0.701	9.31
59.5 deg. cent	25	8.98	0.096	0.102	18.85	3.18	0.676	11.78
		11.97	0.130	0.178	20.00	3.23	0.686	11.40
		14.97	0.163	0.280	19.95	3.24	0.688	11.48
		17.98	0.195	0.413	19.47	3.24	0.688	11.78
		20.99	0.229	0.591	18.52	3.24	0.688	12.88
		24.01	0.265	0.814	17.61	3.27	0.695	12.82
		27.05	0.308	1.100	16.54	3.38	0.718	13.21
		30.11	0.355	1.461	15.40	3.50	0.744	13.68
		33.21	0.420	1.931	14.20	3.75	0.797	13.84
		36.34	0.521	2.680	12.27	4.26	0.905	14.14
23.7 deg. cent	60	9.00	0.215	0.111	181.0	2.87	0.631	0.574
		11.99	0.288	0.198	180.2	2.91	0.639	0.574
		14.99	0.360	0.310	180.2	2.90	0.637	0.574
		17.98	0.435	0.674	119.4	2.92	0.641	0.862
		20.97	0.509	1.389	78.8	2.93	0.643	1.304
		23.95	0.586	2.542	56.0	2.96	0.651	1.811
		26.94	0.669	4.110	44.0	2.99	0.658	2.280
		29.93	0.750	6.250	35.6	3.03	0.666	2.780
		32.91	0.830	9.190	29.2	3.05	0.670	3.360
		35.90	0.921	12.900	24.8	3.10	0.682	3.900
23.7 deg. cent	25	8.98	0.092	0.0070	291.0	3.06	0.649	0.840
		11.97	0.124	0.0124	288.0	3.10	0.659	0.835
		14.97	0.155	0.0193	289.5	3.10	0.659	0.830
		17.98	0.187	0.0302	266.0	3.12	0.663	0.900
		20.99	0.219	0.0581	189.0	3.12	0.663	1.268
		24.01	0.252	0.1024	139.8	3.15	0.668	1.690
		27.05	0.292	0.1661	109.5	3.24	0.688	2.100
		30.11	0.336	0.2718	83.0	3.34	0.710	2.685
		33.21	0.400	0.4230	64.8	3.61	0.767	3.190

a minimum voltage of 15 kv., which corresponds to a stress at the conductor, g_{max} = 12.95 kv. per cm. Since the cable is supposed to operate at 19.5 kv. per cm. it is evident that it would soon give trouble. The thickness of gas space estimated for this sample in the paper on internal ionization previously referred to[2] was found to be seven times that in average good cable. It is interesting to note that at voltages below ionization the watts loss per cm.[3] and the power factors are lower than those in the better cables (Nos. 1 and 2). The permittivity is also lower. This is caused by poor filling and gas spaces, since no loss is experienced in the entrapped gases. A cable made up of unfilled paper would have even less loss if well dried and provided the voltage was below ionization. This characteristic is often deceptive. The watts loss, alone, does not indicate the grade of material, unless there is reasonable assurance that it is compact and well filled.

PREDETERMINATION OF DIELECTRIC LOSS

Dielectric loss characteristics (up to the voltage at which ionization starts) can be predetermined for any cross section of cable with a fair degree of accuracy, provided sufficient measurements are available on cables made of the same materials and by the same process to obtain a good average of the dielectric power factor and permittivity. The loss variation in different batches of well made cable is not over 25 per cent which allows a difference of 12.5 per cent one way or the other from the estimated average loss.

Sixty- and 25-cycle curves of power factor and permittivity vs. temperature, for both single- and three-conductor paper cable filled with mineral hydro-carbon base compound, are given in Figs. 9 and 10. To be on the safe side, the power factor curves given are actually 10 per cent higher than the averages of a large number of tests. They are what might be termed "high average" curves. It will be noticed that the power factor is about 10 per cent higher for three-conductor cable than for single-conductor. We believe that this difference is due to the jute filler rather than three-phase stress, for we have been unable to detect any difference between the three-phase loss calculated from single-phase measurements on three-conductor cable, as described in a previous paper[5] and that actually measured with three-phase voltage.

The permittivity of three-conductor cable is also higher.

The jute filler doubtless has something to do with this. The greater part of the difference, however, we believe to be due to error in the approximate formulas (6 and 7 in Appendix A) used in calculating K_{av} from the measured values of c_1 and c_4 This error cancels itself out when K_{av} is converted back again to c_1 and c_4, as it must be in preliminary calculations, and is therefore of no importance. Knowing the power factor and permittivity, the loss and all other dielectric characteristics in any cross section of cable can be estimated from the formulas in Appendix A.

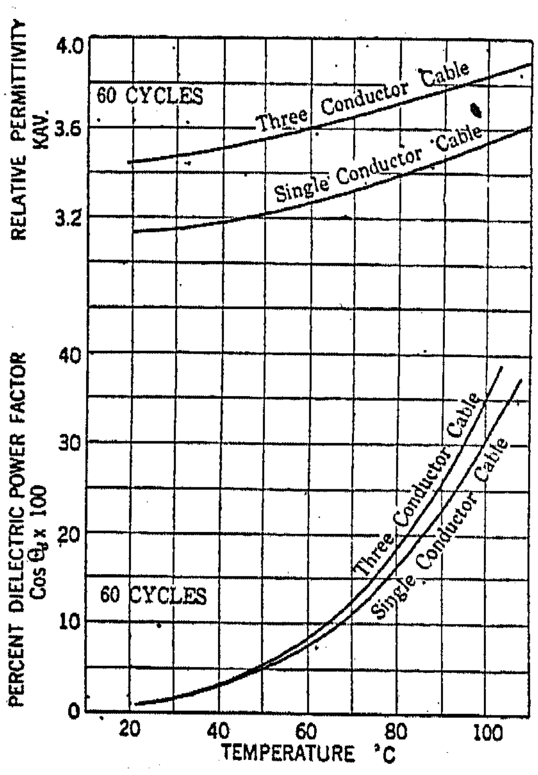


FIG. 9—PAPER CABLE FILLED WITH MINERAL HYDRO-CARBON BASE COMPOUND

These curves are 10 per cent higher than average found from large numbers of tests on different batches—they apply at all voltages below internal ionization

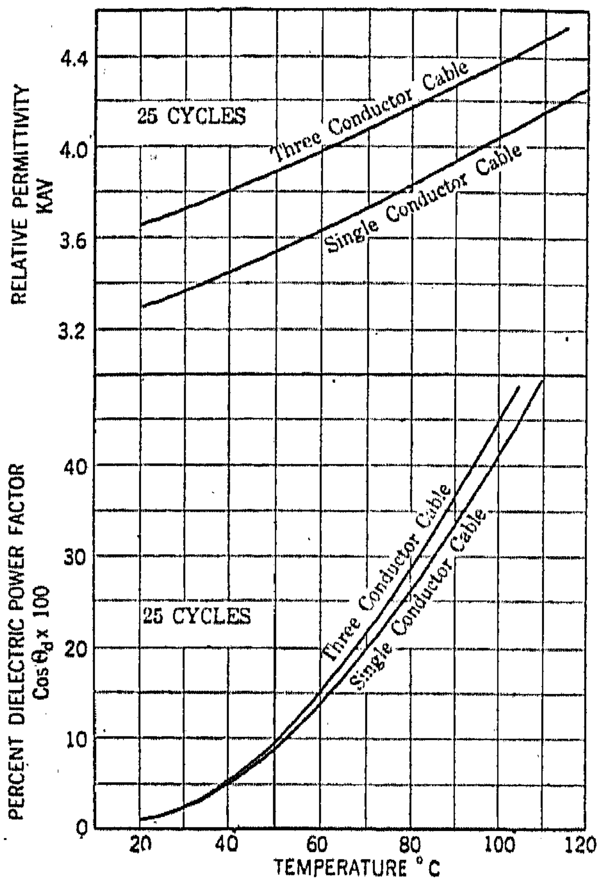


FIG. 10—PAPER CABLE FILLED WITH MINERAL HYDRO-CARBON BASE COMPOUND

These curves correspond to the 60-cycle curves in Fig. 9

INDUCTIVE CHARACTERISTICS OF SINGLE-CONDUCTOR CABLE

Dielectric characteristics are essentially functions of line voltage, and inductive characteristics of line current. The induced sheath current when sheath is short-circuited, or induced voltage when open-circuited, brings in a complication in dealing with which we had considerable trouble during the initial stages of the work with single-conductor cable.

The formulas for calculating the inductive characteristics developed in Appendix B give very accurate results. In fact, the accuracy reached is greater than can be obtained by actual measurements, even when these are taken with extreme care.

The formulas are completely developed theoretically without approximations. Berg,[8] in dealing with the same subject gave short approximate formulas, but they apply only within certain limits not met with by the cable cross sections and spacings considered in this paper.

The theoretical work in Appendix B was checked in the laboratory. A number of tests were made with which we can deal only briefly. Fig. 11 gives a diagram of the test circuit. The two parallel lengths of cable were each 50 ft. in length, laid out straight on wooden horses 3½ ft. high. The horses were made without iron nails and there was no iron within their height of the cables. A uniform magnetic field resulted. A flexible loop of cable connected the far ends of the conductors and the ends of the sheaths were bonded by a straight conductor of low resistance perpendicular to the cable lengths (a switch was inserted in the sheath bond so that external resistance could be added to the sheath circuit when desired). By extending the flexible loop out far enough its inductive effect on the sheath bond was eliminated. The drop leads were connected as shown, with corresponding pairs twisted together and arranged to reduce inductive effect to a minimum.

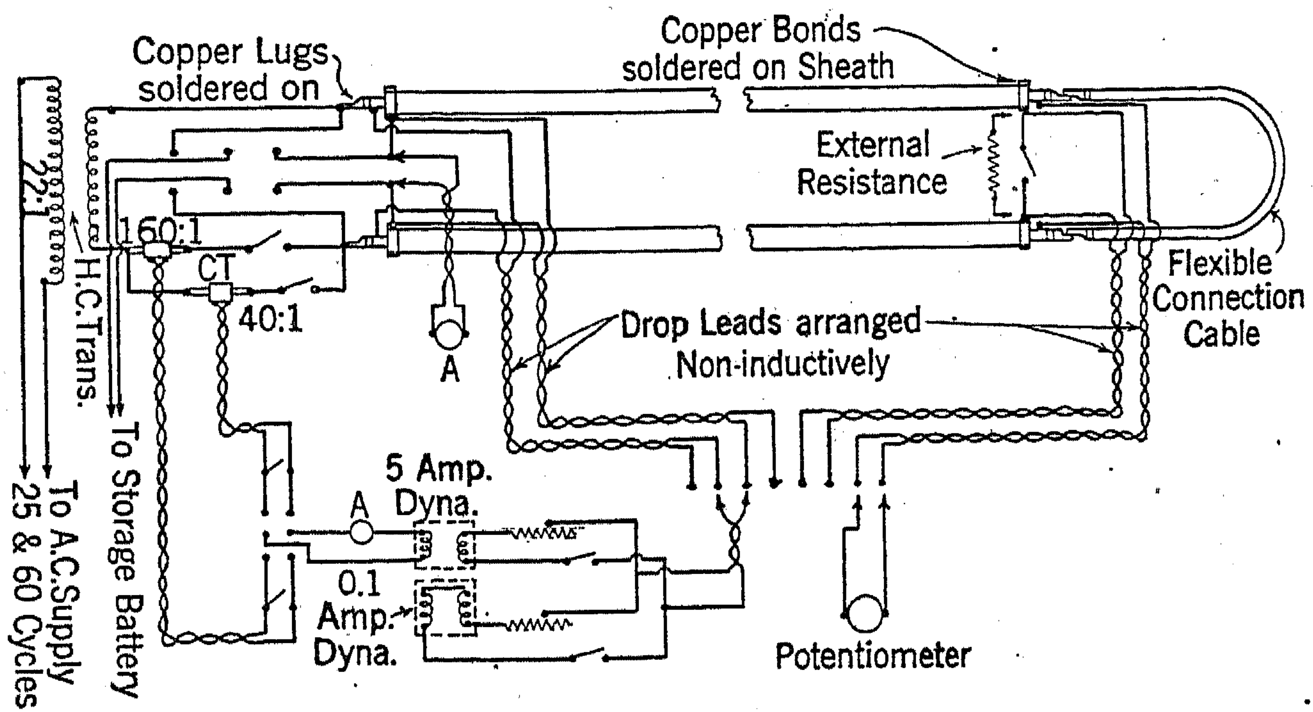


FIG. 11—CONNECTION DIAGRAM OF HIGH-CURRENT SINGLE-CONDUCTOR CABLE TEST

Current transformers stepped the current down for ammeter and wattmeter readings. Watts were read with a dynamometer wattmeter having a constant of 0.27×10^{-6}. Another very sensitive dynamometer (constant 0.013×10^{-6}) was used as a voltmeter. Induced sheath current and wattmeter current were read with carefully calibrated portable ammeters. By a double-throw switch arrangement either a-c. or d-c. readings could be taken, the d-c. voltage drop being measured with a potentiometer. The accuracy of d-c. resistance readings was greatly facilitated by the opportunity to check readings of the total circuit against the sum of the different sections.

---

[8] "Constants of Cables and Magnetic Conductors" by E. J. Berg, TRANS. A. I. E. E., Vol. 26, p. 555.

The general procedure was first to measure the d-c. resistance for determination of copper and sheath temperatures. Then a-c. readings of current, watts and voltage drop were taken over a range of current that would give seven-point curves. The conductor readings were taken on the total circuit (two cable lengths plus connecting loop at far end) and then on the flexible connecting loop only, which is subtracted out. The induced sheath voltage, or current, depending upon whether open- or closed-circuited, was also measured. Finally, the d-c. resistance measurements were repeated, to make sure that no change in temperature had taken place.

The a-c. readings were taken over a fairly wide range of current. It was found that watts varied exactly as the square of the current, and voltage drop directly with the current. In other words, the line constants, such as effective resistance and inductance, were independent of current density, provided of course, there was no change in temperature. This would be expected from theoretical considerations and it is fortunate that it is so, for it allows accurate and complete calculations of line characteristics as will be shown later.

There are two sources of error in this method of test;

a. phase-angle errors, due to inductance in the current transformer and wattmeter circuit.

b. distorted wave shape.

The power factors at which the measurements were taken were fairly high. In the worst case the phase-angle error introduced was only about 4 per cent. By measurement of the current transformer phase angle and inductance of wattmeter circuit it could be largely corrected for. This source of error can be considered negligible when proper care is taken. Errors from distorted wave shape are much more difficult to overcome. The only way they can be corrected for is by using a perfect sine wave of current, which is difficult of attainment with such high values of current. Our wave shape was of the ordinary commercial type. Errors in the measurements given in the following tables were due to variances from a sine wave.

TABLE VI.—CABLE NO. 2 (See Table III.)
ALL VALUES PER 1000 FT. OF ONE CONDUCTOR TO NEUTRAL.
SHEATHS CLOSED-CIRCUITED (GROUNDED IN TWO PLACES.)

Temp deg. cent.		Inches spacing D	Freq.	Measured d-c. resistance			r_c	r_{eff}		$L_{eff} \times 10^{-3}$		Z_{eff}		Ratio I_s/I_c	
Cond.	Sheath			Sheath	r_s	Cond.		Meas.	Calc.	Meas.	Calc.	Meas.	Calc.	Meas.	Calc.
26.0	26.0	24.0	60	0.1161	0.1835	0.017295	0.01736	0.03950	0.03736	0.2140	0.2155	0.0899	0.0895	0.330	0.3305
25.0	25.0	24.0	60	0.1160	1.5990	0.017210	0.01731	0.02123	0.01987	0.2383	0.2340	0.0925	0.0904	0.0415	0.0401
27.0	27.0	24.0	25	0.1146	0.1869	0.01738	0.01738	0.02180	0.02102	0.2242	0.2290	0.0411	0.0405	0.1455	0.1415
25.0	25.0	24.0	25	0.1153	1.9560	0.01727	0.01727	0.01759	0.01736	0.2378	0.2341	0.0412	0.0408	0.01395	0.01364
26.0	26.0	6.15	60	0.1150	0.1951	0.01728	0.01738	0.02390	0.02278	0.1480	0.1490	0.0606	0.0606	0.1710	0.1669
25.0	25.0	6.15	60	0.1140	1.981	0.01724	0.01734	0.01881	0.01789	0.1531	0.1508	0.0608	0.0596	0.01725	0.01665
22.0	22.0	6.15	25	0.1150	0.5890	0.01699	0.01699	0.01807	0.01731	0.1512	0.1509	0.0289	0.0293	0.02255	0.0232
23.0	23.0	6.15	25	0.1146	1.937	0.01705	0.01706	0.01751	0.01716	0.1510	0.1509	0.0295	0.0292	0.00735	0.00706

SHEATHS OPEN-CIRCUITED (GROUNDED IN ONE PLACE ONLY.)

Temp. deg. cent.		Inches spacing D	Freq.	$r_{d\text{-}c}$	r_c		$L_{eff} \times 10^{-3}$.		$M \times 10^{-3}$		Z_{eff}		Ratio e_s/E	
Cond	Sheath				Meas.	Calc.	Meas.	Calc.	Meas.	Calc.	Meas.	Calc.	Meas.	Calc.
27.4	26.7	24.00	60	0.01738	0.01790	0.01746	0.2398	0.2341	0.1759	0.1701	0.0921	0.08995	0.720	0.714
24.6	24.7	24.00	25	0.01718	0.01741	0.01718	0.2382	0.2341	0.1737	0.1701	0.04115	0.04055	0.658	0.657
24.0	24.0	6.15	60	0.01714	0.01784	0.01722	0.1539	0.1509	0.0899	0.0869	0.0607	0.05950	0.560	0.552
23.3	23.7	6.15	25	0.01710	0.01730	0.01710	0.1518	0.1509	0.0895	0.0869	0.02947	0.02920	0.476	0.467

A comparison between measured and calculated values of resistance and inductance for two different cross sections of cables No. 2 and 4 are given in Tables V and VI. The method of calculation and nomenclature are in Appendix B. Before the various characteristics can be calculated the resistances of the sheath and conductor must be known for any assumed temperature. These are given in Fig. 12 for the cable considered in Table V. A 12-ft. length of the cable was placed in an electric oven and d-c. measurements taken of conductor and sheath resistance over the temperature range indicated. Ignoring the spirality effect of the strands the resistance of the conductor is closely represented by copper of 98 per cent conductivity. This value of conductivity is used in Formula 4, Appendix B, for calculating the 25- and 60-cycle effective resistances r_c as given in Fig. 12.

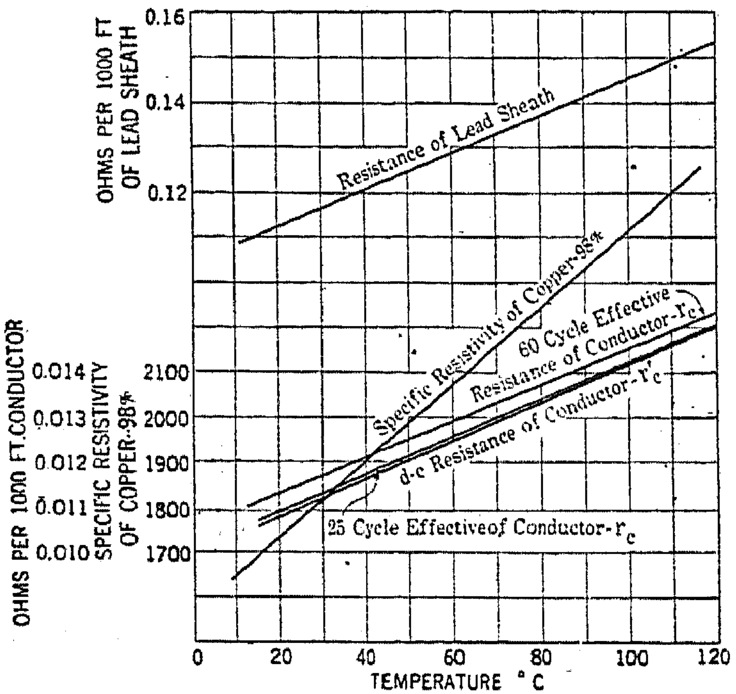


FIG. 12—D-C. AND A-C. RESISTANCE OF CONDUCTOR AND SHEATH AT VARIOUS TEMPERATURES FOR CABLE NO. 4 (TABLE V)

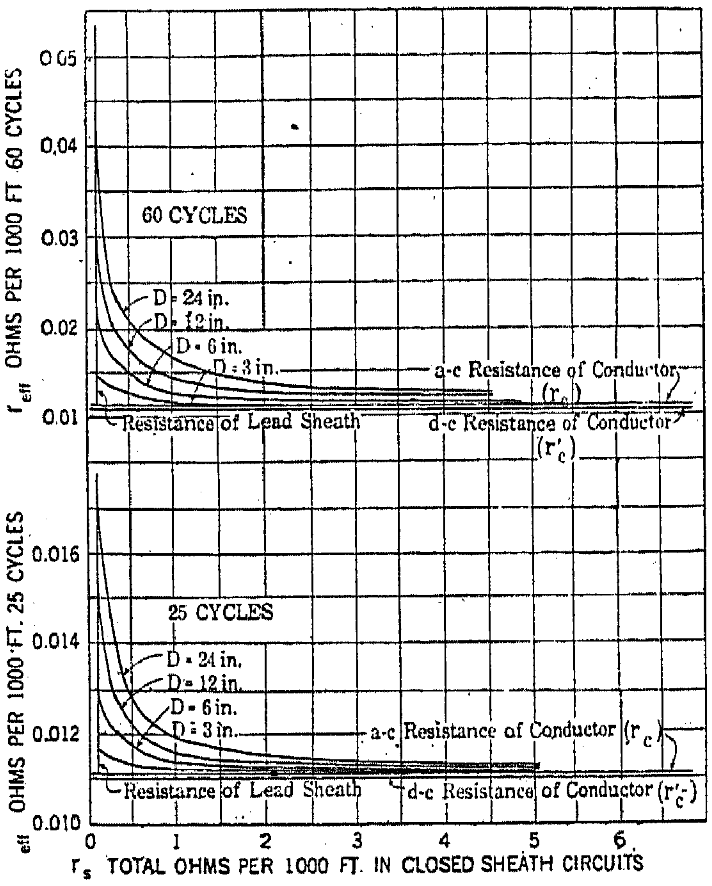


FIG. 13—RELATION BETWEEN EFFECTIVE RESISTANCE AND TOTAL RESISTANCE OF CLOSED SHEATH CIRCUIT—ONE CABLE TO GROUNDED NEUTRAL (TABLE V) 25 DEG. CENT.

The calculated values of effective resistance r_{eff} and effective inductance L_{eff} in Table V for cable No. 4 are given in the form of curves in Figs. 13, 14 and 15. Before plotting the curves the temperature was corrected to 25 deg. cent. from Fig. 12 and by Formulas 2 and 3, Appendix B.

The curves of r_{eff} vs. r_s in Fig. 13 are very interesting. They

Copper area 1,007,000 cm.
Dia. of hemp core....... 0.
Dia. of copper........... 1.

NO. 4)
CONDUCTOR TO NEUTRAL
ED IN TWO PLACES.)

Temp. deg. cent.		Spacing inches		Ratio I_s/I_c	
Cond.	Sheath	D	Calc.	Meas.	Calc.
26	26	24	0.08310	0.4290	0.4310
25	25	24	0.08300	.....	0.0912
25	25	24	0.08800	......	0.0518
23	26	24	0.08800	0.03840	0.0370
25	25	24	0.08800	0.01860	0.0190
28	28	24	0.03880	0.2060	0.1935
25	25	24	0.03815	......	0.0381
25	25	24	0.03800	......	0.0216
25	25	24	0.03795	......	0.01512
25	25	24	0.03780	0.00532	0.00579
25	27	12	0.07050	0.3460	0.3480
25	25	12	0.07205	......	0.0665
25	25	12	0.07205	......	0.0381
23	25.5	12	0.07205	0.02780	0.0281
25	25	12	0.07205	......	0.01461
27	27	12	0.03175	0.1470	0.1450
25	25	12	0.03176	......	0.0290
25	25	12	0.03165	......	0.01662
25	25	12	0.03155	......	0.01170
25	25	12	0.03150	......	0.00446
23	26	6	0.05625	0.2630	0.2500
25	28	6	0.05635	0.0483	0.0496
25	27	6	0.05610	0.0224	0.0228
25	28	6	0.05600	0.0157	0.0167
25	25	6	0.05600	0.00715	0.00744
24	27	6	0.02605	0.0960	0.0951
25	25	6	0.02605	......	0.0207
25	25	6	0.02600	......	0.0117
25	25	6	0.02600	......	0.00823
25	25	6	0.02595	0.00122	0.00121
23	24	3.024	0.04115	0.1390	0.1412
25	25	3.024	0.04095	......	0.0288
25	25	3.024	0.04090	......	0.0163
24	26.5	3.024	0.04085	0.00980	0.00969
25	25	3.024	0.04085	......	0.00599
23	23	3.024	0.01985	0.0570	0.0564
25	25	3.024	0.01980	......	0.0120
25	25	3.024	0.01975	......	0.00678
25	25	3.024	0.01975	......	0.00476
25	25	3.024	0.01975	......	0.00182

Deg. cent. temp.	Inches spacing D	F	Ratio e_s/E	
			Meas.	Calc.
25	24		0.798	0.800
25	24		0.750	0.765
25	12		0.729	0.745
25	12		0.712	0.705
25	6		0.683	0.672
25	6		0.629	0.617
25	3.024		0.524	0.533
25	3.024		0.457	0.466

show that at 60 cycles and a spacing between cable centers of 6 in. (15.2 cm.), which is close to the usual spacing in ducts, a resistance equal approximately to ten times that of the lead sheath must be inserted in the ground bonds to reduce the effective resistance with closed sheath circuit to within 10 per cent of that with open sheath circuit. At 25 cycles the same percentage is almost reached without any resistance at all in the sheath bonds, that is, with the sheath directly grounded. Two interesting conclusions are drawn from this:

a. In a 60-cycle line of the above cable it is better to break the continuity of the sheath into relatively short lengths (at every man hole or every other man hole) and directly ground each sheath length only once because of the high loss with short-circuited sheath. This can be conveniently done by having a common neutral wire running the length of the conduit line. The continuity of the sheaths can be broken by a well designed, moisture-proof sleeve unit. Such a unit could easily be turned out in quantity production and would take the place of the ordinary jointing sleeve. This method would be better than using a continuous sheath and grounding it in two or more places through a bond resistance. Nothing is gained and a good deal is lost by grounding through bond resistance. The resistance would absorb over 90 per cent of the induced sheath voltage and the sheath would remain at almost the same potential above ground as it does open circuited. Also, such a high resistance ground is not reliable and does not afford sufficient protection.

b. 25-cycle operation is more favorable to single-conductor cable than 60-cycle. In a 25-cycle line there is the choice of either grounding the sheaths direct, which amounts to the same thing as short-circuiting them, or breaking their continuity and grounding each length only once as described above. With short-circuited sheaths an additional loss of 14 per cent is introduced in cable No. 4. There will be no potential between sheaths under this condition, as the sheaths, themselves, absorb the induced voltage and remain at ground potential over their whole length. If this additional loss is undesirable the continuity can be broken and the induced sheath voltage, being proportional to the frequency, will be only 42 per cent of that at 60 cycles.

The idea, of course, in breaking the continuity of the sheaths is to keep the maximum sheath voltage within reasonable limits, so that leakage, especially in wet or flooded systems, will not give trouble. Just what would constitute a reasonable limit of sheath voltage depends upon the leakage resistance between sheaths and this depends upon the particular conditions and type of conduit and manhole construction used.

As each cable is enclosed in a separate duct the leakage between sheaths over the conduit length is negligible compared with the leakage at manholes, when well constructed tile or fibre conduit is used. It is at the manholes, then, that the greatest care must be taken to reduce leakage. The ducts should spread as they project into the manhole and the sheaths well protected. The usual methods of protection used to reduce fire risk at manholes as described by Carlton[9] should prove sufficient with a little additional care, such as applying a coating of waterproof paint to the bare sheaths before the protective covering is applied. Methods of reducing sheath voltage and leakage will be dealt with further under another heading.

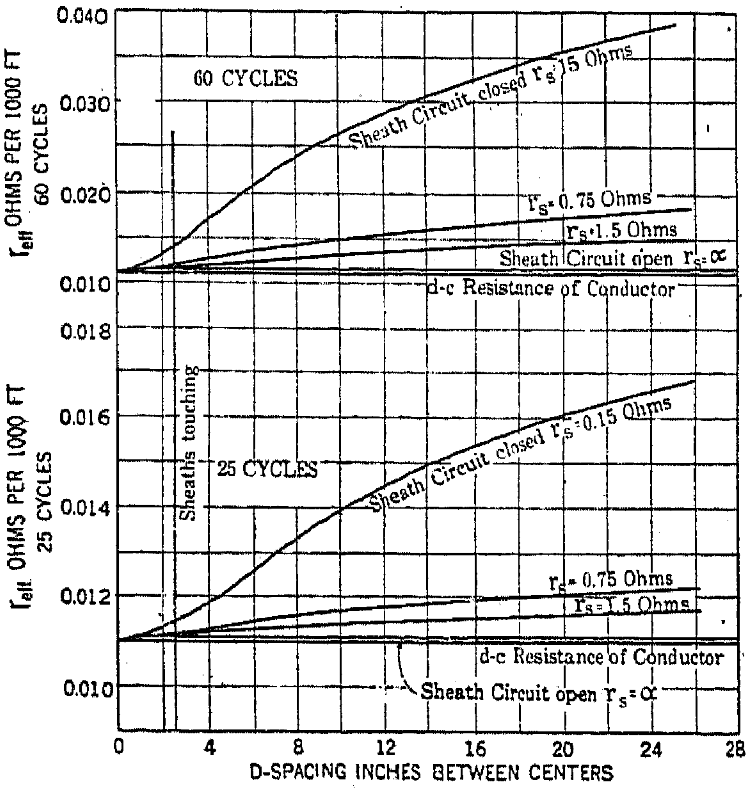


FIG. 14—25 AND 60-CYCLE EFFECTIVE RESISTANCE—ONE CABLE TO NEUTRAL (TABLE V) 25 DEG. CENT.

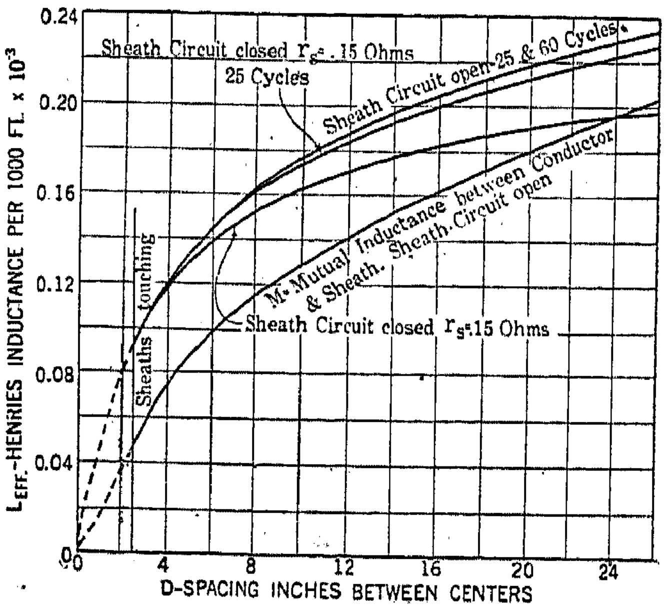


FIG. 15—EFFECTIVE INDUCTANCE—ONE CABLE TO NEUTRAL—(TABLE V) 25 DEG. CENT.

The effective resistance and effective inductance curves of cable No. 4 in Figs. 14 and 15 indicate clearly the importance of keeping the spacing between cables down to a minimum as far as the induced voltage or current in the sheath is concerned.

---

[9] "Protection of Cables from Arcs due to the Failure of Adjacent Cables" by W. G. Carlton, A. I. E. E., 1904, p. 471.

It is surprising, though, to find that the self-inductance between conductors, and therefore the line regulation, compares very favorably with that of ordinary three-conductor cable For instance, cable (c) in Table I has an inductance per conductor to neutral of 0.145×10^{-3} henrys per 1000 ft. with a spacing of 6.74 in., while the equivalent three-conductor cable (a) in Table I, has a corresponding inductance of 0.113×10^{-3} henrys with only 1.38 in. spacing. Formula 8 in Appendix B was used in calculating these values. The difference in conductor cross sectional dimensions accounts for this unexpectedly close comparison.

It must not be assumed from the foregoing discussion that all single-conductor cables have as severe sheath inductive effects as Cable No. 4. This cable represents an extreme case. The sheath inductive effect for any given spacing depends upon the conductor cross section and the distance between sheath and conductor, that is, thickness of insulation. The percentage increase in effective resistance when sheath is short-circuited is tabulated below for three sizes of cable at a spacing of 6 inches.

Frequency	Cable No. 4	Cable No. 2	Cable (c)
25 cycles	14.4 per cent	8.8 per cent	4.5 per cent
60 cycles	96.0 per cent	48.0 per cent	22.0 per cent

These values were calculated by the method described in Appendix B and show clearly that it is perfectly feasible to operate the higher voltage cables at 25 cycles with sheaths short-circuited. Cable (c) might even have its sheath short-circuited at 60 cycles. This question will be taken up later under another heading.

SUMMARY OF SPECIAL INDUCTIVE TESTS

Several special tests other than those dealt with above were made to determine the effects of eddy currents in the open-circuited sheath and conductor, advantage of enameling the individual conductor strands and skin effect in the conductor. The results are outlined briefly:

a. By testing lengths of lead-sheathed cable and then repeating the measurements after the sheath was removed it was clearly proved that the additional loss due to eddy currents in the open-circuited sheath was too small to be detected. Cable No. 4 was the limiting design tested in this way, that is, it had the largest conductor cross section and least thickness of insulation.

b. Lengths of cable having an ordinary conductor of bare copper strands were compared with lengths made up in identically the same way, excepting that the individual copper strands were enameled. Not the slightest difference could be detected, even at the closest possible spacing, and with conductors up to 1,000,000-cir. mil cross section. This shows that eddy currents are not appreciable in ordinary bare stranded conductors. Evidently the thin oxide film that forms naturally on the surface of the strands furnished sufficient insulation to prevent cross currents.

c. The accuracy of our test methods was not great enough to allow a close check of the skin-effect formula (4) in Appendix B. As far as we are able to determine this formula holds nicely. It also checks very closely over the range needed in this work with the results obtained by Kennelly and Affel.[10]

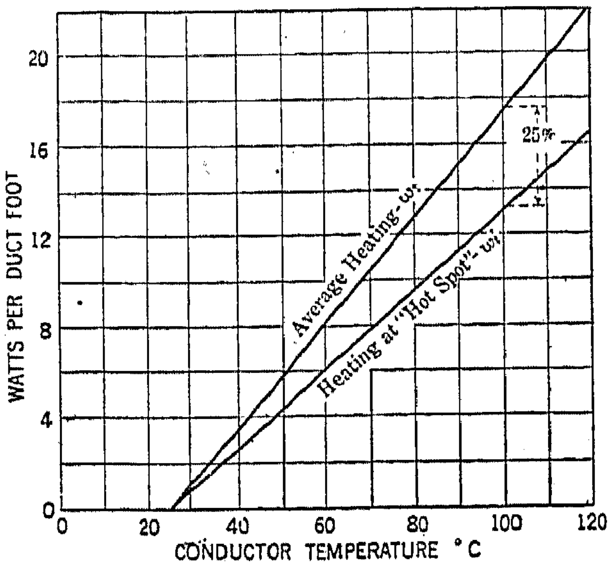


FIG. 16 — REPRESENTATIVE HEATING CHARACTERISTICS IN CONDUIT LINE—25 DEG. CENT. SURROUNDING SOIL TEMPERATURE

NOTE: The line calculations are based on these heating conditions

LINE CHARACTERISTICS OF UNDERGROUND CABLES

Heating in Underground Cable. In determining the characteristics of underground cable lines there is only one factor, the heat dissipating ability of the conduit system, that introduces serious complications. Many papers dealing with the thermal conductivity of soils and the general theory of heating in underground cables have been published, both here and abroad. They have thrown a great deal of light on the subject but it can hardly be said that any method has yet been evolved with which individual systems can be dealt with theoretically.

In calculating line efficiency, regulation, etc. the *average* heat dissipating ability of the whole length of line must be known. In determining the maximum safe load the average is of no value, the *maximum* heating at the "hot spot" section of the line must be considered instead. Fig. 16 shows the average and

---

[10] "Skin Effect Resistance Measurements of Conductors" by A. E. Kennelly and H. A. Affel, *Proc. Inst. of Radio Eng.*, Dec. 1916.

"hot spot" heating characteristics for a representative conduit line, based on the assumption of 25 deg. cent. surrounding soil temperature. The curves represent a very fair average of the conditions usually met with in underground conduit. They are useful in comparing different designs of cable for line efficiency, maximum safe rating, etc.

We would recommend very strongly that some such conditions of heating be adopted as a standard. Cables could then be rated and designs compared on the basis of this standard. Without some such basis to work on it is a very uncertain matter to modify cable ratings to suit individual conduit lines, or even to suit different seasons of the year in any given line. By keeping careful temperature records under actual operating conditions and comparing them with this standard we believe that operating companies could accumulate a valuable mass of empirical data that would clear up many of the uncertainties existing to-day.

Curves similar to those in Fig. 16 can be determined for any given conduit line by a heat run, just as made on any electrical unit such as a transformer or generator. The average temperature is obtained from resistance measurements of the conductor. The maximum temperature at "hot spots" must be found by exploring temperature indicators. Since these curves are practically straight lines only one or two load readings are necessary.

We do not believe that laboratory investigations of the thermal conductivity of soils, duct walls, insulation, etc. and a mathematical study of the heat paths will ever be of practical application. This abstract method of attack has proved and will continue to be a valuable and instructive adjunct to the more direct method outlined above. We are at present conducting a laboratory investigation along these lines and hope to describe the results in a future paper.

CALCULATION OF LINE CHARACTERISTICS

The general method of calculating cable line characteristics is explained in Appendix C. It applies to relatively long lines where the effects of charging current, line regulation, etc., are appreciable. Such elaborate and complete calculations are hardly necessary in dealing with short lines, where the usual approximate methods serve the purpose.

COMPARISON OF SINGLE- AND THREE-CONDUCTOR CABLE LINES

The ultimate purpose of the cable work so far dealt with in this paper has aimed towards placing the reader in a better position to compare the relative merits of single- and three-conductor cable lines.

As a final comparison, two exactly similar four-duct conduit lines will be assumed, placed side by side and each having heating characteristics corresponding to those given in Fig. 16. Three cables of the limiting design of 30-kv. three-conductor cable, given in Table I cable (a) will be placed in one of these conduit lines. The other line will have three 55-kv. single-conductor cables cable (c) designed for approximately the same loss per duct-ft. with open-circuited sheath. The general line characteristics of these two limiting types of cable will be calculated and compared at 60 cycles, following the method in Appendix C. Other details will also be discussed and compared.

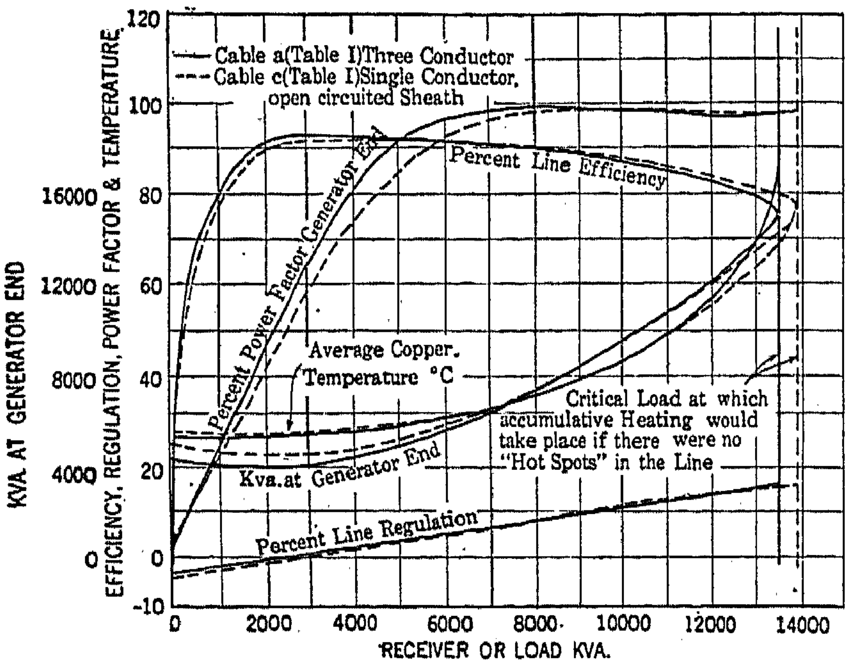


FIG. 17—COMPARISON OF LINE CHARACTERISTICS OF SINGLE- AND THREE-CONDUCTOR CABLES (SEE TABLE VII)—60-CYCLES—ALL VALUES PER CABLE—LENGTH OF LINE 50 MILES

The calculations for a fifty-mile line are tabulated in Table VII. They are based on the assumption of constant voltage at load or receiver end and a load power factor of 85 per cent. The more important characteristics are given in curve form in Fig. 17. There is little to choose between the two cables as far as general line characteristics are concerned. They have practically equal efficiencies, regulation, etc.

A cable line fifty-miles in length is exceptional. This length was chosen partly because it gives a severe comparison between the two cables and partly to show that there is nothing objectionable in such long lines other than the large charging capacity at no load. It amounts to approximately 5000 kv-a. or 100 kv-a. per mile of cable. High-potential testing would require at least 10,000 kv-a. unless made at a lower frequency. Few operating companies would have, or would care to invest, in the necessary a-c. equipment for this testing. A high-potential direct-current testing set would meet the conditions better than anything else.

In long cable lines such as considered the current and voltage, and hence the copper and dielectric losses, vary appreciably over the length of line. Fortunately at heavy loads where heating is a limiting factor the copper and dielectric losses vary in opposite directions, the increase in one being more or less compensated for by decrease in the other. Consequently, if there were no inequalities in the heat dissipating ability of the duct lines the heating would be uniformly distributed and the copper temperature rise would be practically the same over the whole length of line.

The only inequalities we are concerned with are "hot spots," where maximum heating and temperature rise occur. These hot spots may be anywhere in the length of line and due to the complicated relationship between copper and dielectric losses there is always doubt as to just what position of hot spot would cause the severest heating.

In Fig. 18 we have accordingly assumed hot spots for the extreme conditions, that is, at the two ends of the line. The curves in this figure were calculated only for the single-conductor cable (c) by the method explained in Appendix C and the data in Table VII. The empirical formula for the dielectric conductance of cable (c) (Formula 22, Appendix C) found by the $\Sigma \Delta$ method is

$$g_d = 0.079 \times 10^{-9} + 0.306 \times 10^{-13} (T')^{2.65}$$

This formula holds only up to 90 deg. cent. but Fig. 18 shows that accumulative heating is reached at a temperature below this. Fig. 18 also shows that the worst position of hot spot is at receiver or load end. The critical load at which accumulative heating starts here is 11,200 kv-a. If the hot spot is at generator end the critical load is 11,800 kv-a. This difference is due to a greater increase in current, and hence copper loss, at load end over the correspondingly higher voltage and dielectric loss at generator end. A study of Table VII and Figs. 17 and 18 brings out many other interesting features which we will not deal with. It can readily be seen that the worst position of hot spot is not always at receiver end but depends upon the length of line, load power factor and relative dielectric and copper losses.

Since in this particular case the limiting position is at receiver end, the hot spot characteristics of cables (*a*) and (*c*) are compared at this point in Fig. 19. The calculation of these curves is very simple and was first suggested in the paper by Bang and Louis.[11] It furnishes a splendid means of comparing the merits of different cables and would be especially valuable if a standard duct of known heat dissipating ability was adopted, as previously recommended. Fig. 19 does not indicate any appreciable difference between cables (a) and (c). Cable (c) is slightly favored since it has a critical load of 11,200 kv-a., while that of cable (a) is 11,050 kv-a. The introduction of this hot spot at the receiver end, based on the assumption of 25 per cent less heat dissipating ability than the average, has lowered the critical load capacity of cable (a) 18.5 per cent and of cable (c) 19.5 per cent. The safe load ratings are therefore, lowered about the same amount.

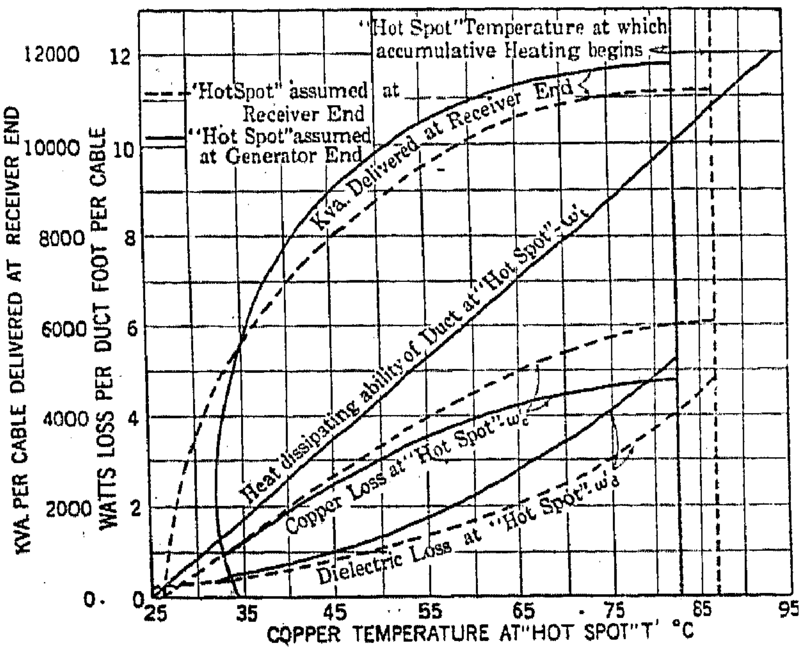


FIG. 18—"HOT SPOT" CHARACTERISTICS OF CABLE C (SEE TABLE VII AND FIG. 17)—60 CYCLES — SURROUNDING SOIL TEMPARATURE 25 DEG. CENT. —OPEN-CIRCUITED SHEATH

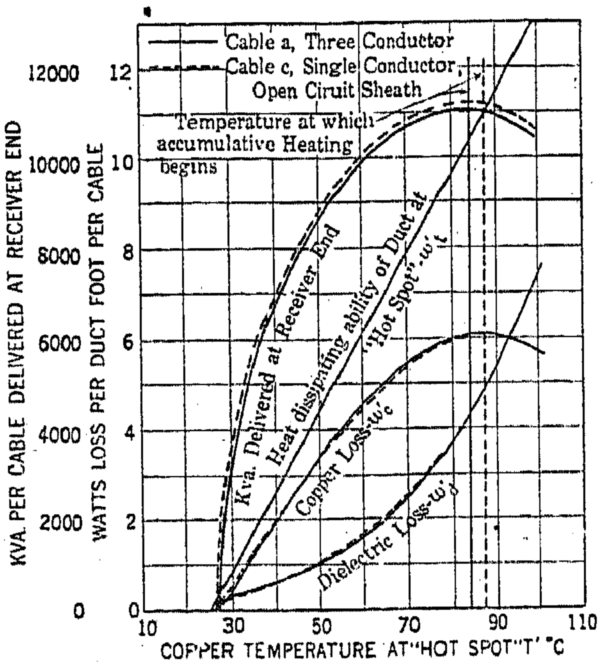


FIG. 19 — COMPARISON OF "HOT SPOT" CHARACTERISTICS OF SINGLE- AND THREE-CONDUCTOR CABLES (SEE TABLE VII) "HOT SPOT" ASSUMED AT RECEIVER END

---

[11] "The Influence of Dielectric Losses on the Rating of High-Tension Cables," by Bang and Louis, A. I. E. E., Feb. 1917.

ADVANTAGES AND DISADVANTAGES OF SINGLE-CONDUCTOR CABLE

Both the line and hot spot characteristics of cables (a) and (c) are so closely similar that they can be neglected in a general comparison of the two types of cables. The comparison is thereby greatly simplified and allows a study of other important factors that might otherwise be over-shadowed.

In changing from (a) to (c) with open-circuited sheath there is a saving in copper of 70 per cent, a tremendous advantage. This is reduced to 66 per cent when the necessary neutral ground wire is added and a line of three cables only is considered. The saving will approach 70 per cent according to the number of cables in the line, as a single neutral wire can be made to serve any number of cables. Other advantages such as greater simplicity and reliability of joints and pot heads, and lesser possibility of insulation deterioration because of uniform radial stressing have already been dealt with.

There are two other advantages we would like to point out before taking up the disadvantages. One of these, and a very important one in large systems of almost unlimited short-circuit power is, that all short circuits from cable failures will be single phase. The chance of short circuits between phases is very remote. The majority of three-conductor cable failures ultimately form between phases and as every one knows they are a much more severe trial on the system.

The other advantage is in the possibility of substituting aluminum conductors. An equivalent aluminum conductor has a cross section ratio to copper of about 1.63 to 1. It would not be feasible to substitute aluminum in the three-conductor cable (a) because the conductor diameter is already the largest practical size. In cable (c) the substitution of aluminum merely means a reduction in the conductor core without otherwise affecting the dimensions. Whether this substitution would be desirable or not would depend upon the existing market price of aluminum and copper, although the difficulty of splicing aluminum conductors would counterbalance to some extent any advantage in cost.

The disadvantages of single-conductor cable that need be seriously considered are: unbalancing of phase inductance and induced sheath voltage or current, according to whether the sheath is open- or short-circuited.

The standard duct used today has a rectangular cross section.

This means that three single-conductor cables installed to transmit three-phase power must be spaced at the corners of a right angle triangle with consequent unbalancing of the phase inductance. In short lines this unbalancing would be of little consequence but in long lines transposition would have to be resorted to, the problem being handled in exactly the same way as in over head transmission.

By using special ducts of round cross section the cables could be spaced equilaterally. If the conduit line consisted of only one three-cable circuit there would be no need of transposition. If, however, more than three cables, six for instance, were closely spaced in one conduit mutual inductance between circuits would cause unbalancing to about the same degree as that caused by unsymmetrical spacing of three cables only.

It is not within the scope of this paper to deal with phase unbalancing or advise when transposition is necessary or when it is not necessary. A theoretical consideration of the subject would involve complicated and tedious mathematics and would be of little practical use. It is a subject that had best be left to the practical experience and judgment of operating engineers. Transposition can be made at manholes without much extra cost and to be on the safe side perhaps it should generally be used, especially so when parallel operation of two or more circuits is attempted.

Under a previous heading it has been shown that single-conductor cable can be operated at 25 cycles with sheaths bonded together and grounded in two or more phases. The short-circuited sheath current introduces an additional loss but this is a small item compared with the great saving in copper accomplished in changing over from three-conductor cable. This form of connection is shown diagrammatically in Fig. 20. (Connection No. 1.) The sheaths remain at ground potential over their whole length with an absence of bothersome leakage between sheaths and the special precautions needed to overcome it. Fig. 20 shows the sheaths bonded together and grounded at the ends in the same way three-conductor cable sheaths are, the only difference being the intermediate bonds, necessary to relieve the sheaths of the charging current that accumulates along their length. If the intermediate bonds are spaced every mile or two the loss due to charging current flowing along sheaths will be negligible. Greater care must be taken to assure reliable and permanent bonding and grounding than with three-conductor cable. If one of the sheaths was accidently freed of its bond connections electromagnetic and electrostatic induction would tend to raise it to a high potential. There would not only be danger of destroying the sheath or causing injury to workmen but there would also be danger of exploding accumulated gases by electrostatic sparking.

The 25-cycle effective resistance of cable (c) with short-circuited sheath and 6.74-in. spacing was calculated (see Formula 2, Appendix B) and found to be 6.3 per cent higher than effective resistance with open-circuited sheath at 25 deg. cent. and 5.3 per cent higher at 50 deg. cent. The decrease at higher temperature is due to increase in sheath resistance. There can be no question concerning the desirability of short-circuiting the sheath of cable (c) for 25-cycle operation. Also, the superiority in every way over cable (a) is very apparent.

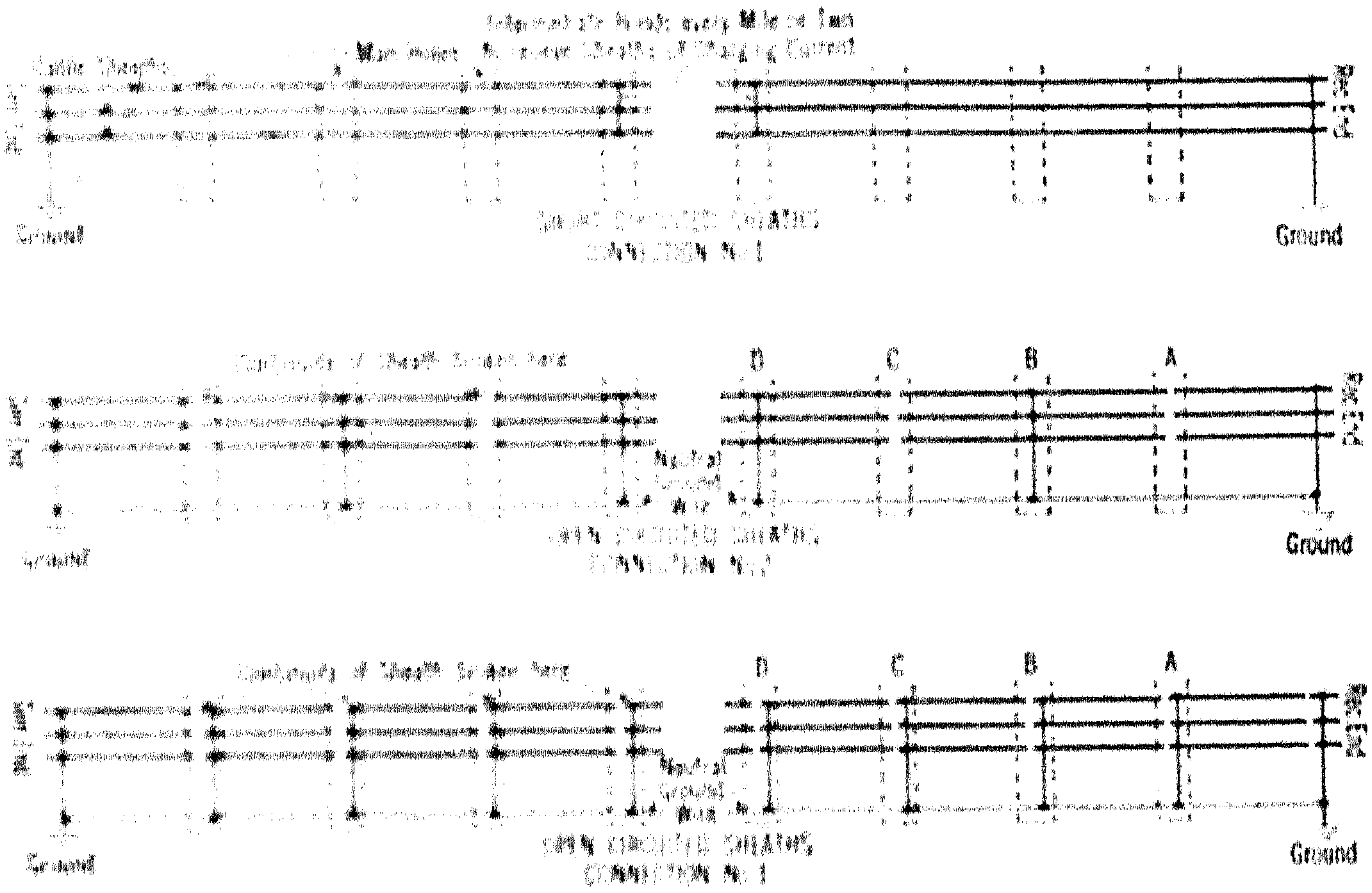


FIG. 20. SHOWING DIFFERENT METHODS OF BONDING AND GROUNDING SINGLE CONDUCTOR CABLE SHEATHS

At 60 cycles, 25 deg. cent., the effective resistance is 30 per cent higher with short-circuited sheath and 26 per cent higher at 50 deg. cent. This means that approximately 28 per cent more copper in cable (c) would give practically the same line loss, efficiency, regulation, etc., with short-circuited sheath as that given in Table VII and Figs. 17, 18 and 19 for open-circuited sheath. Since the neutral ground wire would be eliminated it actually means only 15 per cent more copper for a three cable line. *There would still be a saving in copper* *of* 61 *per cent in changing over from cable* (*a*) *to cable* (*c*) *compared with* 66 *per cent saving when* (*c*) *is operated with open-circuited sheath.* This slight difference in copper saving is a significant and important feature and indicates very convincingly that, in this particular case, short-circuited sheath operation is much more favored even at 60 cycles.

A 28 per cent increase in copper changes the conductor cross section in cable (c) from 275,000 to 350,000 cir. mils. The data in Table VII and Figs. 17, 18 and 19 were calculated for a conductor of 275,000 cir. mils and open-circuited sheath rather than for a conductor of 350,000 cir. mils and short-circuited sheath, not with any intention of emphasizing open-circuited sheath operation but because the data holds, practically, for either case and the assumptions furnish a convenient method of comparing open- and short-circuited sheath operation.

If cable (c) is limited to a temperature of 55 deg. cent. at hot spot, reference to Fig. 19 shows that it will be limited to an output of 9500 kv-a., or a current of 300 amperes. The induced sheath voltage per foot to neutral at receiver end with open-circuited sheath will be

$$e_s = 2\pi f M I_r$$

where M is the mutual inductance as represented in formula 6, Appendix B. Solving the above equation at 60 cycles; $e_s = 0.0000302 \times 300. = 0.00906$ volts per ft. length of cable.

In a fifty-mile line the total induced sheath voltage to neutral would be

$$0.00906 \times 264{,}000 = 2390 \text{ volts.}$$

The problem remains to break the sheath up into the shortest practical sections and arrange the ground connections in such a way that maximum sheath voltage will be reduced to a minimum. Connections Nos. 2 and 3 in Fig. 20 show the two best methods of doing this. In No. 2 the sheath is broken at every other manhole with ground bonds at intermediate manholes. The sheaths are broken and bonded at every manhole in Connection No. 3. Assuming the manholes 500 ft. apart, a maximum distance for this size cable, the sheath voltage along the length of line is given in Fig. 21 for the two different connections.

For connection No. 2 the maximum voltage from sheath to ground is 4.53 volts, therefore the voltage between sheaths of any two cables is $\sqrt{3} \times 4.53 = 7.85$ volts, and the voltage

Number of cables per line..............
Spacing between conductor centers (inch
Length of line (miles)..................
Weight of conductor copper per line (pouD THREE
Weight of copper in No. O, B and S neu
(pounds)........................BLE I.
Total weight of copper per line (pounds)
Total weight of lead per line (pounds)...

60 Cycles—All V

	Line constan			Line Efficiency per cent	Line Regulation per cent
T deg. cent	r_c 10^{-4}	L 10^{-6}	g 1) ^{3}		
27.44	0.1235	0.03765	0150.5	0	− 2.79
27.44	0.1235		0952	92.4	0.46
27.5	0.1235		0066	92.6	0.86
28.0	0.1239		0962	93.6	2.20
35.0	0.1268		1005	91.1	8.40
45.0	0.1315		2945	87.6	12.11
55.0	0.1360		4358	84.5	14.45
75.0	0.1454		10380	78.8	17.32
90.0	0.1521		17478	74.3	17.93
100.0	0.1570		24043	71.3	18.48
28.03	0.4016	0.145	0187.8	0	− 3.86
28.0	0.4016		0283.0	34.8	− 3.72
28.0	0.4016		0311	92.0	0.376
28.03	0.4016		0413	92.4	0.655
30.0	0.4043		0346	92.9	4.05
35.0	0.4125		0968	91.2	8.13
45.0	0.4272		0940	87.7	12.23
55.0	0.4425		1824	85.0	14.62
75.0	0.4730		3829	79.0	17.50
90.0	0.4960		5652	74.4	18.18

[Clark and Shanklin]

*Note: Generator power factor lagging

across the break in continuity of any one sheath is 2 × 4.53 = 9.06 volts. The only advantage of connection No. 3 is that the voltage across break in sheath is reduced by half. If unsymmetrical spacing of cables is used, or there are two or more circuits closely spaced in one conduit, unbalancing of inductance will increase the maximum sheath voltage. The increase will not be more than about 20 per cent.

These potentials are so low that it would not be a difficult matter to insulate the sheaths and reduce leakage to a negligible quantity even in wet or flooded conduit. However, when the 15 per cent increase in copper for short-circuited sheath operation of cable (c) is compared with the complications and expense of open-circuited sheath, such as, addition of neutral ground wire, the many breaks in continuity of sheath and the many bond connections that must be made absolutely secure, the balance in favor of short-circuited sheath stands out clearly.

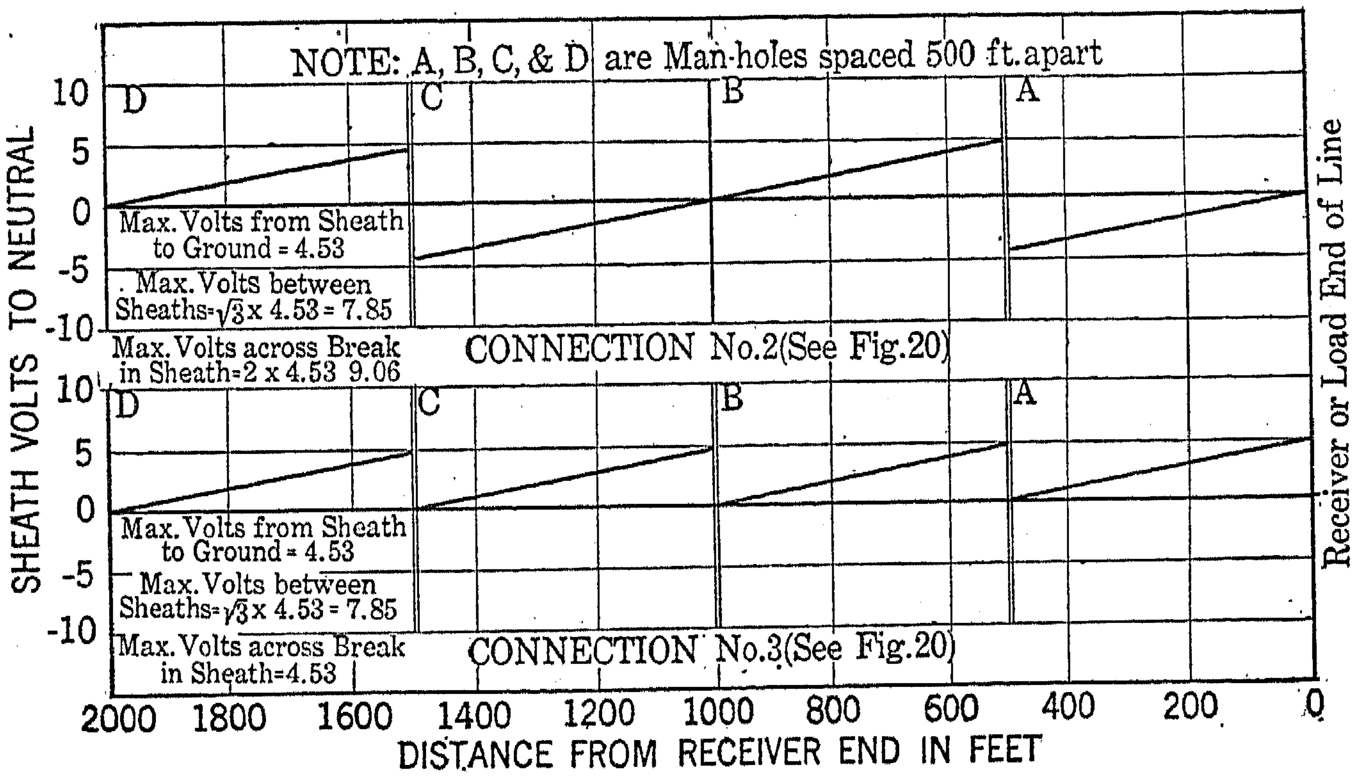


FIG. 21—INDUCED SHEATH VOLTAGE ALONG LENGTH OF SINGLE-CONDUCTOR CABLE—CABLE (c)—60 CYCLES—300 AMPERES—SPACING 6.74 IN.

With lower voltage, heavier current cable, cables No. 2 and 4 for instance, short-circuited sheath would mean an increase in copper of from 50 to 100 per cent, or more, for a 60-cycle line. The balance then turns in favor of open-circuited sheath, although the sheath potential is, unfortunately, somewhat higher than that given for cable (c) in Fig. 21. Potentials as high as 30 volts between sheaths might then have to be considered. The problem of insulating the sheaths would be increasingly difficult, but in some cases it would be the only alternative.

CONCLUSION.

The main features of single-conductor cable operation, as we see them, have now been presented. Some of the work is new, although the greater part is merely a compilation and elaboration of previous work by many different engineers. Where possible credit has been given. It is obviously impossible to give full credit in every particular.

The valuable advice and assistance of the following engineers directly associated with us in this work is deeply appreciated; Messrs. J. J. Matson, D. A. Ballard, W. C. Hayman, J. L. R. Hayden and H. H. Dewey.

APPENDIX A

Nomenclature and Formulas for Dielectric Characteristics

The following nomenclature and formulas are used throughout the paper for single-conductor cable.[12]

e = impressed voltage = $E/\sqrt{3}$ when three phase is considered.
i_d = amperes (dielectric) per ft. length of cable.
w_d = watts loss (dielectric) per ft. length of cable.
$\cos \theta_d$ = dielectric power factor.
c = capacity in farads per ft. length.
K_{av} = relative permittivity (average through thickness.)
K_a = capacity per cm.^{3} of air = 0.08842×10^{-12} farads.
r_d = effective a-c. insulation resistance per ft. length.
ρ_{av} = effective a-c. insulation resistance per cm.^{3}

$$g = \frac{d\,e}{d\,x} = \text{voltage gradient.}$$

g_{max} = voltage gradient at conductor surface.
l = length in cm. (1 ft. = 30.48 cm.)
b = radius in cm. to inner side of sheath.
a = radius in cm. of conductor.
f = frequency.

---

[12] For derivation of these formulas see, "Insulation Characteristics of High-Voltage Cables," TRANS. A. I. E. E., June, 1917.

The dielectric circuit is considered as a parallel circuit, that is, the resistance and capacity are considered as being in parallel Then:

$$\cos \theta_d = \frac{w_d}{e\, i_d} \tag{1}$$

$$c = \frac{i_d \sin \theta_d}{2\, \pi f e} \tag{2}$$

For a single-conductor cable in which the thickness of insulation consists of one kind only:

$$K_{av} = \frac{i_d \sin \theta_d \,.\, \log_\epsilon b/a}{(2\, \pi)^2\, l f e K_a} \tag{3}$$

$$\rho_{av} = \frac{2\pi\, l\, e^2}{w_d \log_\epsilon b/a} \tag{4}$$

$$g = \frac{e}{x \log_\epsilon b/a} \quad \text{or} \quad g_{max} = \frac{e}{a \log_e b/a} \tag{5}$$

THREE-CONDUCTOR CABLE

The calculations of dielectric characteristics for a three-conductor cable are greatly simplified if worked to neutral instead of between conductors.

e = impressed voltage to neutral = $E/\sqrt{3}$

i_d = amperes (dielectric) per con. per ft. length.

w_d = three-phase watts loss (dielectric) per cond. per ft. length.

$3\, w_d$ = total three-phase loss (dielectric) per ft. length of cable.

$\cos \theta_d$ = dielectric power factor (average for cable cross section.)

c_1 = capacity in farads per ft. between one cond. and sheath.

c_4 = capacity in farads per ft. between any two conds.

c_n = capacity in farads per ft. of one cond. to neutral.

K_{av} = relative permittivity (average through cross section.)

l = length in cm. (1 ft. = 30.48 cm.)

a = radius in cm. of conductor.

b = radius in cm. to inner side of sheath.
d = distance in cm. between axis of a and b.

Approximate formulas (**6**) and (**7**) for calculating the capacity of three-conductor cable, as derived by Alex. Russell, are given below:

$$c_1 = K_{av} \left(\frac{0.1693 \times 10^{-10}}{\log_{\epsilon} \frac{b^6 - d^6}{3\,(b^3\,d^2\,a)}} \right) \qquad (\mathbf{6})$$

$$c_4 = K_{av} \left[\frac{0.3384 \times 10^{-10}}{6 \log_{\epsilon} \left\{ \frac{d\sqrt{3}}{a} \cdot \frac{b^2 - d^2}{b^4 + b^2\,d^2 + d^4)^{1/2}} \right\}} - \frac{0.3384 \times 10^{-10}}{6 \log_{\epsilon} \frac{b^6 - d^6}{3\,(b^3\,d^2\,a)}} \right] \qquad (\mathbf{7})$$

It is apparent that the capacity between conductor and sheath c_1 is to ground or neutral. The capacity between conductors c_4 is converted to neutral by multiplying by three.

$$\therefore c_n = c_1 + 3\,c_4 \qquad (\mathbf{8})$$

$$\text{also, } c_n = \frac{i_d \sin\theta\;_d}{2\,\pi\,f\,e} \qquad (\mathbf{9})$$

$$3\,w_d = 3\,(e\,i_d \cos\,\theta_d) \qquad (\mathbf{10})$$

APPENDIX B

Theoretical Solution of Parallel Single-Conductor Cables with Non-Magnetic Sheaths

Since cable conductors are stranded the effect of this stranding on the calculated inductance although fairly small, must be corrected for. Referring to Fig. 22 an accurate way of doing this is to assume a solid conductor of the same cross section as the stranded and having an average radius equal to that of the stranded.

Let a_0 and b_0 be the actual inner and outer radii of the stranded conductor, a_1 and b_1 the radii of the equivalent solid conductor and A the cross section.

$$A = \pi\,(b_1^2 - a_1^2) \quad \text{and } b_0 + a_0 = b_1 + a_1$$

Therefore:
$$\left.\begin{aligned} b_1 &= \frac{b_0 + a_0}{2} + \frac{A}{2\,\pi\,(b_0 + a_0)} \\ a_1 &= \frac{b_0 + a_0}{2} - \frac{A}{2\,\pi\,(b_0 + a_0)} \end{aligned}\right\} \qquad (1)$$

In Fig. 22 let us assume that the two sheaths are short-circuited together and the distance between the two end bonds is length l. (No current flows across the intermediate bonds if the resistance of the end bonds is negligible. If resistance is inserted in end bonds current will flow across intermediate bonds.)

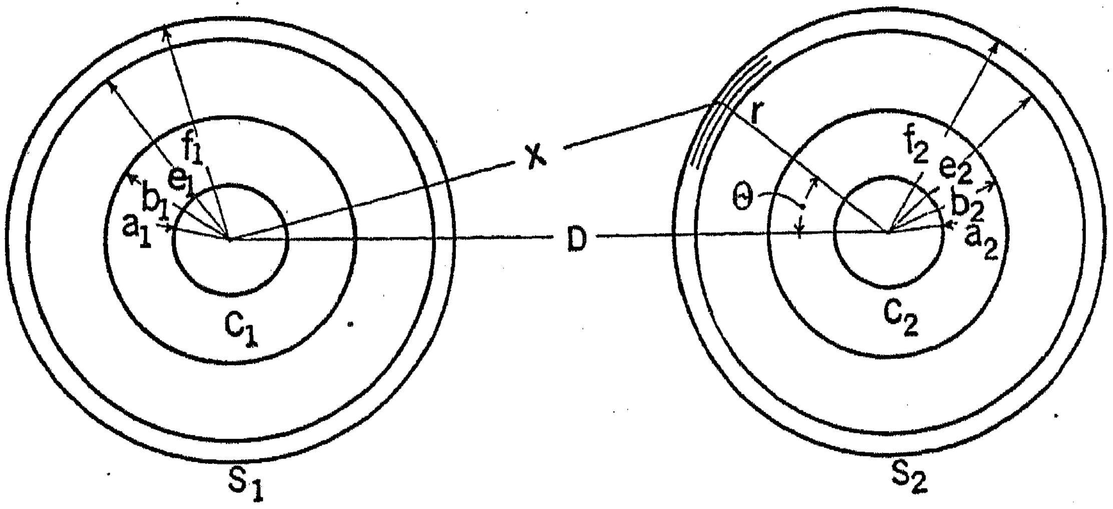


FIG. 22

It can be seen that flux set up by current in the sheath interlinks all of the current in the concentric conductor. Hence the sheath has no self-inductance. The bonds, however, have self-inductance but since they are short compared with length l the inductance is very small in comparison with the sheath resistance between bonds and can be neglected.

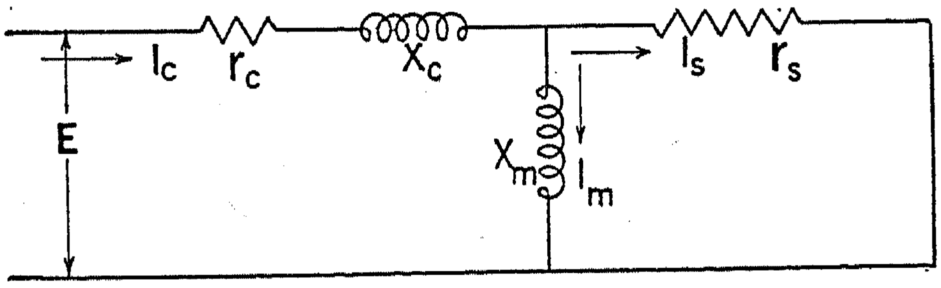


FIG. 23

In deriving the following formulas one cable only will be considered and all values worked to neutral. That is, only one-half of the circuit in Fig. 22 will be considered. The results can then be applied to either single-phase circuits or polyphase circuits having symmetrical arrangement of cables. A unit length of 1000 ft. of cable will also be considered.

Let:

r_c = ohms effective a-c. resistance of copper.
L_c = henries self inductance of conductor to neutral.
M = henries mutual inductance of conductor and sheath to neutral.
r_s = effective a-c. resistance of sheath and bonds to neutral.
i. e. r_s = total resistance of closed-circuit path to neutral.

The arrangement in Fig. 22 is equivalent to an ordinary transformer with short-circuited secondary and with zero reactance in the secondary. The equivalent transformer circuit is shown in Fig. 23.

$$Z_c = r_c - j\,2\,\pi\,f\,L_c$$

$$Y_m = \frac{j}{2\,\pi\,f\,M} \text{ and } Y_s = \frac{1}{r_s}$$

$$Y' = Y_m + Y_s = \frac{X_m + j\,r_s}{X_m\,r_s}$$

$$Z' = \frac{X_m^2\,r_s}{X_m^2 + r_s^2} - j\,\frac{X_m\,r_s^2}{X_m^2 + r_s^2}$$

$$Z_{eff} = Z_c + Z' = \left(r_c + \frac{X_m^2\,r_s}{X_m^2 + r_s^2}\right) - j\left(X_c + \frac{X_m\,r_s^2}{X_m^2 + r_s^2}\right) \qquad (2)$$

Where

$$r_{eff} = r_c + \frac{X_m^2\,r_s}{X_m^2 + r_s^2} \text{ and}$$

$$X_{eff} = 2\,\pi\,f\,L_{eff} = X_c + \frac{X_m\,r_s^2}{X_m^2 + r_s^2}$$

for open-circuited sheath $r_s = \infty$
therefore

$$Z_{eff} = r_c - j\,(X_c + X_m) \qquad (3)$$

The vector diagram of the short-circuit currents and voltages is given in Fig. 24.

If the constants r_c, r_s, M and L_c are known all of the characteristics can be easily calculated.

Effective resistance r_c is the a-c. value that would be measured with open-circuited sheath. It can in most cases be assumed as equal to the measured d-c. value. Eddy currents in stranded cable conductors have been shown both theoretically and by actual measurement to be negligible. In some cases, especially when coreless conductors are used, skin effect must be corrected for. Heaviside's formula for this correction as revised by Russell is given below (see *Philosophical Magazine* April, 1909, p. 545).

$$\frac{r_c}{r_c'} = 1 + \left(\frac{4\pi\mu\omega}{\rho}\right)^2 \left\{ \frac{(7a^2 - b^2)(a^2 - b^2)}{192} + \frac{b^2 a^4}{8(b^2 - a^2)} \log_\epsilon \frac{b}{a} - \frac{b^2 a^6}{4(b^2 - a^2)^2} \left(\log_\epsilon \frac{b}{a}\right)^2 \right\} \quad (4)$$

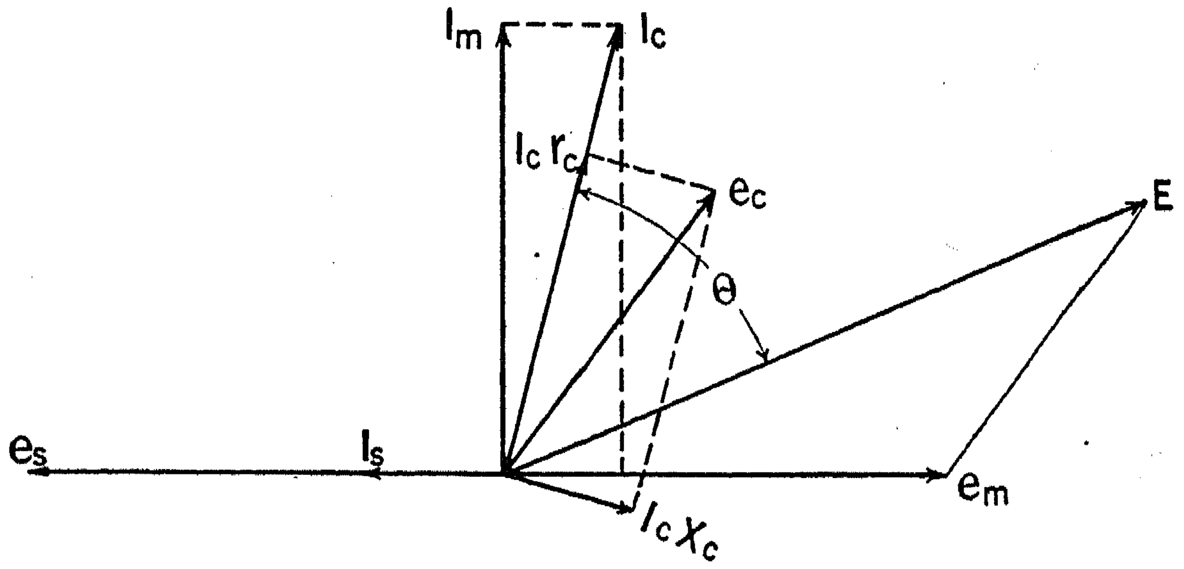


FIG. 24

where:

r_c = effective a-c. resistance.
r_c' = measured d-c. resistance
a = inside radius in cm. (for coreless cond. $a = 0$)
b = outside radius
μ = permeability = 1
ρ = specific resistance in C. G. S. units
$\omega = 2\pi f$

With open-circuit sheath there is a small circulating current in the sheath due to unsymmetrical flux distribution. This also adds to r_c but Berg has shown theoretically (see Berg's *Electric Energy*, p. 57) and actual measurements prove that it is inappreciable. We can, then, ignore the sheath entirely when open-circuited.

Effective resistance r_s is equal to the measured d-c. value. The high resistivity of sheath metal and the fact that the sheath is a comparatively thin shell eliminates skin effect.

Mutual Inductance M can be defined as the sum of the total and partial interlinkages of the flux from the conductor current (unit current) with the filaments of unit sheath current.

The formulas for total and partial interlinkages as used in developing M and L_c were derived by Alex Russell (see Russell's "*Alternating Currents*" Vol. I pp. 55 to 59.

Referring to Fig. 22:

$$-M_0 = M^{\text{I}} + M^{\text{II}} + (M^{\text{III}} + M_{s2}) \tag{5}$$

where M_0 is in C. G. S. units, so that henries per 1000 ft.,

$$M = 0.305 \times 10^{-4} M_0$$

$$M^{\text{I}} = -\frac{1}{I_c I_s} \int_{e_1}^{f_1} \phi c_1 \, i_{s_1} \text{ partial interlinkages.}$$

and:

$$\phi c_1 = \frac{2 I_c}{x} \, dx, \; - i_{s_1} = - \frac{x^2 - e_1^2}{f_1^2 - e_1^2} \, I_s$$

integrating:

$$M^{\text{I}} = -1 + \frac{2 e_1^2}{f_1^2 - e_1^2} \log_\epsilon \frac{f_1}{e_1}$$

$$M^{\text{II}} = -\frac{1}{I_c I_s} \int_{D\ f_1}^{D - f_2} \phi_{c_1} \, i_{s_1} \text{ total interlinkages}$$

where:

$$\phi_{c_1} = \frac{2 I_c}{x} \, dx \text{ and } - i_{s_1} = - I_s$$

integrating:

$$M^{\text{II}} = -2 \log_\epsilon \frac{D - f_2}{f_1}$$

$$M^{\text{III}} = -\frac{1}{I_c I_s} \int_{D - f_2}^{D + f_2} \phi_{c_1} i_{s_1} \text{ total interlinkages}$$

where;

$$\phi_{c_1} = \frac{2\,I_c}{x}\,dx \text{ and } -\,i_{s_1} = -\,I_s$$

integrating:

$$M^{\prime\text{II}} = -\,2\log_\epsilon \frac{D+f_2}{D-f_2}$$

$$M_{s_2} = \frac{1}{I_c\,I_s}\int_{D-f_2}^{D+f_2} \phi_{c_1}\, i_{s_2} \text{ partial interlinkages}$$

In the sheath cross section S_2 consider a concentric ring of radius r and thickness dr. Then

$x^2 = r^2 + D^2 - 2\,D\,r\cos\theta$, keeping r constant

$x\,dx = D\,r\sin\theta\,.\,d\,\theta$

The current i_{s_2}' in this cylinder is equal to

$$\frac{2\,\pi\,r\,dr\,I_s}{\pi\,(f_2{}^2 - e_2{}^2)}\,.$$

Hence the part of $\Sigma\,\phi_{c_1}\,i_{s_2}$ due to the current in cylinder is

$$\int_{D-r}^{D+r} \frac{\theta}{\pi}\,i_{s_2}'\,.\,\frac{2\,I_c}{x}\,dx$$

for $\dfrac{\theta}{\pi}\,i_{s_2}'$ is the number of tubes of current in this elementary cylinder intercepted by a circle of radius x and

$$\frac{2\,I_c}{x}\,dx = \phi_{c_1}$$

Since $x\,dx = r\,D\sin\theta\,.\,d\,\theta$
this may be written

$$\frac{I_c\,i_{s_2}'}{\pi}\int_0^\pi \frac{2\,r\,D\sin\theta\,.\,d\,\theta}{r^2 + D^2 - 2\,r\,D\cos\theta}$$

$= 2\;I_c\,i_{s_2}'\log\,(r + D) - 2\;I_c\,i_{s_2}'\log D$, for the

$$\text{part} - \frac{I_c\,i_{s_2}'}{\pi}\int_0^\pi \log\left(1 - \frac{2\,r}{D}\,\cos\theta + \frac{r^2}{D^2}\right) d\,\theta = 0$$

Therefore up to $D + r$ this elementary cylinder contributes;

$$2\,I_c\,is_2'\,[\log\,(r + D) - \log D\,] \qquad \textbf{(a)}$$

to $\Sigma\,\phi_{c1}\,is_2$. It also contributes from $D + r$ to $D + f_2$

$$\int_{D+r}^{D+f_2} is_2'\,\frac{2\,I_c}{z}\,d\,z = 2\,I_c\,is_2'\,[\log\,(D + f_2) - \log\,(D + r)] \quad \textbf{(b)}$$

The total contribution by this elementary cylinder is $(a) + (b)$

$$= 2I_c\,is_2'\log\,(D + f_2) - 2\,I_c\,is_2'\log D$$

Substituting $\dfrac{2\,r\,dr\,I_s}{f_2^2 - e_2^2}$ for is_2' and integrating for $r = e_2$ to $r = f_2$ we get

$$Ms_2 = 2\log\frac{D + f_2}{D}$$

Adding all the components in (**5**) and simplifying, we get:

$$M_0 = \left(2\log_\epsilon\frac{D}{f_1}\right) - \left(\frac{2\,e_1^2}{f_1^2 - e_1^2}\;.\log_\epsilon\frac{f_1}{e_1}\right) + 1 \text{ or,}$$

$$M = \left[\left(2\log_\epsilon\frac{D}{f_1}\right) - \left(\frac{2\,e_1^2}{f_1^2 - e_1^2}\,.\log_\epsilon\frac{f_1}{e_1}\right) + 1\right]$$

0.305×10^{-4} henries per 1000 ft. of conductor (to neutral) (**6**)

Self Inductance L_c. The most convenient way of obtaining the short-circuit self inductance L_c of the conductors is to subtract M from the open circuit effective inductance L. When the sheath is open circuited it has no effect of any kind as already described. L is then actually the open circuit self inductance of the conductors and reference to Figs. 22 and 23 show $L = L_c + M$ where L is in henries per 1000 ft. and $= 0.305 \times 10^{-4} L_0$

$$L_0 = Lc_1^{\text{I}} + Lc_1^{\text{II}} + (Lc_1^{\text{III}} + Mc_2) \qquad \textbf{(7)}$$

The integral equations for the above components of L_0 work out in exactly the same way as those for the corresponding components of M_0, always giving the interlinkages $\phi\,i$ their proper sign. Thus:

$$Lc_1^{\mathrm{I}} = \frac{1}{I_c^2}\int_{a_1}^{b_1} \phi c_1\, ic_1 \text{ partial interlinkages, where:}$$

$$\phi c_1 = \frac{2\,I_c}{b_1^2 - a_1^2}\left(x - \frac{a_1^2}{x} \right) dx \text{ and } ic_1 = \frac{x^2 - a_1^2}{b_1^2 - a_1^2} I_c$$

hence:

$$Lc_1^{\mathrm{I}} = \frac{2\,a_1^4}{(b_1^2 - a_1^2)^2}\log_\epsilon \frac{b_1}{a_1} + \frac{1}{2}\;\frac{b_1^2 - 3\,a_1^2}{b_1^2 - a_1^2}$$

$$Lc_1^{\mathrm{II}} = \frac{1}{I_c^2}\int_{b_1}^{D - b_2} \phi c_1\, ic_1 = 2\,\log_\epsilon \frac{D - b_2}{b_1}$$

$$Lc_1^{\mathrm{III}} = \frac{1}{I_c^2}\int_{D - b_2}^{D + b_2} \phi c_1\, ic_1 = 2\log_\epsilon \frac{D + b_2}{D - b_2}$$

$$Mc_2 = -\frac{1}{I_c^2}\int_{D - b_2}^{D + b_2} \phi c_1\, ic_2 \text{ partial interlinkages where the}$$

method of integrating is the same as that for Ms_2.
Therefore:

$$Mc_2 = -\,2\log_\epsilon \frac{D + b_2}{D}$$

Adding all components in (7) and simplifying we get:

$$L_0 = \left(2\log_\epsilon \frac{D}{b_1} \right) + \left(\frac{2\,a_1^4}{(b_1^2 - a_1^2)^2} \,.\, \log_\epsilon \frac{b_1}{a_1} \right) + \left(\frac{1}{2}\;\frac{b_1^2 - 3\,a_1^2}{b_1^2 - a_1^2} \right), \text{ or}$$

$$L = \left[\left(2\log_\epsilon \frac{D}{b_1} \right) + \left(\frac{2\,a_1^4}{(b_1^2 - a_1^2)^2} \,.\, \log_\epsilon \frac{b_1}{a_1} \right) + \left(\frac{1}{2}\;\frac{b_1^2 - 3\,a_1^2}{b_1^2 - a_1^2} \right) \right] 0.305 \times 10^{-4}$$

henries per 1000 ft. of conductor (To neutral) (8)

Since there is no self inductance of the conductor beyond the sheath the value L_c can be independently obtained.

$L_{c0} = Lc_1^{I} + Lc_1^{IV} + (Lc_1^{V} + M^{I})$,

where Lc_1^{I} and M^{I} are the same components as defined above.

$$L^{IV}_{c1} = \frac{1}{I_c^2} \int_{b_1}^{e_1} \phi c_1 \, ic_1 = 2 \log_\epsilon \frac{e_1}{b_1}$$

$$L^{V}_{c1} = \frac{1}{I_c^2} \int_{e_1}^{f_1} \phi c_1 \, ic_1 = 2 \log_\epsilon \frac{f_1}{e_1}$$

The values of L_c worked out by these two different methods check exactly.

APPENDIX C

Line Characteristics of Underground Cables

The work is greatly simplified if only one cable is considered in the calculations of line characteristics, whether single- or three-conductor cables are used in the conduit line. Values can then be worked out per cable per duct. Further simplicity is obtained by considering all values such as voltage, inductance, capacity, etc. to neutral instead of between lines.

Single-Conductor Cable. When single-conductor cable is used the inductance depends upon the spacing between cables. Usually the duct arrangement requires that they be spaced at the corners of a right angle triangle. The phase inductance is then unbalanced. When this unbalancing is appreciable it can be conveniently compensated for by transposition at man holes. The equivalent spacing to be used in calculating inductance is;

$$D = \sqrt[3]{a\,b\,c} \qquad (1)$$

where a, b and c are the respective sides of the triangle. With proper transposition the proximity of other single-conductor cable lines can be ignored.

Three-Conductor Cable. The spacing between conductors is fixed and the conductors are symmetrically arranged, D is therefore the actual distance between conductor centers. Since all values are to be calculated per cable the assumed conductor cross section, line current, dielectric loss and capacity to neutral will be three times the actual values per conductor, and the assumed resistance and inductance to neutral will be one-third the actual values per conductor.

Formulas (**2**) and (**3**) below were developed by Dr. Steinmetz* for a line having distribution impedance and admittance.

$$\underset{\cdot}{E_1} = \underset{\cdot}{E_2}\left(1 + \frac{Z\,Y}{2}\right) \pm Z\,\underset{\cdot}{I_2}\left(1 + \frac{Z\,Y}{6}\right) \qquad (\mathbf{2})$$

$$\underset{\cdot}{I_1} = \underset{\cdot}{I_2}\left(1 + \frac{Z\,Y}{2}\right) \pm Y\,\underset{\cdot}{E_2}\left(1 + \frac{Z\,Y}{6}\right) \qquad (\mathbf{3})$$

The plus sign supplies when $\underset{\cdot}{E_1}$ and $\underset{\cdot}{I_1}$ are at or towards generator end and $\underset{\cdot}{E_2}$, $\underset{\cdot}{I_2}$ at or towards receiver load end. The negative sign applies when the directions are reversed.

In practically all cases the voltage is held constant at receiver end and value of load current and power factor assumed, letting the generator voltage, current and power factor come what they will. The above formulas then take the form:

$$\underset{\cdot}{e_g} = \underset{\cdot}{e_r}\left(1 + \frac{Z\,Y}{2}\right) + Z\underset{\cdot}{I_r}\left(1 + \frac{Z\,Y}{6}\right) \qquad (\mathbf{4})$$

$$\underset{\cdot}{I_q} = \underset{\cdot}{I_r}\left(1 + \frac{Z\,Y}{2}\right) + Y\,\underset{\cdot}{e_r}\left(1 + \frac{Z\,Y}{6}\right) \qquad (\mathbf{5})$$

Where:

$\underset{\cdot}{e_r} = e_r' + o\,j =$ receiver end voltage to neutral (constant).

$\underset{\cdot}{I_r} = i_r' + i_r''\,j =$ assumed receiver line current (lagging p. f).

$\underset{\cdot}{e_g} = e_g' \pm e_g''\,j =$ generator end voltage to neutral.

$\underset{\cdot}{I_g} = i_g' \pm i_g''\,j =$ line current at generator end.

$Z = l\,(r_c - j\,x)$ and $x = 2\,\pi\,f\,L$

$Y = l\,(g_d - j\,b_d)$ and $b_d = 2\,\pi\,f\,c$.

where;

l = length of line considered in ft.

r_c = resistance per ft. of conductor in ohms.

L = inductance to neutral per ft. of cond. in henries.

g_d = conductance to neutral of insulation per ft. in mhos.

c = capacity to neutral per ft. in farads.

---

*"Engineering Mathematics" p. 204, McGraw-Hill Co.

Inductance L is the only line constant that is independent of duct temperature. The formulas for L (or L_{eff} when single-conductor cable with short-circuited sheath is used) are given in Appendix B, Formulas 8 and 2.

Resistance r_c varies with duct temperature. It is the effective a-c. resistance discussed in Appendix B, and therefore includes skin effect when this is appreciable. To be on the safe side the copper conductivity should be taken as 98 per cent in calculating skin effect by formula (**4**), Appendix B. Actually, the copper conductivity is usually between 99 and 100 per cent. In the majority of cases skin effect is negligible and the d-c. resistance sufficiently representative (with single-conductor cable and short-circuited sheath the resistance is represented by r_{eff}.) The standard formula for d-c. copper resistance is:

$$r = r_{25} \left[1 + 0.00378 \, (T - 25) \right] \qquad (\mathbf{6})$$

where;

r_{25} = resistance at 25 deg. cent. (98 per cent copper).
T = copper temperature in deg. cent.

Conductance g_d of the cable insulation is an important line constant because of its rapid increase with temperature.

$$g_d = 1/r_d = w_d/e^2 \quad \text{(see Appendix A).}$$

It is much better to measure g_d for the particular cable used. However, if sufficient data are available from other cables of the same type g_d can be approximated, as described in the body of the paper and Appendix A. A small error is brought in by assuming the temperature, and consequently g_d, uniform through cable cross section. Usually, in duct operation, the sheath temperature is about 10 per cent lower than the conductor temperature. If necessary this error can be corrected for. In the majority of cases it can be ignored.

Capacity c should also be measured when possible, although it can be estimated closely by the method referred to above in connection with g_d. The variation of c with temperature is small in good grade impregnated paper, about 15 per cent from 25 deg. cent to 100 deg. cent. It is large enough to be considered if accurate results are desired.

HEATING CHARACTERISTICS OF DUCT SYSTEM

Referring to Fig. 17, let;

w_d = *average* dielectric loss per duct. ft.
w_c = *average* copper loss per duct ft.
$w_t = w_d + w_c$ = av. total loss per duct ft.
T_1 = temperature of surrounding soil
T = *average* copper temperature as measured by by resistance drop.

Since w_t is a straight line function of T the formula is:

$$l\, w_t = A\, l\, (T - T_1) \tag{7}$$

where A is a constant and l is length of line in feet.

RELATION BETWEEN LOAD AND AVERAGE TEMPERATURE

For any assumed value of temperature T there is a certain value of continuous load that will satisfy the conditions. The problem remains to determine this load.

Impedance Z and admittance Y are calculated at the chosen temperature T and inserted in Formulas **4** and **5**. The receiver voltage e_r and power factor $\cos \theta_r$ are known and assumed to be constant at all loads. Then, in Formulas **4** and **5** the only unknown values are $\underset{\cdot}{e}_g$, $\underset{\cdot}{I}_g$ and the numerical value I_r, since;

$$\underset{\cdot}{I}_r = I_r \cos \theta_r + j\, I_r \sin \theta_r \tag{8}$$

We can, therefore, solve for $\underset{\cdot}{e}_g$ and $\underset{\cdot}{I}_g$ in terms of I_r.

Simplifying (**4**) and (**5**) they take the form:

$$\underset{\cdot}{e}_g = (a_1 + a_2\, I_r) + j\, (b_1 + b_2\, I_r) \tag{9}$$

$$\underset{\cdot}{I}_g = (a_3 + a_4\, I_r) + j\, (b_3 + b_4\, I_r) \tag{10}$$

where;

$$e_g' = a_1 + a_2\, I_r, \text{ and } e_g'' = b_1 + b_2\, I_r$$
$$i_g' = a_3 + a_4\, I_r, \text{ and } i_g'' = b_3 + b_4\, I_r$$

The power at generator end is:

$$W_g = e_g'\, i_g' + e_g''\, i_g'' \tag{11}$$

or, taking care to give the numerical values of the constants a_1, b_1, etc. their correct sign;

$$W_g = (a_1 + a_2\, I_r)\,(a_3 + a_4\, I_r) + (b_1 + b_2\, I_r)\,(b_3 + b_4\, I_r) \tag{12}$$

The power at receiver end is:

$$W_r = e_r\, i_r'$$

or

$$W_r = e_r I_r \cos \theta_r \tag{13}$$

The total line loss is:

$$l w_t = W_g - W_r \tag{14}$$

or, combining (**7**), (**12**) and (**13**) we have:

$$A l (T - T_1) = (a_1 + a_2 I_r) (a_3 + a_4 I_r) + (b_1 + b_2 I_r) (b_3 + b_4 I_r) - e_r I_r \cos \theta_r \tag{15}$$

in which I_r, being the only unknown can be solved for.

$$\frac{W_r}{W_g} \times 100 = \text{per cent line efficiency} \tag{16}$$

$$\frac{e_g - e_r}{e_g} \times 100 = \text{per cent line regulation} \tag{17}$$

RELATION BETWEEN LOAD AND HOT SPOT TEMPERATURE

Referring again to Fig. 17 let;

w_d' = dielectric loss per duct ft. at hot spot.
w_c' = copper loss per duct ft. at hot spot.
$w_d' + w_c' = w_t'$ = total loss per duct ft. at hot spot.
T_1 = temp. of surrounding soil
T' = copper temp. at hot spot.

Since w_t' is a straight line function of T' the formula is;

$$w_t' = A' (T' - T_1) \tag{18}$$

It now remains to represent w_d' and w_c' in terms of temperature, voltage and line current so that, for assumed values of any two of these the other can be solved for.

The current and voltage vary over the length of line, therefore, the hot spot temperature will depend upon its position in the line. If it is at receiver end where the voltage and power factor are constant and independent of current the solution is simple, as load current only must be solved for. Thus, for assumed values of T';

$$w_d' = e_r^2 g_d \tag{19}$$

$$w_c' = I_r^2 r_c \tag{20}$$

combining (**18**), (**19**) and (**20**):

$$A' (T' - T_1) = e_r^2 g_d + I_r^2 r_c \tag{21}$$

in which I_r, being the only unknown, can be solved for.

If the hot spot is at distance l from receiver end the solution is more complicated, for line voltage, power factor and current all depend upon the general line characteristics, as shown, for example, in Table VII. Assuming that e_g and I_g are known, temperature T' can be solved for as follows:

The empirical formula for g_d, found by the $\Sigma\triangle$ method, takes the form;

$$g_d = B + D\,(T')^n \tag{22}$$

where B, D and n are constants;

$$\therefore w_d' = e_g^2\,[B + D\,(T')^n] \tag{23}$$

From formulas (**6**)[14] and (**20**);

$$w_c' = I_g^2\,\{r_{25}\,[1 + .00378\,(T' - 25)\,]\,\} \tag{24}$$

combining (**18**), (**23**) and (**24**)

$$A'\,(T' - T_1) = e_g^2\,[\,B + D\,(T')^n\,] + I_g^2\,\{r_{25}\,[1 + 0.00378\,(T' - 25)\,]\,\} \tag{25}$$

in which T' being the only unknown can be solved for.

---

[14] Formula (6) does not apply when single-conductor cable with short-circuited sheath is used. In that case a modification of Formula (**2**) in Appendix B, for r_{eff} must be used.

For discussion of this paper, see page 1017 of Volume II.

INDEX.

NOTE: For complete topical and synoptical index see end of Part II

PAPERS

Abnormal Voltages Within Transformers (Illustrated) (*L. F. Blume and A Boyajian*) 577
Absolute Measurement of the Intensity of Sound, The (*Arthur Gordon Webster*) 701
Address by President on Cooperation, (*C. A. Adams*) 783.
Aeronautics in the United States, 1918. (Illustrated) (*George O. Squier*) 1
Annual Reports of Technical Committees:
  Power Stations Committee 871
  Transmission and Distribution Committee 876
  Industrial and Domestic Power Committee 881
  Lighting and Illumination Committee 885
  Committee on Economics of Electric Service 891
  Protective Devices Committee 893
  Electrochemistry and Electrometallurgy Committee 895
  Electrophysics Committee 896
  Telegraphy and Telephony Committee 897
  Marine Committee 905
  Committee on Use of Electricity in Mines 908
  Iron and Steel Industry Committee 909
  Instruments and Measurements Committee 912
  Educational Committee 915
Dielectric Strength of Air Films Entrapped in Solid Insulation and a Practical Application of the Problem for Alternator Coils and Cables (Illustrated) (*F. Dubsky*) 537
Electromagnetic Theory of the Telephone Receiver (Illustrated) (*A. E. Kennelly and H. Nukiyama*) 651
Fusion in Arc Welding, (Illustrated) (*O. H. Eschholz*) 121
General Equations of the Electric Circuit, The (Illustrated) (*Charles P. Steinmetz*) 191
Grounded Neutral Transmission Line (Illustrated) (*W. E. Richards*) 833
Grounding the Neutral of Generating and Transmission Systems (Illustrated) (*H. R. Woodrow*) 827
High Tension Single-Conductor Cable for Polyphase Systems (Illustrated) (*W. S. Clark and G. B. Shanklin*) 917
Ionization of Occluded Gases in High Tension Insulation (Illustrated) (*G. B. Shanklin and J. J. Matson*) 489
Present Status of Industrial Lighting Codes (*G. H. Stickney*) 725
Radio Telephony (Illustrated) (*E. B. Craft and E. H. Colpitts*) 305
Review of Work of Sub-Committee on Wave Shape Standard of the Standards Committee (Illustrated) (*Harold S. Osborne*) 261
Some Possibilities of Steam Railroad Electrification as Affecting Future Policies (*Calvert Townley*) 621
Telephone Circuits with Zero Mutual Induction (Illustrated) (*William W. Crawford*) 429
Theory of the Transient Oscillations of Electrical Networks and Transmission Systems (Illustrated) (*John R. Carson*) 345
Transmission Line Relay Protection (Illustrated) (*H. R. Woodrow, D. W. Roper, O. C. Traver and P. Mac Gahan*) 795
Utilizing the Time Characteristics of Alternating Current (Illustrated) (*Henry E. Warren*) 767
Welding as a Process in Ship Construction (*S. V. Goodall*) 113
Welding Mild Steel (Illustrated) (*H. M. Hobart*) 63

INDEX TO AUTHORS

Adams, C. A., Paper 783; Discussion......131,298,562,569,615,616, 743
Adams, P. H., Discussion...... 843
Albrecht, H. C., Discussion......850, 855, 860
Alexander, D. C., Jr., Discussion...... 135
Andrews, W. S., Discussion. 143
Argersinger, R. E., Discussion...... 564
Armor, J. C., Discussion...... 136
Atkinson, Ralph W., Discussion...... 566
Auel, C. B., Discussion...... 746
Behrend, B. A., Discussion...... 713
Blackwell, W. T., Discussion...... 757
Blume, L. F., Paper 577; Discussion...... 619
Boyajian, A., Paper...... 577
Candy, A. M., Discussion...... 131
Carle, N. A., Discussion......852, 857, 865
Carson, John R., Paper 345; Discussion...... 485
Chapman, A. G., Discussion...... 481
Chubb, L. W., Discussion......297, 560, 568
Churchward, J., Discussion...... 164
Clark, W. S., Paper...... 917
Clewell, C. E., Discussion...... 744
Colpitts, E. H., Paper...... 305
Conover, H. T., Discussion...... 839
Conwell, R. N., Discussion......843, 851, 856 860
Copley, A. W., Discussion......844, 861
Corbett, Capt., Discussion...... 131
Cowles, Alfred H., Discussion...... 462
Craft, E. B., Paper...... 305
Craighead, J. R., Discussion......839, 840, 855
Cravath, James R., Discussion...... 761
Crawford, Wm. W., Paper...... 429
Creighton, E. E. F., Discussion......294, 571, 839, 866
Dana, A. S., Discussion...... 466
Darling, Chas. R., Discussion...... 163
Davis, E. W., Discussion...... 572
Don Carlos, H. C., Discussion......848, 857
Du Bois, Delafield, Discussion...... 567
Dubsky, F., Paper 537; Discussion......574, 617
Eschholz, O. H., Paper 121; Discussion...... 176
Faccioli, G., Discussion...... 615
Farmer, F. M., Discussion...... 140
Fisher, Henry W., Discussion...... 564
Foltz, T. F., Discussion...... 759
Foster, W. J., Discussion......201, 563
Gibbs, George, Discussion...... 645
Goodall, S. V., Paper 113; Discussion...... 138
Hanker, F. C., Discussion......848, 867
Harding, C. F., Discussion......299, 565
Harrison, Ward, Discussion...... 753
Heaton, T. T., Discussion...... 161
Hester, E. A., Discussion......850, 858, 859, 864
Hill, W. H., Discussion......130, 131
Hobart, H. M., Paper...... 63
Holslag, C. J., Discussion...... 133
Hornor, H. A., Discussion...... 130
Hunt, L. F., Discussion......848, 867
Hurley, W. P., Discussion...... 757
Jackson, R. P., Discussion......140, 647
Jeffries, Zay, Discussion...... 170
Johnson, J. A., Discussion......839, 840, 841, 863
Jones, R. L., Discussion...... 713
Kennelly, A. E., Paper 651; Discussion......289, 296, 618, 722

166
Kicklighter, Charles H., Discussion.......... 172
Kinsey, A. S., Discussion.......... 173
Knox, H. G., Discussion..........839, 857
Lawton, A. H., Discussion.......... 145
Lemp, Herman, Discussion..........143, 845, 864
Lincoln, J. C., Discussion.......... 293
Linebaugh, Jesse J., Discussion.......... 750
Lloyd, M. G., Discussion.......... 795
Mac Gahan, P., Paper.......... 174
Martin, John, Discussion.......... 571
Matson, J. J., Paper 489; Discussion..........848, 852, 856, 858, 859, 860, 864
McKenzie, D. A., Discussion.......... 851
Merrick, E. G., Discussion.......... 134
Merrill, W. L., Discussion.......... 572
Middleton, W. I., Discussion.......... 846
Moore, E. T., Discussion.......... 472
Morecroft, J. H., Discussion.......... 148
Morton, Harry D., Discussion.......... 646
Murphy, John, Discussion.......... 630
Murray, W. S., Discussion.......... 865
Newbury, F. D., Discussion.......... 651
Nukiyama, H., Paper..........300, 481, 719, 844, 852
Osborne, Harold S., Paper 261; Discussion.......... 847
Parker, J. C., Discussion..........568, 617
Peek, F. W., Jr., Discussion.......... 758
Perrott, E. G., Discussion.......... 616
Peters, J. F., Discussion.......... 466
Pinto, A., Discussion..........846, 856, 860, 867
Pollard, N. L., Discussion.......... 641
Potter, W. B., Discussion.......... 722
Press, A., Discussion..........459, 464
Pupin, M. I., Discussion.......... 559
Reist, Henry G., Discussion.......... 833
Richards, W. E., Paper.......... 298
Robinson, L. T., Discussion.......... 842
Roper, D. W., Paper 795; Discussion.......... 745
Rosa, E. B., Discussion..........846, 856, 865
Schuchardt, R. F., Discussion.......... 635
Schwartz, Carl, Discussion..........289, 644
Scott, C. F., Discussion..........643, 645
Sever, George F., Discussion..........489, 917
Shanklin, G. B., Paper.......... 713
Shea, T. E., Discussion.......... 628
Shepard, F. H., Discussion.......... 867
Silver, A. E., Discussion.......... 136
Sparagen, W., Discussion.......... 458
Squier, George O., Paper 1; Discussion..........290, 295, 845
Steinmetz, Chas. P., 191; Discussion.......... 762
Stickney, G. H., Paper 725; Discussion.......... 637
Storer, N. W., Discussion..........615, 616
Summerhayes, H. R., Discussion..........292, 636, 713
Taylor, John B., Discussion.......... 841
Torchio, P., Discussion.......... 347
Townley, Calvert Paper 621; Discussion..........840, 849, 854, 863
Traver, O. C., Paper 795; Discussion.......... 289
Underhill, Chas. R., Discussion..........855, 858
Vincent, H. B., Discussion.......... 755
Vogt, John, Discussion.......... 157
Wagner, R. E., Discussion.......... 767
Warren, Henry E., Paper..........718, 722
Webster, Arthur Gordon, Paper 701; Discussion..........291, 561
Whitehead, J. B., Discussion..........853, 857, 868
Woodrow, H. R., Paper 795, 827; Discussion..........